UNITED STATES DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
Washington, D. C. 20250

September 13, 1966

Advisory ENG-36

To: State Conservation Engineers
Heads, Engineering and Watershed Planning Units

From: C. J. Francis, Director, Engineering Division

Re: PUBLICATIONS-Technical Release No. 34

A copy of Technical Release No. 34, "Application of Statistics to Concrete Quality Control," is attached. Knowledge of its content is vital to improvement of our administration of contracts. It merits careful study by your design engineers and construction engineers.

A specification note pertaining to related subjects will be issued in the near future.

C.E. Ghormley

ACTING

Attachment

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U. S. Department of Agriculture
Soil Conservation Service
Engineering Division

Technical Release No. 34
Design
August 24, 1966

APPLICATION OF STATISTICS TO CONCRETE QUALITY CONTROL[1]

Introduction

The importance of concrete strength as an indicator of not only the load-carrying capacity of a structure but also other characteristics such as durability and permeability is well known. Provisions for insuring adequate strength are included in specifications in the form of either a specified strength requirement or a procedure for closely controlling the concrete mix. In the application of either type of provision, the sampling and testing of the concrete becomes a major element of inspection.

Standardized specification provisions and standard methods of sampling and testing have been developed to aid in such control. However, in the administration of contracts involving concrete structures, the interpretation of acceptance requirements varies considerably. Methods for analyzing test results are apparently not widely understood. Even when the basis of acceptance is reasonably interpreted, the accuracy and validity of the test data often are not known. As a result confusion often develops whenever the concrete appears to fail any element of the acceptance requirement. Lack of understanding of methods of analysis has led to both erroneous rejection and erroneous acceptance of concrete.

The resolution of such problems is impossible unless: (1) sufficient numbers of samples are tested, (2) standard methods of sampling and testing are followed, (3) test results are analyzed by valid statistical methods, and (4) the accuracy of test results can be demonstrated.

The American Concrete Institute's committee on evaluation of compression test results has warned that:

> "Because of the possible disparity between the strength of test specimens and the load carrying capacity of a structure, it is dangerous to place too much reliance on inadequate strength data. It is also wrong to conclude that the strength of a structure is in jeopardy when a single test fails to meet specified strength requirements. Random variations and occasional failures to comply with strength requirements are inevitable. Accordingly inflexible strength requirements are unrealistic and <u>control of the pattern of results rather than individual values is the most appropriate basis for both specifications and the general assessment of results</u>."

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[1] This technical release was prepared by H. L. Cappleman, Jr., Assistant Chief, Design Branch.

Their report (ACI Standard 214) presents an excellent discussion of statistical analysis of concrete test results and the statistical meaning of various specification requirements.

The purpose of this technical release is to amplify those portions of that report that are most directly applicable to Service operations. A review of pertinent statistical relationships is given followed by a discussion of their practical application.

Review of Statistical Relationships

DISPERSION AND STANDARD DEVIATION

If the values of a series of strength tests of a given concrete mix at a given age (for example, 28-day strength) are plotted versus the frequency of occurrence of each value in the series, a considerable dispersion of the points about the average strength value will be noted. The order of occurrence of the various values in the series is unpredictable. No single test value can be considered to be the true strength of the batch it represents; however, within the limits of accuracy of the test series, the average value can be considered to indicate the strength of the concrete produced during the period of time represented by the test series. The amount of dispersion must be considered in the determination of the probable accuracy of the computed average value. An increase in dispersion indicates a decrease in probable accuracy of the mean.

The probable accuracy of the mean can be deduced by comparing the relative frequency of small errors with the relative frequency of large errors (as indicated by the dispersion of the plotted points about the average value). Mathematically, this can be done by determining the standard deviation (or root-mean-square deviation) of the series. The standard deviation (S) is computed by extracting the square root of the average of the squares of the deviations of the individual test values from their average ($\bar{X}$), or:

$$S = \sqrt{\frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 \;.....\; (X_n - \bar{X})^2}{n - 1}} = \sqrt{\frac{\Sigma X^2 - \bar{X}\Sigma X}{n - 1}} \quad \ldots (1)$$

Where: X_1, X_2, -- X_n = individual test values
ΣX = the sum of all test values
n = the number of tests

FREQUENCY DISTRIBUTION

The curve drawn to envelop the points on the plot of strength versus frequency of occurrence is called the frequency-distribution curve. If the plotted values are free from appreciable systematic errors (that is, if uniformity of materials, batching, mixing, handling and sampling are reasonably controlled), then the dispersion is caused mainly by accidental errors in the manufacturing and testing process and the frequency-distribution curve will conform closely to the shape of a normal probability curve.

As shown in Figure 1, a normal probability curve (or normal frequency-distribution curve) is a symetrical, bell-shaped curve having the apex located at the average test value. If y is the frequency of any test value, X, the equation of the normal frequency-distribution curve is stated as:

$$y = y_0 e^{-\frac{1}{2}t^2} \quad \text{..........................} \quad (2)$$

Where: y_0 = frequency corresponding to the mean test value, $\bar{X}$

e = 2.7183

$t = \frac{(\bar{X} - X)}{S}$ = the significance ratio

S = standard deviation

LEVEL OF CONFIDENCE

The usefulness of the frequency-distribution curve as an indicator of the quality of the material represented by the test series is derived from a unique characteristic: The ratio of the area under the normal frequency-distribution curve between $\bar{X}$ and a given value of X to the total area under the curve equals the probability that a test value selected at random will lie between $\bar{X}$ and X. The probabilities corresponding to various values of the significance ratio, t, have been computed and are available in statistical tables. The probabilities are usually expressed in percent. In that form a probability is usually called the level of confidence. In other words, the level of confidence is the percent chance that any randomly selected value in a series will lie within the range of $\bar{X} \pm tS$.

Of equal importance in concrete specifications is the expression of the chance of tests falling below a specified minimum value. This method of expression is simply a variation of the expression of the level of confidence. It is important to remember that the two means of expression are intimately related. Equivalent expressions are summarized in Table 1, below.

TABLE 1. Equivalent Ways of Expressing Level of Confidence

Percentage of Tests Falling Within the Limits of $\bar{X} \pm tS$	Chances of Tests Falling Below the Limit of $\bar{X} - tS$
50	2.5 in 10
60	2 in 10
70	1.5 in 10
80	1 in 10
90	1 in 20
95	1 in 40
99.9	1 in 2000

SIGNIFICANCE RATIO

At the same level of confidence the significance ratio, t, will vary with the number of samples in the series. With good control in sampling technique, the reliability of the test data increases with the number of samples. In other words, the degree of conformance (or fit) of the frequency-distribution curve to the normal probability curve depends on the number of samples in the series--the more samples, the better fit. Usually, the fit is good if the test series contains at least 25 samples. When the series contains less than 25 samples, the assumption of normal frequency distribution is not valid. When less than 25 samples are involved, the value of "t" must be corrected to allow for some degree of variance from normal frequency distribution.

A table of "t" values corresponding to various levels of confidence and numbers of samples is contained in ACI Standard 214. A partial abstract of the values consistent with the more commonly used specification requirements is contained in Table 2.

TABLE 2. Values of "t"

No. of Samples Minus 1	Chances of Tests Falling Below $\bar{X} - tS$			
	2 in 10	1 in 10	1 in 200	1 in 2000
4	0.941	1.533	4.604	12.94
5	0.920	1.476	4.032	8.61
6	0.906	1.440	3.707	6.86
7	0.896	1.415	3.499	5.96
8	0.889	1.397	3.355	5.40

TABLE 2. Values of "t" (cont.)

No. of Samples Minus 1	Chances of Tests Falling Below $\bar{X}$ - tS: 2 in 10	1 in 10	1 in 200	1 in 2000
9	0.883	1.383	3.250	5.04
10	0.879	1.372	3.169	4.78
15	0.866	1.341	2.947	4.07
20	0.860	1.325	2.845	3.88
25	0.856	1.316	2.787	3.75
30	0.854	1.310	2.750	-
∞	0.842	1.282	2.576	3.29

COEFFICIENT OF VARIATION

In statistical analysis it is often convenient to express the measure of dispersion of test results in dimensionless terms. For this purpose, the coefficient of variation, V, is used as a means of expressing the ratio of the standard deviation, S, to the average strength, $\bar{X}$.

$$V = \frac{S}{\bar{X}} \quad \text{.............................(3)}$$

For convenience, "V" is often expressed in terms of percentage. Whenever "V" is expressed as a whole number, it must be recognized as a percentage value and not the ratio.

CUMULATIVE DISTRIBUTION

The cumulative distribution curve is determined by plotting the cumulative numbers of tests having values below various given strengths. Cumulative distribution curves for general statistical use can be plotted in terms of percentage of tests and percentage of average strength. When plotted on a probability scale such curves appear as straight lines. Figure 2 (a copy of Figure 3 of ACI Standard 214) contains a group of cumulative distribution curves for different values of coefficients of variation, V. For any point on a given V-line, the ordinate shows the percentage of tests (in a long series) that can be expected to exhibit values greater than the value shown on the abscissa (expressed as a percentage of the average test value).

WITHIN-TEST VARIATION

When each test value in a series is determined by averaging the values of several samples, the range in values of the sample set can be used as an indicator of the accuracy of the sampling and testing technique.

The within-test standard deviation may be determined as:

$$S_1 = \frac{1}{d^2} \cdot \bar{R} \quad \text{..............................(4)}$$

Where: S_1 = within-test standard deviation
$\bar{R}$ = average range in test value of groups of companion cylinders
$\frac{1}{d^2}$ = a constant depending on the number of samples in each set (for 2 samples, $\frac{1}{d^2} = 0.88$; for 3 samples $\frac{1}{d^2} = 0.59$)

The within-test coefficient of variation is determined as:

$$V_1 = \frac{S_1}{\bar{X}} \quad \text{..............................(5)}$$

Where: $\bar{X}$ = average value of the tests in the series

To produce a reliable value of V_1, the analysis should be based on at least 10 sets of samples. For convenience, "V_1" is also often expressed in percent.

Practical Application

Only rarely is the frequency-distribution curve actually plotted in concrete quality control procedures. In normal construction operations, continuous inspection and supervision insure a reasonable amount of control (and suppression of systematic errors). The normal frequency distribution can usually be assumed as a practical basis for analysis of test results. The index factors for measuring the characteristics of the frequency-distribution curve corresponding to a given set of conditions can be determined and applied without plotting the curve itself.

The previously discussed factors of level of confidence, significance ratio (t), standard deviation (S), average strength ($\bar{X}$), and number of samples (n) must all be considered in the analysis of results. How these factors are applied in practical design and control of concrete mixes is discussed in the following paragraphs.

STANDARDS OF CONCRETE CONTROL

The coefficient of variation, V, can be used as a measure of the overall degree of control being achieved on a given project. A high coefficient indicates poor overall control of concrete manufacturing and testing operations; a low coefficient indicates good control. For general construction operations, the degree of overall control may be judged as shown in Table 3.

TABLE 3. Standards of Concrete Control

Value of "V" (%)	Indicated Degree of Control
Below 10	Excellent
10 to 15	Good
15 to 20	Fair
Above 20	Poor

In the application of these standards the value of "V," to be considered reliable, should be based on analysis of at least 10 tests.

APPLICATION TO MIX DESIGN

From the foregoing discussions it is apparent that for concrete to be acceptable its average strength must exceed the specified minimum strength by a margin adequate to compensate for the probable statistical variation in strength. The required margin of strength will vary with: (1) the level of confidence required by the specification and (2) the degree of control exercised by the manufacturer. A means of predicting the required design strength of a concrete mix can be derived from the equation for the significance ratio, t, as follows:

1. In Equation (2), the significance ratio is defined as:

$$t = \frac{\bar{\bar{X}} - X}{S}$$

2. If the specified strength, f_c'', is substituted for X and the required average strength, f_{cr}, is substituted for $\bar{\bar{X}}$, the equation for "t" becomes:

$$t = \frac{f_{cr} - f_c''}{S}$$

(Note: In this expression the term f_c'' is used to denote specified strength to prevent confusion with the design strength which is denoted by f_c'.)

3. From Equation (3), it can be derived that:

$$S = V\bar{X} = Vf_{cr}$$

4. Substituting for "S," the equation for "t," becomes:

$$t = \frac{f_{cr} - f_c''}{Vf_{cr}}$$

$$\text{or, } tVf_{cr} = f_{cr} - f_c''$$

$$\text{or, } f_{cr} - tVf_{cr} = f_c''$$

$$\text{or, } f_{cr}(1 - tV) = f_c''$$

$$\text{or, } f_{cr} = \frac{f_c''}{1 - tV} \qquad \text{.............(6)}$$

By use of Equation (6), the design strength of a concrete mix that is required to meet any given specification requirement can be predicted. The value of "V" must be estimated according to the anticipated degree of control (see Table 3). The value of "t" can be determined from Table 2 and must be commensurate with: (1) the level of confidence required by the specification and (2) the estimated number of samples to be used as a basis for determining compliance with strength requirements.

The level of confidence to be assumed depends on the wording of the specification. It is usually expressed in terms of the number of tests that can fall below a specified strength value, as demonstrated in the second column of Table 1. The level of confidence required by the specification has a considerable effect on mix design requirements. The effect of the wording of the specification may be demonstrated by considering four cases commonly encountered in practice:

<u>Case I</u>. "Not more than 20 percent of the strength tests shall have values less than the specified strength." (ACI Standard 318, for working stress design.)

<u>Case II</u>. "Not more than 10 percent of the strength tests shall have values less than the specified strength." (ACI Standard 318, for ultimate strength design and prestressed structures; and ASTM-C94.)

<u>Case III</u>. "No strength test shall have a value less than 80 percent of the specified strength." (Federal Specification SS-C-618.)

Case IV. "No strength test shall have a value less than the specified strength."

The required average strength, f_{cr}, for Case I, Case II and Case IV can be computed directly from Equation (6). To put Equation (6) in a form compatible with Case III, it is necessary to express it in the following manner:

$$f_{cr} = \frac{0.8f''_c}{1 - tV} \qquad \text{.......................... (7)}$$

Within the limits of practical precision, the values of "t" given in the fifth column of Table 2 may be used for Cases III and IV even though they represent a frequency-distribution curve that includes only 99.9 percent of the test values in the series instead of 100 percent.

As a base of reference, assume that each analysis concerns Class 2500 concrete and a test series consisting of 16 tests. From Table 2, values of "t" corresponding to 16 samples are: (1) for Case I, 0.87; (2) for Case II, 1.35; and (3) for Cases III and IV, 4.07. The computed values of "f_{cr}" for the various cases for various degrees of control are compared in Table 4. Further, assuming an air-entrained mix designed for 3-inch to 4-inch slump and 1½ inch (maximum size) aggregate, the cement content requirements can be compared as in Table 5.

The comparisons serve to show the relative difficulty of meeting various specification requirements. The importance of control varies considerably with the statistical conditions imposed by the wording of the specification. It appears obvious that, for Case III and Case IV, poor control is unacceptable under any conditions. Furthermore, since the means for compensating for it are technically undesirable and economically impractical, even "fair" control is not adequate for Case III and neither "fair" nor "good" control is adequate for Case IV.

TABLE 4

COMPARISON OF AVERAGE STRENGTH REQUIRED FOR 2500 P.S.I. CONCRETE

	Coefficient of Variation (%)			
Case	V = 10	V = 15	V = 20	V = 25
I	2750	2875	3070	3265
II	2900	3120	3420	4380
III	3390	5130	10500*	∞ *
IV	4230	6400*	13150*	∞ *

*Impractical

TABLE 5

COMPARISON OF APPROXIMATE CEMENT CONTENT (BAGS/CU.YD.)
REQUIRED FOR 2500 P.S.I. CONCRETE
(AIR-ENTRAINED, 1½" AGGREGATE, 3" to 4" SLUMP)

	Coefficient of Variation (%)			
Case	V = 10	V = 15	V = 20	V = 25
I	4.8	4.9	5.2	5.4
II	4.9	5.2	5.6	7.2
III	5.5	8.6	*	*
IV	6.9	9.5*	*	*

*Impractical

DETERMINING QUALITY OF INSPECTION

The quality of results of any analysis depends in large part on the quality of data used. To determine the quality of the sampling and testing phase of the inspection program, the within-test coefficient of variation (V_1) should be determined whenever there are at least 10 tests in the inspection test series. For general construction operations, the degree of control of sampling and testing operations may be judged as shown in Table 6.

TABLE 6

QUALITY OF SAMPLING AND TESTING

Value of "V_1" (%)	Indicated Degree of Control
Below 4	Excellent
4 to 5	Good
5 to 6	Fair
Above 6	Poor

When the value of V_1 exceeds 6 percent, the value of the test series can be seriously questioned. Further statistical analysis of data may lead to misleading results. In such cases the cylinder test results should be verified by impact hammer studies or by the taking and testing of core samples, or both. If such studies do not verify the cylinder test results, enough core samples should be taken to assure good representation, and the core test results should be used as a basis of acceptance or rejection.

DETERMINING COMPLIANCE WITH SPECIFICATIONS

The same statistical methods may be used to advantage in the inspection phase. The manner of their application is different from that used in the design phase, since emphasis is on comparison of test results with specification requirements. Some of the frequency-distribution factors (such as V, $\bar{\bar{X}}$ and t) can be estimated with more precision by analysis of test data.

Cumulative distribution curves (see Figure 2) are very useful in estimating the degree of compliance with specification requirements. This is particularly true when the number of tests is small (say 10 to 25) and compliance appears to be marginal. This can best be demonstrated by means of some examples, as follows:

1. Example 1

   a. Given:

      (1) The specification calls for a 28-day strength of 3000 p.s.i. and states that: "The average of all test values as well as the average of any 5 consecutive tests shall exceed the specified strength and not more than 20 percent of the tests can have values less than the specified strength."

      (2) From the results of 12 strength tests representing the quantity of concrete furnished, the following data are derived:

         (a) The overall average strength, $\bar{\bar{X}}$ equals 3220 p.s.i.

         (b) The average of any 5 consecutive tests exceeds 3000 p.s.i.

         (c) The values of 3 tests are less than 3000 p.s.i.

         (d) The computed overall coefficient of variation, V, is 0.18 (or 18%).

         (e) The within-test coefficient of variation, V_1, is 0.04 (or 4%).

b. Problem:

From the above listed data, it is apparent that 25 percent of the tests have values less than the specified strength. Test control appears to be good (see Table 6). Overall control is only fair (see Table 3). With only 12 tests to use as a basis of measurement, there is some question as to how significant the percentage of low test values may be. (For example, what is the probability that in a longer test series the percentage of low tests might be reduced.) Can the concrete be accepted as meeting specifications?

c. Solution:

The data should be compared with theoretical cumulative distribution data to determine its probable significance, as follows:

(1) Express the specified strength as a percentage of the average strength:

$$f_c'' = \frac{3000}{3220} \cdot \bar{X} = 93\% \text{ of } \bar{X}$$

(2) Enter the cumulative distribution chart (Figure 2) on the curve for V = 18% and find the point on the curve that corresponds to 93 % of average strength on the abscissa.

(3) On the ordinate scale corresponding to the point thus plotted read out the percentage of tests that can be expected to have values exceeding 93% of the average strength. In this case the ordinate scale shows that 65% of the tests will probably exceed the specified strength; 35% of the tests will probably have values less than 3000 p.s.i.

(4) Conclude that the concrete does not meet the requirements of the specification with the indicated degree of control (V = 18%).

2. Example 2

a. Given:

The same conditions as given in Example 1, except that the overall coefficient of variation, V, is 0.08 (or 8%).

b. Solution:

(1) Enter the cumulative distribution chart (Figure 2) on the curve for V = 8% and find the point on the curve that corresponds to 93% of average strength on the abscissa.

(2) On the ordinate scale, determine that 80 percent of the samples can be expected to have values exceeding the specified strength.

(3) Conclude that the concrete does meet specification requirements with the indicated degree of control (V = 8%).

3. Example 3

a. Given:

(1) A structure subject to severe exposure conditions is designed with a conservative design strength, f_c', of 3000 p.s.i. To increase the durability of the structure, the specified strength, f_c'', is stated as 4000 p.s.i.

(2) The specification requires that: "The average of all test values as well as the average of any 5 consecutive tests shall exceed the specified strength and not more than 20 percent of the tests can have values less than the specified strength."

(3) From the results of 16 strength tests representing the quantity of concrete furnished, the following data are derived:

(a) The average strength, $\bar{\bar{X}}$, equals 4300 p.s.i.

(b) The average of one set of 5 consecutive tests is less than 4000 p.s.i.

(c) The values of 4 tests (not consecutive) are less than 4000 p.s.i.

(d) The computed overall coefficient of variation, V, is 15%.

(e) The within-test coefficient of variation is 3.4%.

b. Problem:

It is apparent that 25% of the tests have values less than f_c'' and the average of one set of 5 consecutive tests is low. Test control is excellent (see Table 6). Overall control is fair (see Table 3). The problem is two-fold. Can the concrete be accepted as meeting specifications? If not, can the concrete be accepted at a reduced payment or must it be removed and replaced?

c. Solution:

(1) The data should be compared with theoretical cumulative distribution data to determine the probable degree of conformance with specification requirements, as follows:

(a) Express f_c'' in terms of $\bar{X}$:

$$f_c'' = \frac{4000}{4300} \cdot \bar{X} = 93\% \text{ of } \bar{X}$$

(b) From Figure 2, determine that 67% of the tests can be expected to exceed f_c''.

(c) Conclude that the concrete does not meet the specification requirements.

(2) Determine whether or not the concrete has adequate strength for structural requirements. (Note: For general working stress design, the concrete can be considered to have adequate strength when 80% of the test values can be expected to exceed the design strength, f_c'.)

(a) Express f_c' in terms of $\bar{X}$:

$$f_c' = \frac{3000}{4300} \cdot \bar{X} = 70\% \text{ of } \bar{X}$$

(b) From Figure 2, determine that 97.5% of the tests can be expected to exceed f_c'.

(c) Conclude that: (1) the strength is adequate for structural requirements and (2) the concrete may be accepted at a lesser price if such a procedure is considered more advantageous to the owner, considering the probable increase in maintenance cost.

4. Example 4. When small numbers of samples are involved, marginal data should be analyzed even when the concrete apparently meets all of the specified requirements. The reason for this can be illustrated as follows:

a. Given:

(1) The specification requires a 28-day strength of 3000 p.s.i. and states that: "The average of all tests as well as the average of any 5 consecutive tests shall exceed the specified strength and no test shall have a value less than 80 percent of the specified strength."

(2) From the results of 12 tests all requirements appear to be met.

(3) The average strength, $\overline{X}$, equals 3150 p.s.i.

(4) The coefficient of variation, V, equals 17%.

(5) The within-test coefficient of variation, V_1, equals 3.6%.

(6) Design strength, f_c', equals 3000 p.s.i.

b. Problem:

It is apparent that the concrete nominally meets the specified strength requirement. Testing control is excellent. However, the overall control is only fair and the average strength is quite close to the specified strength. Because of the small number of samples, check the results using cumulative distribution data.

c. Solution:

(1) Express $0.8f_c''$ as a percentage of $\overline{X}$.

$$f_c'' = \frac{2400}{3150} \cdot \overline{X} = 76\% \text{ of } \overline{X}$$

(2) From Figure 2, determine that 92 percent of the tests can be expected to exceed $0.8f_c''$ (instead of 100%, as specified). This casts doubt on the significance of the nominal results.

(3) Check to see if 80% of tests can be expected to exceed the design strength.

(a) $f_c' = \frac{3000}{3150} \cdot \bar{X} = 95\%$ of $\bar{X}$

(b) From Figure 2, determine that only 62 percent of tests can be expected to exceed the design strength.

(4) Conclude that, although the concrete nominally meets the strength requirement, a conference with the design engineer is needed. Core tests may be needed as a basis for final decisions on the need for remedial action. Under the circumstances, the cost of coring and any needed repair would have to be borne by the owner.

Many variations of these basic problems may be analyzed by use of the cumulative distribution curves.

NOTATION

d	-	a factor for computing within-test deviation
e	-	base of Naperian logarithms
f'	-	concrete strength assumed in structural design
f_c''	-	specified concrete strength
f_{cr}	-	average concrete strength required to insure compliance with specified requirements
n	-	number of samples or tests
S	-	standard deviation
S_1	-	within-test standard deviation
t	-	significance ratio
V	-	overall coefficient of variation
V_1	-	within-test coefficient of variation
X	-	the value of an individual test
$\overline{X}$	-	the average value of a test series
y	-	frequency of occurrence
y_0	-	frequency of occurrence corresponding to the average value

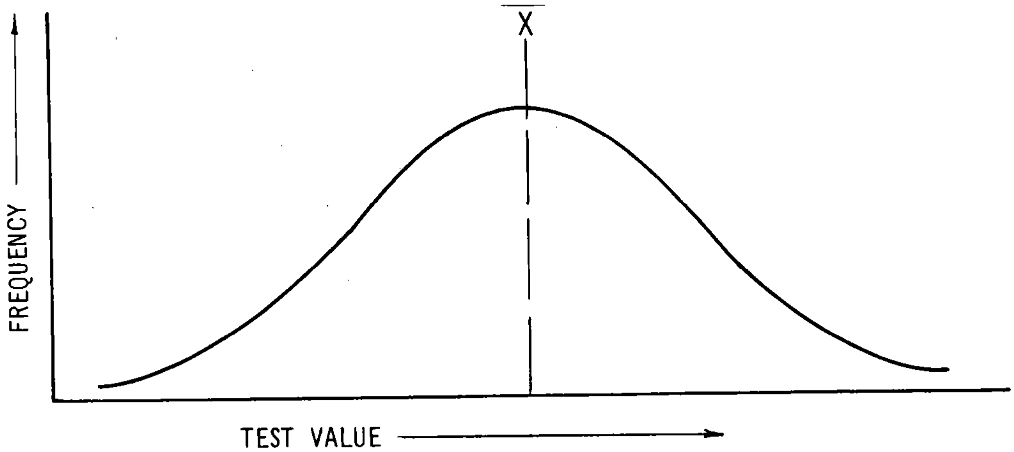

FIGURE I. – Frequency - Distribution Curves

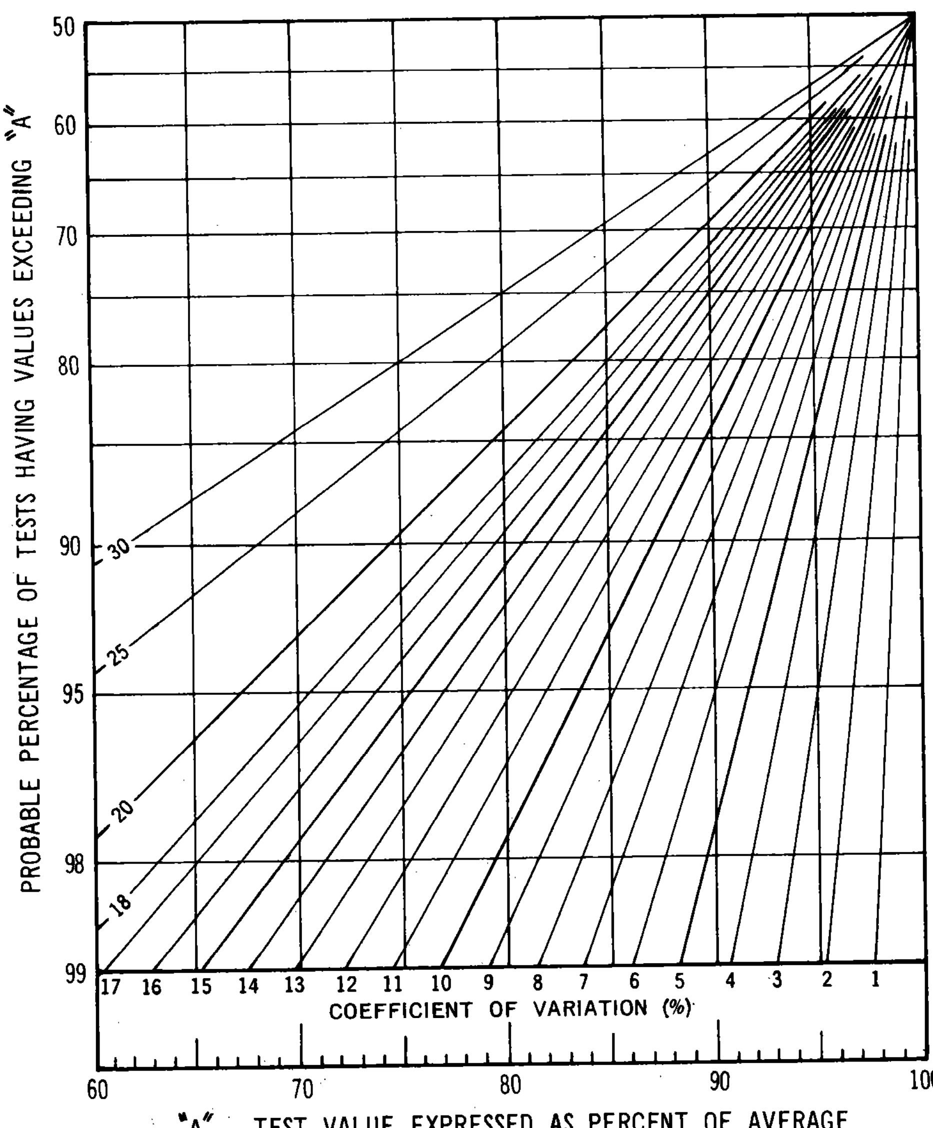

FIGURE 2. – Cumulative Distribution Curves