United States Department of Agriculture
Soil Conservation Service
Engineering Division

Technical Release No. 5
Design Section
November 17, 1958

THE STRUCTURAL DESIGN OF UNDERGROUND CONDUITS

The objectives of this Technical Release are (a) to present the subject of loads on underground conduits and their supporting strengths, and (b) to furnish working aids and procedures for the computation of loads and strengths. The scope of the subject is limited to phases which pertain to the needs of the Soil Conservation Service. Some phases of the subject are relatively new and further experience and knowledge may result in revisions of the release from time to time.

This Technical Release has been written by Messrs. Gerald E. Oman and Paul D. Doubt. Mr. M. M. Culp recommended the values used for X_p for cradles and proposed the method of constructing Type B1 beddings. Mr. Doubt developed the theory and procedure for evaluating the settlement ratio δ. Messrs. H. J. Goon, Richard M. Matthews, Norman P. Hill, and A. R. Gregory assisted in the preparation of this release. Mrs. Joan Robison typed the manuscript. This work was done under the general administrative direction of Mr. M. M. Culp, Chief, Design and Construction Branch, and Mr. Paul D. Doubt, Head, Design Section.

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THE STRUCTURAL DESIGN OF UNDERGROUND CONDUITS

NOMENCLATURE

$a \quad = \frac{2K\mu}{b}$

$a_f \quad = \frac{2K_f\mu_f}{b}$

b = bottom width of cradle or rigid bedding, ft ($b \geqq b_c$)

b_c = outside width of conduit, ft

b_d = width of ditch at the top of the conduit, ft

b_d' = value of the width of ditch at which the load on a conduit as computed by the ditch conduit formula is equal to the load on the conduit as computed by the positive projecting conduit formula, ft

C_d = load coefficient for ditch conduits

C_n = load coefficient for negative projecting conduits, ditch condition

C_p = load coefficient for positive projecting conduits

c = distance from the neutral axis to the outer fiber

D = load per foot of diameter per foot length of pipe for the three-edge bearing test

d = inside diameter of pipe, inches

E' = Young's modulus of elasticity

E = modulus of consolidation of the embankment or backfill material, tons/ft^2

E_f = modulus of consolidation of the foundation material, tons/ft^2

e = 2.7183 = base of natural logarithms

f_{ec} = maximum fiber stress in a pipe having a given load which produces 0.01-inch crack in a R/C pipe or ultimate fiber stress for other types of pipes, lbs/ft^2

f_{eb} = maximum fiber stress for three-edge bearing load of R_{eb}, lbs/ft^2

f_s = allowable stress in reinforcing steel

$F_{sp} = \frac{1.431\ R_{eb}}{s\gamma {b_c}^2}$ = provided strength factor

$F_{sr} = C_pX_p - KT$ = required strength factor

H = for the complete condition--vertical distance from top of backfill to a horizontal element of fill material having a height of dH, ft
for the incomplete condition--vertical distance from plane of equal settlement to a horizontal element of fill material having a height dH, ft

H_c = vertical distance from top of backfill or embankment to top of conduit, ft

H_{ca} = allowable vertical distance from top of backfill or embankment to top of conduit, ft

H_e = vertical distance from the plane of equal settlement to top of conduit, ft

H_e' = distance between the top of the conduit and the upper plane of equal settlement when the interior prism has a width b

H_f = distance between the bottom of the cradle or rigid bedding (bottom of the pipe if no cradle or rigid bedding is used) and the nonyielding foundation

H_ℓ = distance between the bottom of the cradle and the lower plane of equal settlement, ft. When no cradle is used, it is the distance between the bottom of the conduit and the lower plane of equal settlement.

h = vertical distance from any point in the embankment to the upper surface of the fill, ft

I = moment of inertia, in^4

K = ratio at a point of active lateral pressure to vertical pressure for the backfill or embankment material

K_f = ratio at a point of active lateral pressure to vertical pressure for the foundation material

L_f = load factor

$\Delta\ell$ = length of a differential element of the pipewall (see Fig. B-2, page B-3)

M = moment with upper case subscripts denoting location

N = bursting pressure of the pipe, lbs/in^2

P = total vertical pressure on a horizontal plane within the interior prism, lbs/ft length of pipe

P' = vertical pressure on a horizontal plane within the interior prism when the embankment height is equal to the height of equal settlement, lbs/ft length of pipe

P" = additional vertical pressure on a horizontal plane within the interior prism due to the weight of the material above the plane of equal settlement, lbs/ft length of pipe

P_c = total vertical pressure in the width b_d at the top of the conduit, lbs/ft length of pipe

p = intensity of lateral pressure, lbs/ft^2

p_i = internal pressure in a pipe, lbs/in^2

R = reaction with upper case subscripts denoting location

R_c = supporting strength of pipe for a stated load pattern, lbs/ft length of pipe

R_d = safe supporting strength of pipe, lbs/ft length of pipe

R_{eb} = supporting strength of pipe for three-edge bearing load, lbs/ft length of pipe

R'_{eb} = value of the reduced supporting strength of a pipe having positive internal pressure, lbs/ft length of pipe. It is used in supporting strength formulas in place of the three-edge bearing strength R_{eb}.

r = mean radius of pipe, ft

S = allowable shearing stress, lbs/in^2

s = a safety factor

s_c = additional deformation of the conduit, ft (positive and negative projecting conduits)

s_d = additional consolidation of the backfill material between the top of the conduit and the critical plane (negative projecting conduits)

s_f = additional settlement of the bottom of the conduit (i.e., the surface of the natural ground beneath the conduit) due to the consolidation of the foundation, ft (positive and negative projecting conduits)

s_g = additional settlement of the natural ground surface below the exterior prism due to the consolidation of the foundation, ft (positive and negative projecting conduits)

s_m = additional consolidation of the embankment material between the critical plane and the natural ground surface in the exterior prism, ft (positive projecting conduits)

T = $\rho X_a \left(\frac{H_c}{b_c} + \frac{\rho}{2}\right)$ = a parameter used in expressing active lateral earth pressure

t = thickness of pipewall, ft

U = $\frac{2K\mu F_{sp} + K\mu K\rho^2 X_a + X_p}{X_p}$

W_c = total vertical load on the top of an underground conduit, lbs/ft length of conduit

X_a = a function of the projection onto a vertical plane of the area of the pipe over which the lateral loads are assumed to be distributed

X_p = a function of the distribution of the vertical load and vertical reaction

Y = a factor (Eq. B-12, page B-7)

y = distance from the neutral axis to a differential element in the pipewall

Z = a factor (Eq. B-13, page B-7)

α = one-half of the central angle subtended by the arc of the pipe over which no lateral loads are acting on the pipe

β = one-half of the central angle subtended by the arc of the pipe over which the upward vertical reactions are acting

γ = unit weight of backfill or embankment material, lbs/ft^3

γ_f = unit weight of the foundation material, lbs/ft^3

δ = $\dfrac{(s_m + s_g) - (s_f + s_c)}{s_m}$ = settlement ratio for positive projecting conduits

∂ = unit strain

δ' = $\dfrac{s_g - (s_c + s_f + s_d)}{s_d}$ = settlement ratio for negative projecting conduits

θ = angle used in deriving Eq. B-8 (see Fig. B-4, page B-5)

κ_t = $\dfrac{\rho K}{C_p}\left(\dfrac{H_c}{b_c} + \dfrac{\rho}{2}\right)$ = ratio of total lateral load to total vertical load

λ_e = additional consolidation of the embankment material in the exterior prism between the critical plane and the plane of equal settlement, ft (positive projecting conduits)

λ_i = additional consolidation of the embankment material in the interior prism between the top of the conduit and the plane of equal settlement, ft (positive projecting conduits)

λ_{en} = additional consolidation of the embankment material in the exterior prism between the natural ground and the plane of equal settlement, ft (negative projecting conduits)

λ_{in} = additional consolidation of the embankment material in the interior prism between the critical plane and the plane of equal settlement, ft (negative projecting conduits)

μ = tan ϕ = tangent of the angle of internal friction of the backfill or embankment material

μ' = tangent of the angle of sliding friction between the backfill material and the material in the ditch wall

μ_f = tan ϕ_f = tangent of the angle of internal friction of the foundation material

ξ = angle of rotation (see Fig. B-4, page B-5)

ρ = projection ratio for positive projecting conduits = ratio of the distance between the natural ground surface and the top of the conduit (when $H_c = 0$) to the outside width of the conduit

ρ' = projection ratio for negative projecting conduits = ratio of the distance between the natural ground surface and the top of the conduit (when $H_c = 0$) to the width of the ditch b_d

σ = unit stress

ϕ = angle of internal friction of the backfill or embankment material

ϕ_f = angle of internal friction of the foundation material

ψ = ratio of the distance ψb_c (ES-114, page 3-41) to the outside width of the conduit

ω = $2K\mu \dfrac{H_e'}{b}$

GREEK ALPHABET

Α α	Alpha - ăl'få	Ν ν	Nu - nū (new)
Β β	Beta - bā'tå	Ξ ξ	Xi - zī
Γ γ	Gamma - găm'å	Ο ο	Omicron - ŏm'ĭ krŏn
Δ δ	Delta - dĕl'tå	Π π	Pi - pī (pie)
Ε ε	Epsilon - ĕp'sĭ lŏn	Ρ ρ	Rho - rō
Ζ ζ	Zeta - zā'tå	Σ σ	Sigma - sĭg'må
Η η	Eta - ā'tå	Τ τ	Tau - ta (taw)
Θ θ	Theta - thā'tå	Υ υ	Upsilon - ūp'sĭ lŏn
Ι ι	Iota - ī ō'tå	Φ φ	Phi - fī or fē
Κ κ	Kappa - kăp'å	Χ χ	Chi - kī
Λ λ	Lambda - lăm'då	Ψ ψ	Psi - sī or psē
Μ μ	Mu - mū (mew)	Ω ω	Omega - ō mĕg'a

TECHNICAL RELEASE

NUMBER 5

THE STRUCTURAL DESIGN OF UNDERGROUND CONDUITS

The structural design of any structure requires a determination of the loads on the structure and the proportioning of the structure to resist the loads. An understanding of the structural design of underground conduits is facilitated by dividing the subject into the following chapters:

Chapter 1 - Loads on Underground Conduits

Chapter 2 - Supporting Strength of Conduits

Although the structure can be designed by the use of these two divisions, a third chapter is added to facilitate the solution of positive projecting conduit design problems. It also contains examples and charts.

Messrs. Anson Marston, M. G. Spangler, and W. G. Schlick of the Iowa Engineering Experiment Station, Iowa State College, have developed the theory and performed the research on which this technical release is based.* This is a compilation of their theory and data for use by the Soil Conservation Service. Their analysis has been rearranged and additional charts have been prepared.

Since most conduits designed by engineers of the Soil Conservation Service will be classed as ditch conduits or positive projecting conduits, only these two classes are considered in Chapters 1 and 3. The derivation of load equations for ditch conduits, positive projecting conduits, negative projecting conduits, and ditch conduits with compacted backfill is given in Appendix A. A brief discussion of other classes is also given in Appendix A.

The supporting strength of rigid circular conduits is considered in Chapters 2 and 3. Appendix B gives the derivation of the supporting strength formulas for rigid circular pipes. The method of design for concrete monolithic box culverts is given in the National Engineering Handbook, Section 6, Structural Design.

The design of the usual underground conduit installations may be accomplished without a complete understanding of the subject by the use of the procedure charts and computation aids given in the ES drawings, pages 3-37 to 3-87.

CHAPTER 1 - LOADS ON UNDERGROUND CONDUITS

A classification of conduits based on the sets of factors listed below is required for load determinations. The manner in which these factors are used to classify underground conduits is given in ES-116, page 3-53.

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*Numerical references refer to the bibliography following page C-12.

Construction Methods

The term construction methods is used for classification purposes only. It includes site conditions and design requirements, as well as construction methods, and involves

1. The width of the ditch (sometimes infinitely wide) in which the conduit is placed;
2. The compressibility of the backfill relative to the compressibility of the earth in which the ditch was excavated;
3. The compressibility of the materials on which the conduit rests relative to the adjacent foundation materials;
4. The elevation of the top of the backfill or embankment relative to the natural ground line;
5. The elevation of the top of the conduit with respect to the natural ground line; and
6. The compressibility of the material directly above the conduit relative to the compressibility of the adjacent material.

This set of factors is used to determine whether the conduit is classed as a ditch conduit, a positive projecting conduit, a negative projecting conduit, a ditch conduit with dense backfill, an imperfect ditch conduit, or a conduit on compressible bedding.

Relative Settlements

The term relative settlements means the settlement of the top of the conduit relative to the settlement of the critical plane. This involves

1. Deformation of the conduit; and
2. Settlements or consolidations of the backfill, embankment, and foundations. These depend on the soil characteristics.

This set of factors is used to determine whether the positive projecting conduit or the negative projecting conduit is classed as the ditch condition or the projection condition.

Relative Height of Embankment

The term relative height of embankment means the height of embankment relative to the height of the plane of equal settlement. This factor is used to determine whether the conduit is classed as the complete condition or the incomplete condition.

Loads on Ditch Conduits

When a conduit is placed in a ditch and covered with backfill material, the backfill material tends to settle downward. This tendency of the backfill material above the top of the conduit to move produces vertical friction forces or shearing stresses along the sides of the ditch. These shearing stresses give support to the backfill material. (See Fig. 1-1.)

The proportion of the total vertical pressure that is carried by the conduit will depend on the relative rigidity of the conduit and of the fill material between the sides of the conduit and the sides of the ditch. For rigid pipes (see footnote, page 2-1) such as clay, concrete, or cast-iron pipe, the side fills may be relatively compressible and the pipe itself will carry practically all of the load. If the pipe is a relatively flexible thin-walled pipe and the side fills have been thoroughly tamped, the load on the conduit will be reduced by the amount of load the side fills carry.

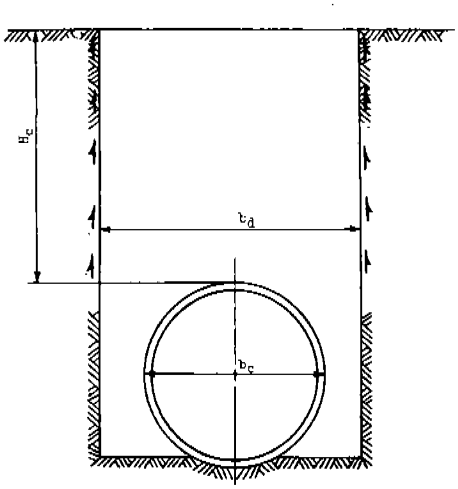


Fig. 1-1 Ditch conduit

The total vertical load on rigid ditch conduits with relatively compressible side fills is

$$W_c = C_d \gamma b_d^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1\text{-}1)$$

where $$C_d = \frac{1 - e^{-2K\mu'(H_c/b_d)}}{2K\mu'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1\text{-}1a)$$

W_c = total vertical load on the top of the underground conduit, lbs/ft length

C_d = load coefficient for ditch conduits

b_d = width of the ditch at the top of the pipe, ft. The width of the ditch b_d is the actual width of a vertical walled ditch in which the pipe is installed. When the ditch is constructed with sloping sides or the conduit is placed in a subditch at the bottom of a wider ditch, experimental results indicate that the proper width b_d is at or slightly below the top of the conduit (see ES-114, page 3-41.

μ = tangent of the angle of internal friction of the backfill material

$K = \dfrac{\sqrt{\mu^2 + 1} - \mu}{\sqrt{\mu^2 + 1} + \mu}$ = ratio at a point of active lateral pressure to vertical pressure

μ' is the tangent of the angle of sliding friction between the backfill material and the material in the ditch wall. μ' is the value of μ for the material in which the ditch is dug if the excavated material is used for backfill. When the backfill material differs from the material in the ditch walls, the smaller of the two values of μ for these two materials should be used for the value of μ'.

γ = unit weight of the backfill material, lbs/ft^3

e = 2.7183 = base of Naperian logarithms

H_c = vertical distance from top of backfill to top of conduit, ft

The total vertical load W_c on flexible pipes with thoroughly compacted side fills is

$$W_c = C_d \gamma b_c b_d \qquad (1\text{-}2)$$

where b_c = outside width of the conduit, ft

The above formulas will give maximum vertical loads to be expected on ditch conduits throughout the life of the structure if the proper physical factors involved in their solution have been selected.

Loads on Positive Projecting Conduits

In the discussion of loads on positive projecting conduits, several terms are used (see Fig. 2-2)

1. The interior prism is that prism of embankment which is bounded by the critical plane at the top of the conduit, the plane of equal settlement, and the vertical planes which are tangent to the sides of the conduit.
2. The exterior prisms are the two masses of embankments exterior to the conduit and having for their boundaries the vertical tangent planes, the natural ground, and the plane of equal settlement.
3. The critical plane is that film of particles of embankment materials which was originally lying in the horizontal flat plane at the top of the conduit when $H_c = 0$.
4. The plane of equal settlement is that film of particles of embankment materials which lies in the lowest horizontal plane that remains as a plane as settlement takes place. This necessitates that the settlement of a particle of embankment at any elevation above the interior prism will be equal to the settlement of any particle having the same elevation above the exterior prism. Thus, there are no vertical shearing forces existing between particles of embankment materials above the plane of equal settlement.
5. The projection ratio ρ is the ratio of the distance between the natural ground surface adjacent to the conduit (when $H_c = 0$) and the top of the conduit (when $H_c = 0$) to the outside width of the conduit.

6. Deformation, consolidation, and settlement. It will be important to differentiate between the meaning of the words deformation, consolidation, and settlement. Deformation is the change in the length of structural materials due to stress. The word deformation is reserved here to denote the change in the vertical dimension of the conduit. If the conduit is rigid the deformation of the conduit is assumed to be negligible or zero. Consolidation is the change in the vertical length of embankment or foundation materials caused by loads above the materials. It is used exactly in the same sense as deformation except the word deformation is reserved for structural materials and consolidation is reserved for embankment and foundation materials. Settlement is the change in elevation of a particle of embankment or foundation material as a result of consolidation of materials or deformation of structures below the particle.

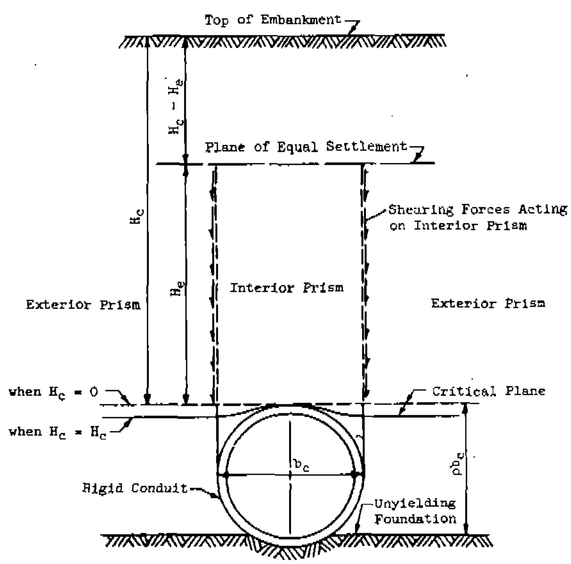


Fig. 1-2 Basic case for considering the action of an embankment over a positive projecting conduit

7. Additional consolidations, additional deformations, and additional settlements. The weight of the embankment materials above the plane of equal settlement will cause consolidations and deformations in addition to those consolidations and deformations due to the weight of the embankment material below the plane of equal settlement. These consolidations and deformations (due to the weight of the material above the plane of equal settlement) will be referred to as "additional consolidations" and "additional deformations" and the corresponding settlements as "additional settlements." The convenience of the use of "additional" instead of "total" becomes apparent in the derivation of load formulas (see Appendix A).

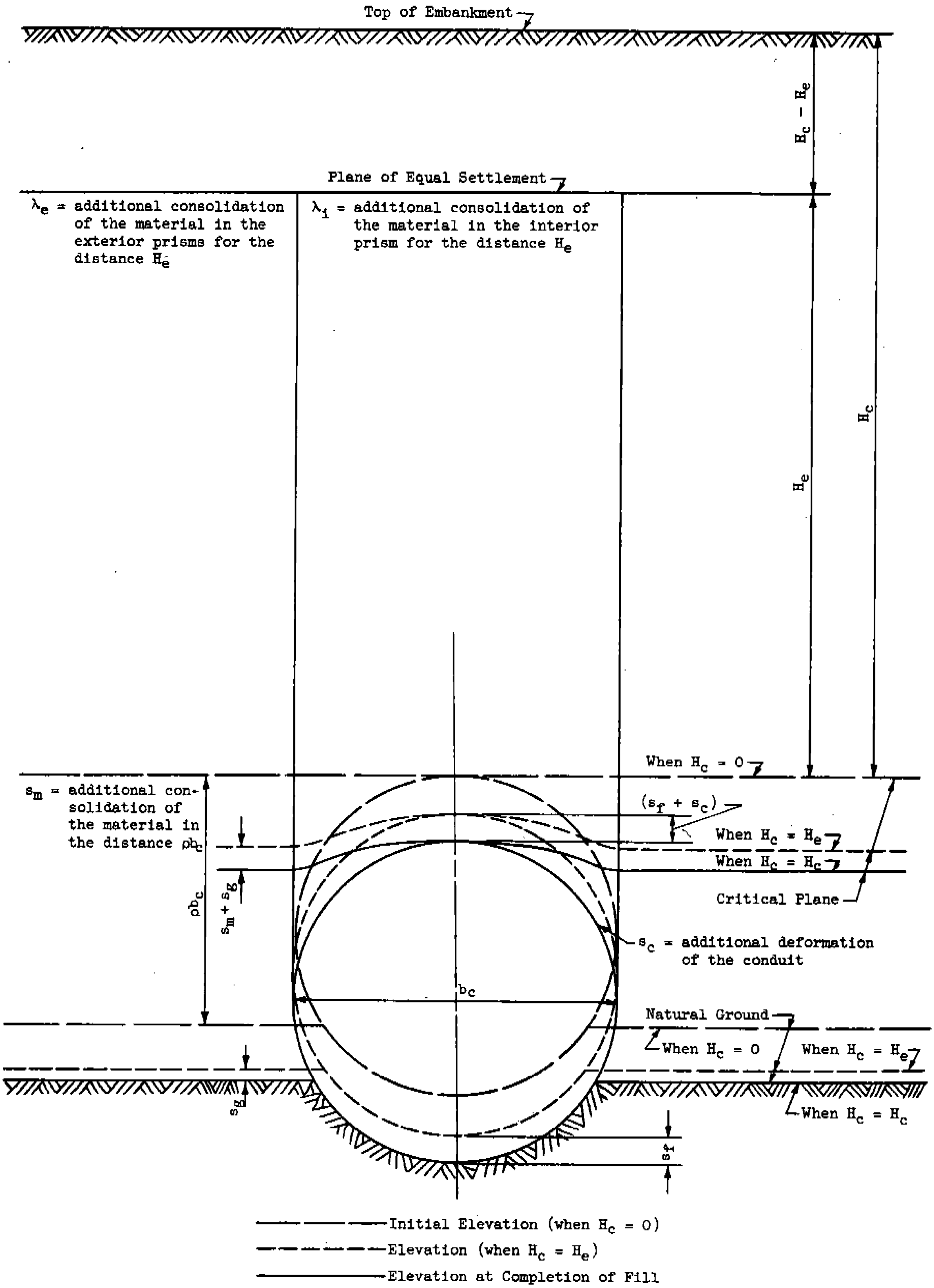


Fig. 1-3 Settlements which influence loads on positive projecting conduits

The symbols used to evaluate the additional settlement of the top of the exterior prism are (see Fig. 1-3)

λ_e = additional consolidation of the embankment material between the critical plane and the plane of equal settlement, ft
s_m = additional consolidation of the embankment material between the critical plane and the natural ground surface, ft
s_g = additional settlement of the natural ground surface below the exterior prism due to the consolidation of the foundation, ft
$s_m + s_g$ = additional settlement of the critical plane, ft

The symbols used to evaluate the additional settlement of the top of the interior prism are

λ_i = additional consolidation of the embankment material between the top of the conduit and the plane of equal settlement, ft
s_c = additional deformation of the conduit, ft
s_f = additional settlement of the bottom of the conduit (i.e., the surface of the natural ground beneath the conduit) due to the consolidation of the foundation, ft
$s_f + s_c$ = additional settlement of the top of the conduit, ft

8. Settlement ratio δ. The settlement ratio is the ratio of the difference of the additional settlement of the top of the conduit and the additional settlement of the critical plane in the exterior prism to the additional consolidation of the embankment material below the critical plane.

$$\delta = \frac{(s_m + s_g) - (s_f + s_c)}{s_m} \quad \ldots\ldots\ldots\ldots (1\text{-}3)$$

The value of δ determines whether the projection or the ditch condition exists. The projection condition occurs when $\delta > 0$. The ditch condition exists when $\delta < 0$.

Projection and Ditch Condition

The projection condition is defined as the condition in which the critical plane in the exterior prism settles more than the top of the conduit (see ES-116, page 3-53). When this condition exists, loads are transferred from the exterior prism to the interior prism. It is obvious that the load on a conduit for the projection condition is always greater than the weight of embankment material above the top of the conduit.

If the conduit is sufficiently flexible, the settlement of the top of the conduit will be greater than the settlement of the critical plane in the exterior prism. This is called the ditch condition. Since loads are being transferred from the interior prism to the exterior prism, the load in the interior prism is less than the load in the exterior prism. Again it is obvious that the load on the conduit for the ditch condition is less than the weight of the embankment material above the conduit.

A neutral condition exists when there is no transfer of loads between the interior and exterior prisms, and the load on the conduit is the weight of the embankment material above the top of the conduit.

The relation between the rigidity of the conduit and the degree of compressibility of the adjacent fill material as well as the modulus of consolidation of the foundation material will determine whether the ditch condition or the projection condition occurs for any given installation. These two conditions represent subclassifications of projecting conduits which are determined by relative settlements or consolidations.

It is important to observe the difference of the meaning in the unfortunate terminology of "ditch conduits" and "ditch conditions." The term "ditch conduits" pertains to a classification of underground conduits based on construction methods. The term "ditch condition" pertains to a subclassification of projecting conduits based on relative settlements.

Likewise, the difference between the terms "projecting conduits" and "projection condition" should be understood. The term "projecting conduits" (positive or negative) pertains to a classification of underground conduits based on construction methods. The term "projection condition" pertains to a subclassification of projecting conduits based on relative settlements.

Complete and Incomplete Conditions

The complete condition exists when the embankment height H_c is less than or equal to the height of the plane of equal settlement H_e (see ES-116, page 3-53). The shearing stresses between the interior and exterior prisms extend completely to the top of the embankment. Thus, $\frac{H_c}{b_c} \leq \frac{H_e}{b_c}$.

The incomplete condition exists when the embankment height H_c is greater than the height of the plane of equal settlement H_e. The shearing stresses between the interior and exterior prisms do not extend completely to the top of the embankment. Hence, $\frac{H_c}{b_c} > \frac{H_e}{b_c}$. It is possible to have complete and incomplete conditions for both the ditch and projection condition.

Height of Equal Settlement H_e

The value of H_e is determined for two reasons.

1. The comparison of $\frac{H_e}{b_c}$ values with $\frac{H_c}{b_c}$ values define whether the complete or incomplete condition exists.

   When $\frac{H_c}{b_c} \leq \frac{H_e}{b_c}$, the complete condition exists.

   When $\frac{H_c}{b_c} > \frac{H_e}{b_c}$, the incomplete condition exists.

2. The value of $\frac{H_e}{b_c}$ is required to determine the load on a projecting conduit for the incomplete condition.

The derivation shown in Appendix A in the determination of H_e is that originally developed by A. Marston.[4] Marston's assumption for determining H_e yields an expression which gives slightly greater values for loads on conduits than those assumptions used by M. G. Spangler.[9]

The expression for the determination of H_e is

$$e^{\pm 2K\mu(H_e/b_c)} \mp 2K\mu(H_e/b_c) = \pm 2K\mu\delta\rho + 1 \quad . . . . . \quad (1\text{-}4)$$

The solution of Eq. 1-4 is facilitated by the use of ES-117, page 3-57.

Load Formulas for Positive Projecting Conduits
(See Appendix A for derivation). The equation for the load on a positive projecting conduit, complete condition, is

$$W_c = C_p \gamma b_c^2 \quad . . . . \quad (1\text{-}5)$$

where the load coefficient for positive projecting conduits, complete condition, is

$$C_p = \frac{e^{\pm 2K\mu(H_c/b_c)} - 1}{\pm 2K\mu} \quad . . . . \quad (1\text{-}5a)$$

In Eq. 1-5a the top sign in the (±) symbol is used for the projection condition, and the bottom sign is used for the ditch condition. This convention applies wherever double signs appear.

Since Eqs. 1-5 and 1-5a are applicable for both the complete ditch condition and the complete projection condition, they may be used to determine loads on both rigid and flexible conduits.

The equation for the load on a positive projecting conduit, incomplete condition, is

$$W_c = C_p \gamma b_c^2 \quad . . . . \quad (1\text{-}6)$$

where the load coefficient for positive projecting conduits, incomplete condition, is

$$C_p = \frac{e^{\pm 2K\mu(H_e/b_c)} - 1}{\pm 2K\mu} + \left[\frac{H_c}{b_c} - \frac{H_e}{b_c}\right] e^{\pm 2K\mu(H_e/b_c)} \quad . . . . \quad (1\text{-}6a)$$

Since Eqs. 1-6 and 1-6a are applicable for both the incomplete ditch condition and the incomplete projection condition, they may be used to determine loads on both rigid and flexible conduits.

The solution of Eqs. 1-5a and 1-6a is facilitated by the use of ES-118, pages 3-63 and 3-65.

It is necessary to determine the value of H_e/b_c by the use of Eq. 1-4 to solve Eq. 1-6a since Eq. 1-6a contains H_e/b_c as one of its variables.

Effect of Width of Ditch

No definite width of ditch is specified to define whether a conduit is classed as a ditch conduit or as a positive projecting conduit. In the analysis of loads on ditch conduits, it is assumed that a rigid conduit resists all of the pressure at the top of the conduit for the width b_d. This pressure which is equal to the load on the rigid conduit is less than the weight of the backfill material because a portion of the weight is transferred by shear into the ditch walls. Increasing the value of b_d for ditch conduits increases the load on the conduit. As the width of a ditch is increased, a value of b_d is reached where the total pressure is no longer resisted by the conduit and the classification changes to a positive projecting conduit.

In the analysis of positive projecting conduit, projection condition, the load on the conduit is the pressure at the elevation of the top of the conduit distributed over the width b_c rather than b_d. This load is greater than the weight of the prism directly above the conduit because a portion of the weight of the exterior prism is transferred by shear to the interior prism. By increasing b_d, the load is increased on ditch conduits to a value b_d' which is equal to the load computed by the formulas for positive projecting conduits, projection condition. It is this value of b_d' which defines the boundary between ditch conduits and positive projecting conduits. Conduits for a given value of H_c/b_c in a ditch having a width $b_d < b_d'$ are ditch conduits. Those conduits in which $b_d > b_d'$ are positive projecting conduits. The value of b_d' can be determined from the chart in ES-117, page 3-59.

Figure 1-4 is a plot showing two curves which gives the load on a 2-foot diameter conduit ($b_c = 2.5$ ft) when $H_c = 10$ ft and $H_c = 15$ ft for various values of b_d. When $b_d = b_c$, the load on the conduit computed by the positive projecting conduit formulas is greater than the load computed by the ditch conduit formula because of the transfer of weights by shear. As b_d is increased, holding b_c constant, the load computed by the positive projecting conduit formulas remains constant while the load computed by the ditch conduit formula increases. At some value of b_d the loads computed by both formulas become equal. Observe in this plot how the value of b_d' increases from $b_d' = 5.5$ ft to $b_d' = 6.2$ ft as the value of H_c is increased from 10 ft to 15 ft.

The Effect of Surface Loads on Underground Conduits

Underground conduits may also be subjected to a load that is transmitted through the soil from traffic loads applied to the surface of the backfill. The effect of these loads may be determined by the use of the Boussinesq solution[14] or by the approximate method given in ES-25, National Engineering Handbook, Section 6, Structural Design. Such loads will be of major importance when the conduit is under a shallow fill and subjected to heavy traffic loads. Most conduits designed by the Soil Conservation Service are subjected to heavy traffic loads only during construction. If the construction equipment is not permitted to cross over the conduit until at least 2 feet of cover has been placed over the conduit and the conduit is designed to carry a minimum earth load of 10 feet, the effect of loads due to construction equipment may be neglected.

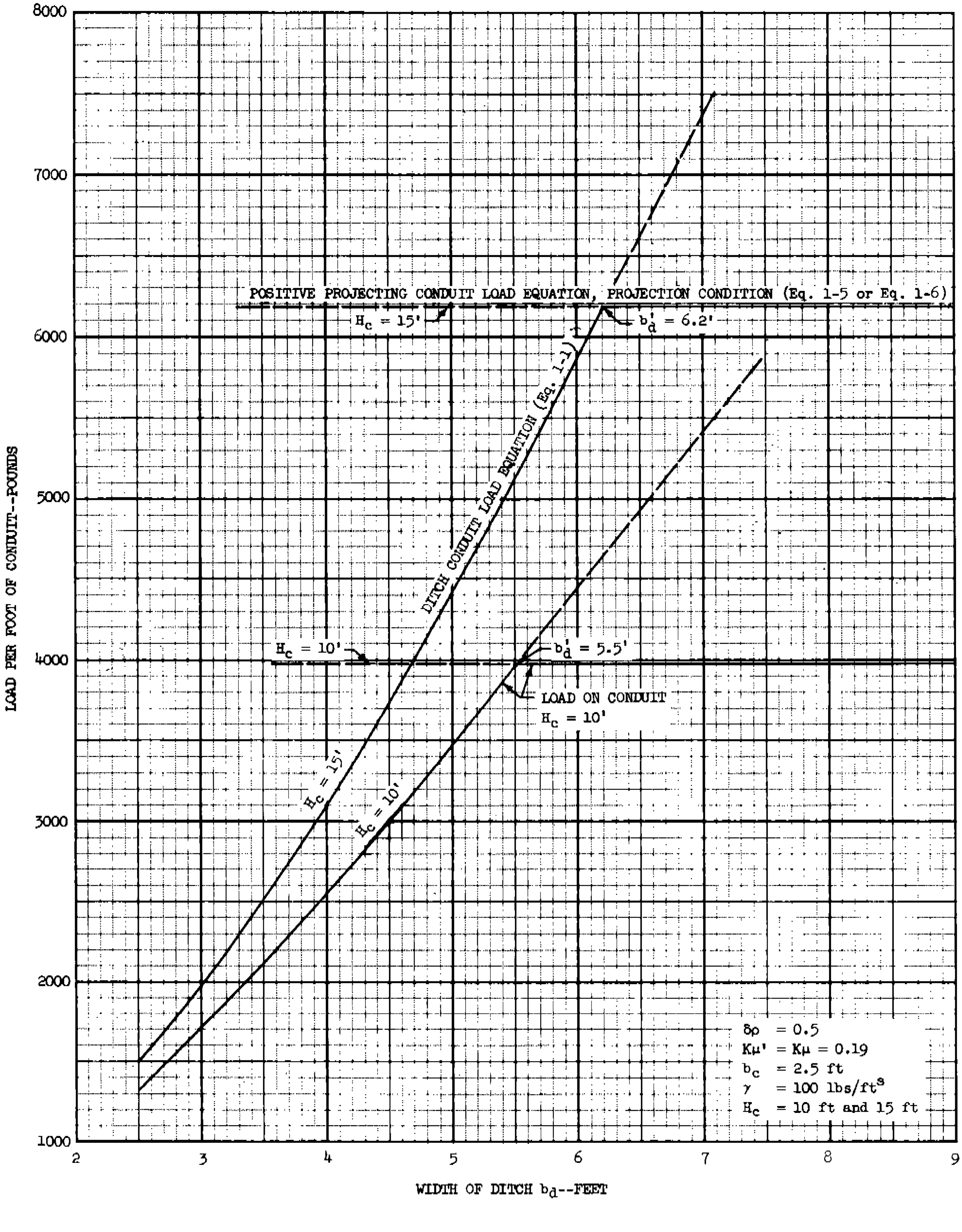


Fig. 1-4 Effect of increasing the width of ditch on the load on a conduit.

Internal Fluid Pressures

The determination of internal fluid pressure in a conduit is a hydraulic problem (see National Engineering Handbook, Section 5, Hydraulics). The magnitudes of internal fluid pressures encountered in Soil Conservation Service work is usually of minor importance in determining loads on underground conduits.

Hydrostatic Loads

When an underground conduit is below the water table, there is an external hydrostatic load acting on the conduit. The determination of hydrostatic pressures is also a hydraulic problem (see National Engineering Handbook, Section 5, Hydraulics).

Procedure for Determining Loads

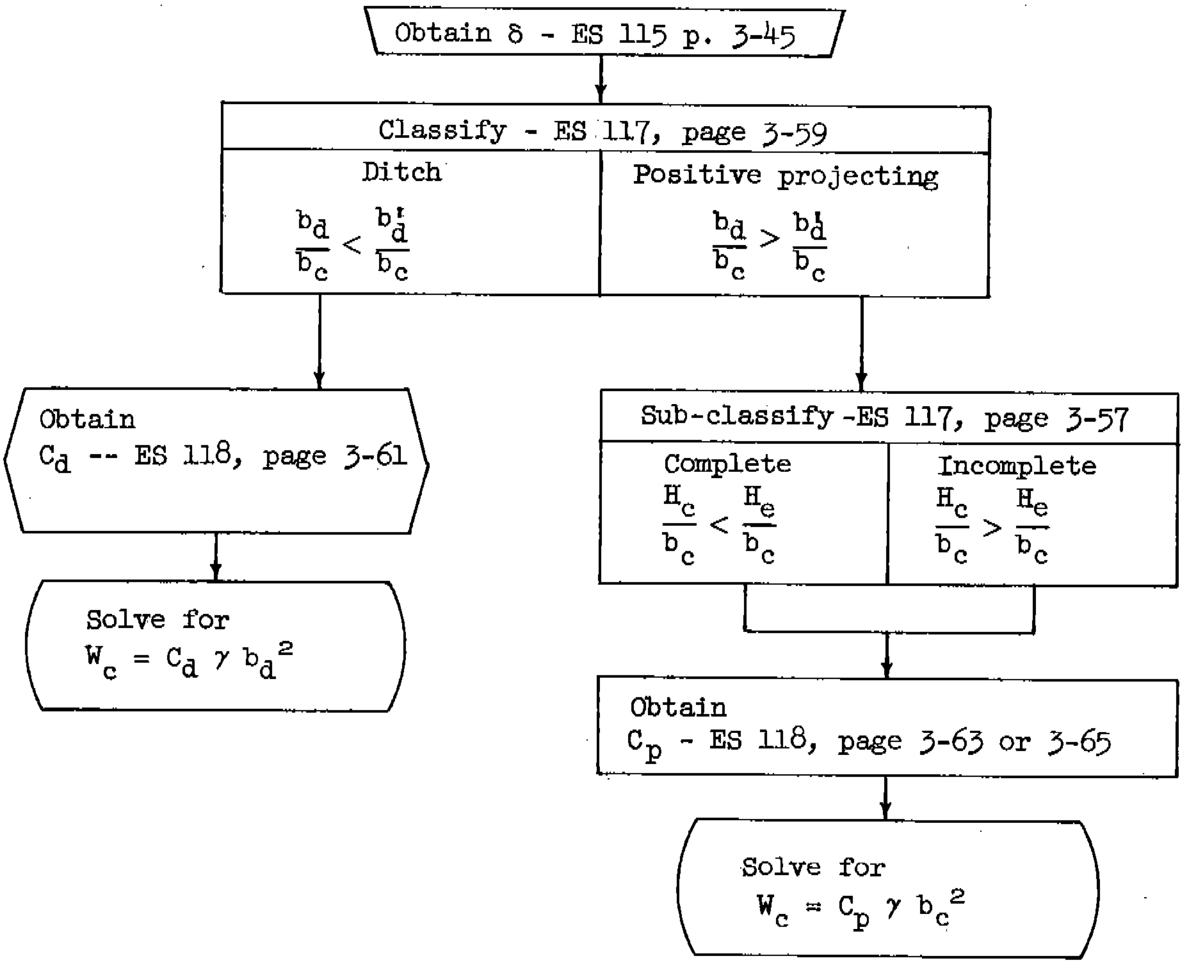


Fig. 1-5. Procedure for determining the load on a ditch conduit or positive projecting conduit.

Example - Determining Loads

Given: 1. A 36 in. reinforced concrete pipe (b_c = 3.5 ft.) is proposed for installation on a cradle (b = 5.0 ft).

2. The height of fill above the conduit is 30.0 ft.

3. The backfill material has a unit weight (γ) of 120 lbs/ft^3 and an angle of internal friction (ϕ) of 30°.

4. The ditch wall and foundation material has an angle of internal friction of 20°.

5. The value of $\left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right] = 1.0$.

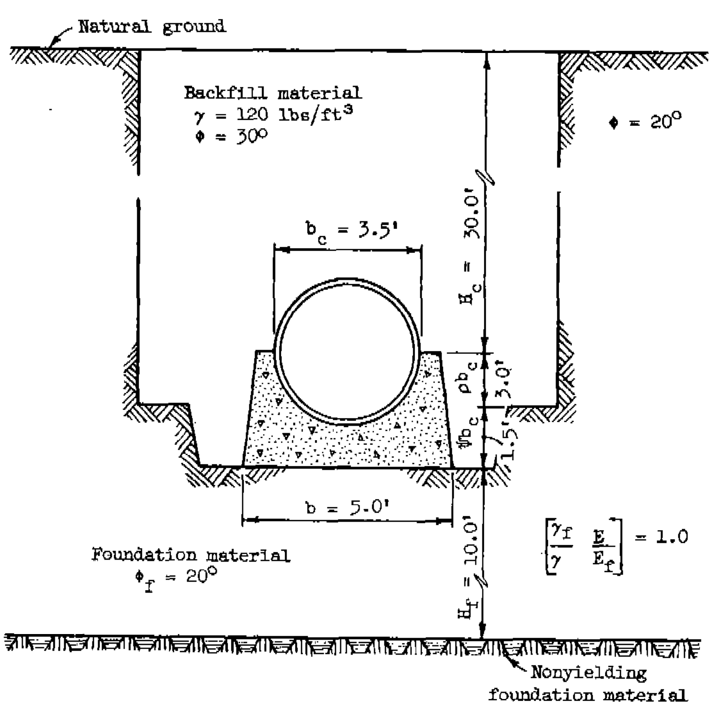
6. The distance ρb_c = 3.0 ft. and the distance ψb_c = 1.5 ft.

7. The distance from the bottom of the cradle to non-yielding foundation material is 10.0 ft.

Determine:

1. The load on the conduit when b_d = 6.0 ft.

2. The load on the conduit when b_d = 15.0 ft.



Solution:

1. Follow the procedure chart in Fig. 1-5.

Obtain δ. ES 115

From ES 115, page 3-45 either case c or d exists

b = 5.0 ft.

$$\delta(\text{case c}) = \frac{1+\left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right]\frac{\psi}{\rho}}{1+\left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right]\frac{K\mu}{K_f\mu_f}}$$

From ES 114, page 3-43, $K\mu = 0.19$, $K_f\mu_f = 0.178$

$$\psi = \frac{1.5}{3.5} = 0.429, \quad \rho = \frac{3.0}{3.5} = 0.857$$

$$\delta(\text{case c}) = \frac{1+(1)\ \left(\frac{0.429}{0.857}\right)}{1+(1)\ \left(\frac{0.190}{0.178}\right)} = \frac{1.5}{2.067} = 0.726$$

$$\delta\rho = (0.726)(0.857) = 0.622$$

From ES 115, page 3-51, $\frac{H_e'}{b} = 1.63$

$$H_e' = (1.63)(5.0) = 8.15 \text{ ft.}$$

$$H_e' \frac{K\mu}{K_f\mu_f} = (8.15)\ \frac{0.190}{0.178} = 8.696$$

$$H_f = 10.0 \text{ ft.}$$

Since $H_f > H_e' \frac{K\mu}{K_f\mu_f}$ case c exists.

$$\delta = 0.726$$

Classify (ES 117, page 3-59)

$$2K\mu\delta\rho = (2)(0.19)(0.622) = 0.236$$

$$2K\mu\ \frac{H_c}{b_c} = 2(0.19)\ \frac{30.0}{3.5} = 3.257$$

$$\frac{b_d'}{b_c} = 2.95$$

$$\frac{b_d}{b_c} = \frac{6.0}{3.5} = 1.714$$

Since $\frac{b_d}{b_c} < \frac{b_d'}{b_c}$ the conduit is classed as a ditch conduit

Obtain C_d from ES 118, page 3-61.

$$\frac{H_c}{b_d} = \frac{30.0}{6.0} = 5.0$$

From ES 114, page 3-43, K = 0.333

μ for backfill = 0.58, μ for ditch wall = 0.36.

The smallest value of μ is 0.36 therefore $\mu' = 0.36$

$K\mu' = (0.333)(0.36) = 0.120$

$C_d = 2.92$

Solve for $W_c = C_d \gamma b_d^2 = (2.92)(120)(6.0)^2 = 12{,}614$ lbs/ft.

2. Find the load on the conduit when $b_d = 15.0$ ft.

The values of δ computed in part 1 will remain unchanged.

$\delta = 0.726$

Determine classification from ES 117, page 3-59

$$2K\mu\delta\rho = (2)(0.19)(0.622) = 0.236$$

$$2K\mu \frac{H_c}{b_c} = (2)(0.19) \frac{30.0}{3.5} = 3.257$$

$$\frac{b_d'}{b_c} = 2.95$$

$$\frac{b_d}{b_c} = \frac{15.0}{3.5} = 4.286$$

Since $\frac{b_d}{b_c} > \frac{b_d'}{b_c}$ the conduit is classed as a positive projecting conduit.

Sub-classify (ES 117, page 3-51). Since the conduit is rigid, it is classed as a projection condition.

$\delta\rho = (0.726)(0.857) = 0.622$

$\frac{H_e}{b_c} = 1.62$

$\frac{H_c}{b_c} = \frac{30.0}{3.5} = 8.571$

Since $\frac{H_c}{b_c} > \frac{H_e}{b_c}$ the conduit is classed as the incomplete condition

Obtain C_p from ES 118, page 3-63

$2K\mu\delta\rho = (2)(0.19)0.622 = 0.236$

$2K\mu \frac{H_c}{b_c} = (2)(0.19)(8.571) = 3.257$

$2K\mu C_p = 5.75$

$C_p = \frac{5.75}{(2)(0.19)} = 15.13$

$W_c = C_p \gamma b_c^2 = (15.13)(120)(3.5)^2 = 22,240$ lbs/ft.

CHAPTER 2 - SUPPORTING STRENGTH OF RIGID PIPE CONDUITS*

The supporting strength of a pipe is the maximum total load the pipe is capable of restraining without failure. The safe supporting strength of an underground rigid pipe depends on three major factors; strength of pipe, load factor, and the safety factor.

Strength of Pipe R_{eb}

The supporting strengths of short lengths of pipes are determined by direct test for certain standard load distributions. The most common method of determining supporting strength of pipes is the three-edge bearing test[1] (Fig. 2-1 and Fig. 2-2a).

The American Society for Testing Materials (ASTM) has set forth specifications or standards for manufactured pipe of various materials. These specifications give the pipe dimensions and their supporting strength R_{eb} for the three-edge bearing test. For reinforced concrete pipe the supporting strength is expressed as the ultimate load and also as the load to produce 0.01-inch crack in the pipewall. For reinforced concrete pipe the value of R_{eb} to produce 0.01-inch crack is used in this Technical Release. For other types of rigid pipes it is the ultimate load.

The supporting strength as determined by the three-edge bearing test is used for the derivation of the safe supporting strength of underground conduits. The use of the supporting strength as determined by other standard tests requires a revision of formulas.

The values of R_{eb} for various types of rigid pipes meeting ASTM specifications are given in ES-119, pages 3-67 to 3-71. The values of R_{eb} for rigid pipes not included in ES-119 may be obtained from ASTM specifications. Some ASTM specifications express the strength of the pipe in terms of D loads. The value of R_{eb} for these pipes is

$$R_{eb} = Dd \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2\text{-}1)$$

where d = nominal diameter of the pipe, ft

Load Factor L_f

In a field installation the supporting strength of a pipe is greater than that determined by the three-edge bearing test. A more favorable load distribution exists on pipes in the field installation than that of the three-edge bearing test.

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* Rigid pipe conduits are defined as pipes whose cross-sectional shape cannot be distorted sufficiently to change their vertical and horizontal dimensions more than 3 percent without causing materially injurious cracks in the pipe.

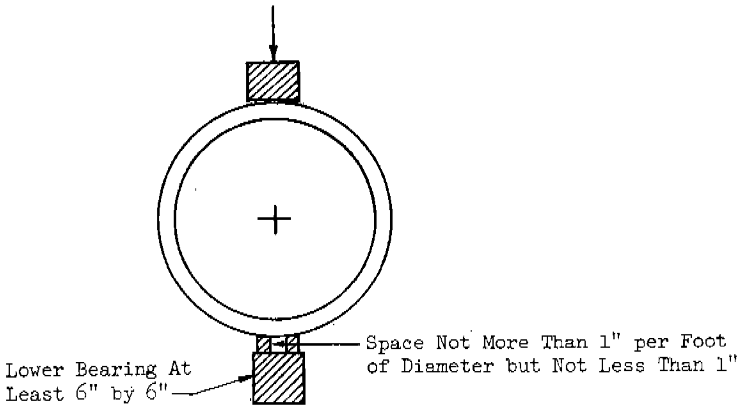


Fig. 2-1 Arrangement of conduit and bearing blocks for standard three-edge bearing laboratory tests of culvert pipe

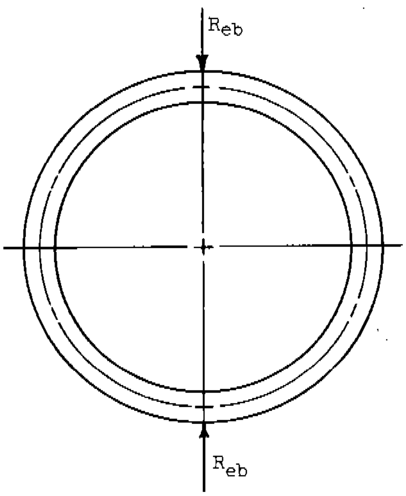


Fig. 2-2a Assumed load distribution for the case of three-edge bearing

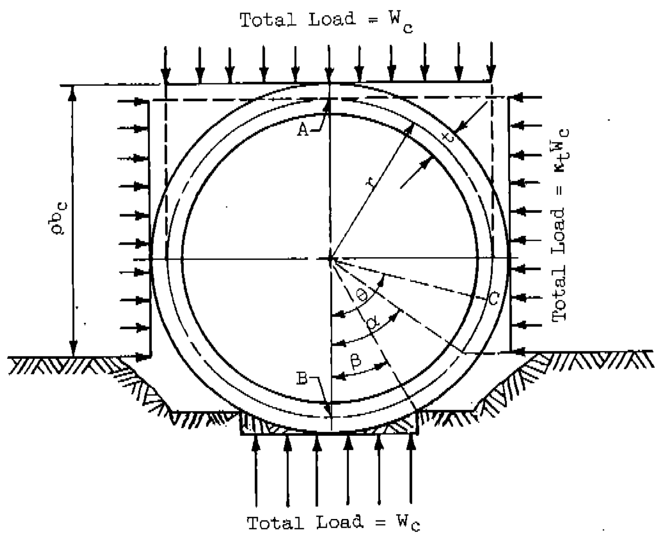


Fig. 2-2b Assumed load distribution on an underground conduit installed on a projecting bedding

The load factor is the ratio of the supporting strength of the pipe in any stated loading condition R_c to the supporting strength of pipe in three-edge bearing R_{eb}.

$$L_f = \frac{R_c}{R_{eb}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}2)$$

where L_f = load factor
R_c = supporting strength of pipe for a stated load pattern, lbs/ft length of pipe
R_{eb} = supporting strength of pipe for three-edge bearing load, lbs/ft length of pipe

The value of a load factor depends on the type of cradle or bedding associated with the conduit together with the classification of the underground conduit. Beddings are a type of construction which provide a distribution of vertical reaction along the lower surfaces of the conduit. Cradles furnish a lateral support as well as a distribution of vertical reaction.

Load Factors for Ditch Cradles and Beddings

Ditch cradles and beddings are associated with those underground conduits for which no lateral loads from the adjacent materials are assumed to act on the conduit. Only two classifications of underground conduits are associated with ditch cradles and beddings. They are ditch conduits and negative projecting conduits, ditch condition. For these two classifications, the backfill adjacent to the conduit is more compressible than the material in the ditch walls. Since none of the vertical loads are assumed to be transmitted through the material adjacent to the conduit, no lateral forces are considered to exist on the conduit.

The load factors for ditch cradles and beddings are determined experimentally for various types of cradles or beddings. Because of the wide variety of cradles and beddings, it is practical to group them and assign load factors which will give safe supporting strengths.

Values of L_f for various ditch beddings and cradles are given in ES-120, page 3-73.

Load Factors for Projecting Cradles and Beddings

Projecting cradles and beddings are associated with those underground conduits for which lateral loads are assumed to act on the conduit. All classifications of underground conduits except ditch conduits and negative projecting conduits, ditch condition, are associated with projecting cradles and beddings. The value of the load factor for projecting cradles and beddings depends on the magnitude and distribution of vertical and horizontal loads.

Figure 2-2b is a generalized load pattern consisting of five variables (β, α or ρ, r, κ_t, and W_c). This pattern was prepared from a study of experimental data of loads on underground conduits. Because of the variety of values of these five variables, it is impractical to obtain load factors by actual test. An expression of the load factor in terms of these variables has been analytically derived.[5] It is determined by the following procedure:

The maximum allowable fiber stress in a pipe for the three-edge bearing test is derived (Fig. 2-2a). This is set equal to the derived maximum fiber stress for the assumed load pattern given in Fig. 2-2b. The resulting equation is rearranged to give R_c/R_{eb} or the load factor L_f. The derivation is given in Appendix B.

The five variables of the load pattern are

1. κ_t = ratio of total lateral load to vertical load. An expression for the value of κ_t is obtained by the following procedure: Rankine's formula for active lateral pressure is

$$p = \gamma h K$$

where p = intensity of lateral pressure, lbs/ft^2
h = vertical height from any point within the embankment to the upper surface of the fill, ft
K = ratio of active horizontal pressure at a point to vertical pressure
γ = unit weight of embankment material, $lbs./ft.^3$

The total active lateral pressure on a rigid pipe is

$$\kappa_t W_c = \gamma \left[H_c + \frac{\rho b_c}{2}\right] K \rho b_c$$

Substituting the value of W_c from Eq. 1-5 page 1-9

$$\kappa_t = \frac{\rho K}{C_p}\left[\frac{H_c}{b_c} + \frac{\rho}{2}\right] \quad \ldots\ldots\ldots\ldots \quad (2\text{-}3)$$

2. r = mean radius of pipe, ft

3. β = one-half of the central angle subtended by the arc of the pipe over which the upward vertical reactions are acting. This value depends on the type of cradle or bedding.

4. α = one-half of the central angle subtended by the arc of the pipe over which no lateral loads are acting on the pipe. Thus,

$$\rho = \frac{1}{2}\left[1 - \sin(\alpha - 90^\circ)\right] \quad \ldots\ldots \quad (2\text{-}4)$$

5. W_c is evaluated in accordance with Chapter 1.

The load factor for projecting cradles and bedding, derived in Appendix B, is

$$L_f = \frac{1.431}{X_p - \kappa_t X_a} \quad \text{. . . . . (2-5)}$$

where X_p is a function of the distribution of the vertical load and vertical reaction (see Appendix B for the equation for X_p). The values of X_p depend on the type of projecting cradle and bedding, and were determined by experimentation, analysis, or approximation. Values of X_p for various cradles and beddings are given in ES-120, pages 3-74 to 3-76.

X_a is a function of the projection onto a vertical plane of the area of the pipe over which the lateral loads are assumed to be distributed (see Appendix B for the equation for X_a).

For uncradled pipe the lateral loads are assumed to act on that portion of the pipe that is above the natural ground. Considering a length of pipe of 1 foot, this area is ρb_c. When ρ is greater than unity ($\rho > 1$) the height of the projected area over which the lateral loads are distributed is only b_c and the value of X_a will correspond to a value of $\rho = 1.0$. The lateral forces on cradled pipes are assumed to be transmitted through the cradle to the pipe, thus, the loads are assumed to be acting on the same area as for uncradled pipe. Values of X_a for pipes on cradles and beddings for various values of ρ may be obtained from the curves in ES-120, page 3-77.

Safety factor s

The safe supporting strength of pipe is the supporting strength of a pipe divided by the safety factor. Safety factors are recommended for various types of materials. Since the value of R_{eb} for rigid pipes other than reinforced concrete are specified as strengths for ultimate loads, a factor of safety of 1.5 or 2.0 may be used. Values of R_{eb} for reinforced concrete are based on the load to produce 0.01-inch crack in the pipe. A factor of safety of 1.0 is considered satisfactory for reinforced concrete pipe for the following reasons:[8]

1. The three-edge bearing load to produce the 0.01-inch crack is the minimum load that random samples of the pipe must meet, thus, in practice samples usually withstand more than the minimum required.

2. Within time limitations concrete pipe increases in strength with age.

3. Other design variables are generally chosen to produce a safe structure, thus making the conservative choices additive.

4. Failure is reached only at a load considerably in excess of the cracking three-edge bearing load.

Equations for the Safe Supporting Strength of Pipes R_d

The symbol R_d is used to represent the safe supporting strength of the pipe in pounds per foot length.

1. Ditch cradles and beddings.

$$R_d = \frac{R_c}{s} = \frac{L_f\ R_{eb}}{s} \quad \ldots\ldots\ldots\ldots \quad (2\text{-}6)$$

2. Projecting cradles and beddings.

   a. Beddings

$$R_d = \frac{R_c}{s} = \frac{1.431\ R_{eb}}{s(X_p - \kappa_t X_a)} \quad \ldots\ldots\ldots \quad (2\text{-}7)$$

   where κ_t, X_a, and X_p are defined by Eqs. 2-3, B-15, and B-16.

   b. Cradles

$$R_d = \frac{R_c}{s} = \frac{1.431\ R_{eb}}{s(X_p - \kappa_t X_a)} \quad \ldots\ldots\ldots \quad (2\text{-}7)$$

   where κ_t, X_a, and X_p are defined by Eqs. 2-3, B-20, and B-21.

Internal Fluid Pressures

Positive internal fluid pressure causes tensile stresses in the pipe wall. These tensile stresses add algebraically with the flexural stresses caused by the external loads. The conduit is not capable of supporting as large an external load as it would support when no internal fluid pressure exists. When positive internal fluid pressure exists, the value of three-edge bearing strength R_{eb} used in the supporting strength formulas is reduced. The adjusted value of R_{eb} is[6]

$$R'_{eb} = R_{eb} \sqrt{\frac{N - p_i}{N}} \quad \ldots\ldots\ldots\ldots \quad (2\text{-}8)$$

where R'_{eb} = value of the reduced supporting strength of a pipe having positive internal pressure, lbs/ft. It is substituted in the supporting strength formulas for the three-edge bearing strength R_{eb}.

N = bursting pressure of the pipe, lbs/in^2

p_i = internal pressure in the pipe, lbs/in^2

The bursting pressure of a reinforced concrete pipe is

$$N = \frac{af_s}{6d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}9)$$

where a = cross sectional area of the circumferential steel in one foot of pipe, in^2

f_s = allowable stress in reinforcing steel, lbs/in^2

d = inside diameter of the pipe, inches

The operating internal pressure is usually small compared to the bursting strength of the pipe. The effect of internal pressures on the supporting strength of rigid pipes installed as underground conduits is usually neglected except for extremely high internal pressures.

Effect of Hydrostatic Load

A hydrostatic load has a similar effect on stresses in a pipe wall as negative internal pressure. The compressive stresses resulting from a hydrostatic load reduce the critical tensile stresses created by earth loads. Therefore, hydrostatic loads are usually neglected in calculating loads on rigid circular pipes.

Procedure for Determining Safe Supporting Strength

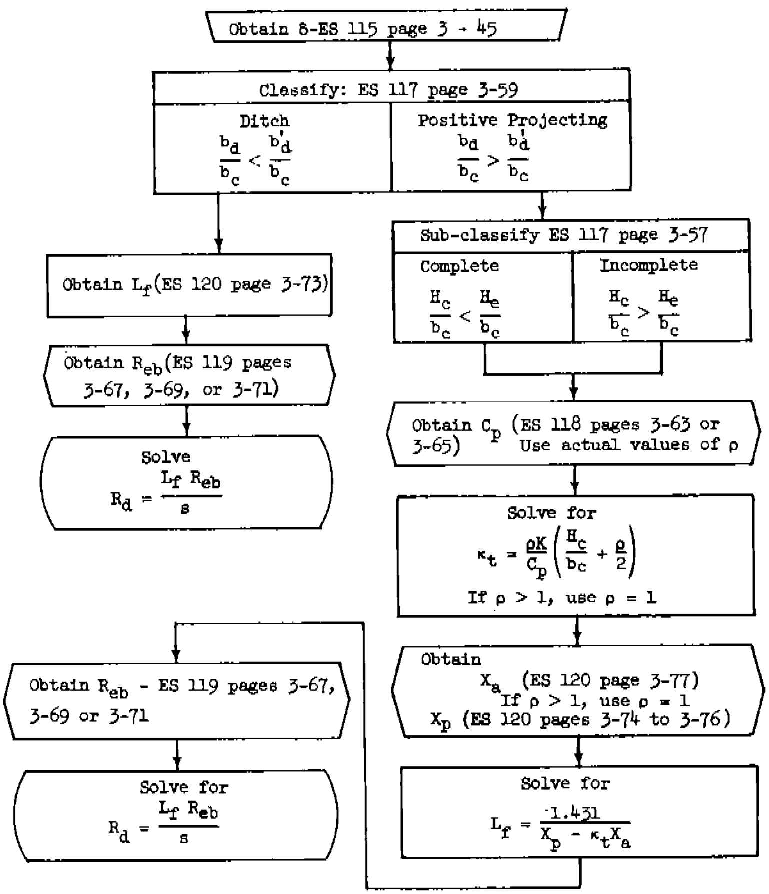


Fig. 2-3. Procedure for determining the safe supporting strength of a conduit installed as a ditch conduit or a positive projecting conduit.

Example - Determining Safe Supporting Strength

Given: 1. A 36 " Class III Wall A pipe (ASTM C76-57T) is proposed for installation on a type A2 cradle.

2. The height of fill above the conduit is 35 ft.

3. The distance $\rho b_c = 4.5$ ft.

4. The distance $H_f = 12$ ft.

5. The bottom width of the cradle is 5.0 ft.

6. The backfill material has a unit weight of 100 lbs/ft^3 and an angle of internal friction of 28^o.

7. The ditch wall and foundation material has an angle of internal friction of 30^o.

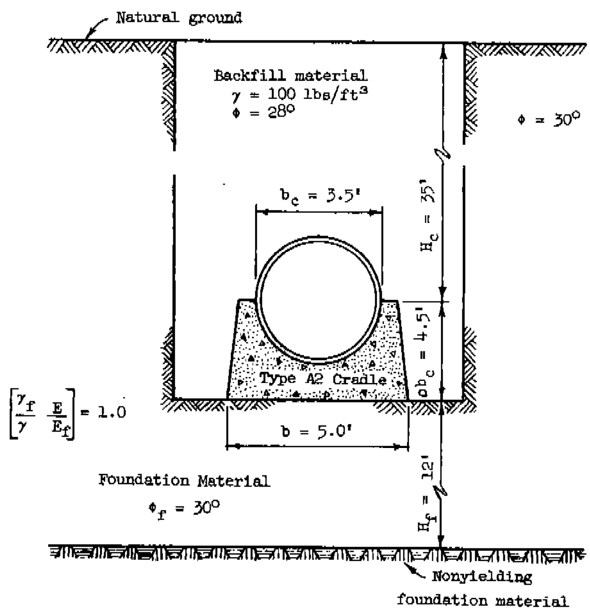
8. The value of $\left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]$ is 1.0.

9. A load factor L_f of 3.0 may be used for this installation if it is classed as a ditch conduit.

10. A safety factor of s = 1 may be used.

Find: 1. The safe supporting strength of the conduit if the ditch in which it is installed is 7 ft wide.

2. The safe supporting strength of the conduit if the ditch which it is installed is 13 ft wide.



<u>Solution</u>: Use the procedure given in figure 2-3.

1. Obtain δ - ES-115, page 3-45
   From ES-115, page 3-47 either case c or case d exists

   $b = 5.0$ ft.

   Assume case c

   $$\rho = \frac{\rho b_c}{b_c} = \frac{4.5}{3.5} = 1.286$$

   Since the natural ground is at the bottom of the cradle $\psi = 0$ (ES-114, page 3-42)
   From ES-114, page 3-43, $K\mu = 0.19$, $K_f\mu_f = 0.19$

   $$\delta \text{ (Case c)} = \frac{1 + (1)\,\frac{0}{1.286}}{1 + (1)\,\frac{0.19}{0.19}} = 0.5$$

   $\delta\rho = (0.5)(1.286) = 0.643$

   From ES-115, page 3-51, $\frac{H_e'}{b} = 1.65$

   $H_e' = (1.65)(5.0) = 8.25$

   $$H_e' = \frac{K\mu}{K_f\mu_f} = (8.25)\ \frac{0.19}{0.19} = 8.25$$

   Since $H_f > H_e' \frac{K\mu}{K_f\mu_f}$ case c exists

   $\delta = 0.5$, $\delta\rho = 0.643$

   Classify - ES-117, page 3-59

   $2K\mu\delta\rho = (2)(0.19)0.643 = 0.244$

   $$2K\mu\,\frac{H_c}{b_c} = (2)(0.19)\,\frac{35}{3.5} = 3.80$$

   $$\frac{b_d'}{b_c} = 3.13$$

   $$\frac{b_d}{b_c} = \frac{7.0}{3.5} = 2.0$$

   Since $\frac{b_d}{b_c} < \frac{b_d'}{b_c}$ the conduit is a ditch conduit.

   Obtain L_f from ES-120, page 3-73.

   $L_f = 3.0$ (given)

   Obtain R_{eb} from ES-119, page 3-67.

   $R_{eb} = 4050$ lbs/ft.

   Solve for $Rd = \frac{L_f R_{eb}}{s} = \frac{(3.0)(4050)}{1} = 12{,}150$ lbs/ft.

2. The value of δ is the same as for part 1
$\delta = 0.5$, $\delta\rho = 0.643$

Classify - ES-117, page 3-59

$$\frac{b_d'}{b_c} = 3.13 \text{ (from part 1)}$$

$$\frac{b_d}{b_c} = \frac{13.0}{3.5} = 3.714$$

Since $\dfrac{b_d}{b_c} > \dfrac{b_d'}{b_c}$ the conduit is classed as a positive projecting conduit. Since a rigid conduit is used, the conduit is sub-classed as the projection condition.

Sub-classify (ES-117, page 3-57)

$$\frac{H_e}{b_c} = 1.65$$

$$\frac{H_c}{b_c} = \frac{35}{3.5} = 10$$

Since $\dfrac{H_c}{b_c} > \dfrac{H_e}{b_c}$ the conduit is classed as the incomplete condition

Obtain C_p from ES-118, page 3-63

$$2K\mu \frac{H_c}{b_c} = 3.80$$

$$2K\mu\delta\rho = 0.244$$

$$2K\mu C_p = 6.83$$

$$C_p = \frac{6.83}{(2)(0.19)} = 17.97$$

Solve for $\kappa_t = \dfrac{\rho K}{C_p}\left(\dfrac{H_c}{b_c} + \dfrac{\rho}{2}\right)$

Since $\rho = 1.286$ is greater than 1 use $\rho = 1$

From ES-114, page 3-43, K = 0.36

$$\kappa_t = \frac{(1)(0.36)}{17.97}\left(\frac{35}{3.5} + \frac{1}{2}\right) = 0.210$$

Obtain X_a from ES-120, page 3-77

Since $\rho > 1$ use $\rho = 1$
$X_a = 0.638$

Obtain X_p from ES-120, page 3-74

$X_p = 0.45$

Solve for $L_f = \dfrac{1.431}{X_p - \kappa_t X_a} = \dfrac{1.431}{0.45-(0.210)(0.638)} = \dfrac{1.431}{0.316} = 4.53$

Obtain R_{eb} from ES-119, page 3-67

$R_{eb} = 4050$ lbs/ft.

Solve for $R_d = \dfrac{R_{eb} L_f}{s} = \dfrac{(4.53)(4050)}{1} = 18{,}347$ lbs/ft.

CHAPTER 3 – FORMULAS OBTAINED BY EQUATING EXPRESSIONS FOR LOAD AND SAFE SUPPORTING STRENGTH OF POSITIVE PROJECTING CONDUITS

The load on an underground conduit can be determined by the use of the equations in Chapter 1. Similarly, the safe supporting strength of a conduit can be determined by the use of the equations in Chapter 2. Sometimes neither of these quantities are specifically desired in practical design problems. Then the question to be answered is one of three types.

1. What are the allowable classes of pipes that can be used for a given embankment height and cradle?

2. What are the types of cradles or beddings that can be used for a given embankment height and pipe?

3. What are the allowable fill heights H_{ca} over the top of the pipe for a given cradle and pipe?

Chapter 3 is written to facilitate the solution of each of the three questions without predetermination of the load or allowable strength of the positive projecting conduit. This is desirable because fewer calculations are required.

The expression for the load on the conduit is equated to the expression for the safe supporting strength of the conduit. This equation is rearranged to permit graphical presentations of certain portions of the equation. Such a rearrangement will usually permit an easy solution without trial and error methods for any one of the three quantities.

The procedures for the determination of each of the three quantities listed above are given on ES-113, pages 3-37 to 3-39.

Positive Projecting Conduits, Complete Projection Condition (Projecting Cradles and Beddings)

Equate Eqs. 1-5 and 2-7.

$$C_p \gamma b_c^2 = \frac{1.431\ R_{eb}}{s\left[X_p - \frac{\rho K X_a}{C_p}\left(\frac{H_c}{b_c} + \frac{\rho}{2}\right)\right]} \quad \ldots\ldots\ldots\ldots \quad (3\text{-}1)$$

where

$$C_p = \frac{e^{2K\mu(H_c/b_c)} - 1}{2K\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}5a)$$

Rearrange

$$\frac{1.431\ R_{eb}}{s\gamma b_c^2} = C_p X_p - K\left[\rho X_a\left(\frac{H_c}{b_c} + \frac{\rho}{2}\right)\right] \quad \ldots\ldots\ldots \quad (3\text{-}2)$$

Let

$$T = \rho X_a \left(\frac{H_c}{b_c} + \frac{\rho}{2}\right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (3\text{-}3)$$

Then

$$\frac{1.431\ R_{eb}}{s\gamma {b_c}^2} = C_p X_p - KT \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (3\text{-}4)$$

Values of C_p are determined from the charts in ES-118, pages 3-63 and 3-65. Values of X_p for various types of beddings and cradles are given in ES-120, pages 3-74 to 3-76. Values of K for various values of ϕ are determined from the chart in ES-114, page 3-43. Values of T for various values of ρ and H_c/b_c are read from the charts in ES-121, pages 3-79 and 3-81.

The expression to the left of the equal sign in Eq. 3-4 represents the strength factor F_{sp} that is provided by a given pipe for a particular installation.

$$F_{sp} = \frac{1.431\ R_{eb}}{s\gamma {b_c}^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (3\text{-}5)$$

Values of F_{sp} for various pipes are determined from ES-119, pages 3-67 to 3-71.

The expression to the right of the equal sign in Eq. 3-4 represents the strength factor F_{sr} that is required for a particular installation.

$$F_{sr} = C_p X_p - KT \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (3\text{-}6)$$

When the allowable height of fill H_{ca} is to be determined, Eq. 3-2 is rearranged

$$e^{2K\mu(H_c/b_c)} - \frac{K\rho X_a}{X_p}\left(2K\mu\ \frac{H_c}{b_c}\right) = U \quad \ldots \ldots \ldots \ldots \quad (3\text{-}7)$$

where

$$U = \frac{2K\mu F_{sp} + K\mu K\rho^2 X_a + X_p}{X_p} \quad \ldots \ldots \ldots \ldots \ldots \quad (3\text{-}7a)$$

The solution of Eq. 3-7 is facilitated by the use of ES-122, page 3-83.

Values of X_a for various values of ρ are read from the curves in ES-120, page 3-77.

In Eq. 3-7 and Eq. 3-7a the value of ρ cannot be greater than one.

Positive Projecting Conduits, Incomplete Projection Condition (Projecting Cradles and Beddings)

Equate Eqs. 1-6 and 2-7.

$$C_p \gamma {b_c}^2 = \frac{1.431\ R_{eb}}{s\left[X_p - \frac{\rho K X_a}{C_p}\left(\frac{H_c}{b_c} + \frac{\rho}{2}\right)\right]} \quad \ldots\ldots\ldots \quad (3\text{-}1)$$

where

$$C_p = \frac{e^{2K\mu(H_e/b_c)} - 1}{2K\mu} + \left[\frac{H_c}{b_c} - \frac{H_e}{b_c}\right] e^{2K\mu(H_e/b_c)} \quad \ldots\ldots \quad (1\text{-}6a)$$

Rearrange Eq. 3-1.

$$\frac{1.431\ R_{eb}}{s\gamma {b_c}^2} = C_p X_p - K\left[\rho X_a\left(\frac{H_c}{b_c} + \frac{\rho}{2}\right)\right] \quad \ldots\ldots \quad (3\text{-}2)$$

Let

$$T = \rho X_a \left[\frac{H_c}{b_c} + \frac{\rho}{2}\right] \quad \ldots\ldots\ldots\ldots \quad (3\text{-}3)$$

Then

$$\frac{1.431\ R_{eb}}{s\gamma {b_c}^2} = C_p X_p - KT \quad \ldots\ldots\ldots\ldots \quad (3\text{-}4)$$

Values of C_p are determined from the charts in ES-118, pages 3-63 and 3-65. Values of X_p for various types of beddings and cradles are given in ES-120, pages 3-74 to 3-76. Values of K for various values of ϕ are determined from the chart in ES-114, page 3-43. Values of T for various values of ρ and H_c/b_c are read from the charts in ES-121, pages 3-79 and 3-81.

The expression to the left of the equal sign in Eq. 3-4 represents the strength factor F_{sp} that is provided by a given pipe for a particular installation.

$$F_{sp} = \frac{1.431\ R_{eb}}{s\gamma {b_c}^2} \quad \ldots\ldots\ldots\ldots \quad (3\text{-}5)$$

Values of F_{sp} for various pipes are determined from ES-119, pages 3-67 to 3-71.

The expression to the right of the equal sign in Eq. 3-4 represents the strength factor F_{sr} that is required for a particular installation.

$$F_{sr} = C_p X_p - KT \quad \ldots\ldots\ldots\ldots \quad (3\text{-}6)$$

When the height of fill H_{ca} is to be determined, Eq. 3-1 is rearranged in the form

$$\frac{H_c}{b_c} = \frac{F_{sp} + 0.5\,K\rho^2\,X_a + \frac{X_p}{2K\mu}\,(x\,e^x - e^x + 1)}{e^x\,X_p - K\rho X_a} \quad . \; . \; (3\text{-}8)$$

where $\quad x = 2K\mu\,\frac{H_e}{b_c}$

Values of X_a for various values of ρ are read from the curves in ES-120, page 3-77.

When $\rho > 1$, use $\rho = 1$ for substitution into Eq. 3-8. Eq. 3-8 is solved by the following procedure:

Find the value of $e^x - x$ from the relation

$e^x - x = 2K\mu\delta\rho + 1$ -- Use the actual value of ρ in this equation.

Obtain x from ES-123, page 3-87.

Obtain e^x from the relation

$$e^x = (e^x - x) + x$$

Substitute these values into Eq. 3-8.

Examples

Examples have been prepared to illustrate the solution of underground conduit design problems, using the procedures and computation aids given in ES-113 through ES-123. The following tabulation permits the selection of examples which illustrate solutions by various methods.

Example Number	Find	Classification	Subclassification	Case of δ	Type of Rigid Pipe	Bedding or Cradle	In a Ditch?
1	Pipe	Ditch	--	c	any	bedding	yes
† 2	Pipe	Pos. Proj.	Incomplete	b	any	cradle	no
3	Pipe	Pos. Proj.	Incomplete	c	any	bedding	no
4	Bedding	Pos. Proj.	Incomplete	d	clay	bedding	yes
5	Cradle	Pos. Proj.	Incomplete	a	R/C	cradle	no
* 6	Bedding or Cradle	Pos. Proj.	Incomplete	c	R/C	either	no
† 7	H_{ca}	Pos. Proj.	Incomplete	d	R/C	bedding	no
8	H_{ca}	Pos. Proj. Ditch	Incomplete --	c	concrete	cradle	yes
9	H_{ca}	Pos. Proj. Ditch	Complete --	a	R/C	bedding	yes
10	H_{ca}	Pos. Proj.	Incomplete	c	R/C	cradle	no

*Example 6 illustrates the type of solution required to determine the unknown quantity at more than one point under a dam.

†Examples 2 and 7 illustrate the effect of changing the value of $\left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]$.

Example No. 1

Given: 1. A 24-inch rigid pipe conduit is proposed for installation in a 42-inch ditch (b_d = 42 in.). The ditch is in a very deep uniform material which has an angle of internal friction ϕ_f of 30°, a unit weight γ_f of 120 lbs/ft^3, and a modulus of consolidation E_f = 11.4 tons/ft^2. The maximum depth of the ditch is 23.6 ft. The distance ρb_c = 1.6 ft.

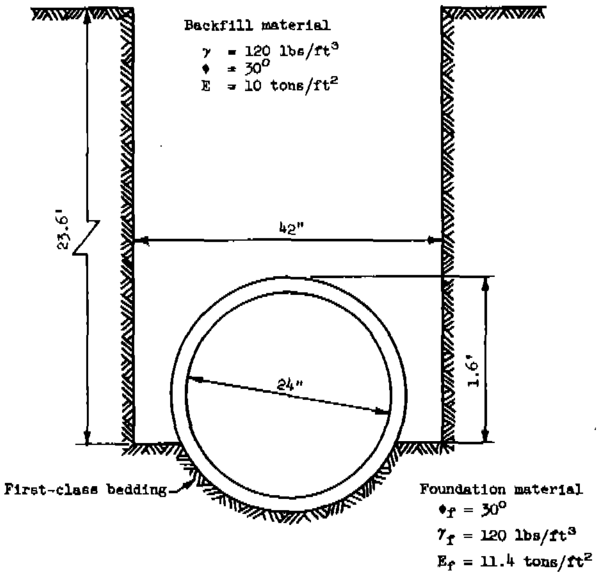
2. The backfill material has the following properties when compacted to required density:

(a) unit weight γ = 120 lbs/ft^3
(b) angle of internal friction ϕ = 30°
(c) modulus of consolidation E = 10 tons/ft^2

3. A first-class bedding (Type B ditch bedding or Type B2 projecting bedding) is proposed (ES-120, pages 3-73 to 3-76).

4. Consider 0.01-inch crack permissible for safe design for R/C pipe and use 1.5 safety factor for clay and non-reinforced concrete pipe.

Determine: A rigid pipe that can be used.



Solution:

List data (ES-114, pages 3-41 and 3-42)

$\gamma = 120 \text{ lbs/ft}^3$

$b_d = \frac{42}{12} = 3.5$ ft

$\mu = \mu'$, therefore, $K\mu = K\mu' = 0.19$ (ES-114, page 3-43)

s = 1 for R/C and 1.5 for other pipes

Follow procedure on ES-113, page 3-37

Assume $b_c = 2.5$ ft

Find δ (Follow procedure on ES-115, page 3-45)

From ES-115, page 3-47, case c applies since the material is very deep.

$b = b_c = 2.5$ ft (ES-114, page 3-41)

Solve for δ

$$\rho = \frac{\rho b_c}{b_c} = \frac{1.6}{2.5} = 0.640$$

$$\psi b_c = b_c - 1.6 = 2.5 - 1.6 = 0.9 \text{ ft}$$

$$\psi = \frac{0.9}{2.5} = 0.360$$

$$\frac{K\mu}{K_f\mu_f} = 1$$

$$\frac{\gamma_f}{\gamma} = 1$$

$$\frac{E}{E_f} = \frac{10.0}{11.4} = 0.877, \quad \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] = 0.877$$

$$\frac{\psi}{\rho} = \frac{0.360}{0.640} = 0.562$$

$$\delta = \frac{1 + (0.877)(0.562)}{1 + 0.877\ (1)} = \frac{1.493}{1.877} = 0.796$$

Check whether classification is ditch conduit or positive projecting conduit (use ES-117, page 3-59).

$$\frac{H_c}{b_c} = \frac{23.6 - 1.6}{2.5} = 8.80, \quad 2K\mu\,\frac{H_c}{b_c} = 3.344$$

$$\delta\rho = (0.796)(0.640) = 0.509, \quad 2K\mu\delta\rho = 0.1934$$

$$\frac{b_d}{b_c} = \frac{3.5}{2.5} = 1.40$$

$$\frac{b_d'}{b_c} = 2.90$$

Since $\frac{b_d}{b_c} < \frac{b_d'}{b_c}$, the conduit is classed as a ditch conduit.

Obtain C_d from ES-118, page 3-61.

$$\frac{H_c}{b_d} = \frac{22}{3.5} = 6.29$$

$$K\mu' = 0.19$$

$$C_d = 2.38$$

Obtain L_f from ES-120, page 3-73, $L_f = 1.9$

Solve for $W_c = C_d\gamma b_d^2 = (2.38)(120)(3.5)^2 = 3500$ lbs/ft

Let $W_c = R_d$. Solve for R_{eb} required.

$$R_{eb} = \frac{sR_d}{L_f} = \frac{(1)(3500)}{1.9} = 1840 \text{ lbs/ft} \quad \text{(for R/C pipe)}$$

$$R_{eb} = \frac{(1.5)(3500)}{1.9} = 2760 \text{ lbs/ft} \quad \text{(for other types of pipes)}$$

From ES-119, pages 3-67, 3-69 , and 3-71, any 24-inch reinforced concrete pipe having a value of R_{eb} equal to or greater than 1840 lbs/ft may be selected. Any 24-inch non-reinforced concrete or clay pipe having a value of R_{eb} equal to or greater than 2760 lbs/ft may be selected.

Select an extra strength clay pipe.

$R_{eb} = 4400, \quad b_c = 2.292$

Obtain δ using new b_c value (ES-115, page 3-45) Case c still exists

$$\rho = \frac{1.6}{2.292} = 0.698$$

$$\psi b_c = 2.292 - 1.6 = 0.69$$

$$\psi = \frac{0.69}{2.292} = 0.301$$

$$\frac{\psi}{\rho} = \frac{0.301}{0.698} = 0.431$$

$$\delta \text{ (case c)} = \frac{1 + (0.877)(0.431)}{1 + (0.877)(1)} = 0.734$$

Classify--ES-117, page 3-57

$$2K\mu\delta\rho = (0.38)(0.734)(0.698) = 0.1947$$

$$2K\mu \frac{H_c}{b_c} = (0.38)\,\frac{22.0}{2.292} = 3.65$$

$$\frac{b_d'}{b_c} = 2.95$$

$$\frac{b_d}{b_c} = \frac{3.5}{2.292} = 1.53$$

Since $\frac{b_d}{b_c} < \frac{b_d'}{b_c}$, classification remains ditch conduit. Selected pipe is satisfactory.

Example No. 2

Given: 1. A 30-inch pipe (ASTM Spec. C76-57T) is proposed for installation through a dam on a type A2 cradle resting on rock. The conduit is not installed in a ditch.

2. The distance ρb_c is 2.10 ft.

3. The distance ψb_c is 1.42 ft.

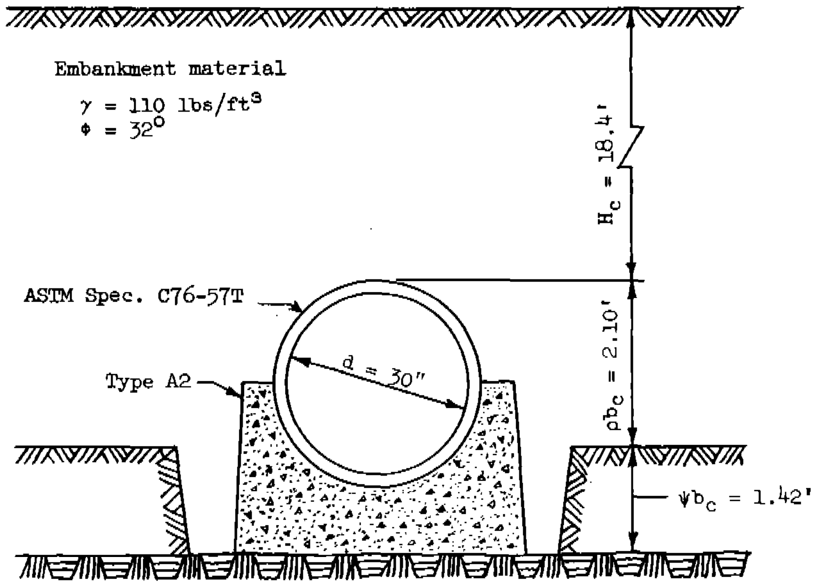
4. Use a factor of safety of 1.

5. The height of fill above the top of the pipe is 18.4 ft.

6. When compacted to required density, the embankment material has a unit weight $\gamma = 110$ lbs/ft^3 and an angle of internal friction ϕ of 32°.

7. Consider that the value $\left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]$ may range from 0.1 to 1.0.

Determine: Class of pipe meeting ASTM Spec. C76-57T that may be used.



Solution:

List data (ES-114, pages 3-41 and 3-42)

$\gamma = 110$ lbs/ft^3

b_c will vary for the various wall thicknesses. The value of b_c for wall B will be used for a preliminary determination.

$b_c = 3.083$ ft

From ES-114, page 3-43, $K = 0.31$, $K\mu = 0.19$

$s = 1$

$$\rho = \frac{2.10}{3.083} = 0.681$$

Follow procedure on ES-113, page 3-37

b_c is assumed as 3.083 ft

Obtain δ. Follow the procedure given on ES-115, page 3-45.

From ES-115, page 3-47, case b applies.

$$\delta = 1 + \left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right] \frac{\psi}{\rho}$$

$$\psi = \frac{1.42}{3.083} = 0.461$$

$$\frac{\psi}{\rho} = \frac{0.461}{0.681} = 0.677$$

When $\left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right] = 0.1$

$$\delta = 1 + 0.068 = 1.068$$

When $\left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] = 1$

$\delta = 1 + 0.677 = 1.677$

Classify--Since $b_d = \infty$, $\frac{b_d}{b_c} > \frac{b_d'}{b_c}$, the conduit is classed as a positive projecting conduit.

Determine subclassification of conduit from ES-117, page 3-57.

$\delta\rho\ (\text{min}) = (1.068)(0.681) = 0.727$ 0.V $1 \times 1.14 = 1.14$

$\delta\rho\ (\text{max}) = (1.677)(0.681) = 1.142$ $1 \times 1.14 = 1.14$

$\frac{H_e}{b_c}\ (\text{min}) = 1.74$

$\frac{H_e}{b_c}\ (\text{max}) = 2.13$

$\frac{H_c}{b_c} = \frac{18.4}{3.083} = 5.968$

Since $\frac{H_c}{b_c} > \frac{H_e}{b_c}$, conduit is classed as incomplete condition.

Obtain C_p from ES-118, page 3-63.

$2K\mu\frac{H_c}{b_c} = 2.27$

$2K\mu\delta\rho = 0.276$

$2K\mu C_p = 4.07$

$C_p\ (\text{min}) = \frac{4.07}{0.38} = 10.71$

$2K\mu\delta\rho = 0.434$

$2K\mu C_p = 4.51$

$C_p\ (\text{max}) = \frac{4.51}{0.38} = 11.87$

Obtain X_p from ES-120, page 3-74, $X_p = 0.450$

Obtain T from ES-121, page 3-79, $T = 3.52$ (for cradles)

Solve for $F_{sr} = C_pX_p - KT$

$F_{sr}\ (\text{min}) = (10.71)(0.45) - (0.31)(3.52) = 3.73$

$F_{sr}\ (\text{max}) = (11.87)(0.45) - (0.31)(3.52) = 4.25$

Note that the range of F_{sr} values are small compared to the large range in $\left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]$ values.

Select pipe (ES-119, page 3-67).

$s\gamma F_{sr}\ (\text{max}) = (1)(110)(4.25) = 468$

A class III, wall B, pipe is selected. $s\gamma F_{sp} = 508.1$

When the actual value of b_c for the selected type of pipe is not equal to the assumed value, the solution should be repeated. The actual value of b_c is used in the repeated solution which determines whether F_{sp} is greater than or equal to F_{sr}.

Example No. 3

Given: 1. A 30-inch pipe with type B1 concrete bedding (bottom width of 2.8 ft) is proposed for installation through a dam. The conduit is not installed in a ditch.

2. The embankment material has a unit weight $\gamma = 110$ lbs/ft^3, an angle of internal friction ϕ of 32°, and a modulus of consolidation E of 26 tons/ft^2.

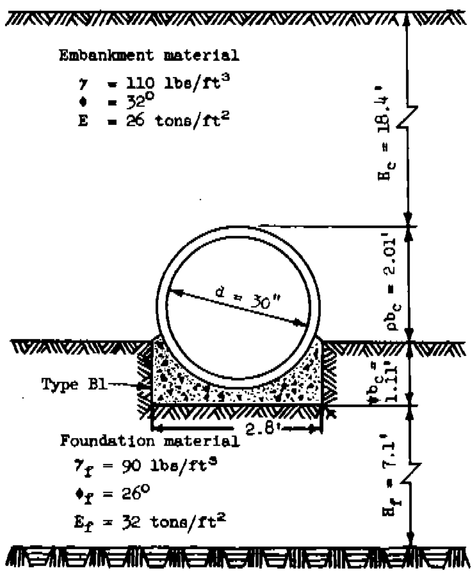
3. The foundation material has an angle of internal friction ϕ_f of 26°, a unit weight γ_f of 90 lbs/ft^3, a modulus of consolidation E_f of 32 tons/ft^2, and extends to a rock layer which is 7.1 ft below the concrete bedding.

4. The distance $\rho b_c = 2.01$ ft and $\psi b_c = 1.11$ ft.

5. The height of fill above the top of the pipe is 18.4 ft.

6. Consider 0.01-inch crack permissible for safe design of reinforced concrete pipe.

Determine: Type of R/C pipe that may be used.



Solution:

List data (ES-114, pages 3-41 and 3-42)

$\gamma = 110$ lbs/ft^3

From ES-114, page 3-43, $K\mu = 0.19$, $K = 0.31$

$s = 1.0$

Follow the procedure on ES-113, page 3-37.

Assume $b_c = 3.083$ (for wall B, ASTM Spec. C76-57T)

Obtain δ. Follow procedure on ES-115, page 3-45.

From ES-115, page 3-47 case c or case d exists.

Since the bottom width of the rigid bedding is less than b_c use $b = b_c$

$b = 3.083$

Solve for

$$\delta \text{ (case c)} = \frac{1 + \left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right] \frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right] \frac{K\mu}{K_f\mu_f}}$$

$$\frac{E}{E_f} = \frac{26}{32} = 0.813$$

$$\frac{\gamma_f}{\gamma} = \frac{90}{110} = 0.818$$

$$\left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right] = (0.818)(0.813) = 0.665$$

$$\frac{\psi}{\rho} = \frac{\psi b_c}{\rho b_c} = \frac{1.11}{2.01} = 0.552$$

From ES-114, page 3-43, $K_f\mu_f = 0.19$, $\frac{K\mu}{K_f\mu_f} = \frac{0.19}{0.19} = 1$

$$\delta \text{ (case c)} \ \frac{1 + (0.665)(0.552)}{1 + 0.665} = 0.821$$

Find $\frac{H_e'}{b}$

$$\rho = \frac{2.01}{3.083} = 0.652$$

$$\delta\rho = (0.821)(0.652) = 0.535$$

From ES-115, page 3-51, $\frac{H_e'}{b} = 1.51$

Is $H_f < H_e' \frac{K\mu}{K_f\mu_f}$?

$H_e' = (1.51)(3.083) = 4.65$

$\frac{K\mu}{K_f\mu_f} H_e' = (1)(4.65) = 4.65$

$H_f = 7.1$ ft

Since $H_f > \frac{K\mu}{K_f\mu_f} H_e'$, depth is sufficiently great and case c exists.

$\delta = 0.821, \quad \delta\rho = 0.535$

Classify--ES-117, page 3-59.

Since $b_d = \infty$, $\frac{b_d}{b_c} > \frac{b_d'}{b_c}$, conduit is classed as positive projecting.

Determine subclassification.

$\frac{H_c}{b_c} = \frac{18.4}{3.083} = 5.97$, From ES-117, page 3-57, $\frac{H_e}{b_c} = 1.51$

Since $\frac{H_c}{b_c} > \frac{H_e}{b_c}$, incomplete condition exists.

Obtain C_p

$2K\mu \frac{H_c}{b_c} = (0.38)(5.97) = 2.27$

$2K\mu\delta\rho = (0.38)(0.535) = 0.203$

From ES-118, page 3-63

$2K\mu C_p = 3.78$

$C_p = \frac{3.78}{0.38} = 9.95$

Obtain X_p from ES-120, page 3-75.

$X_p = 0.65$

Obtain T from ES-121, page 3-81.

$T = 2.28$

Solve for

$$
\begin{aligned}
F_{sr} &= C_p X_p - KT \\
&= (9.95)(0.65) - (0.31)(2.28) \\
&= 5.76
\end{aligned}
$$

Select pipe from ES-119, pages 3-67, 3-69, and 3-71.

$s\gamma F_{sr} = (1)(110)(5.76) = 634$

Any pipe having a value of $s\gamma F_{sp} > s\gamma F_{sr}$ is satisfactory.

A class IV, wall B, pipe is selected.

If a wall A or wall C is selected, the solution is repeated using the correct value of b_c.

Example No. 4

Given: 1. A 36-inch extra strength clay pipe (ASTM Spec. C200-55T) is proposed for installation in a ditch which is 12 ft wide.

2. The depth of the ditch to the top of the conduit is 17 ft.

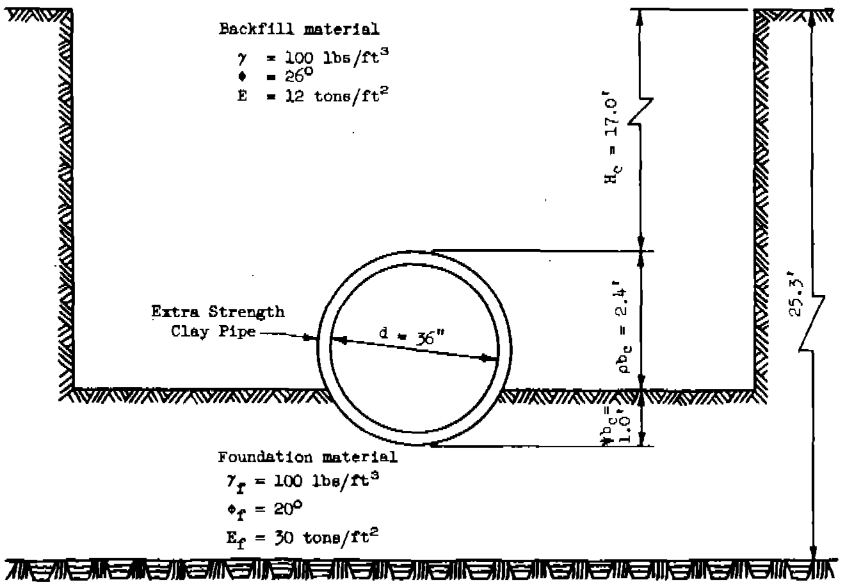
3. The ditch is excavated in a clayey material having a unit weight γ_f = 100 lbs/ft^3, a modulus of consolidation E_f of 30 tons/ft^2, and angle of internal friction ϕ_f of 20°. Borings show that the clayey material is essentially uniform down to a rock layer 25.3 ft below the surface of the ground.

4. The material used for backfill is a loess soil. Tests show that when the backfill material is compacted to the required density it has a unit weight γ of 100 lbs/ft^3, an angle of internal friction ϕ of 26°, and a modulus of consolidation E of 12 tons/ft^2.

5. The distance ρb_c = 2.4 ft and ψb_c = 1.0 ft.

6. Use a factor of safety s of 1.5.

Determine: The type of bedding that may be used.



Solution:

List data (ES-114, pages 3-41 and 3-42)

γ = 100 lbs/ft^3

b_d = 12.0 ft

For the ditch wall material ϕ_f = 20°

From ES-114, page 3-43, μ_f = 0.37

For the backfill material $\phi = 26^{\circ}$

From ES-114, page 3-43, $\mu = 0.48$, $K = 0.38$, $K\mu = 0.19$

Since the ditch wall material has the smallest value of μ, $\mu' = 0.37$ and $K\mu' = (0.38)(0.37) = 0.141$

$s = 1.5$

From ES-119, page 3-71, $b_c = 3.396$ ft

$$\rho = \frac{2.4}{3.396} = 0.707$$

Follow the procedure in ES-113, page 3-38.

Obtain δ. Follow procedure on ES-115, page 3-45.

From ES-115, page 3-46, either case c or case d exists.
$b = b_c = 3.396$ ft

Solve for

$$\delta \text{ (case c)} = \frac{1 + \left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right] \frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right] \frac{K\mu}{K_f\mu_f}}$$

$$\frac{\gamma_f}{\gamma} = \frac{100}{100} = 1$$

$$\frac{E}{E_f} = \frac{12}{30} = 0.40$$

$$\left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right] = 0.40$$

From ES-114, page 3-43, $K_f\mu_f = 0.178$

$$\frac{K\mu}{K_f\mu_f} = \frac{0.19}{0.178} = 1.067$$

$$\frac{\psi}{\rho} = \frac{1.0}{2.4} = 0.417$$

$$\delta \text{ (case c)} = \frac{1 + (0.40)(0.417)}{1 + (1.067)(0.40)} = \frac{1.167}{1.427} = 0.818 \quad = \frac{1}{1.427} = .701$$

Find $\frac{H_e'}{b}$ from ES-115, page 3-51.

$$\delta\rho = (0.818)(0.707) = 0.578$$

$$\frac{H_e'}{b} = 1.57 \qquad 1.71$$

Is $H_f < H_e' \frac{K\mu}{K_f\mu_f}$?

$$H_e' = (1.57)(3.396) = 5.332 \text{ ft} \qquad 1.71 \times 3.4 = 5.814$$

$$\frac{K\mu}{K_f\mu_f} H_e' = (1.067)(5.332) = 5.689$$

$H_f = 4.9$ ft

$H_f < 5.689$ (shallow depth, case d exists)

Compute $2K\mu\rho\left\{1 + \left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]\frac{\psi}{\rho}\right\} = (0.38)(0.707)\left[1 + (0.417)(0.40)\right] = 0.314$ .38

Compute $2K\mu\left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]\frac{H_f}{b} = (0.40)(0.38)\frac{4.9}{3.396} = 0.219$

Obtain ω from ES-115, page 3-49.

$\omega = 0.61$ .67

Compute $\frac{H_e'}{b} = \frac{\omega}{2K\mu} = \frac{0.61}{0.38} = 1.605$ $\frac{.67}{.38} = 1.76$

From ES-115, page 3-51, obtain $\delta\rho = 0.61$ .74

$\delta = \frac{\delta\rho}{\rho} = \frac{0.61}{0.707} = 0.863$ $\frac{.74}{1} = .74$

Determine classification from ES-117, page 3-59.

$\frac{H_c}{b_c} = \frac{17.0}{3.396} = 5.006$

From ES-117, page 3-59, $\frac{b_d'}{b_c} = 2.43$, (2.60) $\frac{b_d}{b_c} = \frac{12.0}{3.396} = 3.53$

$\frac{b_d}{b_c} > \frac{b_d'}{b_c}$, therefore, the conduit is classed as a positive projecting conduit.

Determine subclassification from ES-117, page 3-57.

$\frac{H_e}{b_c} = 1.605$ 1.76

$\frac{H_c}{b_c} > \frac{H_e}{b_c}$, therefore, the conduit is classed as incomplete condition.

Obtain C_p from ES-118, page 3-63.

$2K\mu\delta\rho = 0.2318$, (.2812) $2K\mu(H_c/b_c) = 1.902$

$2K\mu C_p = 3.23$ 3.31

$C_p = 8.49$ 8.71

From ES-119, page 3-71, obtain $s\gamma F_{sp} = 744.5$, $F_{sp} = \frac{744.5}{(100)(1.5)} = 4.963$

From ES-121, page 3-81, obtain $T = 2.24$ (for beddings)

Let $F_{sr} = F_{sp}$

Solve for X_p for bedding

$$X_p = \frac{F_{sr} + KT}{C_p} = \frac{4.963 + (0.38)(2.24)}{8.492} = 0.685$$

Select bedding from ES-120, pages 3-75 and 3-76.

Any bedding having a value of X_p equal to or less than the calculated value is allowable. A type B1 (concrete bedding) may be used.

Example No. 5

Given: 1. A 24-inch class IV, wall A, R/C pipe (ASTM Spec. C76-57T) is proposed for installation on a concrete cradle which rests on rock. There is no foundation material adjacent to the pipe or cradle. The conduit is not installed in a ditch.

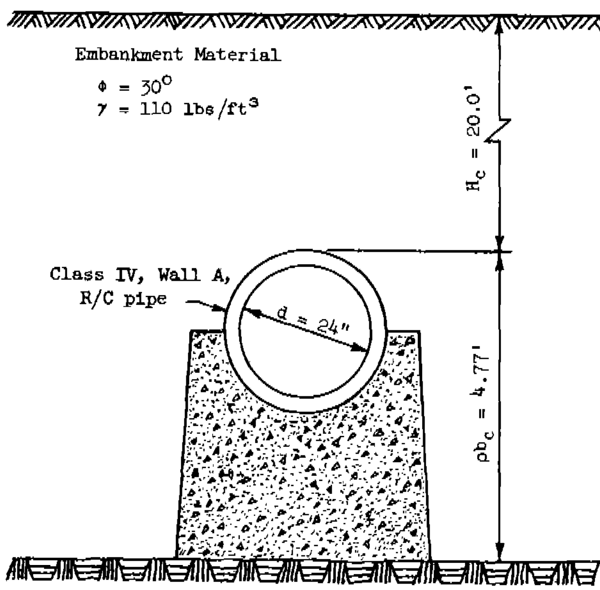
2. The distance ρb_c is 4.77 ft.

3. The embankment material has an angle of internal friction ϕ of 30° and a unit weight γ of 110 lbs/ft^3.

4. The proposed height of fill above the conduit is 20 ft.

5. Consider 0.01-inch crack in the R/C pipe satisfactory for design.

Determine: Type of cradle that can be used.



Solution:

List data (ES-114, pages 3-41 and 3-42)

$\gamma = 110$ lbs/ft^3

From ES-119, page 3-67, $b_c = 2.417$ ft

From ES-114, page 3-43, $K = 0.333$, $K\mu = 0.19$

$s = 1$

$\rho = \frac{4.77}{2.417} = 1.974$ 1.0

Follow procedure on ES-113, page 3-38.

Obtain δ.

Follow procedure on ES-115, page 3-45.

From ES-115, page 3-47, case a exists, and $\delta = 1.0$.

Determine the classification from ES-117, page 3-59.

Since $b_d = \infty$, $\frac{b_d}{b_c} > \frac{b_d'}{b_c}$, conduit is classed as positive projecting conduit.

Determine the subclassification of the conduit from ES-117, page 3-57.

$\delta\rho = (1.974)(1.0) = 1.974$ 1.0

$\frac{H_c}{b_c} = \frac{20.0}{2.417} = 8.27$

Since $\frac{H_c}{b_c} > \frac{H_e}{b_c}$, the conduit is classed as the incomplete condition.

Obtain C_p from ES-118, page 3-63.

$2K\mu\delta\rho = (0.38)(1.974) = 0.750$ .38

$2K\mu(H_c/b_c) = (0.38)(8.27) = 3.143$

From chart

$2K\mu C_p = 7.65$ 6.22

$C_p = \frac{7.65}{0.38} = 20.14$

Obtain F_{sp} from ES-119, page 3-67, $s\gamma F_{sp} = 979.8$, $F_{sp} = \frac{979.8}{(110)(1)} = 8.907$

Obtain T from ES-121, page 3-79, $T = 5.59$ (for cradles)

Let $F_{sp} = F_{sr}$ and solve for $X_p = \frac{F_{sr} + KT}{C_p}$

$X_p = \frac{8.907 + (0.333)(5.59)}{20.14} = 0.535$

Select cradle from ES-120, page 3-74.

Any cradle having a value of X_p less than 0.535 is allowable. Type A3 cradle ($X_p = 0.50$) is satisfactory.

Example No. 6

Given: 1. A 24-inch standard strength R/C (3500 psi) culvert pipe is proposed for installation through an earth dam.

2. The proposed maximum height of embankment above the top of the conduit H_c at Sta 1+50 is 20.8 ft and the distance ρb_c is 0.3 ft. At Sta 1+67 $H_c = 19$ ft and $\rho b_c = 1.8$ ft.

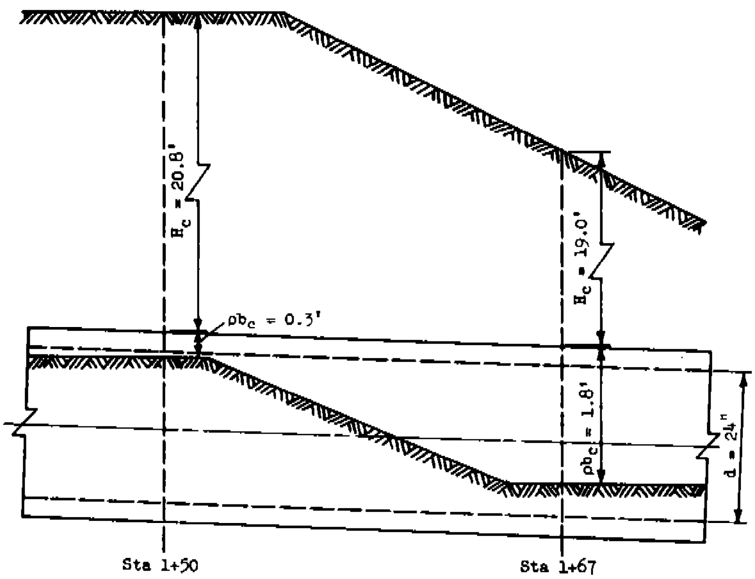
3. Consider that 0.01-inch crack of the R/C culvert is permissible for safe design.

4. The foundation material has an angle of internal friction ϕ_f of 30°, a modulus of consolidation E_f of 30 tons/ft^2, and a unit weight γ_f of 120 lbs/ft^3. The material is uniform down to a rock layer which lies 8.4 ft below the bottom of the conduit.

5. When compacted to required density, the embankment material has a unit weight γ of 120 lbs/ft^3, an angle of internal friction ϕ of 40°, and a modulus of consolidation E of 26 tons/ft^2.

6. The value of b for cradles is 4 ft and for beddings is 2.5 ft.

Determine: Types of beddings or cradles that may be used.



Solution:

List data (ES-114, pages 3-41 and 3-42)

$$\gamma = \gamma_f = 120 \text{ lbs/ft}^3, \quad \frac{\gamma_f}{\gamma} = 1$$

From ES-119, page 3-69, $b_c = 2.5$ ft

From ES-114, page 3-43, $K = 0.22$, $K\mu = 0.183$ for $\phi = 40^{\circ}$

$s = 1.0$

$$\rho = \frac{0.3}{2.5} = 0.12 \text{ for Sta 1+50}$$

$$\rho = \frac{1.8}{2.5} = 0.72 \text{ for Sta 1+67}$$

Follow procedure on ES-113, page 3-38.

Obtain δ.

Follow procedure on ES-115, page 3-45.

From ES-115, page 3-47, either case c or case d exists.

Solve for

$$\delta \text{ (case c)} = \frac{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] \frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] \frac{K\mu}{K_f\mu_f}}$$

$$\frac{E}{E_f} = \frac{26}{30} = 0.867$$

From ES-114, page 3-43, $K_f\mu_f = 0.19$ when $\phi = 30^o$

$$\frac{K\mu}{K_f\mu_f} = \frac{0.183}{0.19} = 0.963$$

$$1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] \frac{K\mu}{K_f\mu_f} = 1 + (1)(0.867)(0.963) = 1.834$$

Since several values of δ (case c) must be determined, prepare columns 1 to 13 of table on page 3-22.

Column 1 lists the station considered.

Column 2 lists arbitrarily selected types of bedding or cradle under consideration.

Column 3 lists values of ψ. Values of ψ are obtained from the distance ψb_c. For type B1 bedding and for all cradles, it is the distance between the natural ground and the bottom of the bedding or cradle. Types B2, C, and D beddings would not be used for a conduit through a dam. Use minimum values for thickness of bedding and cradles under the conduit (ES-120, pages 3-74 and 3-75).

Column 4 lists values of ρ.

Column 5 lists values of H_f. The distance H_f is measured from the bottom of the cradle or rigid bedding to the nonyielding foundation material. H_f = 8.233 ft for B1 bedding. H_f = 7.90 ft for cradles (ES-120, pages 3-74 and 3-75).

Column 6 $\left(\frac{\psi}{\rho}\right)$ is column 3 divided by column 4.

Column 7 $\left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] \frac{\psi}{\rho}$ is the product of column 6 and $\left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]$.

Column 8 lists δ (case c).

Column 9 ($\delta\rho$) is the product of column 8 and column 4.

Column 10 lists values of H'_e/b obtained from ES-115, page 3-51, for values of $\delta\rho$ shown in column 9 and for the given value of $K\mu$.

Column 11 (H'_e) is the product of column 10 and b = 2.5 ft or b = 4.0 ft.

Column 12 $\left(\frac{K\mu}{K_f\mu_f} H'_e\right)$ is the product of column 11 and $\frac{K\mu}{K_f\mu_f}$.

Column 13 lists the case for the determination of δ.

When $\frac{K\mu}{K_f\mu_f} H'_e \leq H_f$, case c exists.

When $\frac{K\mu}{K_f\mu_f} H'_e > H_f$, case d exists.

When case c is determined in column 13, the desired values of δ are given in column 8. If case d is determined in column 13, the desired values of δ are determined by the procedure for case d.

The conduit is not installed in a ditch, therefore, it is a positive projecting conduit.

The subclassification is determined by preparing columns 14 and 15 of the table on page 3-22.

Column 14 lists values of H_c/b_c.

Column 15 lists classification by comparing column 14 with column 10 since $\frac{H_e}{b_c} = \frac{H'_e}{b}$.

When $\frac{H_c}{b_c} \leq \frac{H_e}{b_c}$, complete condition exists.

When $\frac{H_c}{b_c} > \frac{H_e}{b_c}$, incomplete condition exists.

The remainder of the solution is made by preparing columns 16 through 24 of the table on page 3-22.

From ES-119, page 3-69, $F_{sp} = 5.724$.

Column 16 lists the value of T (ES-121, pages 3-79 and 3-81)

Column 17 lists values of $2K\mu\delta\rho$.

Column 18 lists values of $2K\mu(H_c/b_c)$.

Column 19 lists values of $2K\mu C_p$ (ES-118, page 3-63)

Column 20 (C_p) is column 19 divided by $2K\mu$.

Column 21 (KT) is the product of column 16 and K.

Column 22 (F_{sr} + KT) is F_{sr} plus column 21 (when $F_{sr} = F_{sp}$).

Column 23 (X_p) is column 22 divided by column 20.

Column 24 gives permissible beddings and cradles. A bedding or cradle is permissible when the value of X_p in column 23 is equal to or greater than the value of X_p for the particular bedding or cradle (ES-120, pages 3-74 to 3-76).

If it is desirable to use the same type of cradle throughout the length of the conduit, a type A1 cradle is required.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
					$\frac{(3)}{(4)}$	0.866 (6)	$\frac{1+(7)}{1.834}$	(4) (8)	ES-115	b (10)	0.963 (11)	ES-115		
Sta.	Bedding or Cradle	ψ	ρ	H_f	$\frac{\psi}{\rho}$	$\left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right]\frac{\psi}{\rho}$	δ	$\delta\rho$	$\frac{H'_e}{b}$	H'_e	$\frac{K\mu}{K_f\mu_f}H'_e$	Case	$\frac{H_c}{b_c}$	Class.
1+50	B1	0.947	0.12	8.15	8.17	7.07	4.40	0.528	1.53	3.83	3.69	c	8.3	inc.
1+50	A1, A2, A3	1.08	0.12	7.90	9.00	7.79	4.79	0.575	1.60	6.40	6.16	c	8.3	inc.
1+67	B1	0.348	0.72	8.15	0.528	0.457	0.793	0.571	1.59	3.98	3.84	c	7.6	inc.
1+67	A1, A2, A3	0.480	0.72	7.90	0.667	0.578	0.860	0.619	1.66	6.60	6.36	c	7.6	inc.

(1)	(2)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
		ES-121	$2K\mu\cdot$(4)	$2K\mu$ (14)	ES-118	$\frac{(19)}{2K\mu}$	0.22 (16)	5.724 + (21)	$\frac{(22)}{(20)}$	ES-120
Sta.	Bedding or Cradle	T	$2K\mu\delta\rho$	$2K\mu(H_c/b_c)$	$2K\mu C_p$	C_p	KT	F_{sr} + KT	X_p	Satisfactory Bedding or Cradle
1+50	B1	0.06	0.193	3.04	5.08	13.88	0.013	5.737	0.413	none
1+50	A1, A2, A3	0.50	0.210	3.04	5.21	14.24	0.110	5.834	0.410	A1
1+67	B1	3.38	0.209	2.78	4.74	12.93	0.744	6.468	0.500	none
1+67	A1, A2, A3	4.61	0.226	2.78	4.80	13.12	1.014	6.738	0.514	A1, A2, A3

Example No. 7

Given: 1. A 24-inch class III, wall B, R/C pipe (ASTM C76-57T) is proposed for installation through an earth dam. The conduit is not installed in a ditch.

2. A type B1 bedding is proposed for use. The bottom width of the bedding is 3.0 ft.

3. The distance ρb_c is 1.75 ft.

4. The distance ψb_c is 1.05 ft.

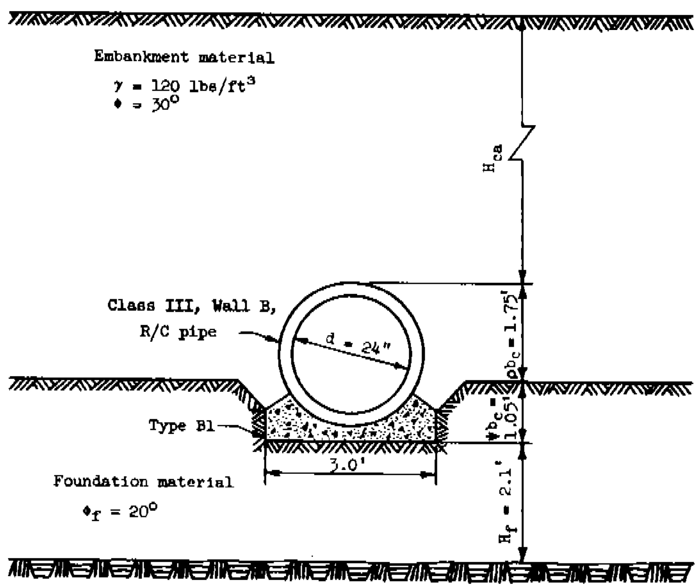
5. The embankment material has a unit weight γ of 120 lbs/ft^3 and has an angle of internal friction ϕ of 30°.

6. The foundation material is a layer of clayey soil over a nonyielding layer. The distance from the bottom of the B1 bedding to the nonyielding layer is 2.1 ft. The angle of internal friction of the clay is $\phi_f = 20°$.

7. Assume that $\left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]$ will fall within the range of $0.20 < \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] < 2.0$.

8. Consider 0.01-inch crack in the conduit permissible.

Determine: The allowable height of fill H_{ca}.



Solution:

List data (ES-114, pages 3-41 and 3-42)

$\gamma = 120$ lbs/ft^3

From ES-119, page 3-67, $b_c = 2.5$ ft

From ES-114, page 3-43, $K = 0.333$, $K\mu = 0.19$

$s = 1.0$

$$\rho = \frac{1.75}{2.5} = 0.70$$

Follow procedure on ES-113, page 3-39.

Obtain δ.

Follow procedure on ES-115, page 3-45.

From ES-115, page 3-47, case c or case d exists.

$b = 3.0$ ft

Solve for δ (case c)

$$\psi = \frac{1.05}{2.5} = 0.42$$

$$\frac{\psi}{\rho} = \frac{0.42}{0.70} = 0.60$$

From ES-114, page 3-43, $K_f\mu_f = 0.178$

$$\frac{K\mu}{K_f\mu_f} = \frac{0.19}{0.178} = 1.067$$

When $\left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right] = 0.2$

$$\delta \text{ (case c)} = \frac{1 + (0.20)(0.60)}{1 + (0.20)(1.067)} = 0.923$$

Find $\frac{H_e'}{b}$ from ES-115, page 3-51

$\delta\rho = (0.923)(0.70) = 0.6461$

$\frac{H_e'}{b} = 1.66$ and $H_e' = (1.66)(3.0) = 4.98$ ft

Is $H_f < H_e' \frac{K\mu}{K_f\mu_f}$?

$$\frac{K\mu}{K_f\mu_f} H_e' = (1.067)(4.98) = 5.31$$

$H_f = 2.1$ ft

$H_f < H_e' \frac{K\mu}{K_f\mu_f}$, therefore, foundation is of shallow depth and case d exists.

Compute $2K\mu\rho\left\{1 + \left[\frac{\gamma_f}{\gamma}\ \frac{E}{E_f}\right]\frac{\psi}{\rho}\right\} = (0.38)(0.7)\left[1 + (0.6)(0.2)\right]$
$= 0.298$

Compute $2K\mu \left[\frac{\gamma_f}{\gamma} \frac{E}{E_f}\right] \frac{H_f}{b} = (0.38)(0.2) \quad \frac{2.1}{3.0} = 0.0532$

Obtain ω from ES-115, page 3-49.

$\omega = 0.66$

Compute $\frac{H_e'}{b} = \frac{\omega}{2K\mu} = \frac{0.66}{0.38} = 1.737$

From ES-115, page 3-51, obtain $\delta\rho = 0.72$

Solve for $\delta = \frac{\delta\rho}{\rho} = \frac{0.72}{0.7} = 1.03$

hen $\left[\frac{\gamma_f}{\gamma} \frac{E}{E_f}\right] = 2.0$

Find δ (case c) $= \frac{1 + (2.0)(0.60)}{1 + (2.0)(1.067)} = 0.702$

Find $\frac{H_e'}{b}$ from ES-115, page 3-51

$\delta\rho = (0.702)(0.70) = 0.491$

$\frac{H_e'}{b} = 1.45$ and $H_e' = (1.45)(3.0) = 4.35$ ft

Is $H_f < H_e' \frac{K\mu}{K_f\mu_f}$?

$\frac{K\mu}{K_f\mu_f} H_e' = (1.067)(4.35) = 4.641$

$H_f = 2.1 < 4.641$, therefore, foundation is of shallow depth and case d exists.

Compute $2K\mu\rho \left\{ 1 + \left[\frac{\gamma_f}{\gamma} \frac{E}{E_f}\right] \frac{\psi}{\rho} \right\} = (0.38)(0.7)\left[1 + (0.6)(2.0)\right] = 0.585$

Compute $2K\mu \left[\frac{\gamma_f}{\gamma} \frac{E}{E_f}\right] \frac{H_f}{b} = (0.38)(2.0) \quad \frac{2.1}{3.0} = 0.532$

Obtain ω from ES-115, page 3-49

$\omega = 0.72$

Compute $\frac{H_e'}{b} = \frac{\omega}{2K\mu} = \frac{0.72}{0.38} = 1.89$

Obtain $\delta\rho$ from ES-115, page 3-51.

$\delta\rho = 0.87$

Compute $\delta = \frac{\delta\rho}{\rho} = \frac{0.87}{0.7} = 1.24$

Assume that classification is positive projecting conduit, incomplete condition.

$$\text{For } \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] = 0.2$$

From ES-119, page 3-67, obtain $F_{sp} = 5.152$

From ES-120, page 3-77, obtain $X_a = 0.588$

From ES-120, page 3-75, obtain $X_p = 0.65$

Solve for

$$e^x - x = 2K\mu\delta\rho + 1 = (0.38)(0.72) + 1 = 1.274$$

From ES-123, page 3-87, obtain $x = 0.659$

Solve for

$$e^x = (e^x - x) + x = 1.274 + 0.659 = 1.933$$

Solve for $\left(\frac{H_{ca}}{b_c}\right)_o$ by Eq. 3-8.

$$\left(\frac{H_{ca}}{b_c}\right)_o = \frac{5.152 + (0.5)(0.333)(0.7)^2(0.588) + \frac{0.65}{0.38}\left[(0.659)(1.933) - 1.933 + 1\right]}{(1.933)(0.65) - (0.333)(0.7)(0.588)}$$

$$= \frac{5.152 + 0.048 + 0.583}{1.256 - 0.137} = 5.168$$

Subclassify

From ES-117, page 3-57, $\frac{H_e}{b_c} = 1.74$

Since $\left(\frac{H_{ca}}{b_c}\right)_o > \frac{H_e}{b_c}$, incomplete condition exists.

Since $b_d = \infty$

$\frac{b_d}{b_c} > \frac{b_d'}{b_c}$ and conduit is classed as positive projecting conduit.

$H_{ca} = (5.168)(2.5) = \underline{12.92 \text{ ft}}$

$$\text{For } \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] = 2.0$$

Values of F_{sp}, X_a, and X_p are unchanged.

Solve for

$$e^x - x = 2K\mu\delta\rho + 1 = 1.331$$

From ES-123, page 3-87, obtain $x = 0.717$

Solve for

$$e^x = (e^x - x) + x = 1.331 + 0.717 = 2.048$$

Solve for $\left(\frac{H_{ca}}{b_c}\right)_o$ by Eq. 3-8.

$$\left(\frac{H_{ca}}{b_c}\right)_o = \frac{5.152 + (0.5)(0.333)(0.7)^2(0.588) + \frac{0.65}{0.38}\left[(0.717)(2.048) - 2.048 + 1\right]}{(2.048)(0.65) - (0.333)(0.7)(0.588)}$$

$$= \frac{5.152 + 0.048 + 0.718}{1.331 - 0.137} = 4.956$$

Subclassify

From ES-117, page 3-57, $\frac{H_e}{b_c} = 1.89$

Since $\left(\frac{H_{ca}}{b_c}\right)_o > \frac{H_e}{b_c}$, incomplete condition exists as assumed.

Since $b_d = \infty$

$\frac{b_d}{b_c} > \frac{b_d'}{b_c}$ and conduit is classed as positive projecting conduit.

$H_{ca} = (4.956)(2.5) = \underline{12.39 \text{ ft}}$

Note the small difference in values of H_{ca} on using values of $\left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]$ of 0.2 and 2.0.

Example No. 8

Given: 1. A 24-inch standard strength concrete sewer pipe (ASTM Spec. C14-55) is proposed for installation in a 5.5 ft ditch.

2. A type A2 cradle with a bottom width b of 3.52 ft is to be used. The type of construction used will give a load factor of 2.6 if the classification is ditch conduit.

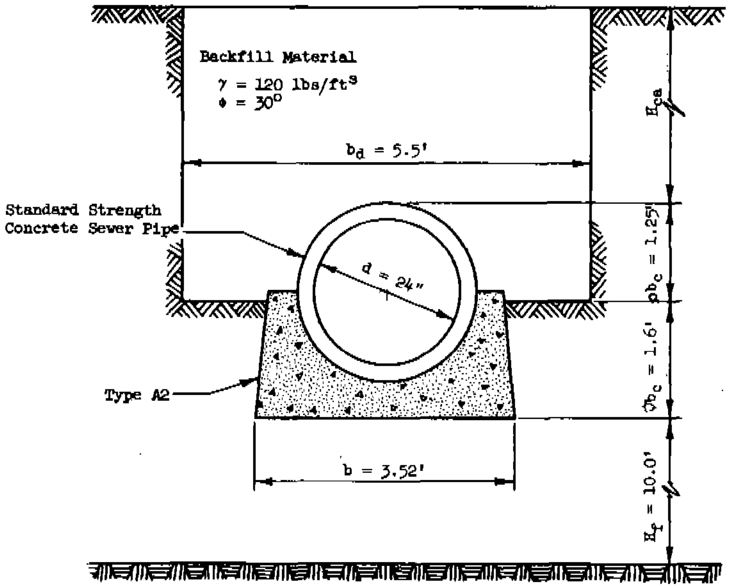
3. The distance ρb_c is 1.25 ft, the distance ψb_c is 1.6 ft, and H_f is 10.0 ft.

4. The excavated material is used for backfill and has a unit weight γ of 120 lbs/ft^3 and an angle of internal friction ϕ of 30°.

5. Use a safety factor s of 1.5.

6. The value of $\left[\frac{\gamma_f}{\gamma}\frac{E}{E_f}\right]$ is 1.

Determine: The allowable fill height H_{ca}.



<u>Solution</u>:

List data (ES-114, pages 3-41 and 3-42)

$\gamma = 120$ lbs/ft^3

From ES-119, page 3-71, $b_c = 2.354$

From ES-114, page 3-43, $K = 0.333$, $K\mu = 0.19$

$s = 1.5$

$\rho = \dfrac{1.25}{2.354} = 0.531$

Follow procedure on ES-113, page 3-39.

Obtain δ.

Follow procedure on ES-115, page 3-45.

From ES-115, page 3-47, either case c or case d exists.

$b = 3.52$ ft

Solve for δ (case c)

$\dfrac{\psi}{\rho} = \dfrac{1.6}{1.25} = 1.28$

δ (case c) $= \dfrac{1 + (1)(1.28)}{1 + (1)(1)} = 1.14$, $\quad \delta\rho = (1.14)(0.531) = 0.605$

From ES-115, page 3-51, obtain $\dfrac{H_e'}{b} = 1.60$

$H_e' = (1.6)(5.32) = 5.632$

$$H_e' \frac{K\mu}{K_f\mu_f} = 5.632$$

$$H_f > H_e' \frac{K\mu}{K_f\mu_f}\text{ , case c exists}$$

$$\delta = 1.14, \quad \delta\rho = 0.605$$

Assume classification is positive projecting conduit, incomplete condition.

From ES-119, page 3-71, obtain $s\gamma F_{sp} = 619.8$

$$F_{sp} = \frac{619.8}{(120)(1.5)} = 3.443$$

From ES-120, page 3-74, obtain $X_p = 0.450$

From ES-120, page 3-77, obtain $X_a = 0.85$

Solve for

$$e^x - x = 2K\mu\delta\rho + 1 = (0.38)(0.605) + 1 = 1.230$$

From ES-123, page 3-87, obtain $x = 0.609$

Solve for $e^x = (e^x - x) + x = 1.230 + 0.609 = 1.839$

Solve for $\left(\frac{H_{ca}}{b_c}\right)_o$ by using Eq. 3-8

$$\left(\frac{H_{ca}}{b_c}\right)_o = \frac{3.443 + (0.5)(0.333)(0.531)^2(0.85) + \frac{0.45}{0.38}\left[(0.609)(1.839) - 1.839 + 1\right]}{(1.839)(0.45) - (0.333)(0.531)(0.85)}$$

$$= \frac{3.443 + 0.040 + 0.333}{0.828 - 0.150} = 5.628$$

Subclassify

From ES-117, page 3-57, $\frac{H_e}{b_c} = 1.60$

Since $\left(\frac{H_{ca}}{b_c}\right)_o > \frac{H_e}{b_c}$, classification is incomplete condition.

$\left(\frac{H_c}{b_c}\right)_o$ is $\left(\frac{H_c}{b_c}\right)_1 = 5.628$

From ES-117, page 3-59, $\left(\frac{b_d'}{b_c}\right)_1 = 2.51$

$$\frac{b_d}{b_c} = \frac{5.5}{2.354} = 2.336$$

$$\frac{b_d}{b_c} < \left(\frac{b_d'}{b_c}\right)_1$$

Assume classification is ditch conduit.

Since the excavated material is used for backfill $\mu' = \mu$.

$L_f = 2.6$

From ES-119, page 3-71, $R_{eb} = 2400$ lbs/ft

Solve for

$$R_d = \frac{L_f\ R_{eb}}{s} = \frac{(2.6)(2400)}{1.5} = 4160$$

Let $R_d = W_{ca}$

Solve for

$$C_d = \frac{W_{ca}}{\gamma b_d^2} = \frac{4160}{(120)(5.5)^2} = 1.146$$

From ES-118, page 3-61, $\frac{H_{ca}}{b_d} = 1.60$

$$\frac{H_c}{b_d} \neq \infty$$

Solve for $\left(\frac{H_{ca}}{b_c}\right)_2 = \frac{H_{ca}}{b_d}\left(\frac{b_d}{b_c}\right) = (1.60)(2.336) = 3.738$

$$\left(\frac{H_{ca}}{b_c}\right)_2 < \left(\frac{H_{ca}}{b_c}\right)_1$$

From ES-117, page 3-59, obtain $\left(\frac{b_d'}{b_c}\right)_2 = 2.25$

$$\frac{b_d}{b_c} > \left(\frac{b_d'}{b_c}\right)_2$$

From ES-117, page 3-59, obtain $2K\mu\ \frac{H_{ca}}{b_c}$ by using $\frac{b_d}{b_c} = 2.336$ for $\frac{b_d'}{b_c}$

$$2K\mu\ \frac{H_{ca}}{b_c} = 1.672$$

$$H_{ca} = 2K\mu\left(\frac{H_{ca}}{b_c}\right)\frac{b_c}{2K\mu} = (1.672)\ \frac{2.354}{0.38} = 10.358 \text{ ft}$$

Example No. 9

Given: 1. A 72-inch standard strength (3500 psi) R/C culvert pipe on type B1 bedding is proposed for installation in a 10 ft ditch ($b_d = 10.0$ ft). If the conduit is classed as a ditch conduit, use $L_f = 1.9$.

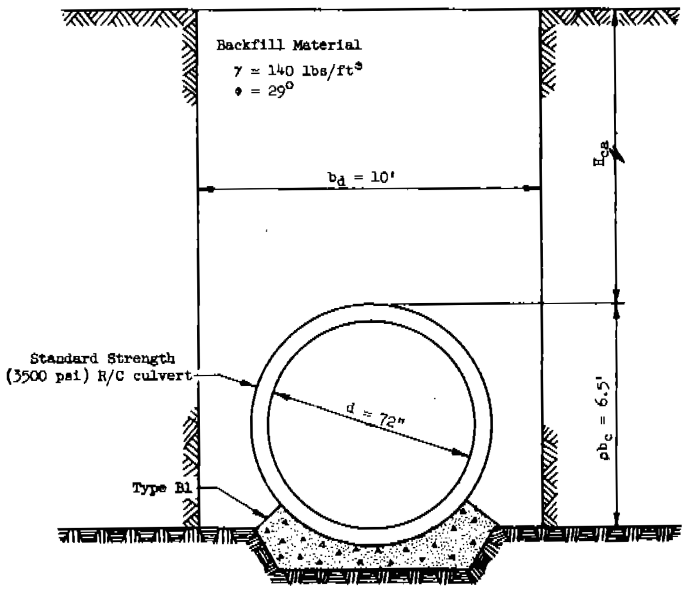
2. The excavated material is used for backfill and has a unit weight γ of 140 lbs/ft^3 and an angle of internal friction ϕ of 29°.

3. The distance ρb_c is 6.50 ft.

4. The foundation material is rock and there is no foundation material adjacent to the pipe.

5. Assume 0.01-inch crack in the culvert is satisfactory for design.

Determine: Allowable height of fill H_{ca}.



Solution:

List data (ES-114, pages 3-41 and 3-42)

$\gamma = 140 \text{ lbs/ft}^3$

From ES-119, page 3-69, $b_c = 7.167$ ft

From ES-114, page 3-43, $K = 0.345$, $K\mu = 0.19$

$$\rho = \frac{6.50}{7.167} = 0.907$$

Follow procedure on ES-113, page 3-39.

Obtain δ.

Follow procedure on ES-115, page 3-45.

From ES-115, page 3-47, case a exists.

$\delta = 1.0$

Assume positive projecting conduit, incomplete condition.

From ES-119, page 3-69, obtain $s\gamma F_{sp} = 183.9$

$$F_{sp} = \frac{183.9}{(1)(140)} = 1.314$$

From ES-120, page 3-77, obtain $X_a = 0.66$

From ES-120, page 3-75, obtain $X_p = 0.650$

Solve for

$$e^x - x = 2K\mu\delta\rho + 1 = (0.38)(0.907)(1) + 1 = 1.345$$

From ES-123, page 3-87, obtain $x = 0.730$

Solve for

$$e^x = (e^x - x) + x = 1.345 + 0.730 = 2.075$$

Solve for

$$\left(\frac{H_{ca}}{b_c}\right)_o = \frac{1.314 + (0.5)(0.345)(0.907)^2(0.66) + \frac{0.65}{0.38}\left[(0.730)(2.075) - 2.075 + 1\right]}{(2.075)(0.65) - (0.345)(0.907)(0.66)}$$

$$= \frac{1.314 + 0.0937 + 0.753}{1.349 - 0.207} = 1.892$$

Subclassify

From ES-117, page 3-57, $\frac{H_e}{b_c} = 1.920$

Since $\left(\frac{H_{ca}}{b_c}\right)_o < \frac{H_e}{b_c}$, conduit is classed as complete condition.

Compute U, Eq. 3-7a

$$U = \frac{(0.38)(1.314) + (0.19)(0.345)(0.907)^2(0.66) + 0.650}{0.650}$$

$$= \frac{0.499 + 0.036 + 0.650}{0.650} = 1.823$$

Compute $\frac{K\rho X_a}{X_p} = \frac{(0.345)(0.907)(0.66)}{0.650} = 0.318$

From ES-122, page 3-83, obtain $2K\mu \left(\frac{H_{ca}}{b_c}\right)_1 = 0.72 \quad \left(\frac{H_{ca}}{b_c}\right)_1 = 1.895$

From ES-117, page 3-59, obtain $\left(\frac{b_d'}{b_c}\right)_1 = 1.77$

Is $\frac{b_d}{b_c} < \left(\frac{b_d'}{b_c}\right)_1$?

$$\frac{b_d}{b_c} = \frac{10}{7.167} = 1.395$$

$$\frac{b_d}{b_c} < \left(\frac{b_d'}{b_c}\right)_1$$

Assume classification is ditch conduit.

Since the excavated material is used for backfill, $K\mu' = K\mu$.

$L_f = 1.9$

From ES-119, page 3-69, obtain $R_{eb} = 6600$ lbs/ft

Solve for

$$R_d = \frac{L_f\ R_{eb}}{s} = \frac{(1.9)(6600)}{1} = 12{,}540 \text{ lbs/ft}$$

Let $R_d = W_{ca} = 12{,}540$ lbs/ft

Solve for

$$C_d = \frac{W_{ca}}{\gamma b_d^2} = \frac{12{,}540}{(140)(10.0)^2} = 0.896$$

From ES-118, page 3-61, obtain $\frac{H_{ca}}{b_d} = 1.20$

$$\frac{H_{ca}}{b_d} \neq \infty$$

Solve for $\left(\frac{H_{ca}}{b_c}\right)_2 = \left(\frac{H_{ca}}{b_d}\right) \frac{b_d}{b_c} = (1.20)\ \frac{10}{7.167} = 1.674$

$$\left(\frac{H_{ca}}{b_c}\right)_2 < \left(\frac{H_{ca}}{b_c}\right)_1$$

From ES-117, page 3-59, obtain $\left(\frac{b_d'}{b_c}\right)_2 = 1.65$

$$\frac{b_d}{b_c} < \left(\frac{b_d'}{b_c}\right)_2$$

Compute

$$H_{ca} = \left(\frac{H_{ca}}{b_c}\right)_2 b_c = (1.674)\ 7.167 = 12.0 \text{ ft}$$

Example No. 10

Given: 1. A 24-inch Class V, Wall B, pipe (ASTM Spec. C76-57T) is proposed for installation on a type A1 concrete cradle. The conduit is not installed in a ditch.

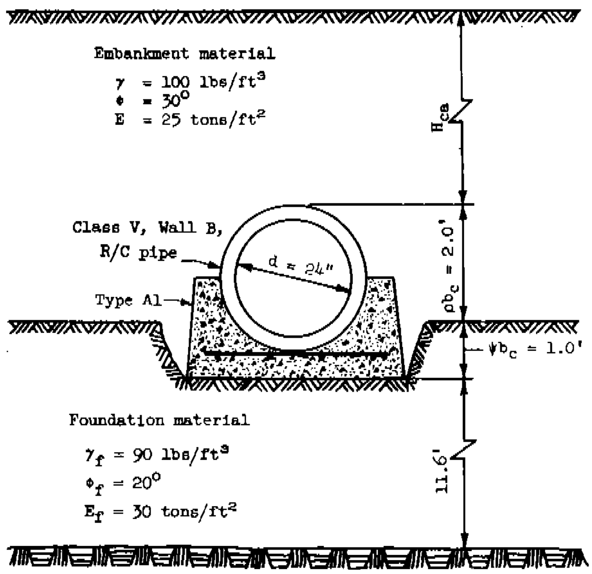
2. The distance ρb_c is 2.0 ft and ψb_c is 1.0 ft.

3. The embankment material has an angle of internal friction ϕ of 30^o, a unit weight γ of 100 lbs/ft^3, and a modulus of consolidation E of 25 tons/ft^2.

4. The foundation material has a unit weight γ_f of 90 lbs/ft^3, an angle of internal friction ϕ_f of 20^o, and a modulus of consolidation E_f of 30 tons/ft^2. There is a layer of nonyielding material a distance H_f of 11.6 ft below the bottom of the cradle.

5. Consider 0.01-inch crack in the pipe as permissible for design.

Determine: The allowable height of fill H_{ca}.



Solution:

List data (ES-114, pages 3-41 and 3-42)

$\gamma = 100 \text{ lbs/ft}^3$

From ES-119, page 3-67, $b_c = 2.5$ ft

From ES-114, page 3-43, $K\mu = 0.19$, $K = 0.333$

$s = 1.0$

$$\rho = \frac{2.0}{2.5} = 0.80$$

Follow procedure on ES-113, page 3-39.

Obtain δ.

Follow procedure on ES-115, page 3-45.

From ES-115, page 3-47, case c or case d exists.

$b = 3.0$ ft

Solve for δ (case c)

$$\frac{\psi}{\rho} = \frac{\psi b_c}{\rho b_c} = \frac{1.0}{2.0} = 0.5$$

$$\frac{\gamma_f}{\gamma} = \frac{90}{100} = 0.9, \quad \frac{E}{E_f} = \frac{25}{30} = 0.833, \quad \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right] = 0.75$$

From ES-114, page 3-43, $K_f\mu_f = 0.178$

$$\frac{K\mu}{K_f\mu_f} = \frac{0.19}{0.178} = 1.067$$

$$\delta \text{ (case c)} = \frac{1 + (0.75)(0.5)}{1 + (1.067)(0.75)} = \frac{1.375}{1.80} = 0.764$$

Find $\frac{H_e'}{b}$

$$\delta\rho = 0.611$$

From ES-115, page 3-51, $\frac{H_e'}{b} = 1.61$

$$H_e' = (1.61)(3.0) = 4.83 \text{ ft}$$

Is $H_f < H_e' \frac{K\mu}{K_f\mu_f}$?

$$H_e' \frac{K\mu}{K_f\mu_f} = (4.83)(1.067) = 5.154$$

$$H_f = 11.6$$

$H_f > H_e' \frac{K\mu}{K_f\mu_f}$, foundation is sufficiently deep, case c exists.

Assume positive projecting conduit, incomplete condition.

From ES-119, page 3-67, obtain $F_{sp} = 13.74$

From ES-120, page 3-77, obtain $X_a = 0.766$

From ES-120, page 3-74, obtain $X_p = 0.40$

Solve for

$$e^x - x = 2K\mu\delta\rho + 1 = (0.38)(0.611) + 1 = 1.232$$

From ES-123, page 3-87, obtain $x = 0.612$

Solve for $e^x = (e^x - x) + x = 1.232 + 0.612 = 1.844$

Solve for $\left(\frac{H_{ca}}{b_c}\right)_o$ by Eq. 3-8

$$\left(\frac{H_{ca}}{b_c}\right)_o = \frac{13.74 + (0.5)(0.333)(0.8)^2(0.766) + \frac{0.40}{0.38}\left[(0.612)(1.844) - 1.844 + 1\right]}{(1.844)(0.40) - (0.333)(0.80)(0.766)}$$

$$= \frac{13.74 + 0.082 + 0.300}{0.738 - 0.204} = 26.44$$

Subclassify

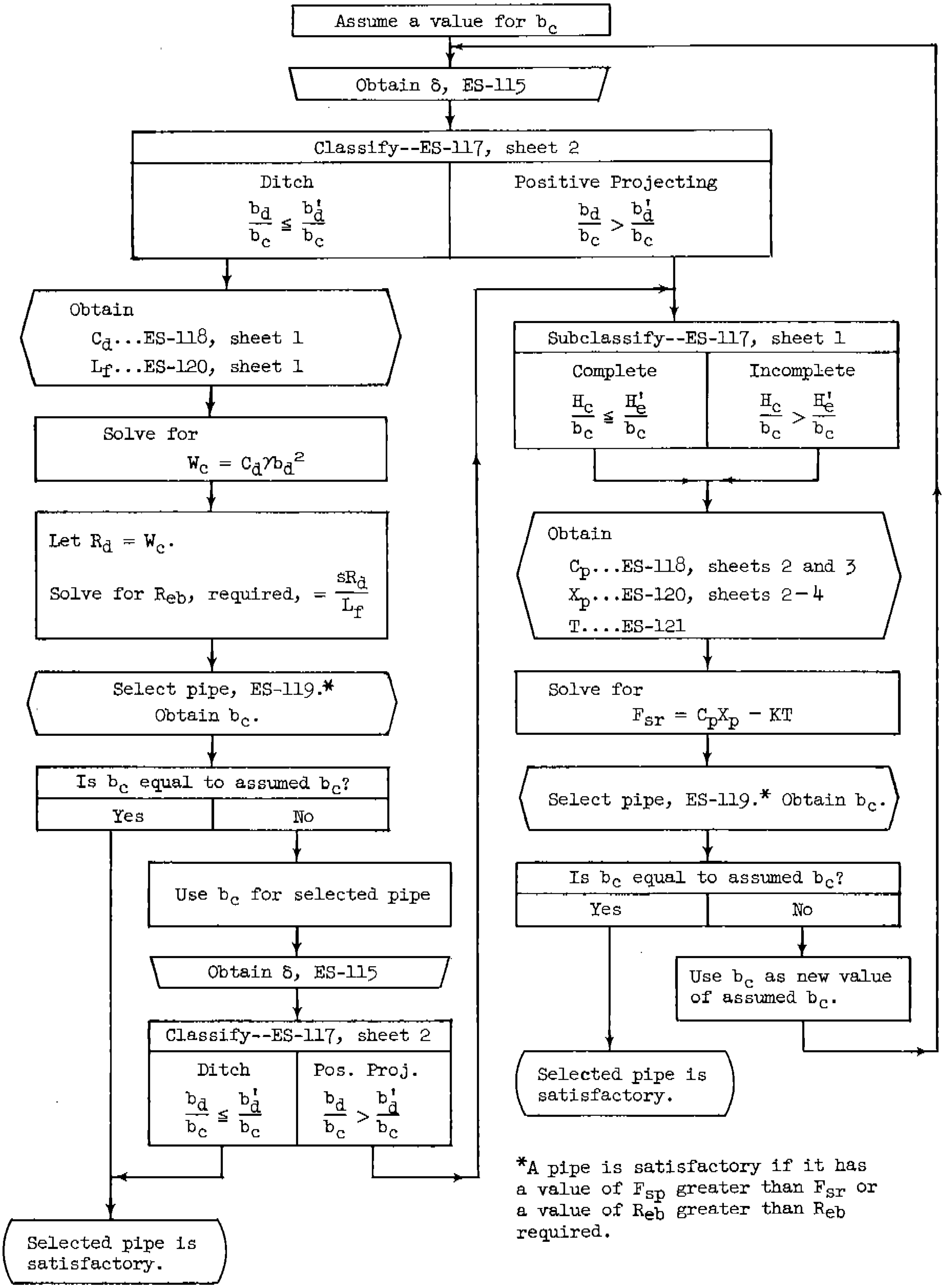
From ES-117, page 3-57, obtain $\frac{H_e}{b_c} = 1.61$

Since $\left(\frac{H_{ca}}{b_c}\right)_o > \frac{H_e}{b_c}$, the incomplete condition exists.

Since the conduit is not installed in a ditch $b_d = \infty$ and $\frac{b_d}{b_c} > \frac{b_d'}{b_c}$. Conduit is classed as positive projecting conduit.

Compute $H_{ca} = \left(\frac{H_{ca}}{b_c}\right)_1 b_c = (26.44)(2.5) = 66.1$ ft

UNDERGROUND CONDUITS: Procedure for ditch conduits or positive projecting conduits; Determination of pipe when H_c and cradle or bedding are known.



*A pipe is satisfactory if it has a value of F_{sp} greater than F_{sr} or a value of R_{eb} greater than R_{eb} required.

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES- 113
SHEET 1 OF 3
DATE 10-18-56
REVISED 6-9-58

UNDERGROUND CONDUITS: Procedure for ditch conduits or positive projecting conduits; Determination of cradle or bedding when H_c and pipe are known.

Obtain δ, ES-115

Classify--ES-117, sheet 2

Ditch	Positive Projecting
$\frac{b_d}{b_c} \leq \frac{b_d'}{b_c}$	$\frac{b_d}{b_c} > \frac{b_d'}{b_c}$

Ditch:

Obtain C_d....ES-118, sheet 1
Obtain R_{eb}...ES-119

Solve for

$$W_c = C_d \gamma b_d^2$$

Solve for $L_f = \frac{sR_d}{R_{eb}}$

(Let $W_c = R_d$)

Select cradle or bedding, ES-120, sheet 1. It is satisfactory if it has a value of L_f equal to or greater than the required value.

Positive Projecting:

Subclassify--ES-117, sheet 1.

Complete	Incomplete
$\frac{H_c}{b_c} \leq \frac{H_e}{b_c}$	$\frac{H_c}{b_c} > \frac{H_e}{b_c}$

Obtain C_p.
ES-118, sheets 2 and 3.

Obtain

F_{sp}..........ES-119
T cradles....ES-121, sheet 1
T beddings...ES-121, sheet 2

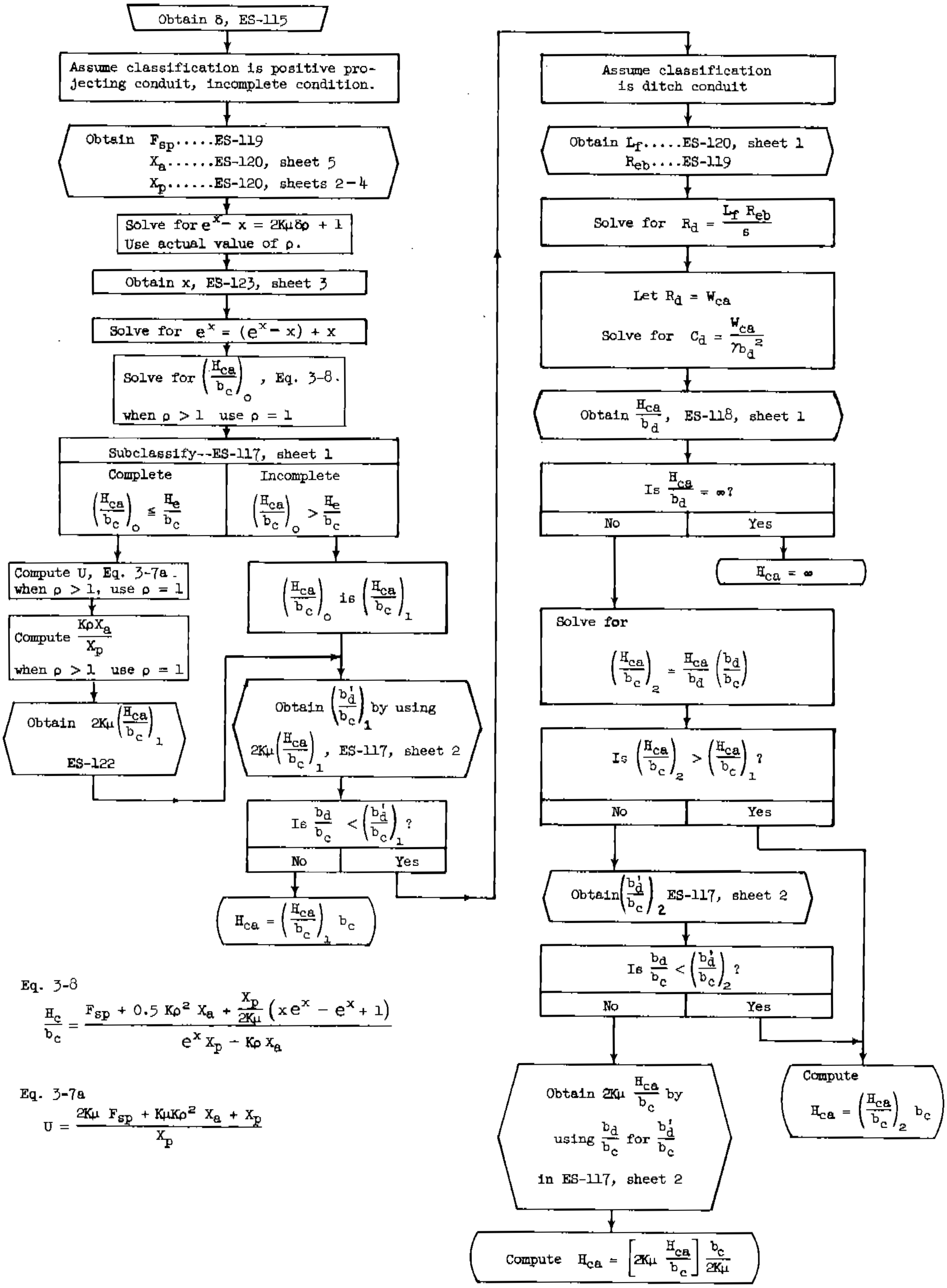
Let $F_{sr} = F_{sp}$

Solve for $X_p = \frac{F_{sr} + KT}{C_p}$ (cradle)

and $X_p = \frac{F_{sr} + KT}{C_p}$ (bedding)

Select cradle or bedding, ES-120, sheets 2 to 4. It is satisfactory if it has a value of X_p equal to or less than the required value.

UNDERGROUND CONDUITS: Procedure for ditch conduits or positive projecting conduits; Determination of allowable fill height H_{ca} when pipe and cradle or bedding are known.



Eq. 3-8

$$\frac{H_c}{b_c} = \frac{F_{sp} + 0.5\,K\rho^2 X_a + \frac{X_p}{2K\mu}\left(x e^x - e^x + 1\right)}{e^x X_p - K\rho X_a}$$

Eq. 3-7a

$$U = \frac{2K\mu\ F_{sp} + K\mu K\rho^2\ X_a + X_p}{X_p}$$

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 113 SHEET 3 OF 3 DATE 10-18-56

REVISED 6-9-58

UNDERGROUND CONDUITS: Required data from site dimensions.

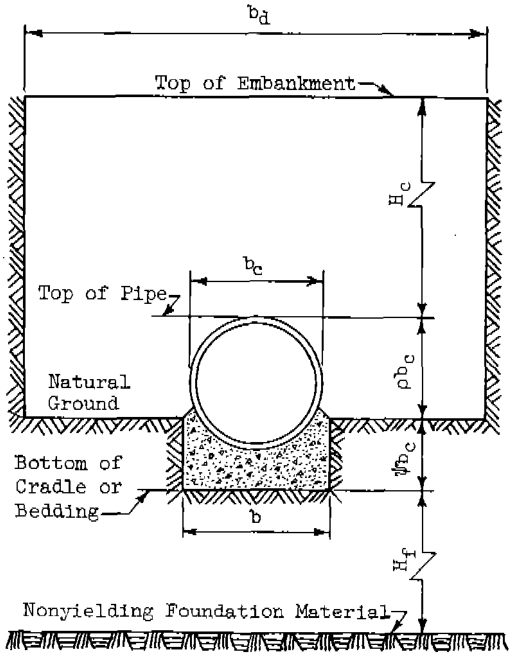
H_c = vertical distance between top of embankment (or backfill) and top of pipe, ft. When the allowable height of embankment (or backfill) H_{ca} is to be determined, the dimension H_{ca} is substituted for H_c in the load formulas.

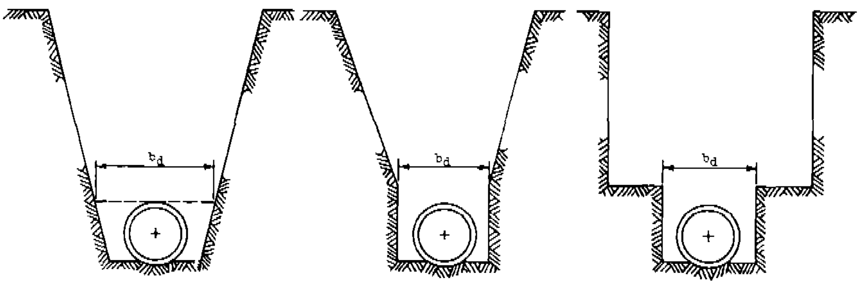
H_f = distance between the bottom of the cradle or rigid bedding (bottom of the pipe if no cradle or rigid bedding is used) and the nonyielding foundation. This may also be considered as the depth of foundation material below the bottom of the cradle.

b = bottom width of cradle or rigid bedding, ft. If $b < b_c$, use $b = b_c$.
If no cradle or rigid bedding is used $b = b_c$.

b_c = outside diameter of pipe (ES-119). When the type of pipe is to be determined, a value for b_c is assumed.

b_d = actual width of a vertical walled ditch in which the pipe is installed. When the ditch is constructed with sloping sides or the conduit is placed in a subditch at the bottom of a wider ditch, experiments have shown that the width of the ditch at or slightly below the top of the conduit is the proper width to use to determine the load on the conduit.





ρb_c = the distance between the top of the conduit and the natural ground surface before any settling has taken place

ρ = projection ratio. Determine ρ from the relation $\rho = \frac{\rho b_c}{b_c}$

ψb_c – For conduits resting on a yielding foundation, the quantity ψb_c is the distance between the surface of the natural ground and the bottom of the cradle or rigid bedding. If the conduit is installed on a flexible bedding, the quantity ψb_c is the distance between the surface of the natural ground and the bottom of the conduit. For conduits installed on a rigid support resting on a nonyielding foundation, the quantity ψb_c is the depth of the yielding foundation material.

ψ – Obtain the value of ψ from the relation $\psi = \frac{\psi b_c}{b_c}$.

The natural ground surface is the surface between the foundation material and the embankment material. If this is not a horizontal surface, values of ρb_c and ψb_c that will provide a conservative design should be selected.

UNDERGROUND CONDUITS: Required data from soil tests-other required data

Data from Soil Tests

ϕ = the angle of internal friction of the backfill or embankment material. The value of ϕ may be obtained by actual measurement or by estimation. Methods for the determination of ϕ are given in the National Engineering Handbook, Section 7, Soil Mechanics. Values of K, μ, and Kμ for various values of ϕ may be obtained from ES-114, Sheet 3.

ϕ_f = the angle of internal friction of the foundation material.

ϕ' = the angle of internal friction of either the backfill material or material in the ditch wall according to whichever is the lesser. When the angle of internal friction of the ditch wall is greater than the angle of internal friction of the backfill material, $\phi' = \phi$ otherwise ϕ' = angle of internal friction of the material in the ditch walls.

$K = \dfrac{\sqrt{\mu^2 + 1} - \mu}{\sqrt{\mu^2 + 1} + \mu}$ = ratio of active lateral pressure to vertical pressure at a point in the backfill material.

$\mu = \tan \phi$

Values of K, μ, and Kμ for various values of ϕ may be determined from ES-114, Sheet 3.

γ = the unit weight of the embankment (or backfill) material, lbs/ft^3. The value of γ varies considerably for different types of embankment (or backfill) materials. It should be measured or closely approximated. Methods for determining the value of γ are given in the National Engineering Handbook, Section 7, Soil Mechanics.

γ_f = the unit weight of the foundation material.

E = modulus of consolidation of the backfill or embankment material.

E_f = modulus of consolidation of the foundation material.

$$\frac{E}{E_f} = \frac{C_c'(1 + e_o)}{C_c(1 + e_o')}$$

where C_c = compression index of the soil

e_o = initial void ratio of the soil

Primed quantities refer to the foundation material

An approximate method of determining C_c is given in Soils Mechanics in Engineering Practice, Terzaghi and Peck, John Wiley and Sons, page 66.

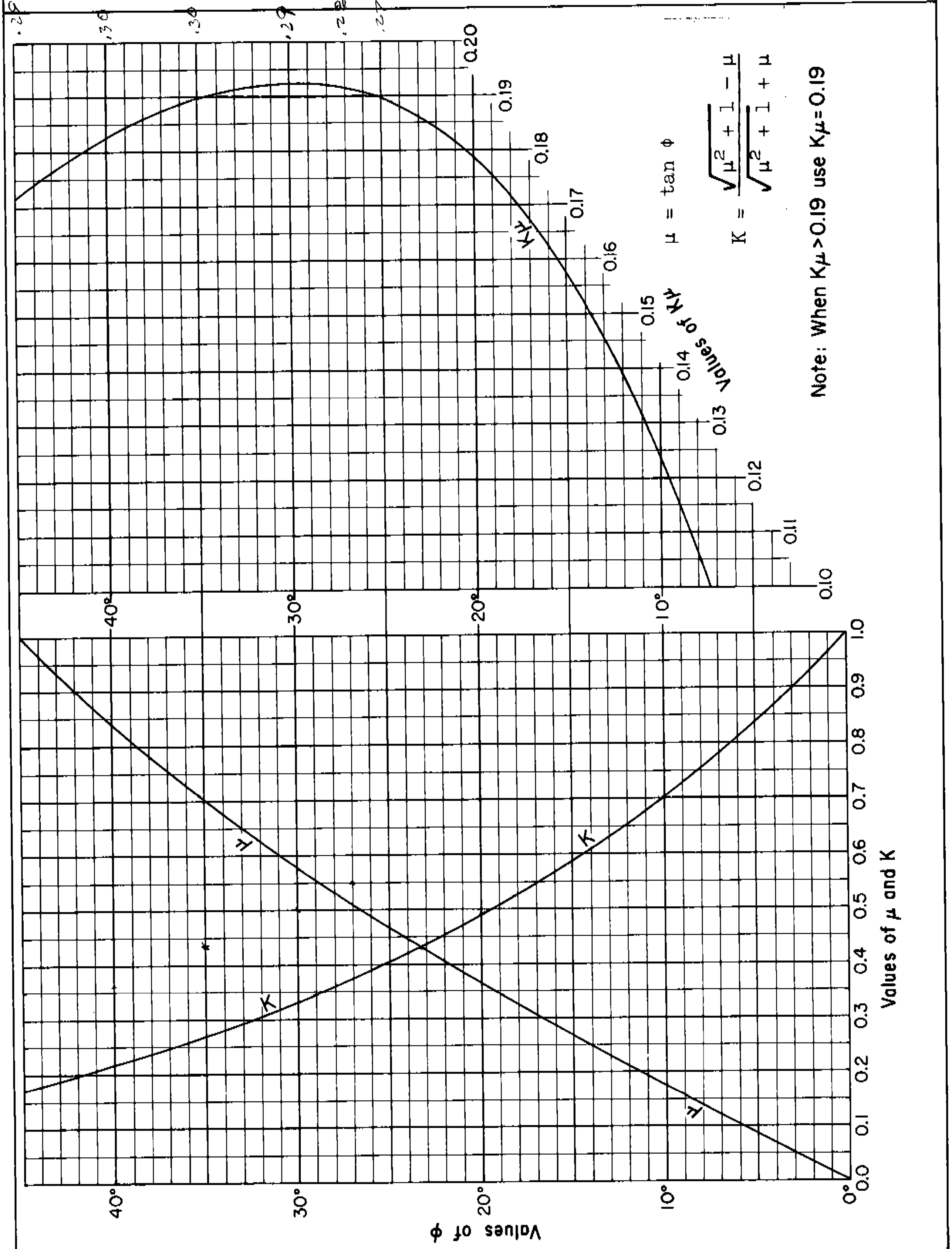
$\left[\dfrac{\gamma_f}{\gamma}\dfrac{E}{E_f}\right]$ Since this parameter does not greatly affect the computed value of δ, approximating its value as 1.0 is usually satisfactory.

Other Data

δ = the settlement ratio. Values of δ for rigid conduits may be obtained by following the procedure on ES-115.

s = a safety factor. The safe supporting strength of pipe is the supporting strength of a pipe divided by the safety factor. Safety factors are recommended for various types of materials. Since the value of R_{eb} for rigid pipes other than reinforced concrete are specified as strengths for ultimate loads a factor of safety of 1.5 or 2.0 may be used. Values of R_{eb} for reinforced concrete are based on the load to produce a 0.01-inch crack in the pipe. A factor of safety of 1.0 is considered satisfactory for reinforced concrete pipe.

UNDERGROUND CONDUITS: Relation of K, μ, and Kμ, in terms of ϕ



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-114 SHEET 3 OF 3 DATE 5-1-58 REVISED

ERGROUND CONDUITS: Determination of settlement ratio δ; Procedure

Select Proper Case, sheet 2

Case a	Case b	Cases c and d

Case a:

$\delta = 1$

Case b:

$$\delta = 1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{\psi}{\rho}$$

Cases c and d:

Obtain b

Assume case c. Solve for δ.

$$\delta\ (\text{case c}) = \frac{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{K\mu}{K_f\mu_f}}$$

Find $\frac{H_e'}{b}$, sheet 4

Is the foundation condition case d?

Is H_f less than $H_e' \frac{K\mu}{K_f\mu_f}$?

No	Yes

No: δ (case c)

Yes:

Compute $2K\mu\rho \left\{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{\psi}{\rho}\right\}$

Compute $2K\mu \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{H_f}{b}$

Obtain ω, sheet 3

Compute $\frac{H_e'}{b} = \frac{\omega}{2K\mu}$

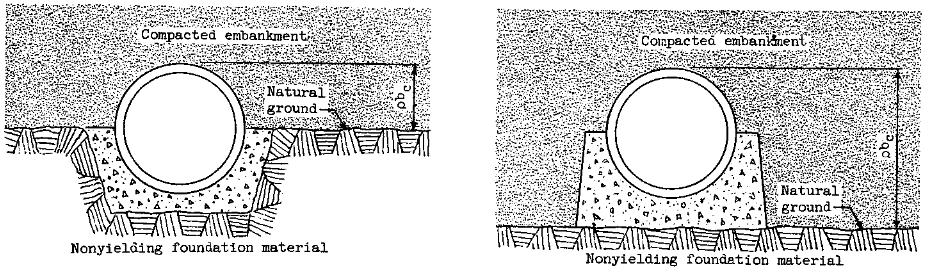
Obtain $\delta\rho$, sheet 4

case d

$\delta = \frac{\delta\rho}{\rho}$

UNDERGROUND CONDUITS: Determination of settlement ratio δ; Cases

Case a



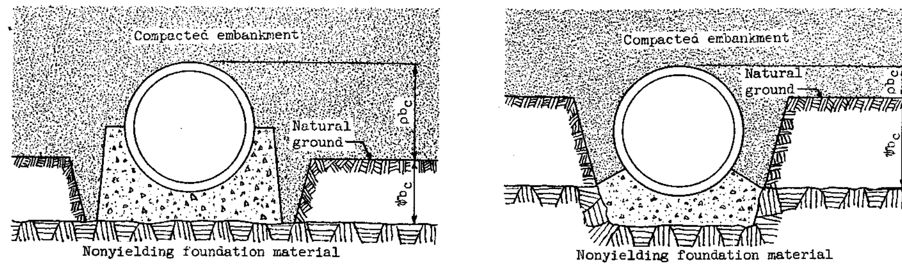
Conduit Resting in or on Nonyielding Foundations with Embankment Extending to the Nonyielding Foundation

The surface of the nonyielding foundation is considered as the natural ground line. The distance between the top of the conduit and the natural ground line is ρb_c.

Since the foundation is nonyielding, the additional settlements s_f and s_g are both zero, and

$$\delta = 1$$

Case b

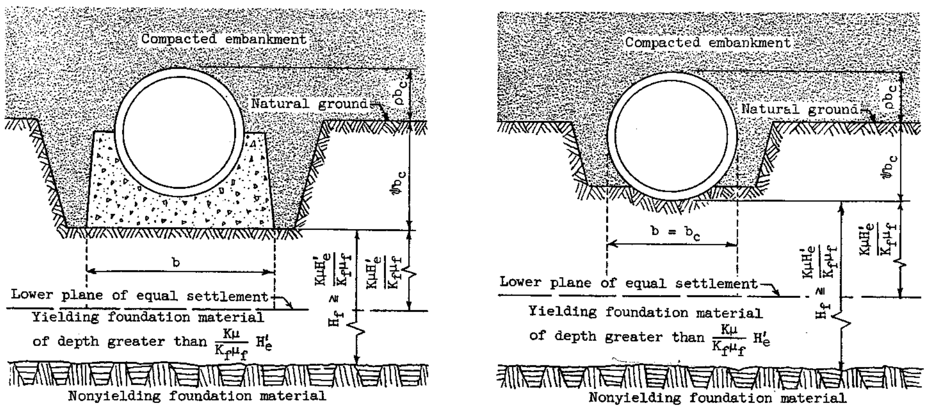


Conduit Resting on Nonyielding Support with Compressible Foundation Materials Adjacent to the Conduit

The value of δ is defined by the following relation

$$\delta = 1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{\psi}{\rho}$$

Case c



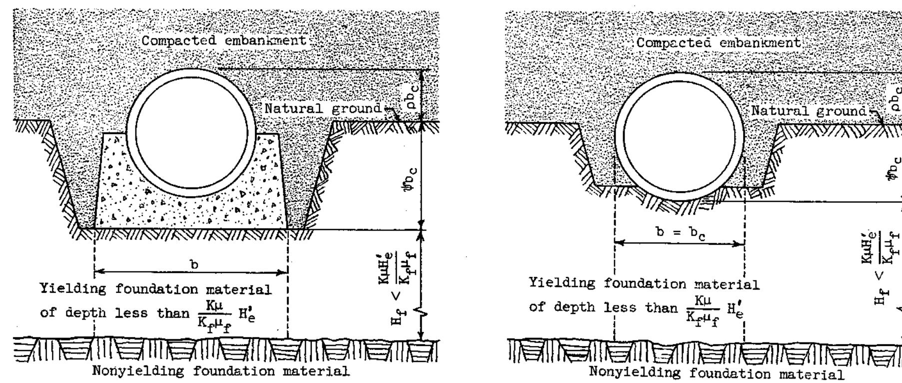
Conduit Resting on Yielding Foundation Material of Sufficiently Great Depth

The values of δ and H_e' are determined from the following two derived relations:

$$\delta = \frac{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{K\mu}{K_f\mu_f}} \quad \text{and} \quad e^{2K\mu(H_e'/b)} - 2K\mu(H_e'/b) = 2K\mu\delta\rho + 1$$

When the value of δ has been determined from the first of these two relations, the value of H_e' may be obtained from the second relation. The values of δ and H_e' determined in this manner are the correct values of δ and H_e' if $H_f \geq \frac{K\mu}{K_f\mu_f} H_e'$. See Procedure, sheet 1.

Case d



Conduit Resting on Yielding Foundation Material of Shallow Depth

The values of δ and H_e' depend, among other parameters, on the value of H_f. They are determined by the simultaneous solution of the following two derived relations:

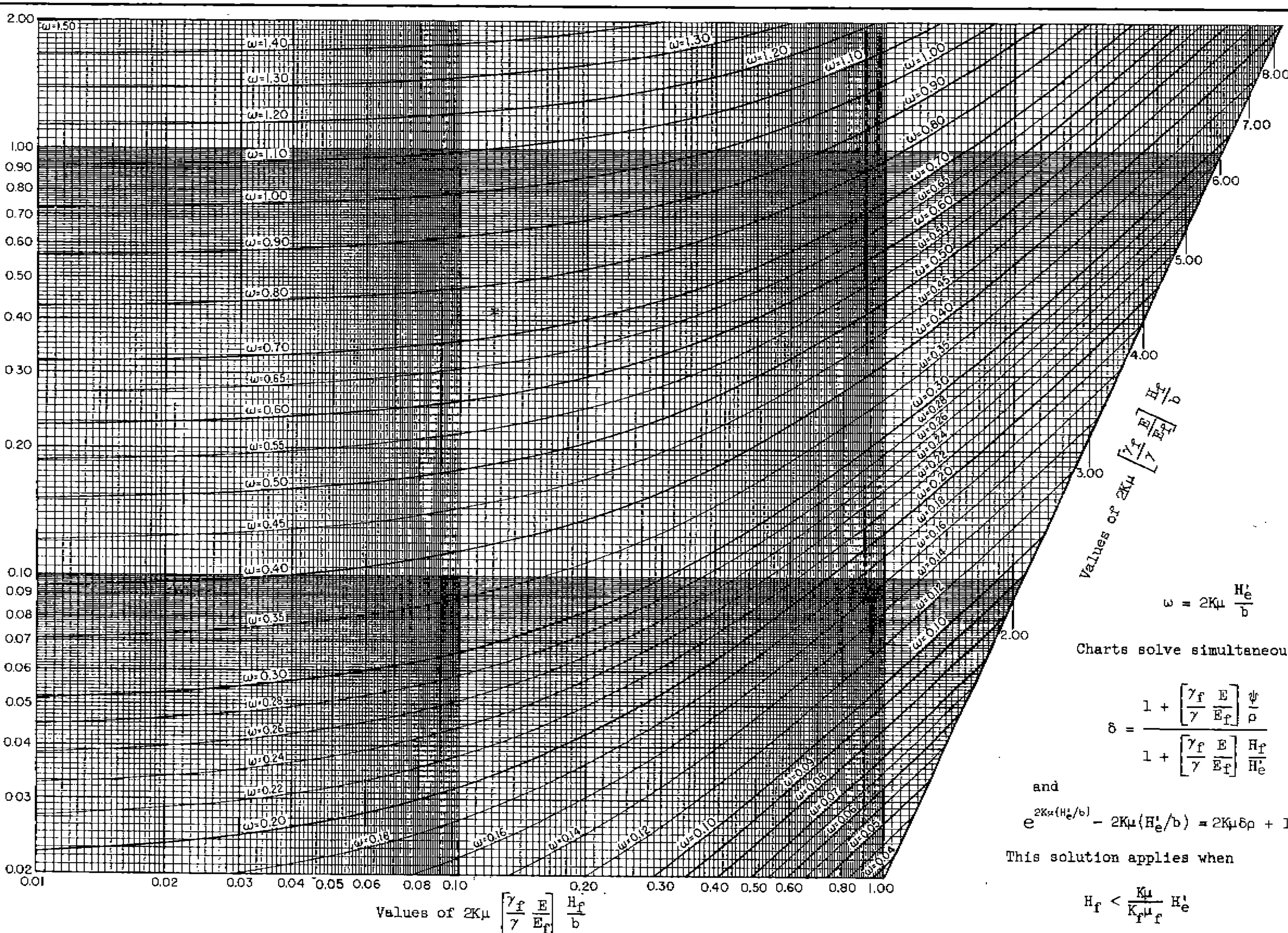
$$\delta = \frac{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{H_f}{H_e'}} \quad \text{and} \quad e^{2K\mu(H_e'/b)} - 2K\mu(H_e'/b) = 2K\mu\delta\rho + 1$$

See Procedure, sheet 1.

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES-115
SHEET 2 OF 4
DATE 5-1-58



$$\omega = 2K\mu \frac{H_e'}{b}$$

Charts solve simultaneously

$$\delta = \frac{1 + \left[\frac{\gamma_f}{\gamma} \frac{E}{E_f}\right] \frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma} \frac{E}{E_f}\right] \frac{H_f}{H_e'}}$$

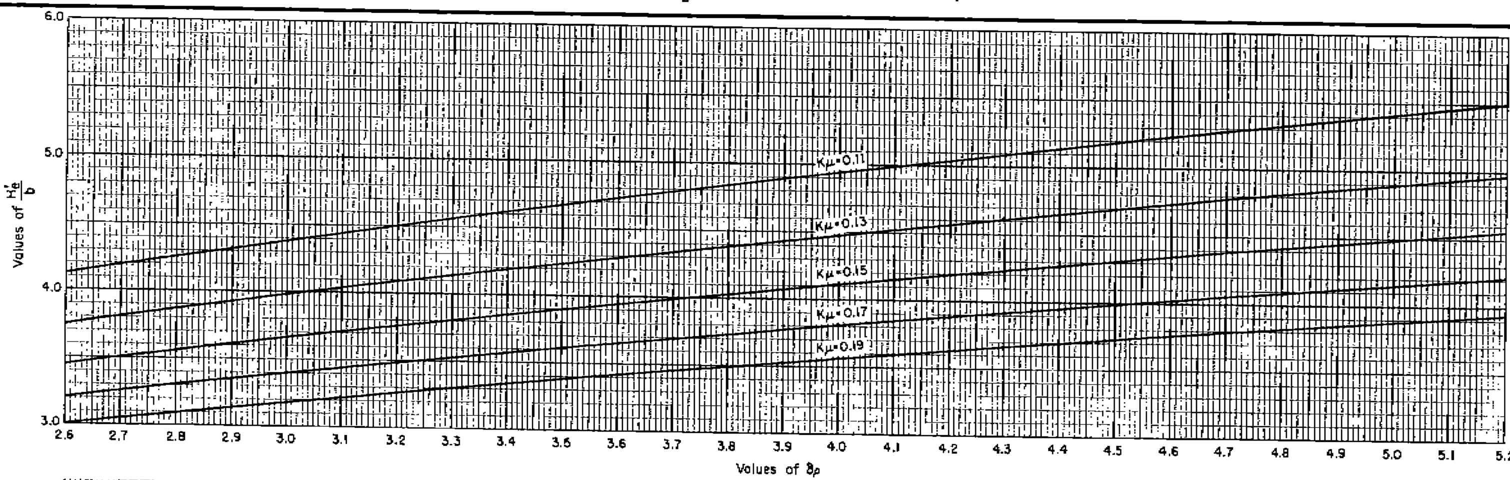
and

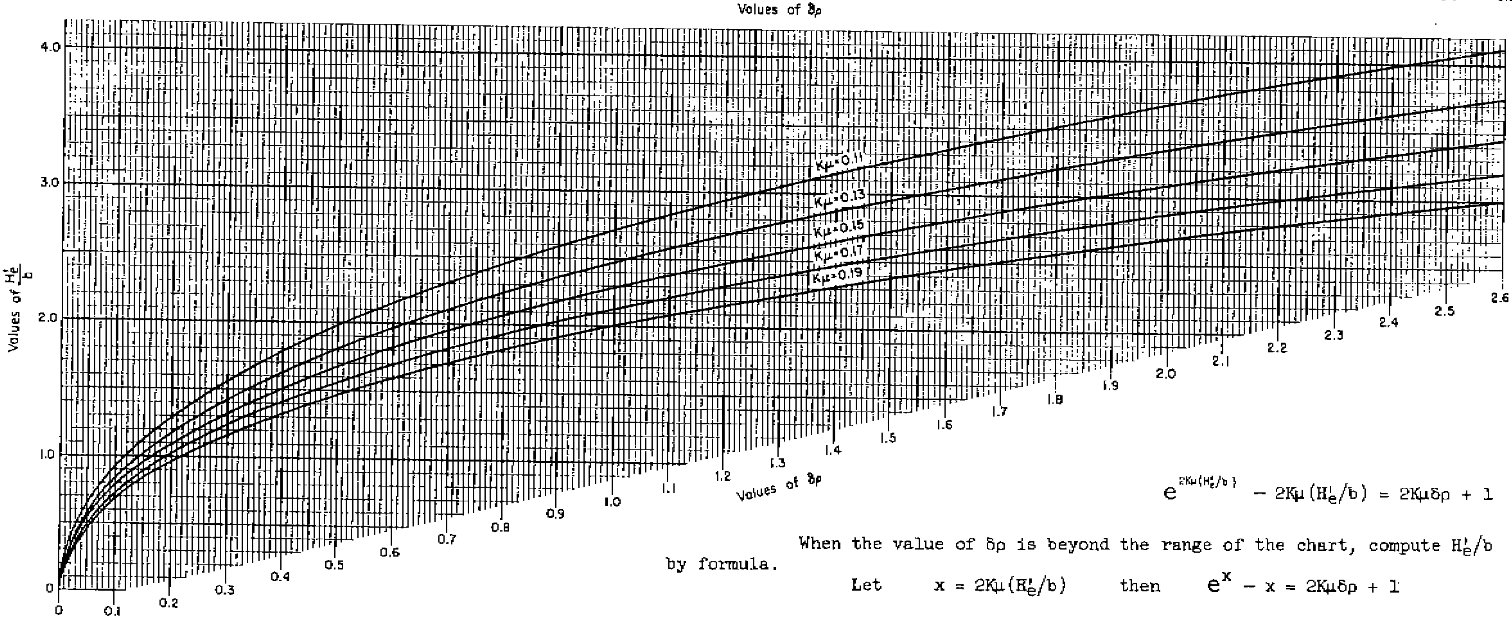
$$e^{2K\mu(H_e'/b)} - 2K\mu(H_e'/b) = 2K\mu\delta\rho + 1$$

This solution applies when

$$H_f < \frac{K\mu}{K_f\mu_f} H_e'$$

UNDERGROUND CONDUITS: Determination of settlement ratio δ, relation of $\frac{H'_e}{b}$ and $\delta\rho$ for various values of $K\mu$.



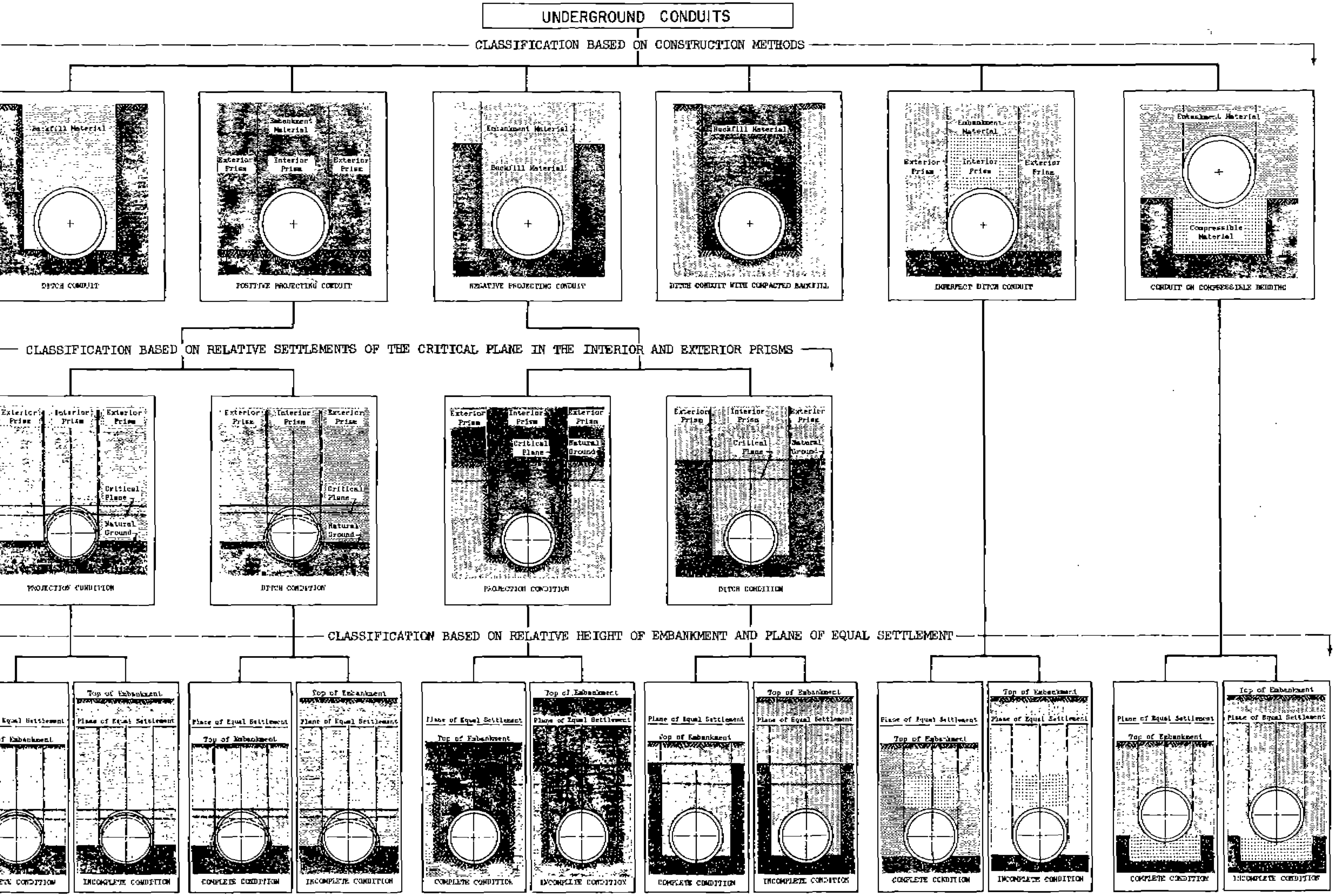


$$e^{2K\mu(H'_e/b)} - 2K\mu(H'_e/b) = 2K\mu\delta\rho + 1$$

When the value of $\delta\rho$ is beyond the range of the chart, compute H'_e/b by formula.

Let $x = 2K\mu(H'_e/b)$ then $e^x - x = 2K\mu\delta\rho + 1$

ERGROUND CONDUITS: Classification used for load determination



U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES- 116
SHEET 1 OF 2
DATE 5-1-58
REVISED

"Construction methods" will class all conduits into 6 primary divisions. It includes site conditions and design requirements as well as construction methods.

DITCH CONDUIT

An underground conduit is classed as a ditch conduit if all of the following conditions exist:

1. The conduit is installed in a sufficiently narrow ditch.

2. The ditch is backfilled to an elevation that is higher than the top of the conduit but not higher than the original ground surface.

3. The backfill is more compressible than the material in the ditch wall.

The determination of whether or not a ditch is sufficiently narrow is given by ES-117, sheet 2.

POSITIVE PROJECTING CONDUIT

An underground conduit is classed as a positive projecting conduit if all of the following conditions exist:

1. The conduit is installed in a sufficiently wide ditch or no ditch.

2. The foundation material under the conduit is approximately the same as the foundation material adjacent to the conduit.

3. The compressibility of the material in the interior prism is approximately the same as the material in the exterior prism.

The determination of whether or not a ditch is sufficiently wide is given by ES-117, sheet 2.

NEGATIVE PROJECTING CONDUIT

An underground conduit is classed as a negative projecting conduit if all of the following conditions exist:

1. The conduit is installed in a sufficiently narrow ditch.

2. The ditch is backfilled to an elevation that is higher than the natural ground.

A ditch is sufficiently narrow if the load on the conduit computed by the negative projecting conduit formula is less than the load on the conduit computed by the positive projecting conduit formula.

DITCH CONDUIT WITH COMPACTED BACKFILL

An underground conduit is classed as a ditch conduit with compacted backfill if all of the following conditions exist:

1. The conduit is installed in a sufficiently narrow ditch.

2. The ditch is backfilled to an elevation that is higher than the top of the conduit but not higher than the original ground surface.

3. The backfill is less compressible than the material in the ditch walls.

A ditch is sufficiently narrow if its width is less than the value of b_d' as computed by the formula

$$b_d' = \frac{C_p b_c^2}{H_c}$$

IMPERFECT DITCH CONDUIT

An underground conduit is classed as an imperfect ditch conduit if the following condition exists:

An unusual method of construction is used to insure that the material in the interior prism immediately above the conduit is sufficiently more compressible than the material in the exterior prisms. An embankment is constructed in the usual manner to a height 1 to 1 1/2 times the width of the conduit above its top. A trench having a width b_c and centered directly above the conduit is dug in this constructed embankment to the top of the conduit. The trench is loosely backfilled to the top of the trench and the embankment completed in the usual manner.

CONDUIT ON COMPRESSIBLE BEDDING

An underground conduit is classed as a conduit on compressible bedding if the following condition exists:

An unusual method of construction is used to insure that the foundation material under the conduit is sufficiently more compressible than the foundation material adjacent to the conduit. This is accomplished by excavating a trench in the foundation material slightly wider than the outside width of the conduit. The trench is backfilled with a compressible material. The conduit is installed on the compressible material.

"Relative settlements" subclassify positive an
negative projecting conduits into either pro-
jection condition or ditch condition.

1. The projection condition exists when the ex-
terior prism settles more than the interior prism
(δ and δ' are positive). For positive projecting con
duits this condition occurs for all rigid pipes. For
negative projecting conduits this condition exists wh
the backfill material around and above the pipe is l
compressible than the material in the ditch wall.

2. The ditch condition exists when the interior
prism settles more than the exterior prism (δ and δ'
are negative). The ditch condition does not occur fo
rigid pipes installed as positive projecting conduits
(It can exist for flexible pipes installed as positiv
projecting conduits.) For negative projecting condui
the ditch condition occurs when the backfill material
around and above the pipe is more compressible than
the material in the ditch walls.

"Relative height of embankment" subclassifies a
classifications except ditch conduits and ditch
conduits with compacted backfill into either th
complete condition or the incomplete condition.

1. The complete condition occurs when $\frac{H_c}{b_c} \leq \frac{H_e}{b_c}$.

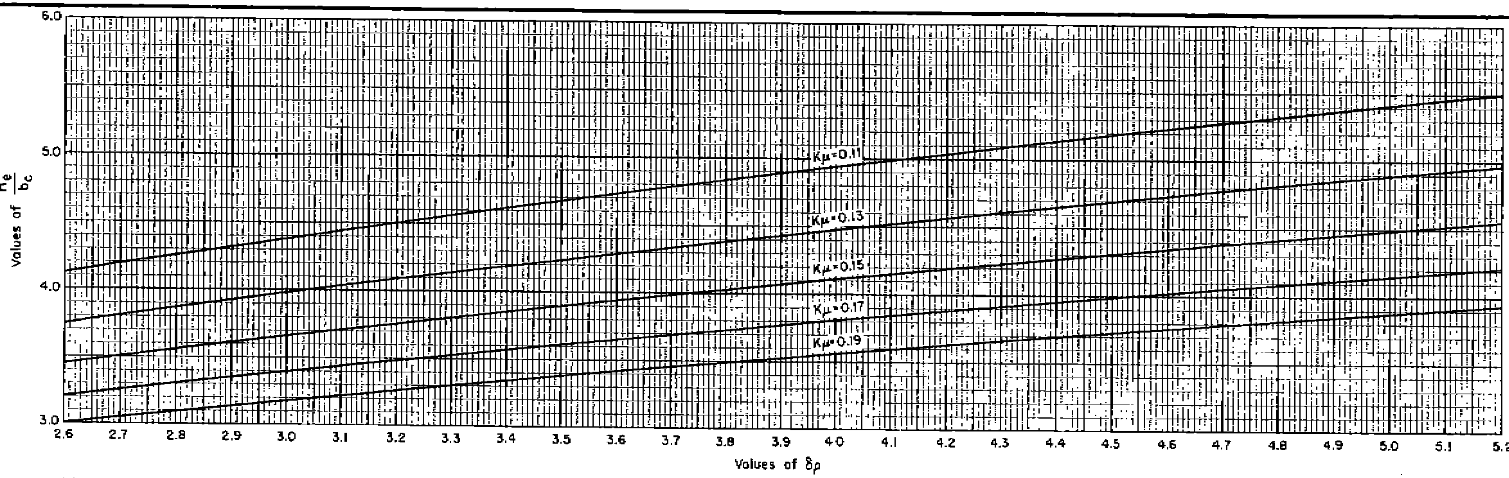
2. The incomplete condition occurs when $\frac{H_c}{b_c} > \frac{H_e}{b_c}$

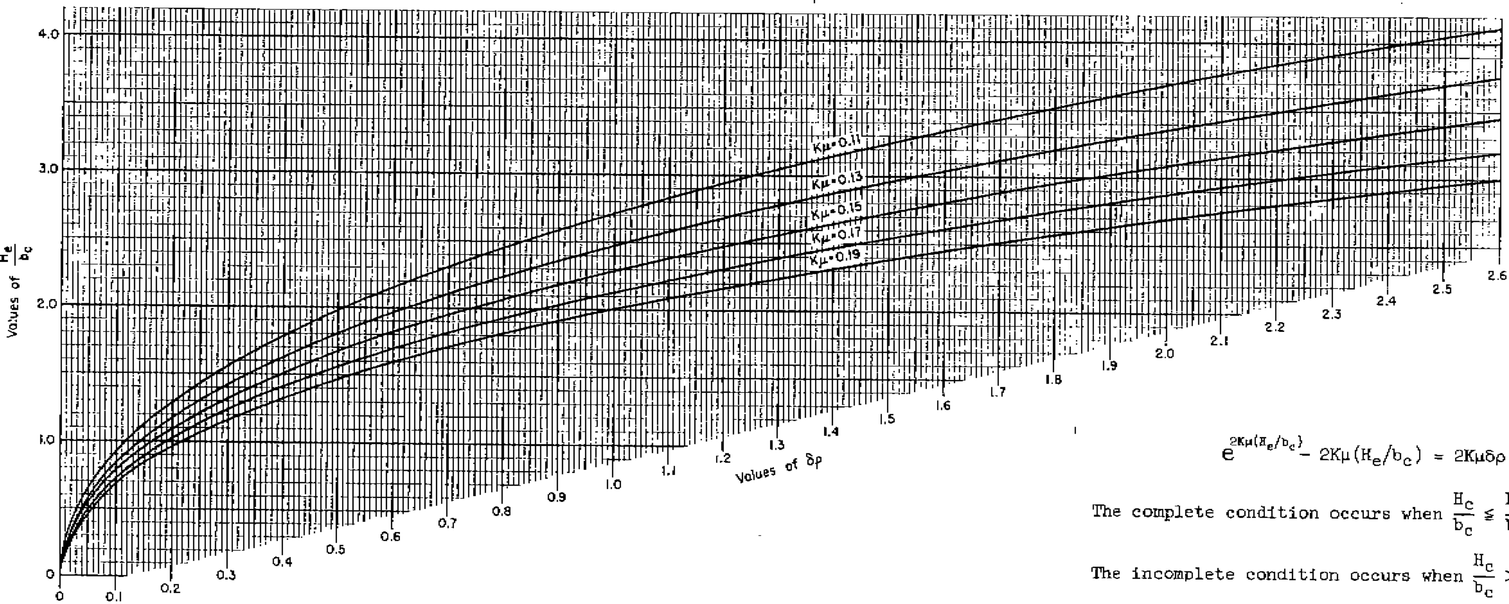
where H_c = height to top of embankment from the to of the conduit

H_e = height to plane of equal settlement fro the top of the pipe

Since no plane of equal settlement exists for ditch c
duits and ditch conduits with compacted backfill, the
are not subdivided.

UNDERGROUND CONDUITS: Categorizing positive projecting conduits; Complete or incomplete condition





$$e^{2K\mu(H_e/b_c)} - 2K\mu(H_e/b_c) = 2K\mu\delta\rho + 1$$

The complete condition occurs when $\frac{H_c}{b_c} \leqq \frac{H_e}{b_c}$

The incomplete condition occurs when $\frac{H_c}{b_c} > \frac{H_e}{b_c}$

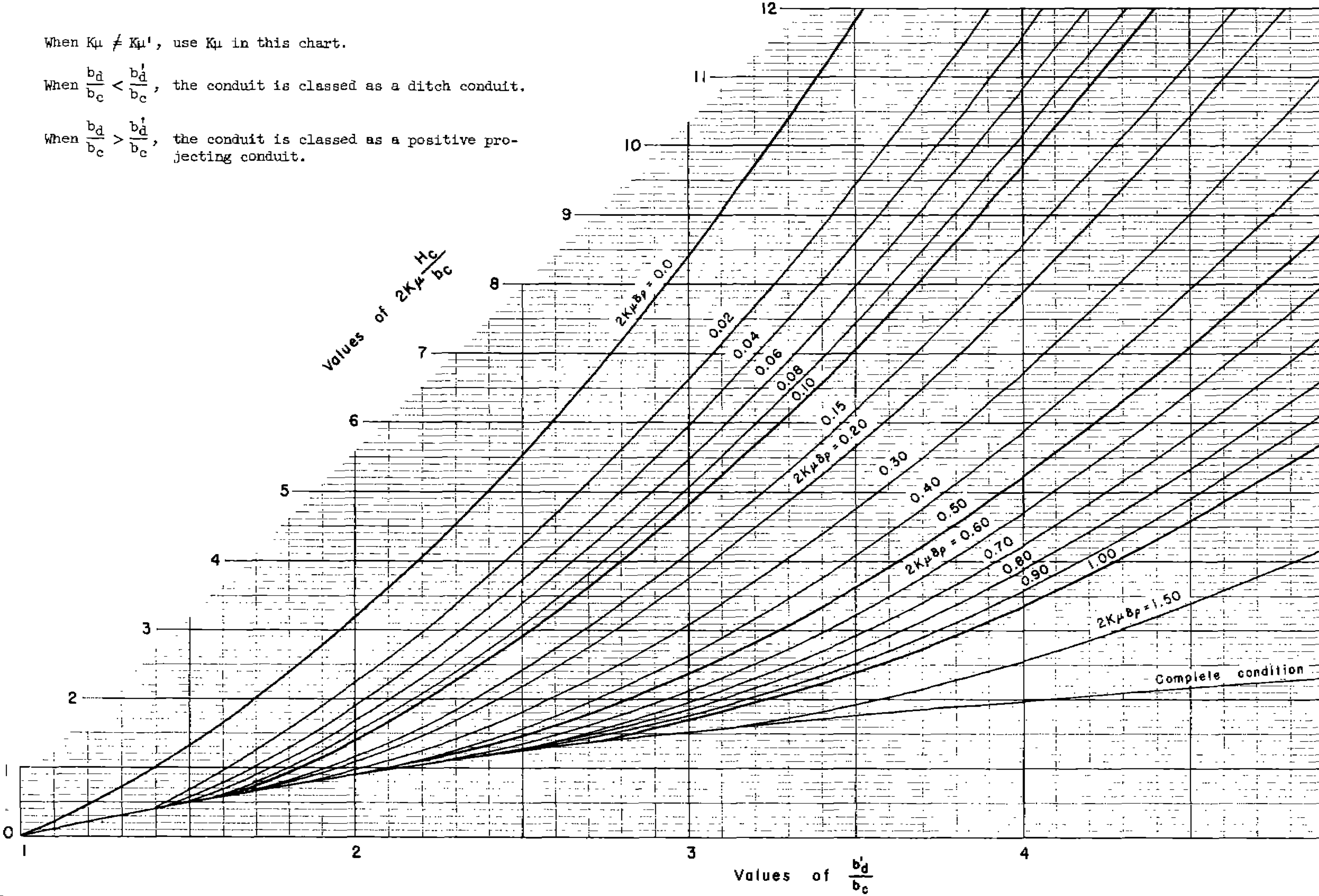
U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG NO. ES- 117 SHEET 1 OF 2 DATE 8-5-58 REVISED

UNDERGROUND CONDUITS: Categorizing conduits; ditch conduits, or positive projecting conduits.

When $K\mu \neq K\mu'$, use $K\mu$ in this chart.

When $\frac{b_d}{b_c} < \frac{b_d'}{b_c}$, the conduit is classed as a ditch conduit.

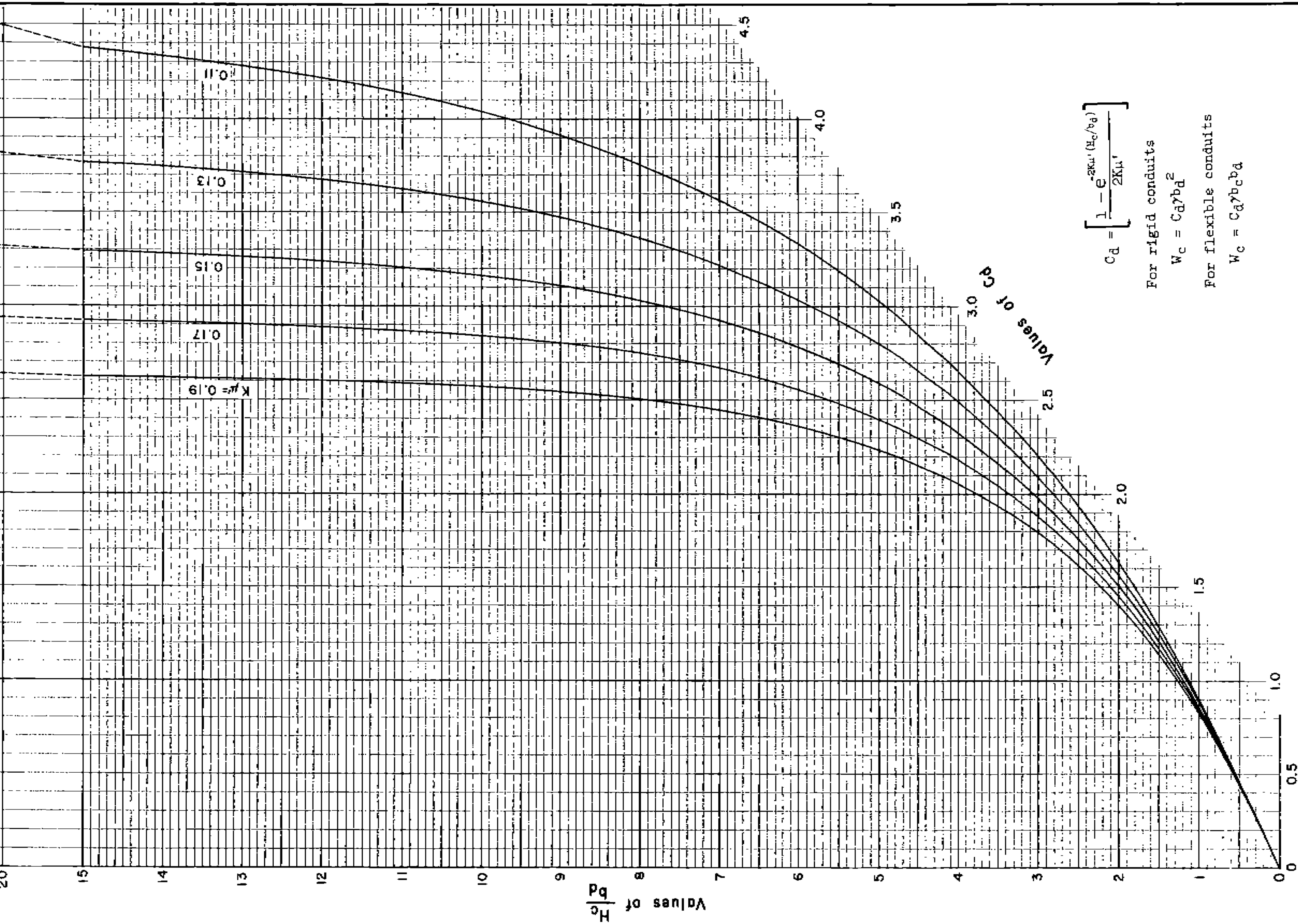
When $\frac{b_d}{b_c} > \frac{b_d'}{b_c}$, the conduit is classed as a positive projecting conduit.



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

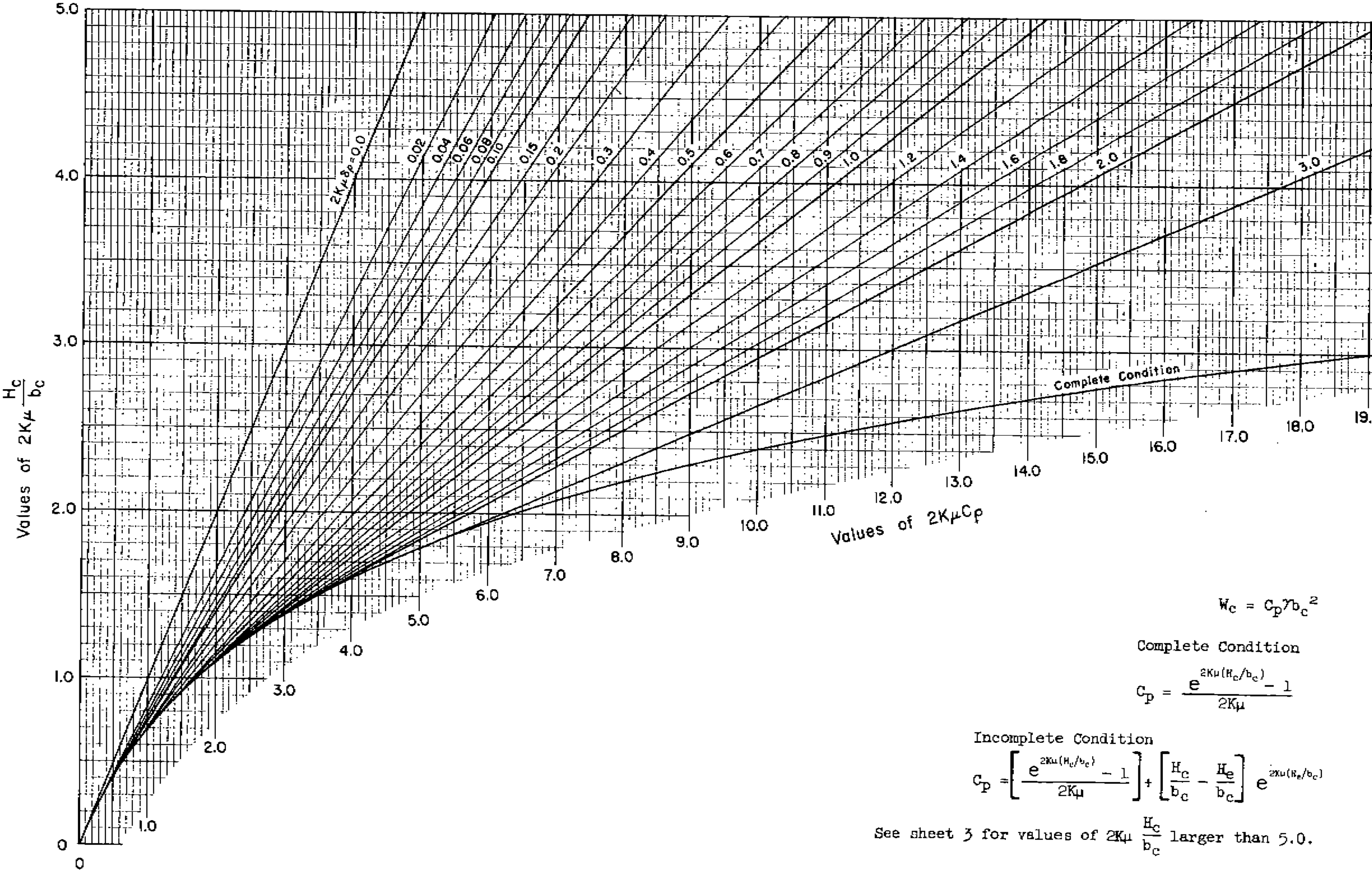
STANDARD DWG. NO.
ES- 117
SHEET 2 OF 2
DATE 4-1-57
REVISED 6-?-58



U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION · DESIGN SECTION

STANDARD DWG. NO.
ES- 118
SHEET 1 OF 3
DATE 10-18-56
REVISED 5-9-58

UNDERGROUND CONDUITS: Loads on positive projecting conduits, projection condition.



$$W_c = C_p \gamma b_c^{\,2}$$

Complete Condition

$$C_p = \frac{e^{2K\mu(H_c/b_c)} - 1}{2K\mu}$$

Incomplete Condition

$$C_p = \left[\frac{e^{2K\mu(H_c/b_c)} - 1}{2K\mu}\right] + \left[\frac{H_c}{b_c} - \frac{H_e}{b_c}\right] e^{2K\mu(H_e/b_c)}$$

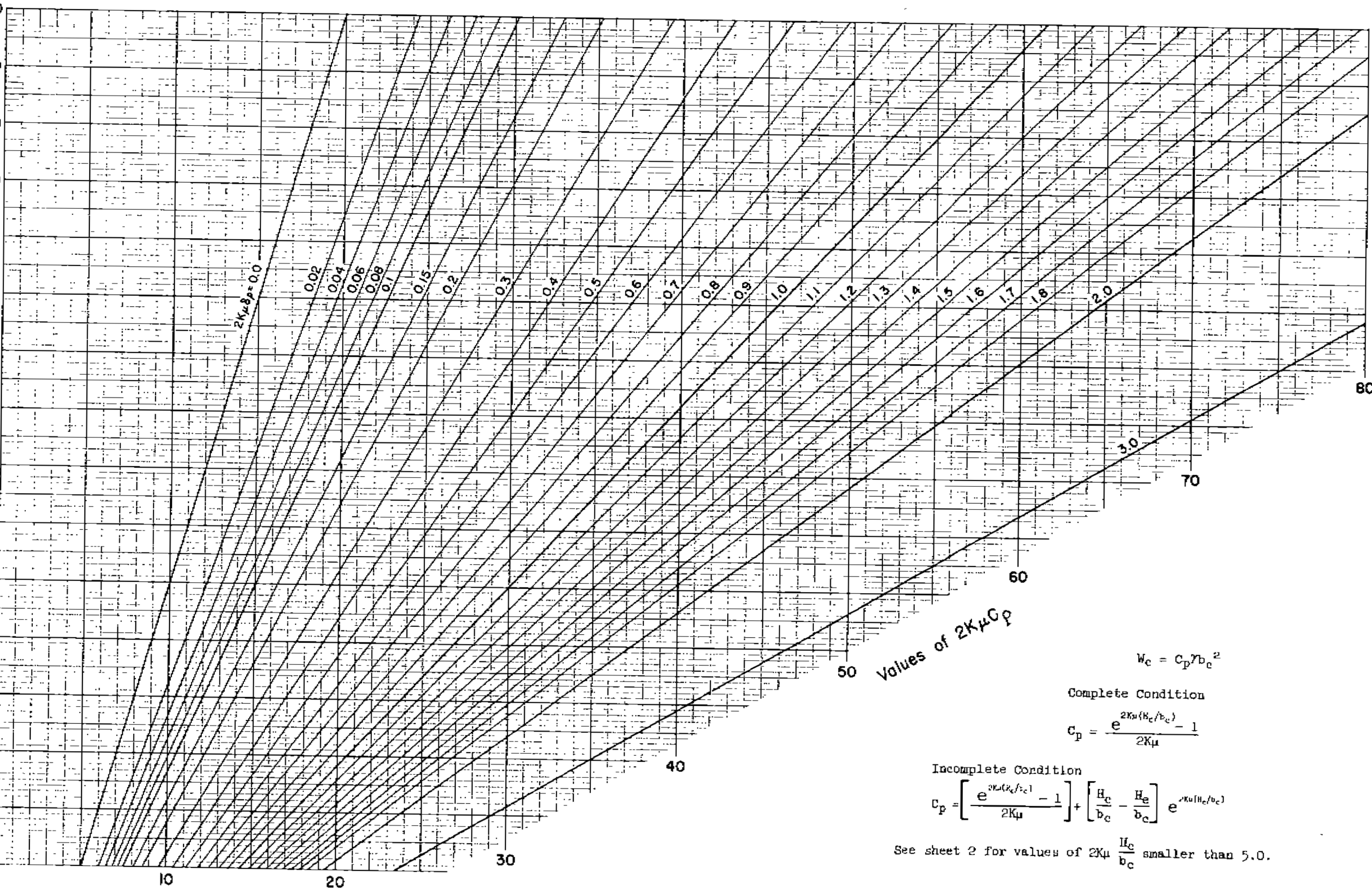
See sheet 3 for values of $2K\mu \frac{H_c}{b_c}$ larger than 5.0.

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. N
ES- 118
SHEET 2 OF
DATE 10-18-5

ERGROUND CONDUITS: Loads on positive projecting conduits, projection condition.



$$W_c = C_p \gamma b_c^2$$

Complete Condition

$$C_p = \frac{e^{2K\mu(H_c/b_c)} - 1}{2K\mu}$$

Incomplete Condition

$$C_p = \left[\frac{e^{2K\mu(H_e/b_c)} - 1}{2K\mu}\right] + \left[\frac{H_c}{b_c} - \frac{H_e}{b_c}\right] e^{2K\mu(H_e/b_c)}$$

See sheet 2 for values of $2K\mu \frac{H_c}{b_c}$ smaller than 5.0.

REINFORCED CONCRETE PRESSURE PIPE

(ASTM Spec. C361-55T)

Internal Diameter d--in.	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
12	1.333	2250	1811
15	1.583	2625	1499
18	1.875	3000	1221
21	2.146	3000	932.2
24	2.417	3000	734.9
30	2.958	3375	552.0
36	3.521	4050	467.5
42	4.125	4725	397.4
48	4.687	5400	351.8
54	5.250	5850	303.7
60	5.833	6000	252.4
66	6.417	6300	218.9
72	7.000	6600	192.7

CONCRETE PRESSURE SEWER PIPE

(ASTM Spec. C362-55T)

Internal Diameter d--in.	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
12	1.333	2250	1811
15	1.583	2625	1499
18	1.875	3000	1221
21	2.146	3000	932.2
24	2.417	3000	734.9
30	2.958	3375	552.0
36	3.521	4050	467.5
42	4.125	4725	397.4
48	4.687	5400	351.8
54	5.250	5850	303.7
60	5.833	6000	252.4

REINFORCED CONCRETE SEWER PIPE

(ASTM Spec. C75-55)

Internal Diameter d--in.	R_{eb}* lbs/ft	3000 psi concrete		3500 psi concrete		4000 psi concrete	
		Outside Diameter b_c--ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	$s\gamma F_{sp}$*
12	1800	1.333	1450	1.292	1543	.....	.....
15	2000	1.625	1084	1.583	1142	.....	.....
18	2200	1.917	856.7	.....	.....	1.833	937.0
21	2400	2.208	704.5	.....	.....	2.125	760.5
24	2400	2.500	549.5	2.437	578.3	2.417	587.9
27	2550	2.750	482.5	2.708	497.6	2.687	505.4
30	2700	3.083	406.5	3.000	429.3	2.958	441.6
33	2850	3.375	358.0	3.292	3.763	3.208	396.3
36	3000	3.667	319.3	3.563	338.2	3.500	350.4
42	3200	4.250	253.5	4.125	269.1	4.062	277.5
48	3400	4.833	208.3	4.708	219.5	4.625	227.5
54	3700	5.417	180.4	5.271	190.6	5.208	195.2
60	4000	6.000	159.0	5.833	168.2	5.750	173.1
66	4250	6.583	140.3	6.396	148.7	6.292	153.6
72	4500	7.167	125.4	6.958	133.0	6.833	137.9

REINFORCED CONCRETE CULVERT PIPE

(ASTM Spec. C76-55)

Internal Diameter d--in.	Standard Strength 3500 psi concrete			Standard Strength 4500 psi concrete			Extra Strength		
	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
12	1.333	2250	1812	1.292	2250	1929	.....	....	.....
15	1.625	2625	1436	1.583	2625	1499	.....	....	.....
18	1.917	3000	1168	1.833	3000	1278	.....	....	.....
24	2.500	3000	686.9	2.417	3000	734.9	2.500	4000	915.8
30	3.083	3375	508.1	3.000	3375	536.6	3.083	5000	752.8
36	3.667	4050	431.0	3.562	4050	456.8	3.667	6000	638.5
42	4.250	4725	374.3	4.125	4725	397.4	4.250	7000	554.6
48	4.834	5400	330.7	4.708	5400	348.6	4.833	8000	490.1
54	5.416	5850	285.3	5.271	5850	301.3	5.417	9000	438.8
60	6.000	6000	238.5	5.833	6000	252.4	6.000	9000	357.8
66	6.583	6300	208.1	6.417	6300	218.9	6.583	9500	313.7
72	7.167	6600	183.9	7.000	6600	192.7	7.167	9900	275.8

* R_{eb} and $s\gamma F_{sp}$ values on this sheet are based on the load to produce 0.01-inch crack.

$$s\gamma F_{sp} = \frac{1.431\ R_{eb}}{b_c^{2}}$$

F_{sp} = provided strength factor

R_{eb} = three-edge bearing strength (0.01-inch crack)

b_c = outside width of conduit

s = safety factor (see ES-114, sheet 2)

γ = unit weight of backfill or embankment material (see ES-114, sheet 2)

REFERENCE

ERGROUND CONDUITS: Clay and non-reinforced concrete pipe; values of b_c, R_{eb}, and $s\gamma F_{sp}$

NDARD STRENGTH CLAY SEWER PIPE
(ASTM Spec. C13-54)

ERAMIC GLAZED CLAY SEWER PIPE
(ASTM Spec. C261-54)

nal ter n.	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
	0.406	1000	8681
	0.589	1000	4125
	0.771	1000	2407
	0.958	1100	1715
	1.146	1200	1308
	1.432	1400	977.0
	1.719	1700	823.3
	2.010	2000	708.4
	2.292	2400	653.8
	2.583	2750	589.8
	2.865	3200	557.9
	3.135	3500	509.6
	3.396	3900	483.9

CONCRETE IRRIGATION PIPE
(ASTM Spec. C118-55)

Internal Diameter d--in.	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
4	0.458	1000	6822
5	0.542	1000	4871
6	0.625	1000	3663
8	0.813	1000	2165
10	1.000	1250	1789
12	1.188	1500	1521
14	1.375	1600	1211
15	1.479	1700	1112
16	1.563	1800	1054
18	1.750	1900	887.8
20	1.979	2000	730.8
21	2.083	2100	692.6
24	2.354	2200	568.1

EXTRA STRENGTH CLAY PIPE
(ASTM Spec. C200-55T)

Internal Diameter d--in.	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
6	0.589	2000	6250
8	0.771	2000	4815
10	0.958	2000	3118
12	1.146	2250	2452
15	1.432	2750	1919
18	1.719	3300	1598
21	2.010	3850	1364
24	2.292	4400	1199
27	2.583	4700	1008
30	2.865	5000	871.7
33	3.135	5500	800.8
36	3.396	6000	744.5

EXTRA STRENGTH CERAMIC GLAZED CLAY PIPE
(ASTM Spec. C278-55T)

Internal Diameter d--in.	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
6	0.589	2000	6250
8	0.771	2000	4815
10	0.958	2000	3118
12	1.146	2250	2452
15	1.432	2750	1919
18	1.719	3300	1598
21	2.010	3850	1364
24	2.292	4400	1199
30	2.865	5000	871.7
36	3.396	6000	744.5

CONCRETE SEWER PIPE
(ASTM Spec. C14-55)

Internal Diameter d--in.	Standard Strength			Extra Strength		
	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
4	0.427	1000	7848	0.458	2000	13644
6	0.604	1100	4315	0.625	2000	7327
8	0.792	1300	2966	0.812	2000	4341
10	0.979	1400	2090	1.000	2000	2862
12	1.166	1500	1579	1.229	2250	2132
15	1.458	1750	1178	1.521	2750	1701
18	1.750	2000	934.5	1.833	3300	1405
21	2.042	2200	755.0	2.125	3850	1220
24	2.354	2400	619.8	2.437	4400	1060

PERFORATED CLAY PIPE
(ASTM Spec. C211-50)

Internal Diameter d--in.	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
4	0.406	1000	8681
6	0.589	1000	4125
8	0.771	1000	2407
10	0.958	1100	1715
12	1.146	1200	1308
15	1.432	1400	977.0
18	1.719	1700	823.3
21	2.010	2000	708.4
24	2.292	2400	653.8

DRAIN TILE (CLAY OR CONCRETE)
(ASTM Spec. C4-55)

rnal eter in.	Standard			Extra Quality		
	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	R_{eb}* lbs/ft	$s\gamma F_{sp}$*
4	No Standards	800	No Standards	No Standards	1100	No Standards
5		800			1100	
6		800			1100	
8		800			1100	
0		800			1100	
2		800			1100	
5		870			1100	
8		930			1200	
1		1000			1400	
4		1130			1600	
7		1230			1800	
0		1330			2000	
3		1430			2200	
6		1530			2400	
2		1730			2800	

* R_{eb} and $s\gamma F_{sp}$ values on this sheet are based on the ultimate load.

$$s\gamma F_{sp} = \frac{1.431\ R_{eb}}{{b_c}^2}$$

F_{sp} = provided strength factor

R_{eb} = three-edge bearing strength (ultimate load)

b_c = outside width of conduit

s = safety factor (see ES-114, sheet 2)

γ = unit weight of backfill or embankment material (see ES-114, sheet 2)

U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 119 SHEET 3 OF 3 DATE 10-9-56

REVISED 5-1-58

UNDERGROUND CONDUITS: Reinforced concrete culvert, storm drain, and sewer pipe, (ASTM spec. C76-57T); values of b_c, R_{eb}, and $s\gamma F_{sp}$

CLASS I

D load to produce 0.01-inch crack = 800 (3-edge bearing test)

Internal Diameter d--in.	R_{eb}* lbs/ft	Wall A		Wall B	
		Outside Diameter b_c--ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	$s\gamma F_{sp}$*
60	4000	5.833	168.2	6.000	159.0
66	4400	6.417	152.9	6.583	145.3
72	4800	7.000	140.2	7.167	133.7
78	5200	7.583	129.4	7.750	123.9
84	5600	8.167	120.1	8.333	115.4
90	6000	8.750	112.1	8.917	108.0
96	6400	9.333	105.1	9.500	101.5
102	6800	9.917	98.9	10.083	95.7
108	7200	10.500	93.4	10.667	90.6

CLASS II

D load to produce 0.01-inch crack = 1000 (3-edge bearing test)

Internal Diameter d--in.	R_{eb}* lbs/ft	Wall A		Wall B	
		Outside Diameter b_c--ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	$s\gamma F_{sp}$*
12	1000	1.292	857.4	1.333	805.3
15	1250	1.563	732.2	1.625	677.3
18	1500	1.833	638.8	1.917	584.1
21	1750	2.125	554.5	2.208	513.7
24	2000	2.417	489.9	2.500	457.9
27	2250	2.688	445.6	2.792	413.1
30	2500	2.958	408.9	3.083	376.4
33	2750	3.229	377.4	3.375	345.5
36	3000	3.500	350.4	3.667	319.3
42	3500	4.083	300.4	4.250	277.3
48	4000	4.667	262.8	4.833	245.1
54	4500	5.250	233.6	5.417	219.4
60	5000	5.833	210.3	6.000	198.8
66	5500	6.417	191.1	6.583	181.6
72	6000	7.000	175.2	7.167	167.2
78	6500	7.583	161.8	7.750	154.9
84	7000	8.167	150.2	8.333	144.3
90	7500	8.750	140.2	8.917	135.0
96	8000	9.333	131.4	9.500	126.8
102	8500	9.917	123.7	10.083	119.6
108	9000	10.500	116.8	10.667	113.2

CLASS III

D load to produce 0.01-inch crack = 1350 (3-edge bearing test)

Internal Diameter d--in.	R_{eb}* lbs/ft	Wall A		Wall B		Wall C	
		Outside Diameter b_c--ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	$s\gamma F_{sp}$*
12	1350	1.292	1158	1.333	1087	.....	.....
15	1688	1.563	988.5	1.625	914.4	.....	.....
18	2025	1.833	862.4	1.917	788.5	.....	.....
21	2363	2.125	748.6	2.208	693.5	.....	.....
24	2700	2.417	661.4	2.500	618.2	.....	.....
27	3038	2.688	601.6	2.792	557.6	.....	.....
30	3375	2.958	552.0	3.083	508.1	.....	.....
33	3713	3.229	509.6	3.375	466.4	.....	.....
36	4050	3.500	473.1	3.667	431.0	.....	.....
42	4725	4.083	405.6	4.250	374.3	.....	.....
48	5400	4.667	354.8	4.833	330.8	.....	.....
54	6075	5.250	315.4	5.417	296.3	.....	.....
60	6750	5.833	283.9	6.000	268.3	.....	.....
66	7425	6.417	258.0	6.583	245.2	.....	.....
72	8100	7.000	236.6	7.167	225.7	7.292	218.0
78	8775	7.583	218.4	7.750	209.1	7.875	202.5
84	9450	8.167	202.7	8.333	194.7	8.458	189.0
90	10125	8.750	189.2	8.917	182.2	9.042	177.2
96	10800	9.333	177.4	9.500	171.2	.....	.....
102	11475	9.917	167.0	10.083	161.5	.....	.....
108	12150	10.500	157.7	10.667	152.8	.....	.....

CLASS IV

D load to produce 0.01-inch crack = 2000 (3-edge bearing test)

Internal Diameter d--in.	R_{eb}* lbs/ft	Wall A		Wall B		Wall C	
		Outside Diameter b_c--ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	$s\gamma F_{sp}$*
12	2000	1.292	1715	1.333	1611	.....	.....
15	2500	1.563	1464	1.625	1355	.....	.....
18	3000	1.833	1278	1.917	1168	.....	.....
21	3500	2.125	1109	2.208	1027	.....	.....
24	4000	2.417	979.8	2.500	915.8	2.667	804.7
27	4500	2.688	891.3	2.792	826.1	2.938	746.0
30	5000	2.958	817.7	3.083	752.8	3.208	695.3
33	5500	.....	.....	3.375	690.9	3.500	642.5
36	6000	.....	.....	3.667	638.5	3.813	590.6
42	7000	.....	.....	4.250	554.6	4.375	523.3
48	8000	.....	.....	4.833	490.1	4.958	465.7
54	9000	.....	.....	5.417	438.9	5.521	422.5
60	10000	.....	.....	6.000	397.5	6.125	381.4
66	11000	.....	.....	6.583	363.2	6.708	349.8
72	12000	.....	.....	7.167	334.3	7.292	322.9
78	13000	.....	.....	.....	.....	7.875	300.0
84	14000	.....	.....	.....	.....	8.458	280.0

CLASS V

D load to produce 0.01-inch crack = 3000 (3-edge bearing test)

Internal Diameter d--in.	R_{eb}* lbs/ft	Wall B		Wall C	
		Outside Diameter b_c--ft	$s\gamma F_{sp}$*	Outside Diameter b_c--ft	$s\gamma F_{sp}$
12	3000	1.333	2416	.....	.....
15	3750	1.625	2032	.....	.....
18	4500	1.917	1752	.....	.....
21	5250	2.208	1541	.....	.....
24	6000	2.500	1374	2.667	1207
27	6750	2.792	1239	2.938	1119
30	7500	3.083	1129	3.208	1043
33	8250	3.375	1036	3.500	963.[illegible]
36	9000	3.667	957.8	3.813	885.[illegible]
42	10500	4.250	831.8	4.375	785.[illegible]
48	12000	4.833	735.2	4.958	698.[illegible]
54	13500	.....	.....	5.521	633.[illegible]
60	15000	.....	.....	6.125	572.[illegible]
66	16500	.....	.....	6.708	524.[illegible]
72	18000	.....	.....	7.292	484.[illegible]

* R_{eb} and $s\gamma F_{sp}$ values on this sheet ar[e] based on the load to produce 0.01-inc[h] crack.

$$s\gamma F_{sp} = \frac{1.431\ R_{eb}}{b_c^{\,2}}$$

F_{sp} = provided strength factor

R_{eb} = three-edge bearing strength (0.01-inch crack)

b_c = outside width of conduit

s = safety factor (see ES-114, shee[t]

γ = unit weight of backfill or emba[nk]ment material (see ES-114, sheet 2)

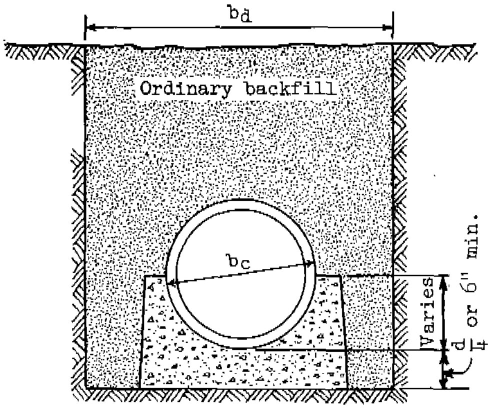
REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG.
ES-119
SHEET 1 OF [illegible]
DATE 10-9-5[illegible]
REVISED 5-1-58

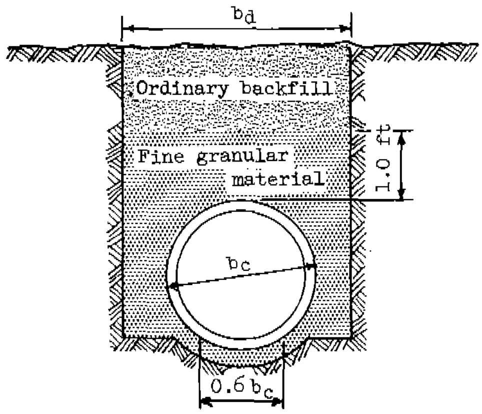
UNDERGROUND CONDUITS: Rigid pipes; ditch cradles and beddings and their load factor values

Type A–concrete cradle
$L_f = 2.2$ to 3.4



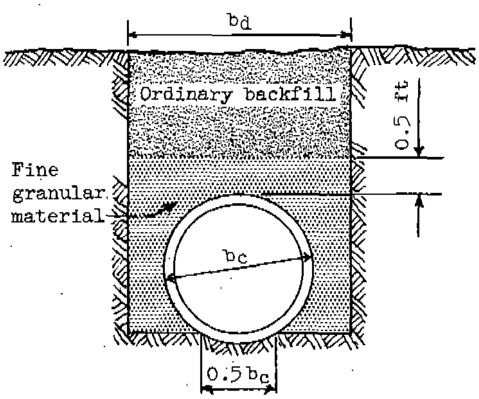
A concrete cradle is obtained by placing the pipe in plain or reinforced concrete having a minimum thickness under the lowest part of the conduit exterior of d/4 or of 6 inches whichever is greater (where d is the nominal inside diameter of the pipe in inches).

Type B–first class bedding
$L_f = 1.9$



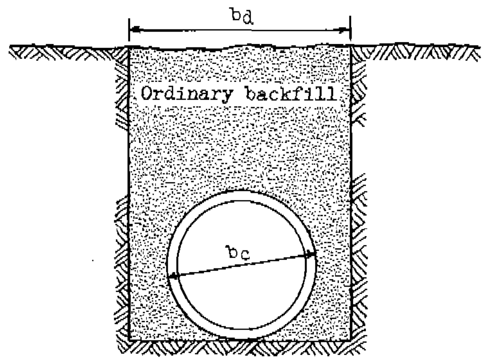
First-class bedding is that type of bedding obtained by placing the pipe on fine granular materials in an earth foundation carefully shaped to fit the lower part of the pipe exterior for a width of at least 60 percent of the pipe breadth. The remainder of the pipe is entirely surrounded to a height of at least 1.0 ft above its top by granular materials carefully placed by hand, to fill completely, all spaces under and adjacent to the pipe. The fill is tamped thoroughly on each side and under the pipe as far as practicable in layers not exceeding 0.5 ft thick.

Type C–ordinary bedding
$L_f = 1.5$



Ordinary bedding is that type of bedding obtained by placing the pipe with "ordinary" care in an earth foundation shaped to fit the lower part of the pipe exterior with reasonable closeness for a width of at least 50 percent of the pipe breadth. The remainder of the pipe is surrounded to a height of at least 0.5 ft above its top by granular materials, shovel placed and shovel tamped to fill completely all spaces under and adjacent to the pipe.

Type D–impermissible bedding
$L_f = 1.1$



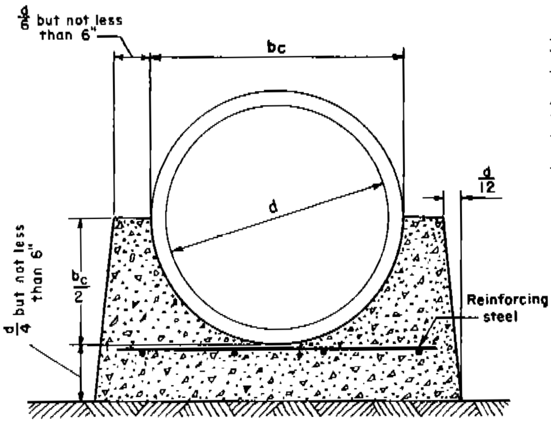
Impermissible bedding is that type of bedding in which little or no care is exercised to shape the foundation to fit the lower part of the pipe exterior and to refill all spaces under and around the pipe.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-120 SHEET 1 OF 5 DATE 11-6-56

REVISED 5-1-58

UNDERGROUND CONDUITS: Rigid pipes; Projecting cradles and their bedding factor values.

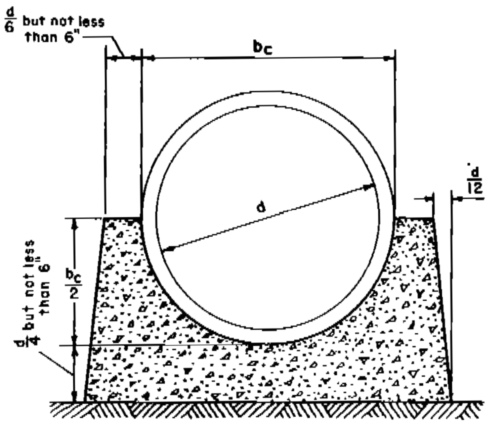
Type A1 Concrete Cradle $X_p = 0.400$



Type A1 is that type of cradle having a minimum thickness of reinforced concrete under the pipe of one-fourth of the nominal interior diameter and extending up the sides of the pipe for a height equal to one-half of the outside diameter. The transverse steel requirement for a type A1 cradle is:

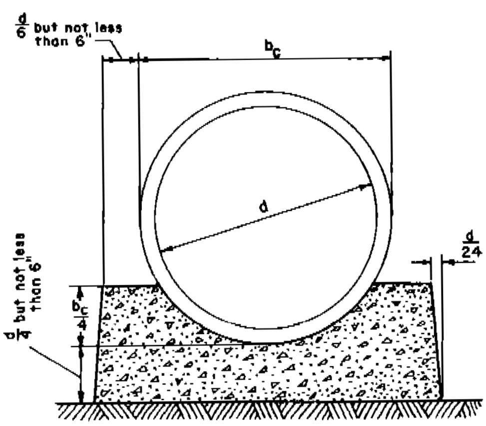
(1) Bar sizes shall not be smaller than number 3.
(2) Bar spacing shall be a maximum of 12 inches.

Type A2 Concrete Cradle $X_p = 0.450$



Type A2 is that type of cradle having a minimum thickness of concrete under the pipe of one-fourth of the nominal interior diameter and extending up the sides of the pipe for a height equal to one-half of the outside diameter.

Type A3 Concrete Cradle $X_p = 0.500$



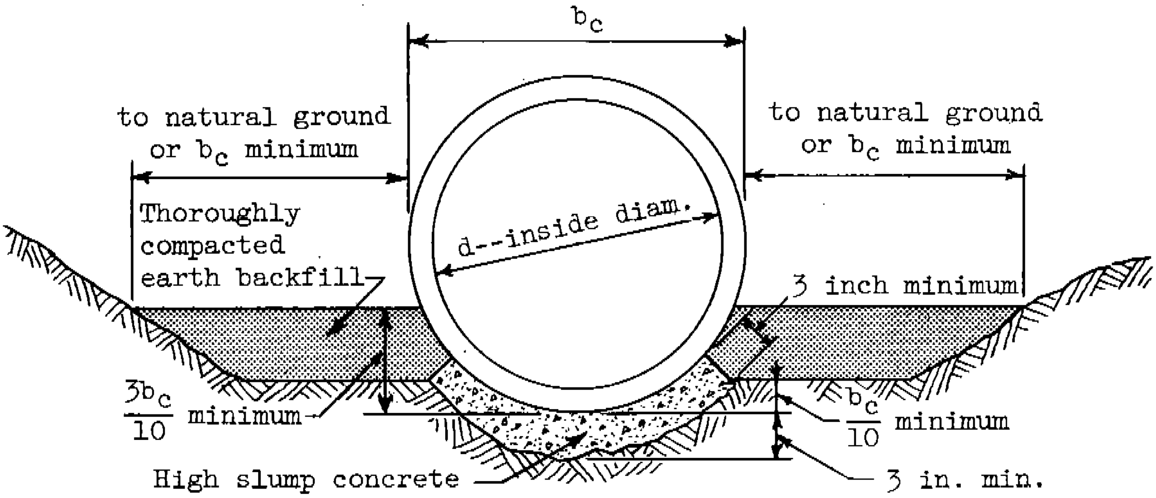
Type A3 is that type of cradle having a minimum thickness of concrete under the pipe of one-fourth of the nominal interior diameter of the pipe and extending up the sides of the pipe for a height equal to one-fourth of the outside diameter.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 120 SHEET 2 OF 5 DATE 11-8-56

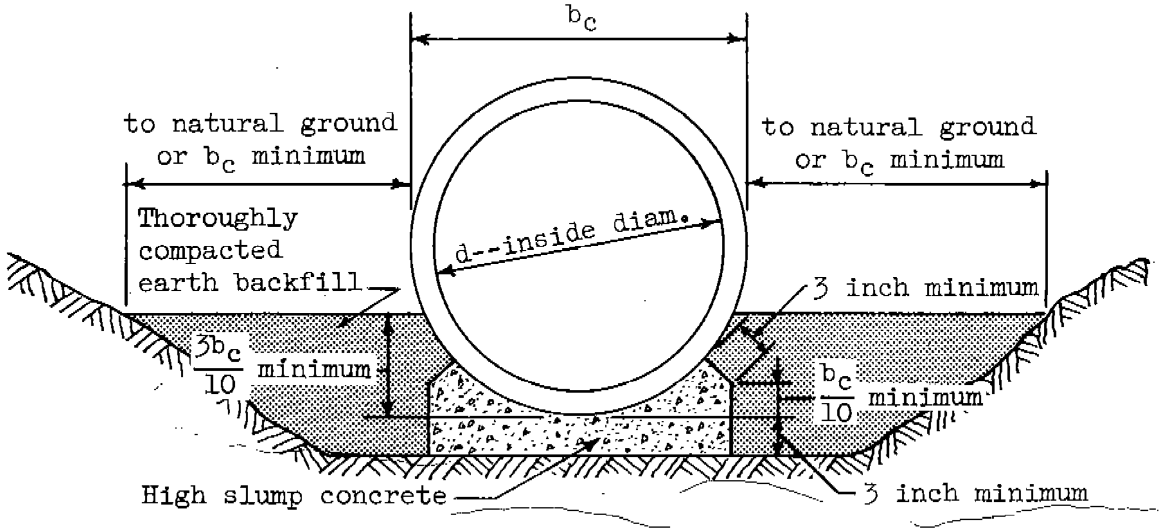
REVISED 5-1-58

UNDERGROUND CONDUITS: Rigid pipes; Projecting beddings and their bedding factor values.

Type B1 Concrete bedding $X_p = 0.650$



CONSTRUCTION METHOD 1



CONSTRUCTION METHOD 2

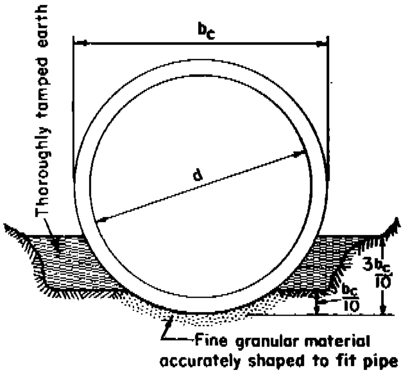
Type B1, concrete bedding, is that type of bedding having the lower part of the pipe bedded in a thin layer of concrete for at least 10 percent of its overall height. The earth filling material is thoroughly rammed and tamped, in layers not more than 6 inches deep, around the pipe for the remainder of the lower 30 percent of its height.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 120 SHEET 3 OF 5 DATE 11-8-56

REVISED 5-1-58

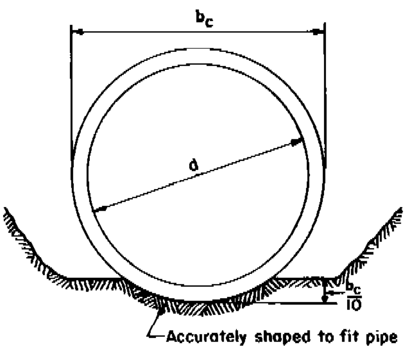
UNDERGROUND CONDUITS: Rigid pipes; Projecting beddings and their bedding factor values.

Type B2 First class bedding $X_p = 0.707$



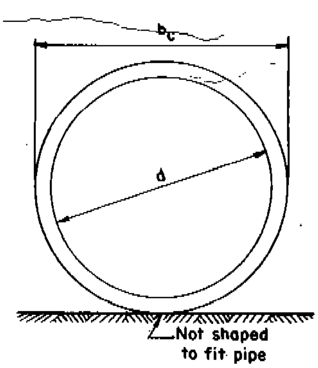
Type B2, first-class bedding, is that type of bedding having a projection ratio not greater than 0.70 in which the pipe is carefully bedded on fine granular materials in an earth foundation carefully shaped to fit the lower part of the pipe exterior for at least 10 percent of its overall height. The earth filling material is thoroughly rammed and tamped, in layers not more than 6 inches deep, around the pipe for the remainder of the lower 30 percent of its height.

Type C Ordinary bedding $X_p = 0.840$



Type C, ordinary bedding, is that type of bedding in which the pipe is bedded with "ordinary" care in an earth foundation shaped to fit the lower part of the pipe exterior with reasonable closeness for at least 10 percent of its overall height. The remainder of the pipe is surrounded by granular materials, placed by shovel to fill all spaces completely under and adjacent to the pipe. In the case of rock foundations, the pipes are bedded on an earth cushion, having a thickness under the pipe of not less than 0.5 inches per foot of height of fill, with a minimum allowable thickness of 8 inches.

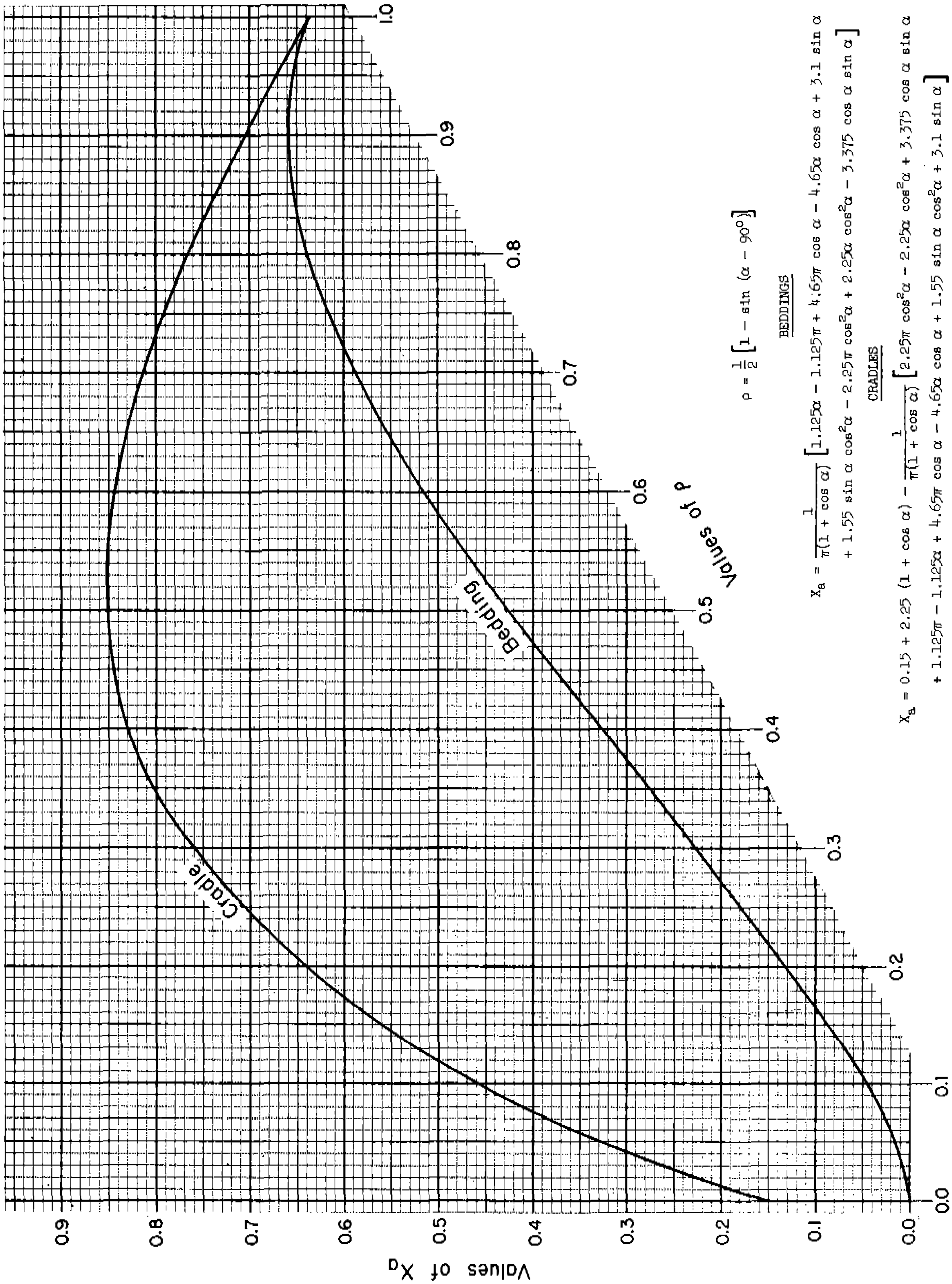
Type D Impermissable bedding $X_p = 1.310$



Type D, impermissible bedding, is that type of bedding in which little or no care is exercised either to shape the foundation surface to fit the lower part of the pipe exterior or to fill all spaces under and around the pipe with granular materials. This type of bedding also includes pipes on rock foundations in which an earth cushion is provided under the pipe, but is so shallow that the pipe, as it settles under the influence of vertical load, approaches contact with the rock.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 120 SHEET 4 OF 5 DATE 5-1-58 REVISED

UNDERGROUND CONDUITS: Rigid pipes; projecting cradles and beddings; relation of X_a and ρ



$$\rho = \frac{1}{2}\left[1 - \sin(\alpha - 90^0)\right]$$

BEDDINGS

$$X_a = \frac{1}{\pi(1 + \cos\alpha)}\Big[1.125\alpha - 1.125\pi + 4.65\pi\cos\alpha - 4.65\alpha\cos\alpha + 3.1\sin\alpha + 1.55\sin\alpha\cos^2\alpha - 2.25\pi\cos^2\alpha + 2.25\alpha\cos^2\alpha - 3.375\cos\alpha\sin\alpha\Big]$$

CRADLES

$$X_a = 0.15 + 2.25(1 + \cos\alpha) - \frac{1}{\pi(1 + \cos\alpha)}\Big[2.25\pi\cos^2\alpha - 2.25\alpha\cos^2\alpha + 3.375\cos\alpha\sin\alpha + 1.125\pi - 1.125\alpha + 4.65\pi\cos\alpha - 4.65\alpha\cos\alpha + 1.55\sin\alpha\cos^2\alpha + 3.1\sin\alpha\Big]$$

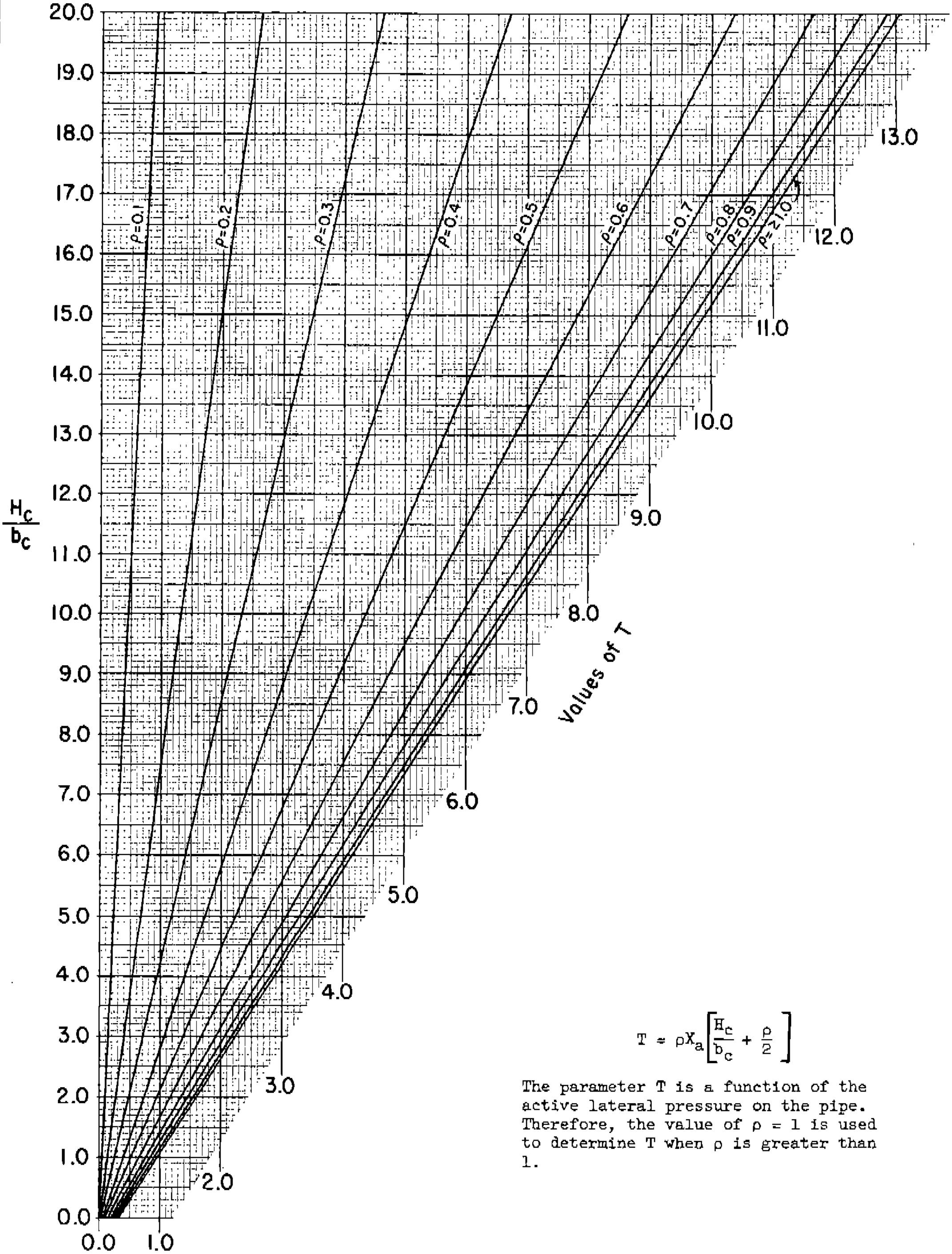
REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 120 SHEET 5 OF 5 DATE 11-26-56

REVISED 5-1-58

REFERENCE

UNDERGROUND CONDUITS; Rigid pipes; relation of T and H_c/b_c for various values of ρ

FOR CRADLES



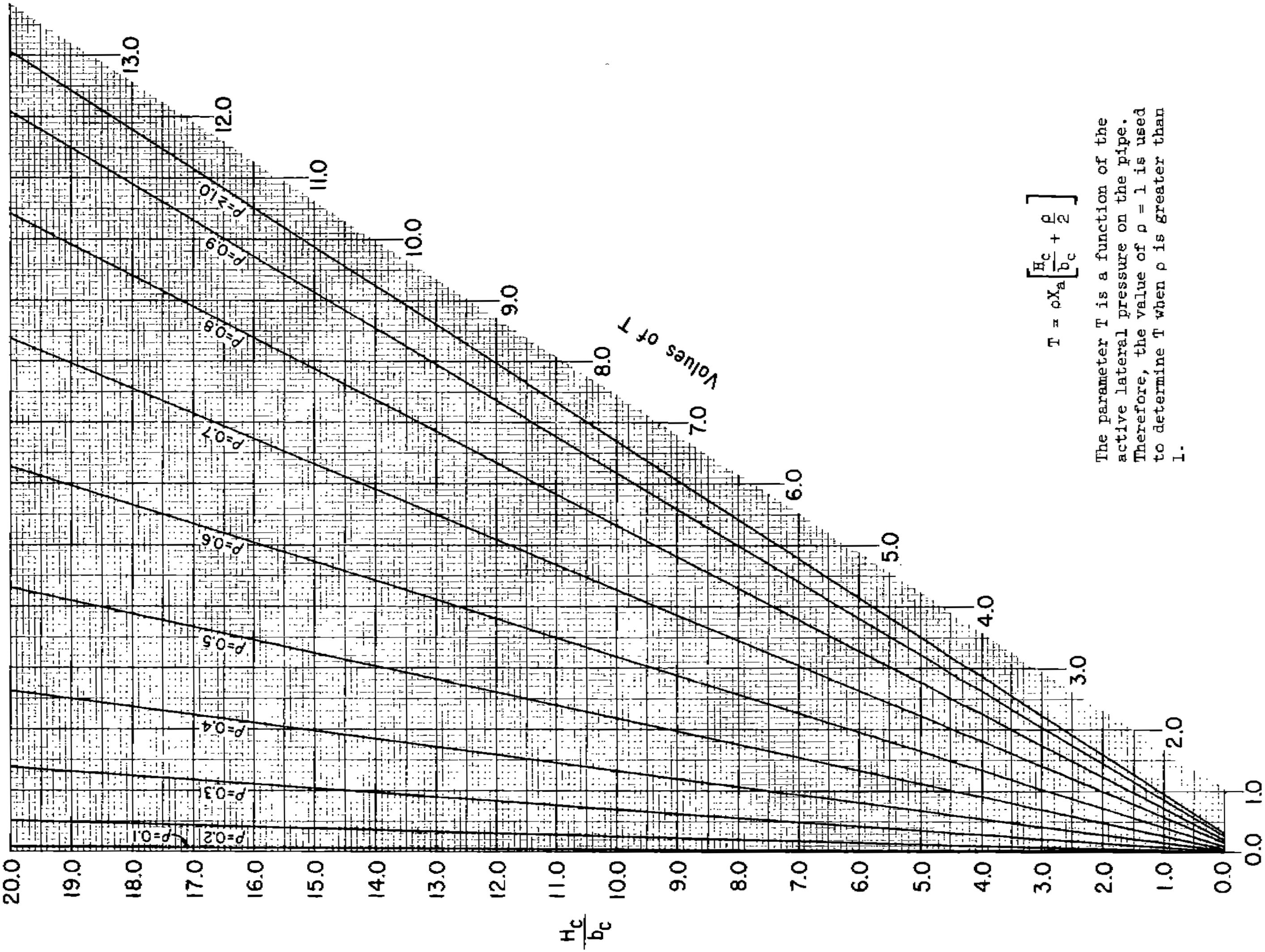
$$T = \rho X_a \left[\frac{H_c}{b_c} + \frac{\rho}{2} \right]$$

The parameter T is a function of the active lateral pressure on the pipe. Therefore, the value of $\rho = 1$ is used to determine T when ρ is greater than 1.

See sheet 2 for values of T for beddings

U.S. DEPARTMENT OF AGRICULTURE
STANDARD DWG. NO.

RGROUND CONDUITS: Rigid pipes; relation of T and H_c/b_c for various values of ρ



$$T = \rho X_a \left[\frac{H_c}{b_c} + \frac{\rho}{2}\right]$$

The parameter T is a function of the active lateral pressure on the pipe. Therefore, the value of $\rho = 1$ is used to determine T when ρ is greater than 1.

See sheet 1 for values of T for cradles

U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-121 SHEET 2 OF 2 DATE 8-5-58

UNDERGROUND CONDUITS: Positive projecting conduits, complete projection condition; Relation of $2K\mu \frac{H_c}{b_c}$ vs U for various values of $\frac{K\rho X_a}{X_p}$

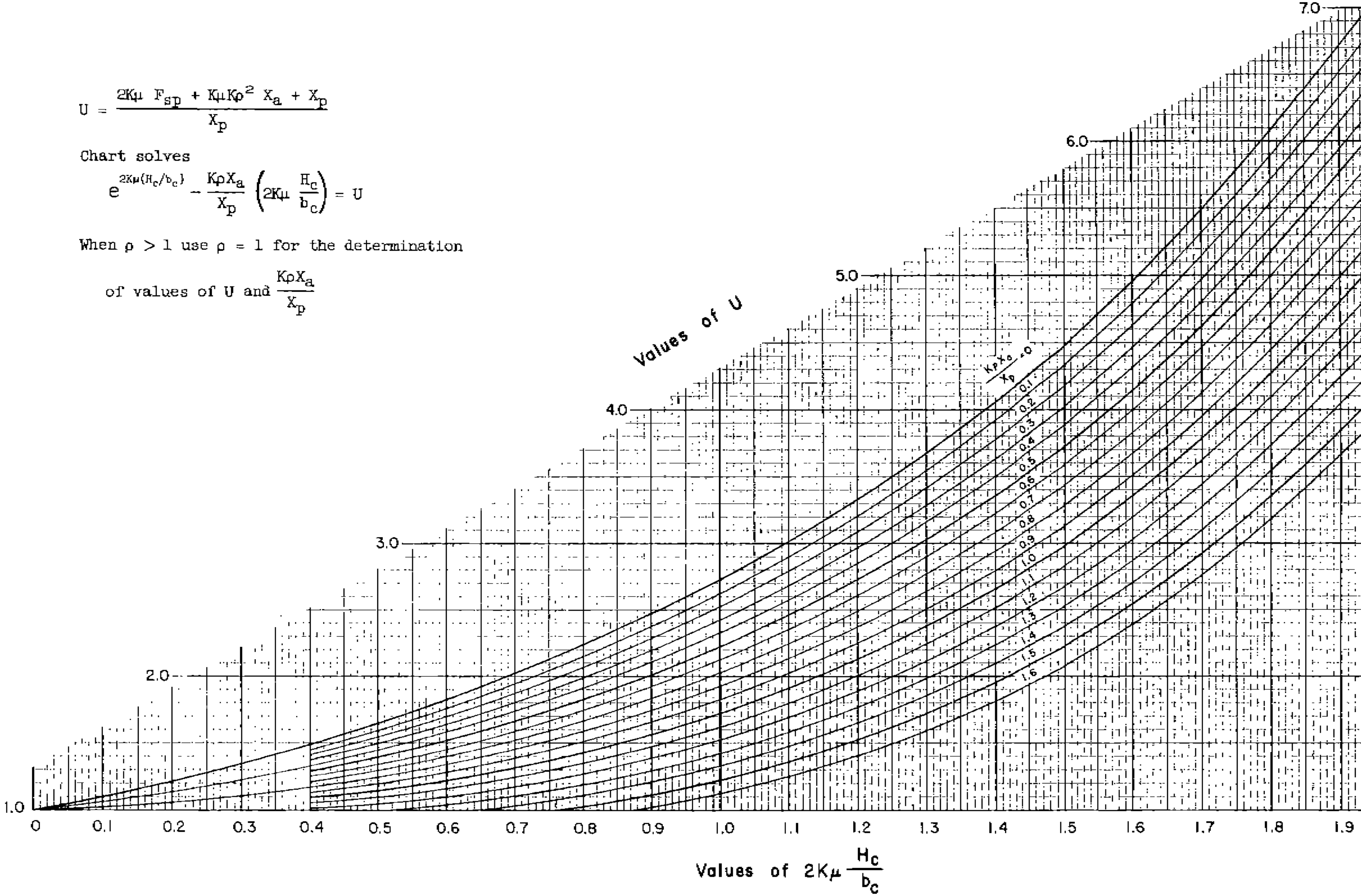
$$U = \frac{2K\mu \; F_{sp} + K\mu K\rho^2 \; X_a + X_p}{X_p}$$

Chart solves

$$e^{2K\mu(H_c/b_c)} - \frac{K\rho X_a}{X_p}\left(2K\mu \; \frac{H_c}{b_c}\right) = U$$

When $\rho > 1$ use $\rho = 1$ for the determination

of values of U and $\frac{K\rho X_a}{X_p}$

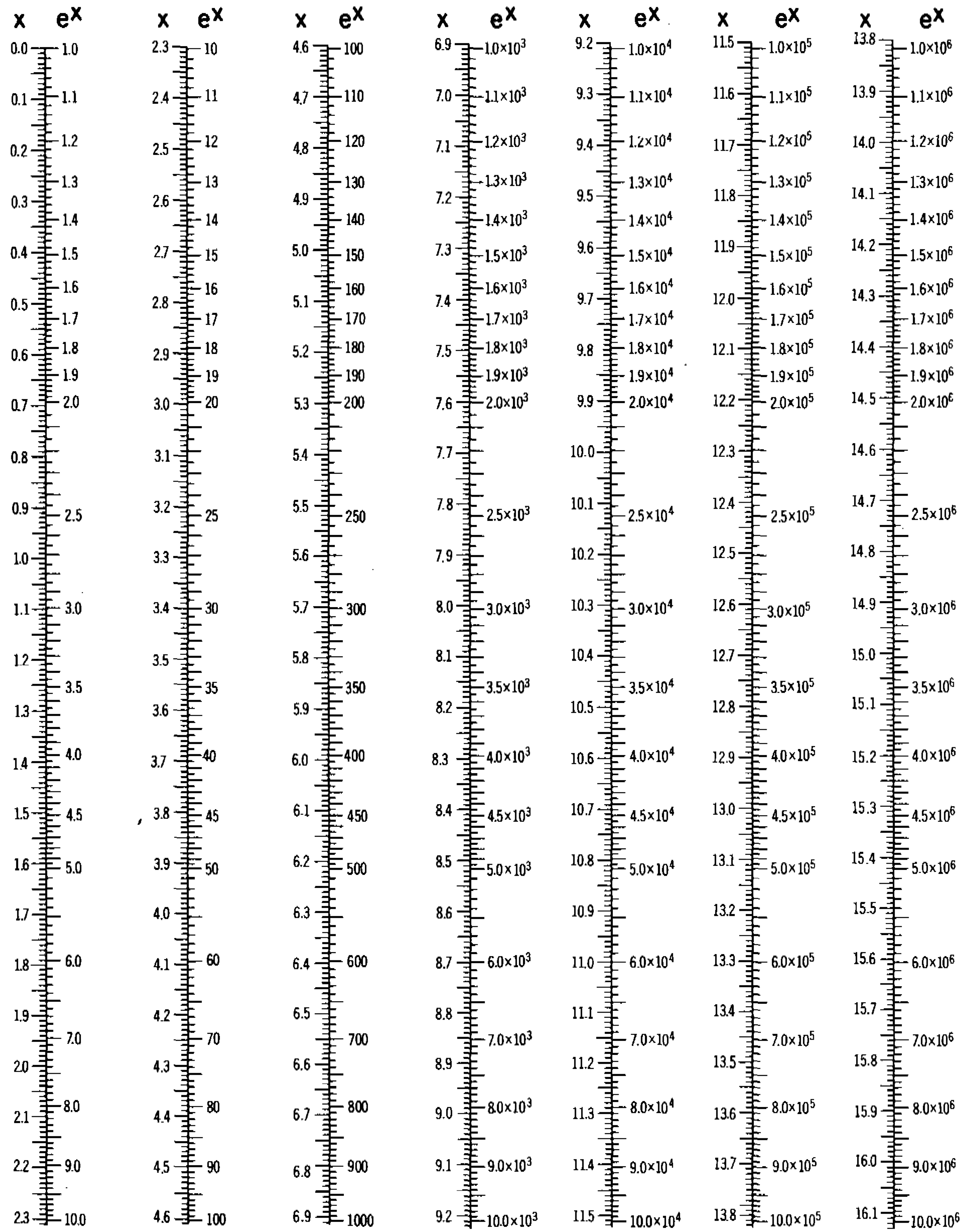


REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG.
ES- 122
SHEET 1
DATE 10-8-5

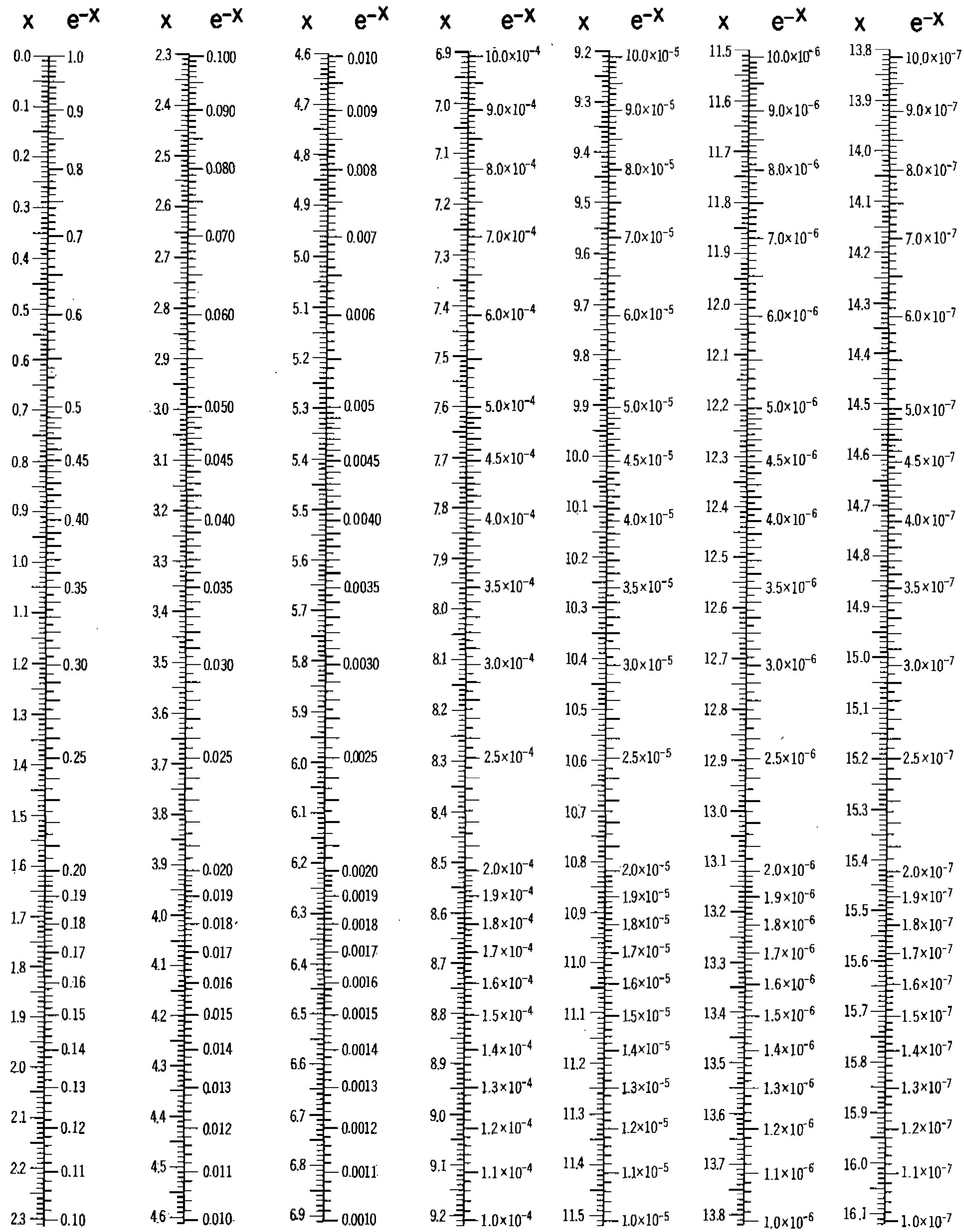
UNDERGROUND CONDUITS: Values of e^x for various values of x



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 123 SHEET 1 OF 3 DATE 1-28-57

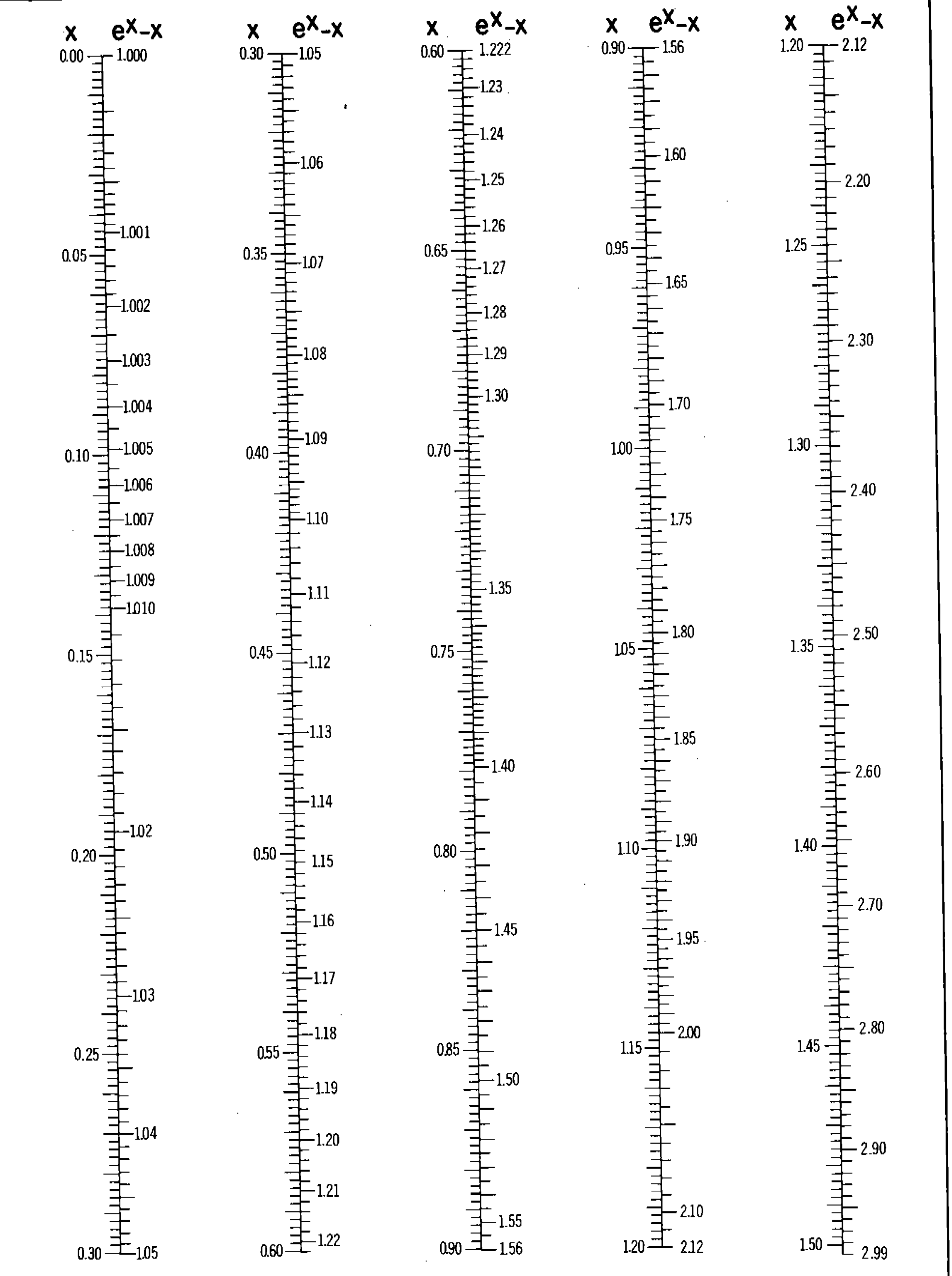
REVISED

UNDERGROUND CONDUITS: Values of e^{-x} for various values of x



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-123 SHEET 2 3 DATE 12-18-56 REVISED

UNDERGROUND CONDUITS: Values of e^x-x for various values of x



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-123 SHEET 3 OF 3 DATE 11-19-56 REVISED

APPENDIX A - DERIVATION OF FORMULAS FOR LOADS ON UNDERGROUND CONDUITS

Loads on Ditch Conduits

Assumptions

The following assumptions are made in the derivation of the formula for loads on ditch conduits:

1. The conduit is installed in a ditch whose walls are vertical.
2. The weight of the backfill material produces uniform vertical pressure at any horizontal plane over the entire width of the ditch.
3. Cohesion is negligible.
4. The ditch walls do not settle.
5. For rigid conduits all of the vertical pressure in the backfill at the elevation of the top of the conduit is carried by the conduit.
6. For flexible conduits part of the vertical pressure in the backfill at the elevation of the top of the conduit is carried by the backfill material between the conduit and the sides of the ditch.
7. The shear at the sides of the ditch is distributed as uniform vertical pressures (by virtue of internal friction in the backfill material) over the entire width of the ditch at any horizontal plane.
8. The magnitude of the vertical shearing stresses between the backfill material and the sides of the ditch is equal to the active lateral pressure exerted by the earth backfill against the sides of the ditch multiplied by the tangent of the angle of friction between the two materials.

Derivation of Load Formula for Ditch Conduits

When a conduit is placed in a ditch and covered with backfill material, the backfill material tends to settle downward. This tendency of the backfill material above the top of the conduit to move produces vertical friction forces or shearing stresses along the sides of the ditch. These shearing stresses give support to the backfill material.

Shearing Stresses. The cross section of a ditch conduit having a length of one foot is shown in Fig. A-1. Consider a horizontal differential element of the backfill material. The distance H is measured from the top of the backfill to the differential element.

The shearing stress exerted on each ditch wall is obtained as follows:

The active lateral pressure of the backfill material on the ditch wall at any point is

$$K \frac{P}{b_d}$$

where P = total vertical pressure on a horizontal plane within the interior prism, lbs/ft length of ditch

The magnitude of the shearing stresses at one end of the horizontal differential element is

$$K\mu' \frac{P}{b_d} dH$$

Differential equation. Equate the vertical forces acting on this element.

$$P + dP + 2K\mu' \frac{P}{b_d} dH = P + \gamma b_d \, dH$$

$$\frac{dP}{dH} + 2K\mu' \frac{P}{b_d} - \gamma b_d = 0$$

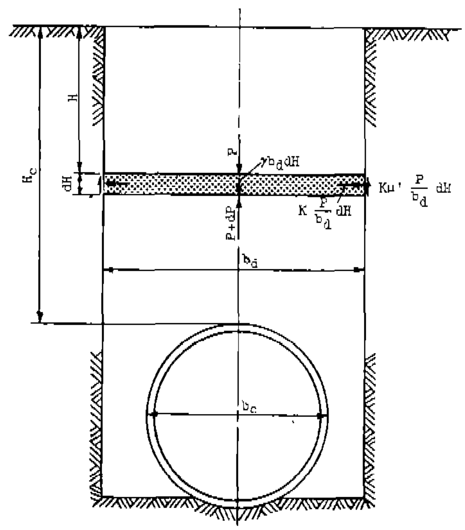


Fig. A-1 Loads on a horizontal element of the backfill material for a ditch conduit

The solution of this linear differential equation gives the total vertical pressure P within the ditch on any horizontal plane a vertical distance H from the top of the backfill in the interval $0 \leq H \leq H_c$. When $H = H_c$, the total vertical pressure within the ditch at the elevation of the top of the conduit is

$$P_c = \gamma b_d^2 \left[\frac{1 - e^{-2K\mu'(H_c/b_d)}}{2K\mu'} \right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(A-1)}$$

where P_c = total vertical pressure in the width b_d at the top of the conduit, lbs per linear foot of conduit

H = vertical distance from top of backfill to a horizontal element of fill material having a height of dH, ft

The proportion of this total vertical pressure that is carried by the conduit will depend on the relative rigidity of the conduit and of the fill material between the sides of the conduit and the sides of the ditch. For rigid pipes such as burned clay, concrete, or cast-iron pipe, the side fills may be relatively compressible and the pipe itself will carry practically all of the load P_c. It the pipe is a relatively flexible thin-walled pipe and the side fills have been thoroughly tamped, the load on the conduit will be reduced by the amount of load the side fills carry.

Load formulas for ditch conduits. The total vertical load on rigid ditch conduits with relatively compressible side fills is

$$W_c = C_d \gamma b_d^2 \quad . . . . (1\text{-}1)$$

where $$C_d = \frac{1 - e^{-2K\mu'(H_c/b_d)}}{2K\mu'} \quad . . . . (1\text{-}1a)$$

The total vertical load W_c on flexible pipes with thoroughly compacted side fills is

$$W_c = C_d \gamma b_c b_d \quad . . . . (1\text{-}2)$$

Loads on Positive Projecting Conduits

Assumptions

The following assumptions are made for deriving the equations for loads on positive projecting conduits:

1. Vertical shearing planes exist between the interior and exterior prisms.
2. Cohesion is negligible.
3. The magnitude of the shearing stresses is equal to the active lateral pressure at the vertical shearing planes multiplied by the tangent of the angle of internal friction of the embankment material.
4. The weight of the embankment material above the top of the conduit produces a uniform vertical pressure over the entire width of the interior prism.
5. The shear at the sides of the interior prism is distributed as uniform vertical pressure (by virtue of internal friction in the embankment material) over the entire width of the interior prism.
6. The shear at the sides of the exterior prism is distributed as uniform vertical pressures throughout the embankment in the infinitely wide exterior prism and its effect on the consolidation in the exterior prism may be neglected.
7. The embankment material has a constant modulus of consolidation.
8. The foundation material has a constant modulus of consolidation.

The modulus of consolidation of a body of soil is the ratio of the unit vertical pressure to the unit consolidation in the vertical direction.

The assumption of vertical shearing planes is employed for convenience, and load measuring experiments indicate that the assumption is valid.

Existence of the Plane of Equal Settlement

Consider a rigid conduit resting on a nonyielding foundation (Fig. A-2). The interior prism at the top of the conduit has not settled because the rigid conduit and its nonyielding foundation have prevented any settling. The exterior prism at the nonyielding foundation has not settled, but a particle in the exterior prism has settled from point A to the point A' because of the consolidation of the embankment material below the particle. Particles in the interior prism resist the settling of particle A and thus a portion of the load above particle A is transferred to the interior prism causing consolidation of the particles in the interior prism.

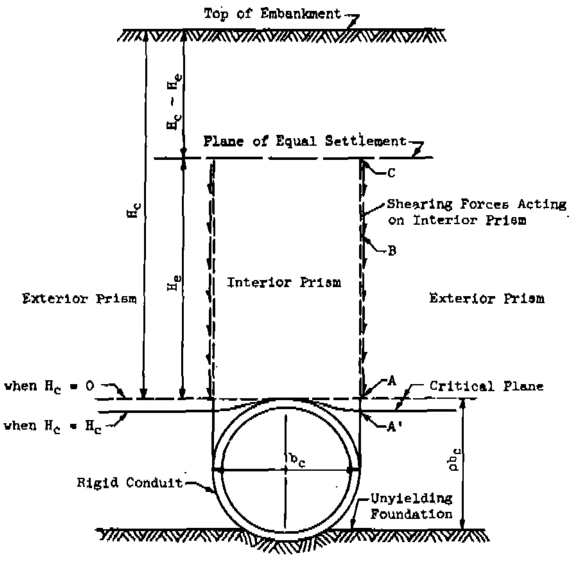


Fig. A-2 Basic case for considering the action of an embankment over a positive projecting conduit.

Consider a higher particle B in the exterior prism which settles more than an adjacent particle in the interior prism. A portion of the load is also transferred from the exterior prism to the interior prism. Since loads are being transferred from the exterior prism to the interior prism, the unit load in the exterior prism is less than the unit load in the interior prism. A greater consolidation exists in the interior prism than in the exterior prism above point A because of the greater unit loads in the interior prism causing consolidation.

At point C the consolidation in the interior prism is greater than the consolidation in the exterior prism above point A by the amount of the settling from A to A'. Therefore at point C the settlement in the interior prism is equal to the settlement in the exterior prism and there is no transfer of loads from the exterior prism to the interior prism at and above the plane of equal settlement.

Determination of the Height of Equal Settlement H_e

To ascertain whether the complete or incomplete condition occurs, it is necessary that H_e be determined. The derivation shown in the determination of H_e is that originally developed by A. Marston[4]. Marston's assumption for determining H_e yields an expression which gives slightly greater values for loads on conduits than those assumptions used by M. G. Spangler[9]. The expression for H_e is obtained by a consideration of the various additional settlements and additional consolidations.

Expression for the value of s_m. By the definition of the average modulus of consolidation of the embankment material subjected to loads resulting from fill heights of H_e to H_c, $E = \frac{\text{change in unit stress}}{\text{change in unit deformation}}$, obtain the result

$$s_m = \frac{\gamma(H_c - H_e)}{E}\,\rho b_c \quad . . . . \quad (A\text{-}2)$$

Expression for the value of λ_e. Likewise, obtain the result on recognizing assumption 6, page A-3

$$\lambda_e = \frac{\gamma(H_c - H_e)}{E}\,H_e \quad . . . . \quad (A\text{-}3)$$

Expression for the value of λ_i. Equating the vertical forces acting on the horizontal differential element of the interior prism, and recognizing, as was shown on page A-1, that

$$\frac{dP'}{dH} \mp 2K\mu\frac{P'}{b_c} - \gamma b_c = 0$$

obtain the differential equation (see Fig. A-3)

$$dP'' = \pm\, 2K\mu\,\frac{P''}{b_c}\,dH \quad . . . . \quad (A\text{-}4)$$

where P' = vertical pressure on a horizontal plane within the interior prism when the embankment height is equal to the height of equal settlement ($H_c = H_e$), lbs/ft length of conduit

P'' = additional vertical pressure on a horizontal plane within the interior prism due to the weight of the material above the plane of equal settlement, lbs/ft length of conduit

In Eq. A-4 the top sign in the ($\pm$) symbol is used for the projection condition and the bottom sign is used for the ditch condition. This convention is used wherever double signs appear.

The solution of this differential equation (Eq. A-4) gives (using the boundary condition $P'' = \gamma b_c(H_c - H_e)$ when $H = 0$)

$$P'' = \gamma b_c(H_c - H_e)\,e^{\pm 2K\mu(H/b_c)} \quad . . . . \quad (A\text{-}5)$$

The value of additional consolidation of the embankment material between the top of the conduit and the plane of equal settlement in the interior prism λ_i is the summation of the consolidation for each differential horizontal element. The consolidation of each differential element is

$$d\lambda_i = \frac{P''}{Eb_c}\, dH \quad \dots\dots\dots\dots\dots\dots\dots \quad \text{(A-6)}$$

The solution of this differential equation is (using the boundary condition $\lambda_i = 0$ when $H = 0$).

$$\lambda_i = \pm \frac{\gamma(H_c - H_e)}{E} \frac{b_c}{2K\mu}\left[e^{\pm 2K\mu(H_e/b_c)} - 1\right] \quad \dots\dots \quad \text{(A-7)}$$

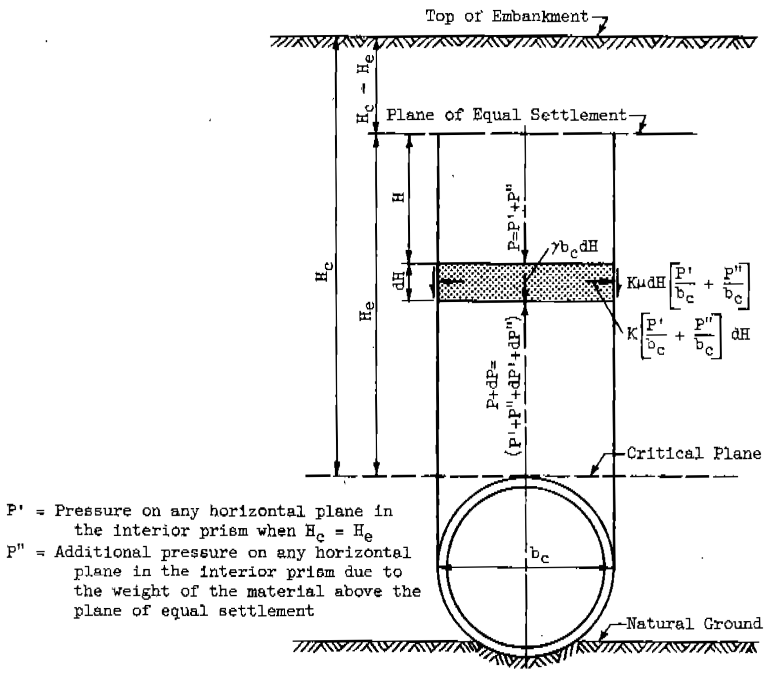


Fig. A-3 Forces used to determine the additional consolidation of the material in the interior prism

Expression for H_e. By the definition of the plane of equal settlement, the additional settlement at the top of the interior prism is equal to the additional settlement at the top of the exterior prism.

$$\pm \frac{\gamma(H_c - H_e)}{E} \frac{b_c}{2K\mu}\left[e^{\pm 2K\mu(H_e/b_c)} - 1\right] + s_f + s_c$$

$$= \left[\frac{\gamma(H_c - H_e)}{E}\right] H_e + s_m + s_g \quad \dots\dots \quad \text{(A-8)}$$

Rearranging, and using Eqs. 1-3 and A-2, obtain

$$e^{\pm 2K\mu(H_e/b_c)} \mp 2K\mu(H_e/b_c) = \pm 2K\mu\delta\rho + 1 \quad \dots\dots \quad \text{(1-4)}$$

Derivation of Load Formulas for Positive Projecting Conduits, Complete Condition

The vertical forces on any horizontal differential element in the interior prism may be equated as follows (see Fig. A-4)

$$P + dP = P + \gamma b_c \, dH \pm 2K\mu \frac{P}{b_c} dH \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (A\text{-}9)$$

The solution of this differential equation (using the boundary condition H = 0, P = 0) is

$$P = \gamma b_c^2 \left[\frac{e^{\pm 2K\mu(H/b_c)} - 1}{\pm 2K\mu} \right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (A\text{-}10)$$

At the top of the conduit $P = W_c$ when $H = H_c$. Therefore, the load on projecting conduits, complete condition, is

$$W_c = C_p \gamma b_c^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1\text{-}5)$$

where

$$C_p = \frac{e^{\pm 2K\mu(H_c/b_c)} - 1}{\pm 2K\mu} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1\text{-}5a)$$

Since Eqs. 1-5 and 1-5a are applicable for both the complete ditch condition and the complete projection condition, they may be used to determine loads on both rigid and flexible conduits.

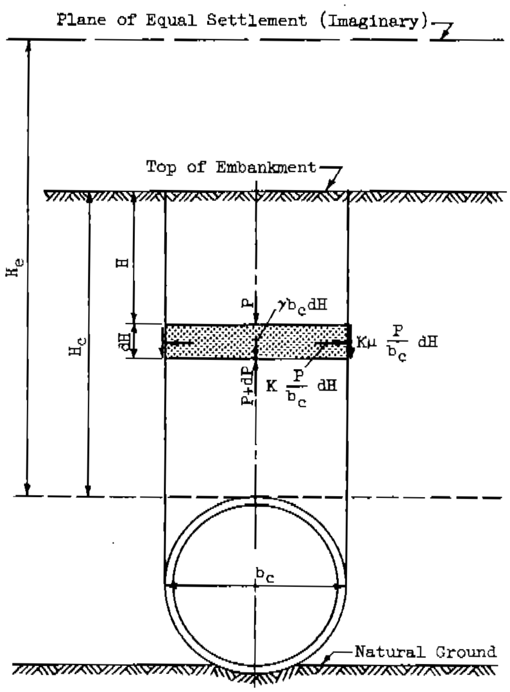


Fig. A-4 Loads on a horizontal element in the interior prism for the complete condition

Derivation of Load Formulas for Positive Projecting Conduits, Incomplete Condition

The vertical forces on a horizontal differential element of the interior prism are equated (see Fig. A-5).

$$P + dP = P + \gamma b_c\, dH \pm 2K\mu \frac{P}{b_c}\, dH \quad . . . . \quad (A\text{-}11)$$

The solution of this differential equation (using the boundary condition $H = 0$, $P = [H_c - H_e]\gamma b_c$) is

$$P = \mp \frac{\gamma b_c^2}{2K\mu} \pm \frac{\gamma b_c^2}{2K\mu}\, e^{\pm 2K\mu (H/b_c)} + (H_c - H_e)\gamma b_c\, e^{\pm 2K\mu (H/b_c)} \quad . . \quad (A\text{-}12)$$

At the top of the conduit (when $H = H_e$) $P = W_c$. Therefore, the load on positive projecting conduits, incomplete condition, is

$$W_c = C_p \gamma b_c^2 \quad . . . . \quad (1\text{-}6)$$

where

$$C_p = \frac{e^{\pm 2K\mu (H_e/b_c)} - 1}{\pm 2K\mu} + \left[\frac{H_c}{b_c} - \frac{H_e}{b_c}\right] e^{\pm 2K\mu (H_e/b_c)} \quad . . . . \quad (1\text{-}6a)$$

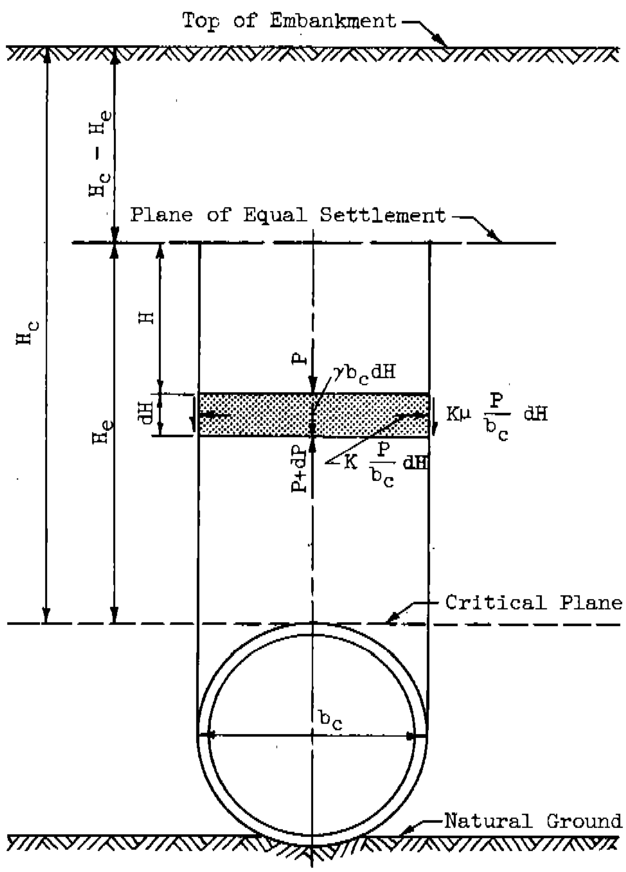


Fig. A-5 Loads on a horizontal element in the interior prism for the incomplete condition

Loads on Negative Projecting Conduits

Classification Requirements

An underground conduit is classed as a negative projecting conduit if all of the following conditions exist:

1. The conduit is installed in a sufficiently narrow ditch.
2. The ditch is backfilled to an elevation that is higher than the natural ground.

A ditch is sufficiently narrow if the load on the conduit as computed by the negative projecting conduit formula is less than the load on the conduit as computed by the positive projecting conduit formula.

Projection Condition

When the backfill material around and above a negative projecting rigid conduit is less compressible than the material in the ditch walls, the projection condition exists. The projection condition exists because loads are transferred from the exterior prisms to the interior prism. The load on a negative projecting conduit, projection condition, will usually not be greater than that obtained by using Eqs. 1-5 and 1-6 when $\rho = 1$. No load formulas will be derived for negative projecting conduits, projection condition.

Ditch Condition

The ditch condition requires that loads be transferred from the interior prism to the exterior prism. The ditch condition exists when the backfill is more compressible than the natural ground.

Definitions used for the ditch condition. In the discussion of loads on negative projecting conduits for the ditch condition, the following terms are redefined:

1. The interior prism is that prism of backfill and embankment materials which is bounded by the vertical planes coincident with the sides of the ditch containing the conduit, the top of the conduit, and the plane of equal settlement.
2. The exterior prism is that prism of embankment material which is bounded by the interior prism, the natural ground, the the plane of equal settlement.
3. The critical plane is that film of particles of backfill materials that was originally lying in the horizontal flat plane at the elevation of the natural ground when $H_c = \rho' b_d$.
4. The plane of equal settlement is that film of particles of embankment materials that lies in the lowest horizontal plane which remains as a plane as settlement takes place. This necessitates that the settlement of a particle of embankment at any elevation above the interior prism will be equal to the settlement of any particle having the same elevation above the exterior prism. Thus there are no vertical shears existing between the particles of embankment materials above the plane of equal settlement.

5. The projection ratio ρ' is the ratio of the distance between the natural ground surface and the top of the conduit (when $H_c = 0$) to the width of the ditch b_d.

Determination of height of equal settlement H_e. Again, to determine whether the complete or incomplete ditch condition occurs, it is necessary to determine H_e. The derivation shown in the determination of H_e is based on the same set of assumptions that A. Marston used for positive projecting conduits and will give slightly greater loads than the derivation developed by M. G. Spangler.[12]

Definitions and symbols for negative projecting conduits, ditch condition, will be discussed before deriving the expression for the height of equal settlement H_e since some are different from those used for positive projecting conduits.

Symbols. The symbols used to evaluate the additional settlement of the top of the exterior prism are (see Fig. A-6)

λ_{en} = additional consolidation of the embankment material between the natural ground and the plane of equal settlement, ft

s_g = additional settlement of the natural ground surface below the exterior prism due to the consolidation of the foundation, ft

The symbols used to evaluate the additional settlement of the interior prism are (see Fig. A-6)

λ_{in} = additional consolidation of the embankment material between the critical plane and the plane of equal settlement, ft

s_c = additional deformation of the conduit, ft

s_f = additional settlement of the bottom of the conduit (i.e., the surface of the natural ground beneath the conduit) due to the consolidation of the foundation, ft

s_d = additional consolidation of the backfill material between the top of the conduit and the critical plane, ft

Definition of settlement ratio δ'. The settlement ratio δ' is the ratio of the difference of the additional settlement of the natural ground in the exterior prism s_g and the additional settlement of the critical plane $(s_c + s_f + s_d)$ to the additional consolidation of the backfill material between the top of the conduit and the critical plane s_d.

$$\delta' = \frac{s_g - (s_c + s_f + s_d)}{s_d} \quad \text{. . . . .} \quad \text{(A-13)}$$

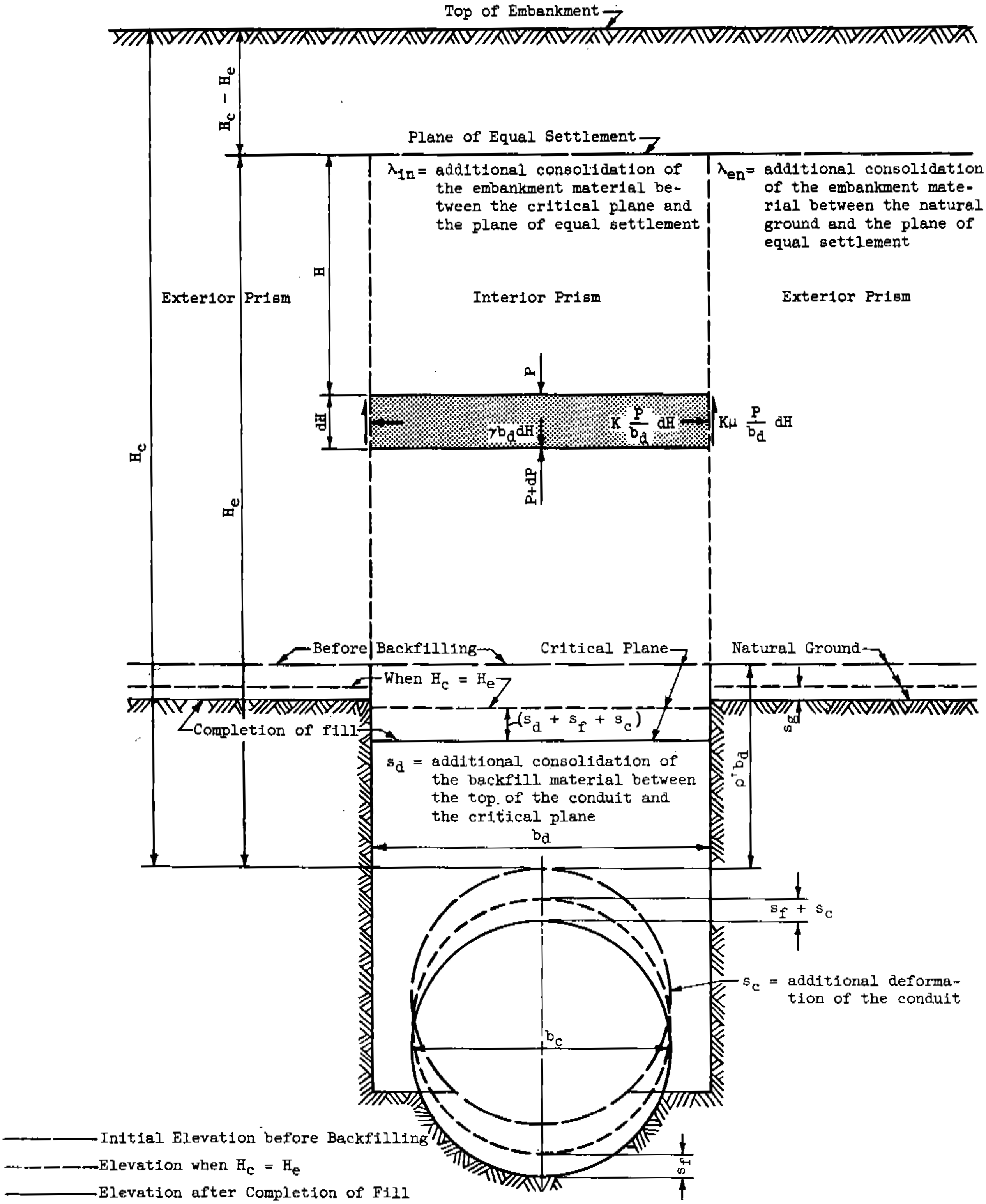


Fig. A-6 Negative projecting conduit

Values of s_d, λ_{en}, λ_{in}. Expressions for the values of s_d, λ_{en}, and λ_{in} are obtained in a manner similar to that used to obtain the additional consolidation for positive projecting conduits.

$$\lambda_{in} = \frac{\gamma(H_c - H_e)b_d}{2K\mu E}\left[1 - e^{-2K\mu(H_e - \rho' b_d)/b_d}\right]$$

$$s_d = \frac{\gamma(H_c - H_e)b_d}{2K\mu E}\left[e^{-2K\mu(H_e - \rho' b_d)/b_d} - e^{-2K\mu(H_e/b_d)}\right] \quad . \; . \; (A\text{-}14)$$

$$\lambda_{en} = \frac{\gamma(H_c - H_e)(H_e - \rho' b_d)}{E}$$

Expression for H_e. By the definition of the plane of equal settlement, the additional settlement at the top of the interior prism is equal to the additional settlement at the top of the exterior prism.

$$\frac{\gamma(H_c - H_e)b_d}{2K\mu E}\left[1 - e^{-2K\mu(H_e - \rho' b_d)/b_d}\right] + s_f + s_c + s_d =$$

$$\frac{\gamma(H_c - H_e)(H_e - \rho' b_d)}{E} + s_g \quad . \; . \; . \; . \; . \; (A\text{-}15)$$

Rearranging and using Eqs. A-13 and A-14, obtain

$$e^{-2K\mu(H_e/b_d)}\left[(\delta' + 1)e^{2K\mu\rho'} - \delta'\right] + 2K\mu(H_e/b_d) - (2K\mu\rho' + 1) = 0 \quad . \; . \; (A\text{-}16)$$

Complete and incomplete conditions. The comparison of H_e/b_d values with H_c/b_d values define whether the complete condition or incomplete condition exists.

When $\frac{H_c}{b_d} \leq \frac{H_e}{b_d}$, the complete ditch condition exists.

When $\frac{H_c}{b_d} > \frac{H_e}{b_d}$, the incomplete ditch condition exists.

The value of H_e/b_d is required to determine the load on a negative projecting conduit for the incomplete ditch condition.

Derivation of load formulas for negative projecting conduit, complete ditch condition. The equations for the load on negative projecting conduits, complete ditch condition, are obtained in a manner similar to that used for obtaining Eqs. 1-5 and 1-5a.

$$W_c = C_n \gamma b_d^2 \quad . \; (A\text{-}17)$$

where

$$C_n = \frac{e^{-2K\mu(H_c/b_d)} - 1}{-2K\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(A-17a)}$$

Derivation of load formulas for negative projecting conduits, incomplete ditch condition. The equations for the load on negative projecting conduits, incomplete ditch condition, are obtained in a manner similar to that used in obtaining Eqs. 1-6 and 1-6a.

$$W_c = C_n \gamma b_d^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(A-18)}$$

where

$$C_n = \frac{e^{-2K\mu(H_e/b_d)} - 1}{-2K\mu} + \left[\frac{H_c}{b_d} - \frac{H_e}{b_d}\right] e^{-2K\mu(H_e/b_d)} \quad \ldots \quad \text{(A-18a)}$$

It is necessary to determine the value of H_e/b_d by the use of Eq. A-16 to solve Eq. A-18a since Eq. A-18a contains H_e/b_d as one of its variables.

Ditch Conduit with Compacted Backfill

An underground conduit is classed as a ditch conduit with compacted backfill if all of the following conditions exist:

1. The conduit is installed in a sufficiently narrow ditch.
2. The ditch is backfilled to an elevation that is higher than the top of the conduit but not higher than the original ground surface.
3. The backfill is less compressible than the material in the ditch walls.

A ditch is sufficiently narrow if its width is less than the values of b_d' as computed by the formula

$$b_d' = \frac{C_p b_c^2}{H_c} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(A-19)}$$

The load on the conduit depends on the degree of compaction of the material adjacent to the conduit. If this material is relatively incompressible, part of the weight of the material in the ditch above the conduit will be transferred through the adjacent material to the foundation. Therefore, the minimum load on the conduit will be equal to the weight of the material directly above the conduit. If the material adjacent to the conduit is relatively compressible, no load will be transferred through the material to the foundation and the load on the conduit approaches a value equal to the weight of the backfill material of width b_d in the ditch above the conduit. For conservative design the latter assumption is used and the load on the ditch conduit with compacted backfill is

$$W_c = \gamma H_c b_d \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(A-20)}$$

If external loads, such as wheel loads, cause consolidtion of the material in the ditch wall, the loads on the conduit are increased and the conduit should be treated as a positive projecting conduit.

Imperfect Ditch Conduit

An underground conduit is classed as an imperfect ditch conduit if the following condition exists:

> An unusual method of construction is used to insure that the compressibility of the materials in the interior prism immediately above the conduit is sufficiently greater than the compressibility of the materials in the exterior prisms. An embankment is constructed in the usual manner to a height 1 to 1 1/2 times the width of the conduit above its top. A trench having a width b_c and centered directly above the conduit is dug in this constructed embankment to the top of the conduit. The trench is loosely backfilled to the top of the trench and the embankment completed in the usual manner.

The method of construction is used to insure that the interior prism will settle more than the exterior prism so that the friction forces acting on the interior prism will reduce the vertical load on the conduit. This type of construction should never be used through an embankment that is used to store or retain water.

The load formulas for an imperfect ditch conduit are the load formulas for negative projecting conduits, ditch condition.

Conduit on Compressible Bedding

An underground conduit is classed as a conduit on compressible bedding if the following condition exists:

> An unusual method of construction is used to insure that the foundation material under the conduit is more compressible than the foundation material adjacent to the conduit. This is accomplished by excavating a trench in the foundation material slightly wider than the outside width of the conduit. The trench is backfilled with compressible material. The conduit is installed on the compressible material.

This type of construction insures that the ditch condition exists. This requires that the interior prism settles more than the exterior prisms. The shearing forces transfer a portion of the weight of the interior prism to the exterior prisms. Conduits on compressible bedding should not be used where the function of the embankment is to store water.

The load formulas for a conduit on compressible bedding are the load formulas for positive projecting conduits, ditch condition.

APPENDIX B - DERIVATION OF SUPPORTING STRENGTH FORMULAS FOR CIRCULAR RIGID PIPES INSTALLED ON PROJECTING CRADLES AND BEDDINGS

In a field installation the supporting strength of a pipe is greater than that determined by the three-edge bearing test. A more favorable load distribution exists on pipes in the field installation than that of the three-edge bearing test.

Load Factor

The ratio of the supporting strength of the pipe in any stated loading condition R_c to the supporting strength of pipe in three-edge bearing R_{eb} is called the load factor.

$$L_f = \frac{R_c}{R_{eb}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}2)$$

The value of a load factor depends on the distribution of the loads on the conduit. The type of cradle or bedding associated with the conduit together with the classification of the underground conduit determines the distribution of the load on the conduit.

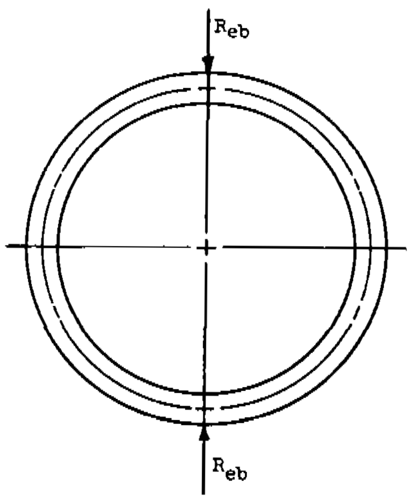


Fig. B-1a Assumed load distribution for the case of three-edge bearing

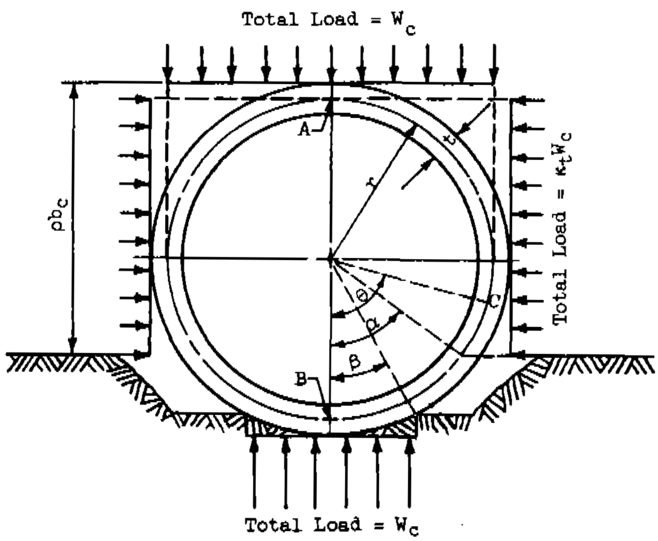


Fig. B-1b Assumed load distribution on an underground conduit installed on a projecting bedding

Figure B-1b is a generalized load pattern consisting of five variables (β, α or ρ, r, κ_t, and W_c). This pattern was prepared from a study of experimental data of loads on underground conduits. Because of the variety of values of these five variables, it is impractical to obtain load factors by actual test. An expression for the load factor in terms of these variables has been analytically derived.[5] The expression for the load factor is determined by the following procedure:

The maximum allowable fiber stress in a pipe for the three-edge bearing test is derived (Fig. B-1a). This is set equal to the derived maximum fiber stress for the assumed load pattern given in Fig. B-1b. The resulting equation is rearranged to give R_c/R_{eb} or the load factor L_f.

The maximum fiber stress for the three-edge bearing load is to be expressed in terms of R_{eb} and r. The maximum moment in the shell of a rigid pipe for a three-edge bearing load occurs at the bottom and the top of the pipe. Since tension is the critical stress for most rigid pipes, the pipe fails at the inside surface.

The horizontal reaction R_B is zero since no horizontal loading is assumed in the three-edge bearing test (see Fig. B-3). By the flexure formula

$$f_{eb} = \frac{M_B c}{I} \quad \text{(B-1)}$$

The value of M_B is not statically determinable. A relation based on the elastic properties of the pipe is required to determine M_B.

Elastic Theory of a Thin Ring

The Bending of a Beam by a Moment M

The application of a moment at any section C of a static beam (see Fig. B-2) will cause compressive stresses in the fibers on one side of the neutral surface and tensile stresses on the opposite side of the neutral surface. This causes bending in the beam and a tangent line to the neutral surface is rotated through an angle $\Delta\theta$. The relation of the angle $\Delta\theta$ and moment M is derived.

Consider a differential element (see Fig. B-2) in compression of a length $\Delta\ell$ a distance y from the neutral axis. The rotation of the tangent through an angle $\Delta\theta$ causes a change in the central angle by the amount $\Delta\theta$. The differential element is shortened by the amount $y\Delta\theta$. The unit deformation is $\partial = y\Delta\theta/\Delta\ell$. By Young's modulus ($E' = \sigma/\partial$) the total stress on the differential element of a cross-sectional area Δa is

$$\Delta S = \sigma\Delta a = E'\partial\ \Delta a = E'y\,\frac{\Delta\theta}{\Delta\ell}\,\Delta a$$

The moment of this differential element with respect to the neutral surface is

$$\Delta M = y\Delta S$$

or

$$dM = E'y^2\,\frac{\Delta\theta}{\Delta\ell}\,da$$

Assuming the modulus of elasticity is the same for both compression and tension obtain, on integrating,

$$M = E'I \frac{d\theta}{d\ell} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (B\text{-}2)$$

where $\frac{d\theta}{d\ell}$ = the instantaneous rate of change of the angle which the tangent to the neutral surface is rotated per unit length of the neutral surface. This rate of rotation is caused by the moment M. The value of $d\theta/d\ell$ is also equal to the instantaneous rate of change (caused by the moment M at section C) in the central angle per unit length of the neutral surface. The central angle θ for a circular ring is measured from any fixed radius to the section C.

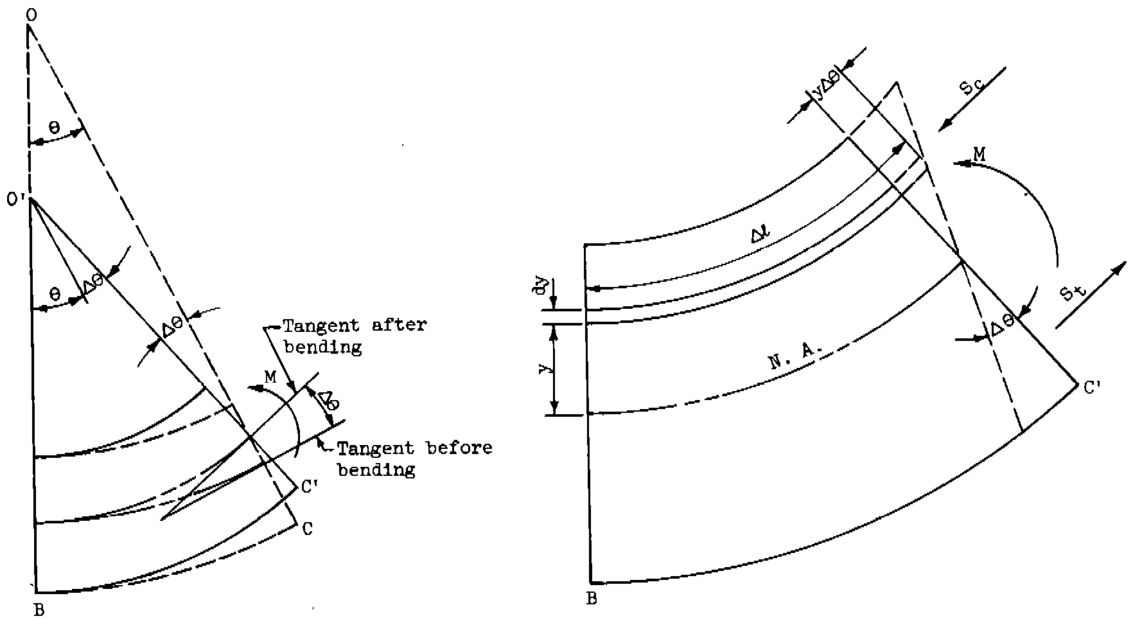


Fig. B-2 Bending of a circular beam caused by a moment at a section

The total change in the central angle $(\theta_2 - \theta_1)$ by application of a variable moment M in the segment of a thin ring subtending the angle θ is (observing that $d\ell = r d\theta$)

$$\int_{\theta=\theta_1}^{\theta=\theta_2} \frac{d\theta}{d\ell}\, d\ell = \int_{\theta=\theta_1}^{\theta=\theta_2} \frac{Mr}{E'I}\, d\theta \quad \ldots\ldots \quad (B\text{-}3)$$

The change in the central angle between sections A and B is zero for a symmetric loading on the ring. Therefore, multiplying the integrand of Eq. B-3 by the constant $E'I/r$, obtain

$$\int_0^{\pi} M\, d\theta = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (B\text{-}4)$$

Maximum Fiber Stress for Three-edge Bearing Load

The moment M at any point C in the ring is contained in the integrand of Eq. B-4. This is to be expressed in terms of R_{eb}, r, and the moment M_B at B. By statics (see Fig. B-3)

$$M = M_B - \frac{R_{eb}}{2} r \sin \theta$$

Substituting into Eq. B-4

$$M_B \int_0^{\pi} d\theta - r \frac{R_{eb}}{2} \int_0^{\pi} \sin \theta \, d\theta = 0$$

obtain

$$M_B = \frac{R_{eb} \, r}{\pi}$$

A factor of 0.75 is applied to allow for the shift in the neutral axis and for the non-linear proportionality of stress and strain near ultimate stress or

$$M_B = \frac{0.75 \, R_{eb} \, r}{\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(B-5)}$$

Observing that $\frac{I}{c} = \frac{t^2}{6}$ and substituting Eq. B-5 into Eq. B-1, obtain

$$f_{eb} = \frac{1.431 \, R_{eb} \, r}{t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(B-6)}$$

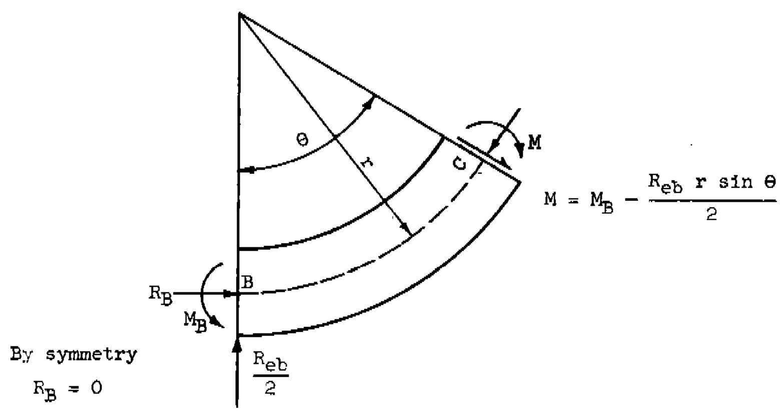


Fig. B-3

Second Relation by Elastic Theory

A second relation based on the elastic theory is required for the derivation of the expression for the maximum fiber stress for the assumed load pattern given in Fig. B-1b

The second relation is found by a consideration of the displacement of the end of a circular beam caused by rotating the beam at a section through a small angle. Rotating a beam at section C through any small angle ξ will cause the free end B to be displaced (see Fig. B-4).

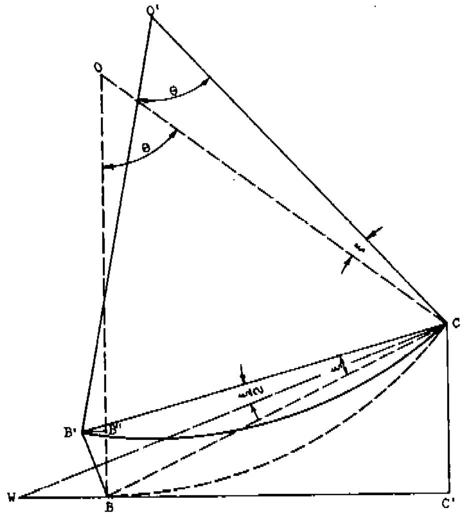


Fig. B-4 Displacement of the end of a circular beam caused by a small rotation at a section

Since CC', C'W, and CW are perpendicular to B'B", BB", and BB' respectively, then

$$\Delta WC'C \approx \Delta BB''B' \quad \text{and} \quad \frac{B'B''}{BB'} = \frac{CC'}{CW}$$

when the angle ξ at point C is small, then approximately

$$BB' = CB\xi$$

$$CB = CW$$

Hence the horizontal displacement B'B" of the point B due to a rotation at section C is

$$B'B'' = CC'\xi \quad \text{but} \quad CC' = r(1 - \cos\theta) \quad \text{then}$$

$$B'B'' = \xi r(1 - \cos\theta) \quad \ldots\ldots\ldots\ldots\ldots \quad (B\text{-}7)$$

The rotation per unit length of beam $\Delta\theta$ at any section C caused by the moment M is given by (see Eq. B-2)

$$\Delta\theta = \frac{M}{E'I}\Delta l$$

The horizontal displacement of point B by the rotation $\Delta\theta$ is obtained by substituting $\Delta\theta$ for ξ in Eq. B-7.

$$B'B'' = \frac{Mr}{E'I}(1 - \cos\theta)\Delta l$$

The sum of all displacements at point B caused by the moment M at all sections C in the interval $0 < \theta < \pi$ is zero for a symmetric loading on the ring. Observe that $\Delta \ell = r\Delta\theta$, then

$$\int_0^{\pi} \frac{Mr^2}{E'I} (1 - \cos\theta)\, d\theta = 0$$

Multiplying by $E'I/r^2$ and subtracting from Eq. B-4

$$\int_0^{\pi} M \cos\theta d\theta = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(B-8)}$$

Maximum Fiber Stress in a Pipe Installed on a Bedding

The fiber stress is obtained for the load pattern given by Fig. B-1b.

The load R_c is that load which causes failure. The maximum fiber stress for the given load pattern is to be expressed in terms of R_c, r, β, α, and κ_t. The maximum moment in the shell of a rigid pipe for a symmetrical loading about a vertical diameter is either at the top or at the bottom of the pipe. Pipes on cradles fail at the top and pipes on bedding fail at the bottom of the pipe.

By the flexure formula and observing that a direct stress R_B is to be considered (see Fig. B-5)

$$f_{ec} = M_B \frac{c}{I} - \frac{R_B}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(B-9)}$$

The values of M_B and R_B are not statically determinable and are determined by the use of Eqs. B-4 and B-8. An expression for M (moment at any section C of the pipe wall) is written for all values of θ in the intervals $0 \leqq \theta \leqq \beta$, $\beta \leqq \theta \leqq \alpha$, $\alpha \leqq \theta \leqq \pi/2$, and $\pi/2 \leqq \theta \leqq \pi$, using the principles of statics. These expressions are substituted into Eqs. B-4 and B-8 with the proper limits. After integration the values of M_B and R_B are determined.

The values of R_B and M_B are

$$R_B = \frac{R_c}{6\pi} (\cos^2\beta + 2Y\kappa_t) \quad \ldots\ldots\ldots\ldots \quad \text{(B-10)}$$

$$M_B = \frac{3r\,R_c}{4\pi}\left[\frac{-\cos^2\beta}{6} - \frac{Y\kappa_t}{3} + \frac{\beta}{8\sin\beta} + \frac{3\cos\beta}{8} + \frac{3\pi}{16} - \frac{\pi\sin\beta}{4} + \frac{\beta\sin\beta}{4} + \frac{\kappa_t Z}{4}\right] \quad \ldots \quad \text{(B-11)}$$

where

$$Y = \frac{3\pi \cos \alpha - 3\alpha \cos \alpha + \sin \alpha \cos^2 \alpha + 2 \sin \alpha}{1 + \cos \alpha} \qquad \text{(B-12)}$$

$$Z = \frac{2\pi \cos^2 \alpha - 2\alpha \cos^2 \alpha + 3 \cos \alpha \sin \alpha + \pi - \alpha}{1 + \cos \alpha} \qquad \text{(B-13)}$$

As before a factor of 0.75 has been applied to the moment M_B.

Observing that $\frac{I}{c} = \frac{t^2}{6}$ and substituting M_B and R_B into Eq. B-9 and letting $t = 0.15r$, obtain

$$f_{ec} = \frac{rR_c}{t^2} (X_p - \kappa_t X_a) \quad \ldots\ldots\ldots\ldots\ldots \text{(B-14)}$$

where

$$X_p = \frac{13.5}{16} - \frac{0.775 \cos^2 \beta}{\pi} + \frac{9\beta}{16\pi \sin \beta} + \frac{13.5 \cos \beta}{8\pi} - \frac{9 \sin \beta}{8} + \frac{9\beta \sin \beta}{8\pi} \quad \ldots\ldots \text{(B-15)}$$

and

$$X_a = \frac{1.55Y}{\pi} - \frac{1.125Z}{\pi} \quad \ldots\ldots\ldots\ldots\ldots \text{(B-16)}$$

where Y and Z are defined by Eqs. B-12 and B-13.

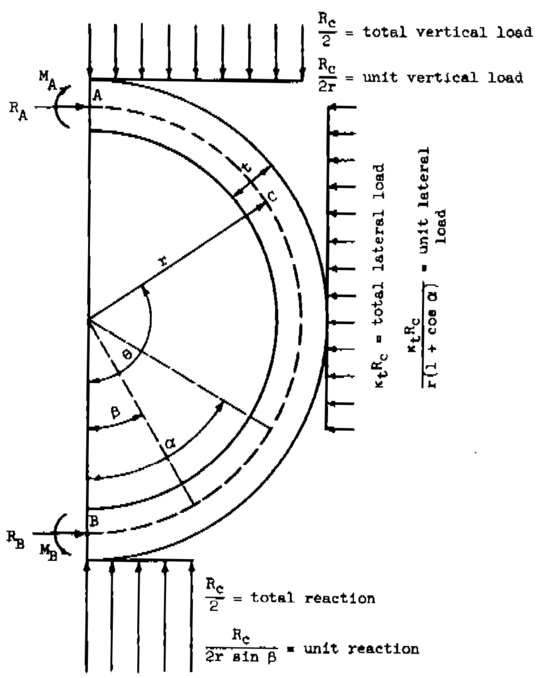


Fig. B-5 Assumed loads for actual field conditions

Maximum Fiber Stress in Pipe Installed on Cradles

Pipes installed on cradles fail at the top.

The maximum fiber stress at the top of the pipe is (see Fig. B-5)

$$f_{ec} = M_A \frac{c}{I} - \frac{R_A}{t} \quad \ldots \quad (B\text{-}17)$$

The values of M_A and R_A are obtained by use of the values of M_B and R_B and the principles of statics. They are

$$M_A = \frac{rR_c}{\pi}\left[\frac{\cos^2\beta}{6} + \frac{Y\kappa_t}{3} + \frac{\beta}{8\sin\beta} + \frac{3\cos\beta}{8} - \frac{\pi}{16} + \frac{\beta\sin\beta}{4} + \frac{\kappa_t Z}{4} - \frac{\pi\kappa_t}{2} - \frac{\pi\kappa_t\cos\alpha}{2}\right] \quad \ldots \quad (B\text{-}18)$$

$$R_A = \frac{R_c}{\pi}\left[\pi\kappa_t - \frac{\cos^2\beta}{6} - \frac{Y\kappa_t}{3}\right] \quad \ldots \quad (B\text{-}19)$$

Substituting M_A and R_A into Eq. B-17 and letting $t = 0.15r$, obtain

$$f_{ec} = \frac{rR_c}{t^2}(X_p - \kappa_t X_a) \quad \ldots \quad (B\text{-}14)$$

where

$$X_p = \frac{0.775\cos^2\beta}{\pi} + \frac{0.5625\beta}{\pi\sin\beta} + \frac{1.6875\cos\beta}{\pi} - 0.28125 + \frac{1.125\beta\sin\beta}{\pi} \quad \ldots \quad (B\text{-}20)$$

and

$$X_a = 2.40 + 2.25\cos\alpha - \frac{1.125Z}{\pi} - \frac{1.55Y}{\pi} \quad \ldots \quad (B\text{-}21)$$

where Y and Z are defined by Eqs. B-12 and B-13.

Load Factor for Projecting Cradles and Beddings

Equating Eqs. B-6 and B-14, obtain

$$\frac{1.431\,R_{eb}\,r}{t^2} = \frac{rR_c}{t^2}(X_p - \kappa_t X_a)$$

or

$$L_f = \frac{R_c}{R_{eb}} = \frac{1.431}{X_p - \kappa_t X_a} \quad \ldots \quad (2\text{-}5)$$

APPENDIX C – EVALUATION OF THE SETTLEMENT RATIO FOR POSITIVE PROJECTING CONDUITS

The settlement ratio δ is a parameter used in load formulas of positive projecting conduits. Its value is required to determine the height of the plane of equal settlement H_e. The settlement ratio is defined as the ratio of the difference of the additional settlement of the top of the conduit and the additional settlement of the critical plane in the exterior prism to the additional consolidation of the embankment material below the critical plane.

$$\delta = \frac{(s_m + s_g) - (s_f + s_c)}{s_m} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1\text{-}3)$$

where s_m = additional consolidation of the embankment material in the exterior prism between the critical plane and the natural ground surface, ft

s_g = additional settlement of the natural ground surface below the exterior prism due to the consolidation of the foundation, ft

$s_m + s_g$ = additional settlement of the critical plane in the exterior prism, ft

s_c = additional deformation of tne conduit, ft

s_f = additional settlement of the bottom of the conduit (i.e., the surface of the natural ground beneath the conduit) due to the consolidation of the foundation, ft

$s_f + s_c$ = additional settlement of the top of the conduit, ft

The equation for determining the height of equal settlement H_e is

$$e^{\pm 2K\mu(H_e/b_c)} \mp 2K\mu(H_e/b_c) = \pm\, 2K\mu\delta\rho + 1 \quad . \; . \; . \; . \; . \quad (1\text{-}4)$$

The value of H_e can be determined from Eq. 1-4 if the values for δ and the other variables are known.

The present method of estimating the value of δ is based on a consideration of test data taken from studies made on existing underground conduits. The values of δ obtained by these tests showed wide variation and could not be definitely correlated with the conditions under which the conduits had been installed.[9] This lack in correlation appears to be the result of the incorrect assumption that values of ρ have minor influences on the value of δ.

An analytical derivation for the expression of δ is presented. This expression for δ is in terms of factors that are readily determined for the particular site and conditions under which the conduit is installed.

The existence of a lower plane of equal settlement can be proved by the same procedure as Marston[4] used to prove the existence of an upper plane of equal settlement.

Assumptions

The following assumptions are made in the derivation:

a. Vertical shearing planes exist adjacent to the cradle (or conduit if no cradle). The shearing plane is taken adjacent to the cradle because the total load on the cradle is to be evaluated. The additional load on the cradle is required to evaluate the additional settlements in the interior prism below the conduit.

b. Cohesion is negligible.

c. The magnitude of the shearing stresses is equal to the active lateral pressure at the vertical shearing planes multiplied by the tangent of the angle of internal friction of the embankment material.

d. The weight of the embankment material above the top of the conduit produces a uniform vertical pressure over the entire width of the interior prism.

e. The load on any horizontal differential element in the interior prism below the bottom of the conduit is a uniform vertical pressure over the entire width of the interior prism.

f. The shear at the sides of the interior prism is distributed as uniform vertical pressure (by virtue of internal friction in the embankment or foundation materials) over the entire width of the interior prism.

g. The shear at the sides of the exterior prism is distributed as uniform vertical pressures throughout the embankment and foundation in the infinitely wide exterior prism and its effect on the consolidation in the exterior prism may be neglected.

h. The embankment and foundation materials have constant moduli of consolidation.

i. The weight of the conduit and cradle are neglected.

j. One mathematical approximation is made in the derivation for Case c and two mathematical approximations are made for Case d.

Symbols

The following additional symbols are used in the derivation:

b = bottom width of cradle, ft. When no cradle is used, $b = b_c$ = outside width of conduit, ft

H_e' = distance between the top of the conduit and the upper plane of equal settlement when the interior prism has a width b

P' = vertical pressure on a horizontal plane within the interior prism when the embankment height is equal to or less than the height of equal settlement, lbs/ft length of conduit

P'' = additional vertical pressure on a horizontal plane within the interior prism due to the weight of the material above the plane of equal settlement, lbs/ft length of conduit

E = modulus of consolidation of the embankment material, tons/ft^2

E_f = modulus of consolidation of the foundation material, tons/ft^2

ϕ_f = angle of internal friction of the foundation material

μ_f = tangent of the angle of internal friction ϕ_f for the foundation material

K_f = ratio at a point of active lateral pressure to vertical pressure on the foundation material

γ_f = unit weight of foundation material, lbs/ft^3

H_ℓ = distance between the bottom of the cradle and the lower plane of equal settlement, ft. When no cradle is used, it is the distance between the bottom of the conduit and the lower plane of equal settlement.

H_f = distance between the bottom of the cradle and the nonyielding foundation material, ft. When no cradle is used, it is the distance between the bottom of the conduit and the nonyielding foundation material.

ψb_c = vertical distance between the natural ground line in the exterior prism and the bottom of the cradle (or the bottom of the conduit if no cradle is used), ft

$$a = \frac{2K\mu}{b}$$

$$a_f = \frac{2K_f\mu_f}{b}$$

Cases

The four cases represented by the drawings shown on ES-115, page 3-47 will be considered separately.

Case a. Value of δ for conduits resting on rock foundation

When the conduit and embankment are on nonyielding foundation, the values of s_g, s_c, and s_f are zero. Thus by Eq. 1-3

$$\delta = \frac{s_m}{s_m} = 1 \quad \text{. . . . .} \quad \text{(C-1)}$$

Case b. Value of δ for conduits resting on rigid support with compressible adjacent foundation and embankment materials.
When the conduit is on nonyielding foundation, the values of s_f and s_c are zero.

$$\delta = \frac{(s_m + s_g)}{s_m} = 1 + \frac{s_g}{s_m}$$

The additional consolidation s_m of the adjacent material between the top of the conduit and the natural ground is that consolidation due to the additional load. The additional load is the weight of the embankment between $H = H_c$ and $H = H_e'$. (See assumption g.)

$$s_m = \frac{\gamma(H_c - H_e')}{E} \rho b_c \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (C\text{-}2)$$

Similarily, the additional consolidation s_g of the material between the natural ground and the bottom of the cradle is (see assumption g)

$$s_g = \frac{\gamma_f(H_c - H_e')}{E_f} \psi b_c \quad \ldots\ldots\ldots\ldots\ldots \quad (C\text{-}3)$$

On substituting these values of s_m and s_g, obtain

$$\delta = 1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{\psi}{\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (C\text{-}4)$$

Case c. Determination of the settlement ratio δ when the foundation material below the top of the conduit is homogeneous material of sufficient depth.
By Eq. 1-3 when $s_c = 0$, the value of δ for rigid conduits is

$$\delta = \frac{s_m + s_g - s_f}{s_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (C\text{-}5)$$

But by definition the upper plane of equal settlement is the lowest horizontal plane at which the additional settlement at the plane for the top of the interior prism is equal to the settlement at the plane for the exterior prism, i.e.,

$$s_f + \lambda_i = s_g + \lambda_e + s_m \quad \ldots\ldots\ldots\ldots\ldots \quad (C\text{-}6)$$

The evaluation of s_f, s_g, s_m, λ_i, and λ_e is made later.

Lower plane of equal settlement. In the derivation of δ for this case, a lower plane of equal settlement is recognized. At this plane the intensities of pressure of the interior prism are equal to those of the exterior prism. Furthermore, the additional consolidation of every portion of each horizontal plane below this plane of equal settlement are equal. When loads are transferred into the upper interior prism, loads are transferred out of the lower interior prism. Thus, shearing forces of the upper interior prism are oppositely directed from those of the lower interior prism. The additional consolidation in the interior prism is equal to the additional consolidation in the exterior prism between the upper and lower planes of equal settlement when a rigid conduit is installed. Hence,

$$\lambda_i + \lambda_i' = \lambda_e + s_m + \lambda_e' \quad \ldots \ldots \ldots \ldots \quad (C\text{-}7)$$

The evaluation of λ involves the summation of the additional consolidations resulting from the variable additional pressures of each horizontal differential element. These pressures are evaluated next. The top sign in all of the following expressions pertains to the projection condition and the bottom sign pertains to the ditch condition.

Expressions for P_c'' and P_ℓ''. Equating the vertical forces on the differential element ΔH (see Fig. C-1) for the interval $(H_c - H_e') < H < H_c$.

$$\frac{dP}{dH} \mp aP = \gamma b \quad \ldots \ldots \ldots \ldots \quad (C\text{-}8)$$

where $P = P' + P''$

For the interval $(H_c + \rho b_c + \psi b_c) < H < (H_c + \rho b_c + \psi b_c + H_\ell)$

$$\frac{dP}{dH} \pm a_f P = \gamma_f b \quad \ldots \ldots \ldots \ldots \quad (C\text{-}9)$$

On observing the existence of the plane of equal settlement and since Eq. C-8 is a linear homogeneous differential equation, it may be written in two components.

$$\frac{dP'}{dH} \mp aP' = \gamma b \quad \ldots \ldots \ldots \ldots \quad (C\text{-}8a)$$

$$\frac{dP''}{dH} \mp aP'' = 0 \quad \ldots \ldots \ldots \ldots \quad (C\text{-}8b)$$

and similarly Eq. C-9 is written

$$\frac{dP_\ell'}{dH} \pm a_f P_\ell' = \gamma_f b \quad \ldots \ldots \ldots \ldots \quad (C\text{-}9a)$$

$$\frac{dP_\ell''}{dH} \pm a_f P_\ell'' = 0 \quad \ldots \ldots \ldots \ldots \quad (C\text{-}9b)$$

The general solution of Eq. C-8b is

$$P'' = c\, e^{\pm aH} \quad \ldots \ldots \ldots \ldots \quad (C\text{-}10a)$$

where c is an arbitrary constant.

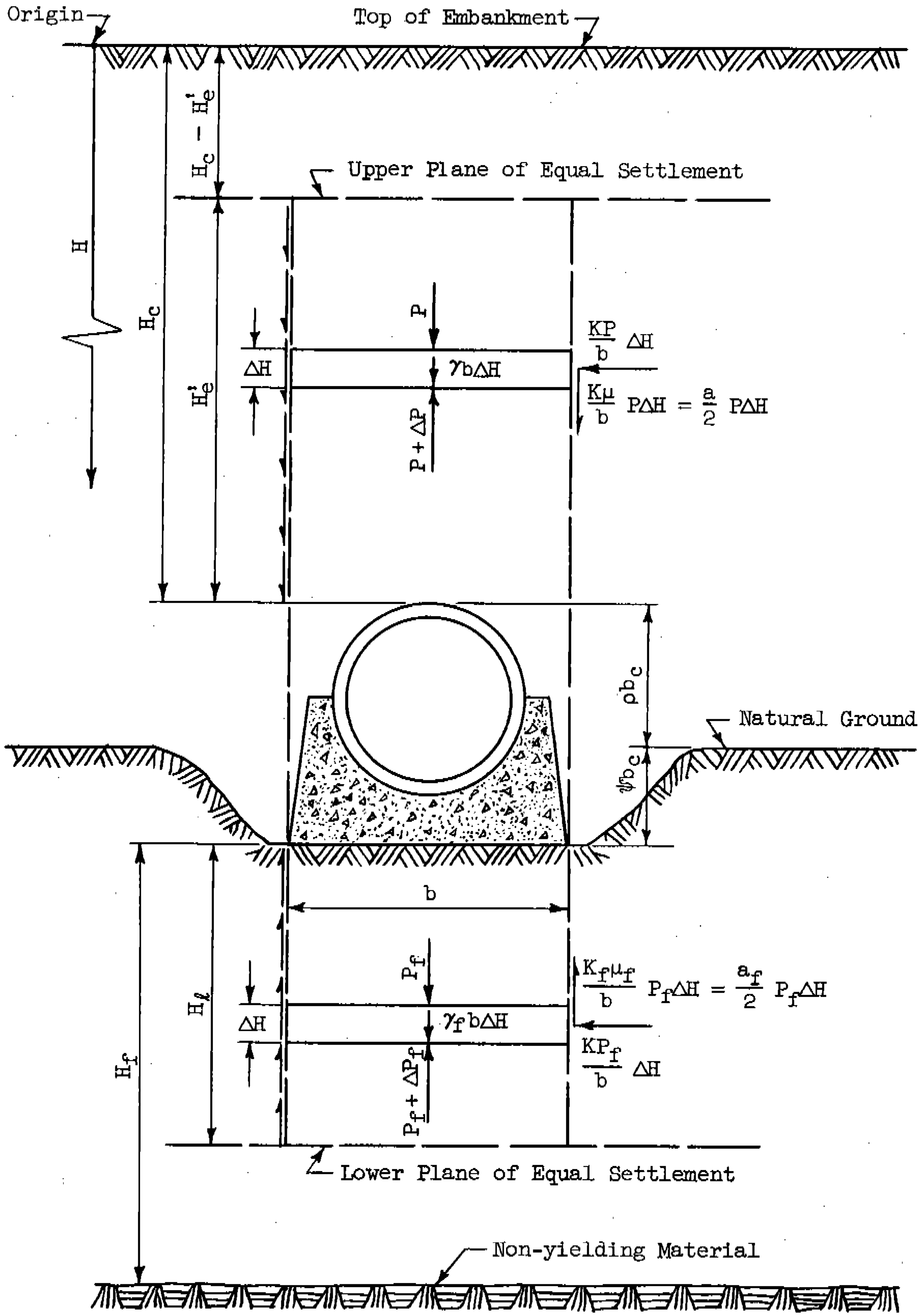

Origin
Top of Embankment
Upper Plane of Equal Settlement
H
H_c
$H_c - H_e'$
H_e'
P
ΔH
$\gamma b \Delta H$
$P + \Delta P$
$\frac{KP}{b}\Delta H$
$\frac{K\mu}{b} P\Delta H = \frac{a}{2} P\Delta H$
ρb_c
ψb_c
Natural Ground
b
H_ℓ
H_f
P_f
$\gamma_f b \Delta H$
$P_f + \Delta P_f$
$\frac{K_f \mu_f}{b} P_f\Delta H = \frac{a_f}{2} P_f\Delta H$
$\frac{KP_f}{b}\Delta H$
Lower Plane of Equal Settlement
Non-yielding Material

Fig. C-1

When $H = H_c - H_e'$, $P'' = \gamma b(H_c - H_e')$, the value of c is

$$c = \gamma b(H_c - H_e')\, e^{\mp a(H_c - H_e')} \quad \text{. . . . . (C-10b)}$$

At the top of the conduit $H = H_c$ and $P'' = P_c''$.

$$P_c'' = \gamma b(H_c - H_e')\, e^{\pm a H_e'} \quad \text{. . . . . (C-11)}$$

where P_c'' = additional vertical pressure on a horizontal plane at the top of the conduit

The general solution of Eq. C-9b is

$$P_\ell'' = c\, e^{\mp a_f H} \quad \text{. . . . . (C-12a)}$$

where c is an arbitrary constant.

When $H = H_c + \rho b_c + \psi b_c$, then $P_\ell'' = P_c''$, the value of c is

$$c = \gamma b(H_c - H_e')\, e^{\pm\left[aH_e' + a_f(H_c + \rho b_c + \psi b_c)\right]} \quad \text{. . . . . (C-12b)}$$

At the lower plane of equal settlement $H = H_c + \rho b_c + \psi b_c + H_\ell$ and $P'' = P_\ell''$.

$$P_\ell'' = \gamma_f b(H_c - H_e')\; e^{\pm(aH_e' - a_f H_\ell)} \quad \text{. . . . . (C-13)}$$

where P_ℓ'' = additional vertical pressure on a horizontal plane at the lower plane of equal settlement

Expressions for λ_e, λ_i, λ_e', and λ_i'. The differential equation expressing the additional consolidation in the interior prism λ_i is

$$d\lambda_i = \frac{P''}{bE}\, dH$$

Substituting the value of P'' previously determined (see Eqs. C-8a and C-8b)

$$d\lambda_i = \frac{\gamma}{E}\,(H_c - H_e')\, e^{\pm a\left[H - (H_c - H_e')\right]} dH$$

The general solution is

$$\lambda_i = \frac{\gamma}{E}\,(H_c - H_e')\; e^{\mp a(H_c - H_e')} \left[\frac{1}{\pm a}\,(\, e^{\pm aH} + c)\right] \quad \text{. . (C-14a)}$$

where c is an arbitrary constant.

When $H = H_c - H_e'$, then $\lambda_i = 0$, and the value of c is

$$c = -e^{\pm a(H_c - H_e')} \quad \text{. . . . . (C-14b)}$$

The total additional consolidation in the interior prism λ_i between $H = H_c - H_e'$ and $H = H_c$ is

$$\lambda_i = \frac{\gamma(H_c - H_e')}{\pm aE}\left(e^{\pm aH_e'} - 1\right) \quad \text{. . . . . (C-15)}$$

The additional consolidation in the exterior prism λ_e for the interval $(H_c - H_e') < H < H_c$ is

$$\lambda_e = \frac{\gamma(H_c - H_e')}{E} H_e' \quad \text{. . . . . (C-16)}$$

The differential equation expressing the additional consolidation in the lower interior prism λ_i' is

$$d\lambda_i' = \frac{P_\ell''}{bE_f}\, dH$$

Substituting the value of P_ℓ'' previously determined, Eqs. C-9a and C-9b, the general solution is

$$\lambda_i' = \frac{\gamma_f(H_c - H_e')}{E_f} e^{\pm\left[aH_e' + a_f(H_c + \rho b_c + \psi b_c)\right]}\left[\frac{1}{\mp a_f}\left(e^{\mp a_f H} + c\right)\right] \quad \text{. . . . (C-17a)}$$

where c is an arbitrary constant.

When $H = H_c + \rho b_c + \psi b_c$, then $\lambda_i' = 0$, and the value of c is

$$c = -e^{\mp a_f(H_c + \rho b_c + \psi b_c)} \quad \text{. . . . . (C-17b)}$$

The total additional consolidation in the lower interior prism λ_i' between $H = H_c + \rho b_c + \psi b_c$ and $H = H_c + \rho b_c + \psi b_c + H_\ell$ is

$$\lambda_i' = \frac{\gamma_f(H_c - H_e')}{\pm a_f E_f} e^{\pm aH_e'}\left(1 - e^{\mp a_f H_\ell}\right) \quad \text{. . . . . . (C-18)}$$

The additional consolidation λ_e' in the lower exterior prism is

$$\lambda_e' = \frac{\gamma_f(H_c - H_e')}{E_f}(H_\ell + \psi b_c) \quad \text{. . . . . (C-19)}$$

The additional settlement s_m of the material adjacent to the conduit is

$$s_m = \frac{\gamma(H_c - H_e')}{E} \rho b_c \quad \text{. . . . . (C-2)}$$

The expression for H_e'. The expression for H_e' is obtained by substituting the evaluations of λ_i and λ_e previously determined, Eqs. C-15 and C-16, into Eq. C-6.

$$s_f + \frac{\gamma(H_c - H_e')}{\pm aE}\left(e^{\pm aH_e'} - 1\right) = s_g + s_m + \frac{\gamma(H_c - H_e')}{E} H_e'$$

Rearranging and using Eq. 1-3, $\delta s_m = s_m + s_g - s_f$

$$\delta \frac{\gamma(H_c - H_e')}{E} \rho b_c = \frac{\gamma(H_c - H_e')}{\pm aE} (e^{\pm aH_e'} - 1) - \frac{\gamma(H_c - H_e')}{E} H_e'$$

which reduces to

$$e^{\pm aH_e'} - 1 = \pm a\delta\rho b_c \pm aH_e'$$

or

$$e^{\pm aH_e'} \mp aH_e' = \pm a\delta\rho b_c + 1 \quad \ldots\ldots\ldots\ldots \quad (C\text{-}20)$$

This relation evaluates the position of the plane of equal settlement for the conduit and cradle. This relation differs from Eq. 1-4 which evaluates the plane of equal settlement for the conduit.

Expression for H_ℓ. By definition the location of the lower plane of equal settlement is determined by observing that the additional vertical pressure at the lower plane of equal settlement is equal to the additional vertical pressure at the upper plane of equal settlement.

$$\gamma(H_c - H_e')b = \gamma_f b(H_c - H_e') \, e^{\pm(aH_e' - a_f H_\ell)}$$

Rewriting

$$\frac{\gamma}{\gamma_f} = e^{\pm(aH_e' - a_f H_\ell)} \quad \ldots\ldots\ldots\ldots \quad (C\text{-}21)$$

or

$$aH_e' - a_f H_\ell = \mp \operatorname{Log} \frac{\gamma}{\gamma_f}$$

and

$$H_\ell = \frac{a}{a_f} H_e' \pm \frac{1}{a_f} \operatorname{Log} \frac{\gamma}{\gamma_f} \quad \ldots\ldots\ldots\ldots \quad (C\text{-}22)$$

If the approximation $\gamma = \gamma_f$ is made, then

$$H_\ell = \frac{a}{a_f} H_e' \quad \ldots\ldots\ldots\ldots \quad (C\text{-}23)$$

Expression for evaluation of δ. Substituting the evaluations of s_m, λ_i, λ_e, λ_i', and λ_e' as given by Eqs. C-2, C-15, C-16, C-18, and C-19 into Eq. C-7, obtain

$$\frac{\gamma(H_c - H_e')}{\pm aE} (e^{\pm aH_e'} - 1) + \frac{\gamma_f(H_c - H_e')}{\pm a_f E_f} \left[e^{\pm aH_e'} - e^{\pm aH_e' \mp a_f H_\ell} \right]$$

$$= \frac{\gamma(H_c - H_e')}{E} (H_e' + \rho b_c) + \frac{\gamma_f(H_c - H_e')}{E_f} (H_\ell + \psi b_c)$$

$$\frac{\gamma(H_c - H_e')}{\pm aE}\left(e^{\pm aH_e'} - 1\right) + \frac{\gamma_f(H_c - H_e')}{\pm a_f E_f}\, e^{\pm aH_e'}\left(1 - e^{\mp a_f H_\ell}\right)$$

$$= \frac{\gamma(H_c - H_e')}{E}\left(\rho b_c + H_e'\right) + \frac{\gamma_f(H_c - H_e')}{E_f}\left(H_\ell + \psi b_c\right)$$

Multiplying by $\frac{\pm a}{H_c - H_e'}$ and substituting Eqs. C-20 and C-23, obtain on rearranging

$$\frac{\pm a\gamma}{E}\left(\delta\rho b_c + H_e'\right) + \frac{\gamma_f}{E_f}\frac{a}{a_f}\left[\pm a\delta\rho b_c \pm aH_e' + 1 - \frac{\gamma_f}{\gamma}\right]$$

$$= \frac{\pm a\gamma}{E}\left(H_e' + \rho b_c\right) \frac{\pm a\gamma_f}{E_f}\left[\frac{a}{a_f}H_e' \pm \frac{1}{a_f}\operatorname{Log}\frac{\gamma}{\gamma_f} + \psi b_c\right]$$

On rearranging

$$\delta = \frac{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{\psi}{\rho} \pm \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{1}{a_f\rho b_c}\left[\operatorname{Log}\frac{\gamma}{\gamma_f} + \frac{\gamma_f}{\gamma} - 1\right]}{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{a}{a_f}} \quad . \quad \text{(C-24)}$$

By approximating the last term to be negligible (i.e., $\frac{\gamma}{\gamma_f} = 1$) obtain

$$\delta = \frac{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma}\,\frac{E}{E_f}\right]\frac{K\mu}{K_f\mu_f}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(C-25)}$$

Equation C-25 gives the expression for δ when the foundation material is homogeneous for a sufficiently great depth. The depth H_f is sufficiently great if

$$H_f \geq H_\ell$$

or from Eq. C-23

$$H_f \geq \frac{K\mu}{K_f\mu_f}H_e'$$

<u>Case d. Determination of the settlement ratio δ when the foundation material is shallow ($H_f < H_\ell$)</u>

The additional settlements of the interior prism and exterior prisms for Case d at $H = H + (\rho + \psi)b_c + H_f$ are equal. By the definition of the plane of equal settlement, the additional consolidations in the interior prism and exterior prisms between the plane of equal settlement and the nonyielding foundation are equal.

This results in the relation

$$\lambda_i' + \lambda_i = \lambda_e' + \lambda_e + s_m$$

The evaluations of the terms s_m, λ_i, and λ_e are given by Eqs. C-2, C-15, and C-16. The evaluations of λ_i' and λ_e' are obtained by substituting H_f for H_ℓ in Eqs. C-18 and C-19. Making these substitutions, obtain

$$\frac{\gamma_f(H_c - H_e')}{\pm a_f E_f} e^{\pm aH_e'} \left[1 - e^{\mp a_f H_f}\right] + \frac{\gamma(H_c - H_e')}{\pm aE}\left[e^{\pm aH_e'} - 1\right]$$
$$= \frac{\gamma_f(H_c - H_e')}{E_f}(H_f + \psi b_c) + \frac{\gamma(H_c - H_e')}{E}(H_e' + \rho b_c)$$

Multiplying by $\dfrac{\pm a_f E_f}{\gamma_f(H_c - H_e')}$ and substituting xaH_e' for $a_f H_f$

$$e^{\pm aH_e'}\left[1 - e^{\mp axH_e'}\right] + \frac{\gamma}{\gamma_f}\frac{a_f}{a}\frac{E_f}{E}\left[e^{\pm aH_e'} - 1\right]$$
$$= \pm(axH_e' + a_f\psi b_c) \pm a_f \frac{\gamma}{\gamma_f}\frac{E_f}{E}(H_e' + \rho b_c)$$

Recognizing that $e^{\pm aH_e'} - 1 = \pm a\delta\rho b_c \pm aH_e'$

$$\left[1 + \frac{\gamma}{\gamma_f}\frac{a_f}{a}\frac{E_f}{E}\right]\left[\pm a\delta\rho b_c \pm aH_e'\right] + \left[1 - e^{\pm a(1-x)H_e'}\right]$$
$$= \pm(axH_e' + a_f\psi b_c) \pm a_f\frac{\gamma}{\gamma_f}\frac{E_f}{E}(H_e' + \rho b_c)$$

or

$$\left[1 + \frac{\gamma}{\gamma_f}\frac{a_f}{a}\frac{E_f}{E}\right](\delta\rho b_c) + \left\{\frac{1}{\pm a}\left[1 - e^{\pm a(1-x)H_e'}\right] + (1 - x)H_e'\right\}$$
$$= \frac{a_f}{a}\psi b_c + \frac{\gamma}{\gamma_f}\frac{a_f}{a}\frac{E_f}{E}\rho b_c$$

Make the approximation

$$\frac{1}{\pm a}\left[1 - e^{\pm a(1-x)H_e'}\right] + (1 - x)H_e' = (x - 1)\delta\rho b_c$$

Obtain

$$\left[x + \frac{\gamma}{\gamma_f} \frac{a_f}{a} \frac{E_f}{E}\right] (\delta\rho b_c) = \frac{a_f}{a} \psi b_c + \frac{\gamma}{\gamma_f} \frac{a_f}{a} \frac{E_f}{E} \rho b_c$$

or

$$\delta = \frac{1 + \left[\frac{\gamma_f}{\gamma} \frac{E}{E_f}\right] \frac{\psi}{\rho}}{1 + \left[\frac{\gamma_f}{\gamma} \frac{E}{E_f}\right] \frac{H_f}{H_e'}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(C-26)}$$

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