

ENGINEERING  
HANDBOOK

**drop  
spillways**

**section**

**11**

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE

PREFACE

SECTION 11

DROP SPILLWAYS

This handbook is intended primarily for the use of Soil Conservation Service engineers. Much of the information will also be useful to engineers in other agencies and in related fields of work.

The aim of the handbook is to present in brief and usable form information on the application of engineering principles to the problems of soil and water conservation. While this information will generally be sufficient for the solution of most of the problems ordinarily encountered, full use should be made of other sources of reference material.

The scope of the handbook is necessarily limited to phases of engineering which pertain directly to the program of the Soil Conservation Service. Therefore, emphasis is given to problems involving the use, conservation, and disposal of water, and the design and use of structures most commonly used for water control. Typical problems encountered in soil and water conservation work are described, basic considerations are set forth, and all of the step-by-step procedures are outlined to enable the engineer to obtain a complete understanding of a recommended solution. These solutions will be helpful in training engineers and will tend to promote nation-wide uniformity in procedures. Since some phases of the field of conservation engineering are relatively new, it is expected that further experience may result in improved methods which will require revision of the handbook from time to time.

This section of the Engineering Handbook has been written by M. M. Culp, Head of the Design Section of the Engineering Division, and C. A. Reese, Design Engineer. Two successive drafts have been submitted to field engineers and others for review. Suggestions received have led to improvements in the text and have been sincerely appreciated.



CONTENTS  
SECTION 11  
DROP SPILLWAYS

<u>Subject</u>	<u>Page</u>
<u>1. General</u>	
Description - - - - -	1.1
Material - - - - -	1.1
Functional Use - - - - -	1.1
Advantages - - - - -	1.1
Disadvantages - - - - -	1.2
<u>2. Layout</u>	
General - - - - -	2.1
Site Selection - - - - -	2.1
Channel Alignment - - - - -	2.1
Foundation Conditions - - - - -	2.2
Other Considerations - - - - -	2.2
Structure Dimensions - - - - -	2.4
Top Width of Earth Embankment - - - - -	2.4
Fill Slopes - - - - -	2.4
Required Height of Earth Fill Above Top of Headwall Extension - - - - -	2.4
Riprap of Approach Channel - - - - -	2.5
<u>3. Hydraulic Design</u>	
Hydrologic Determinations - - - - -	3.1
Discharge Capacity Determinations - - - - -	3.1
Free Discharge - - - - -	3.1
Velocity of Approach - - - - -	3.2
Freeboard - - - - -	3.7
Working Procedures, Tools, and Examples for Free Flow - - - - -	3.10
Submerged Discharge - - - - -	3.15
Examples for Submerged Flow - - - - -	3.18
Layout and Hydraulic Design Criteria - - - - -	3.21
<u>4. Structural Design</u>	
General - - - - -	4.1
Proportions Required for Stability - - - - -	4.1
Horizontal Pressures - - - - -	4.1
Loads on Headwall - - - - -	4.1
Relative Permeability of Foundation and Backfill - - - - -	4.2
Effect of Water Table Elevation - - - - -	4.2
Drainage of Fill Against Headwall - - - - -	4.3

<u>Subject</u>	<u>Page</u>
Loads on Sidewalls and Wingwalls - - - - -	4.9
Loads on Headwall Extensions - - - - -	4.9
Uplift - - - - -	4.9
Contact Pressures - - - - -	4.12
Piping - - - - -	+.14
Overturning - - - - -	4.19
Sliding - - - - -	4.19
Codes and Criteria - - - - -	4.21
Headwall Analysis - - - - -	4.21
Sidewall Analysis - - - - -	4.22
Wingwall Analysis - - - - -	4.26
Apron Analysis - - - - -	4.26
Buttress Analysis - - - - -	4.26
Longitudinal Sill Analysis - - - - -	4.30
Transverse Sill Analysis - - - - -	4.30
Headwall Extension Analysis - - - - -	4.31
<u>5. Type B Drop Spillway</u>	
General - - - - -	5.1
Tailwater - - - - -	5.2
Volumes of Concrete and Steel - - - - -	5.4
<u>6. Structural Design Example</u>	
General - - - - -	6.1
Hydraulic Design - - - - -	6.1
Stability Design - - - - -	6.5
Headwall Design - - - - -	6.16
Sidewall Design - - - - -	6.19
Apron Design - - - - -	6.23
Buttress Design - - - - -	6.32
Longitudinal Sill Design - - - - -	6.35
Transverse Sill Design - - - - -	6.42
Wingwall Design - - - - -	6.45
Design of Headwall Extension - - - - -	6.52
<u>7. Masonry Drop Spillways</u>	
General - - - - -	7.1
Design Procedures and Aids - - - - -	7.1
Reinforced Concrete Apron and Sill Design - - - - -	7.1
Example 7.1--Design of Masonry Drop Spillway - - - - -	7.4

ENGINEERING STANDARD DRAWINGS

SECTION 11

DROP SPILLWAYS

<u>Title</u>	<u>Drawing No.</u>	<u>Page</u>
Required Width of Headwall Extension		
Footings for Type B - - - - -	ES-48	5.10
Apron Design--Moments and Shears - - - - -	ES-56	4.27
Nomenclature and Symbols of Drop Spillway - - - - -	ES-63	1.3
Required Base Width for Gravity Walls with Various		
Loads and Loadings - - - - -	ES-64	7.17
Solution of Equation $Q = (3.1 Lh^{3/2}) \div (1.10 + 0.01F)$ - -	ES-65	3.11
Approximate Volumes of Reinforced Concrete in		
Cubic Yards--Type B - - - - -	ES-66	5.7
Layout and Hydraulic Design Criteria--Type B - - - - -	ES-67	5.3
Dimensionless Coordinates of Water Surfaces for Aerated		
Nappe Over Weir with Level Flush Approach Channel - - -	ES-68	6.3
Typical Layouts - - - - -	ES-70	2.3
Details of Masonry Drop Spillways - - - - -	ES-71	7.16
Minimum Concrete Volumes--Type B - - - - -	ES-74	5.9
Riprap of Approach Channel - - - - -	ES-79	2.6
Details of R/C Drop Spillway - - - - -	ES-80	6.53
Aeration of Weirs - - - - -	ES-81	3.3



SECTION 11  
DROP SPILLWAYS

1. GENERAL

Description. The drop spillway is a weir structure. Flow passes through the weir opening, drops to an approximately level apron or stilling basin, and then passes into the downstream channel. The basic elements of the drop spillway and the nomenclature are shown by drawing ES-63 (page 1.3). Various designs and proportions are in use. Further research and systematic evaluation of experience with existing structures will lead to continued improvement in design criteria.

Material. For most soil conditions, drop spillways may be built of any of the construction materials adapted for use in hydraulic structures. Some or all of the following materials will be available for consideration in any locality: concrete, reinforced or cyclopean; rock masonry; concrete blocks, with or without reinforcing; and steel sheet piling. Reinforced concrete is most widely used and has been very satisfactory for long-life, low-annual-cost structures. In a given case, particularly where a number of structures are involved, the selection of the material to be used should be based on: (1) the required life span of the structures, (2) an annual cost comparison which recognizes all of the costs, including maintenance and replacement, for structures built of the different available materials.

Functional Use. Drop spillways are used for the following purposes:

- (1) To control gradient in either natural or constructed channels.
- (2) To serve as inlet or outlet structures for tile drainage systems in conjunction with gradient control.
- (3) To control tailwater at the outlet of a spillway or conduit.
- (4) To serve as reservoir spillways where the total drop (F) is relatively low.

Experience and comparison of structural and hydraulic characteristics show that drop spillways have certain advantages and disadvantages compared with other structures adapted to similar functional uses. These general advantages and disadvantages should not be regarded as a basis for final selection of the type of structure for a given site, but can be used in deciding whether the drop spillway should be one of the alternate types to be considered.

Advantages

(a) Stability. The drop spillway is very stable, and the likelihood of serious structural damage is more remote than for other types of structures.

(b) Nonclogging of weir. The rectangular weir is less susceptible to clogging by debris than the openings of other structures of comparable discharge capacity.

(c) Low maintenance costs. Drop spillways indicate a definite tendency toward lower maintenance cost as compared with other types of structures for most embankment and foundation soils.

(d) Ease and economy of construction. Drop spillways are relatively easy to construct. When reinforced concrete is used, the flat slabs and straight, plane-surfaced walls simplify the forming and steel setting operations. Standard form panels or reusable sectional forms may be used.

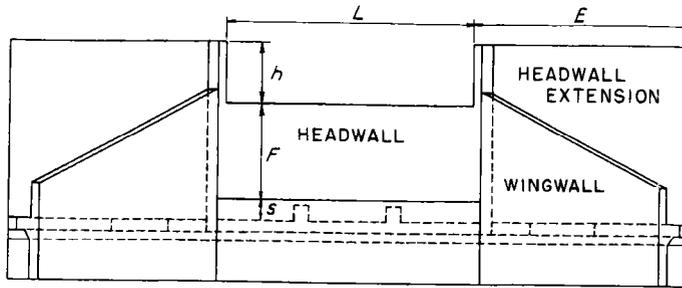
(e) Standardization. Drop spillways may be standardized readily both as to structural design and construction, which results in savings in engineering and construction costs.

#### Disadvantages

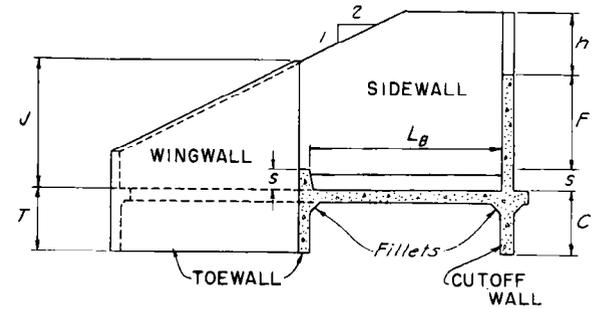
(a) The drop spillway may be more costly than some other types of structures where the required discharge capacity is less than 100 cfs and the total head or drop is greater than 8 or 10 feet.

(b) The drop spillway is not a favorable structure where it is desired to use temporary spillway storage to obtain a large reduction in discharge at and downstream from the structure. Discharge through a weir increases with the 1.5 power of the specific head at the weir ( $H_e$ ) while the discharge through a closed conduit flowing full increases as the 0.5 power of the total drop in hydraulic grade line. The above statements are not to be taken as meaning that significant spillway storage should be neglected in determining the required discharge capacity of a drop spillway.

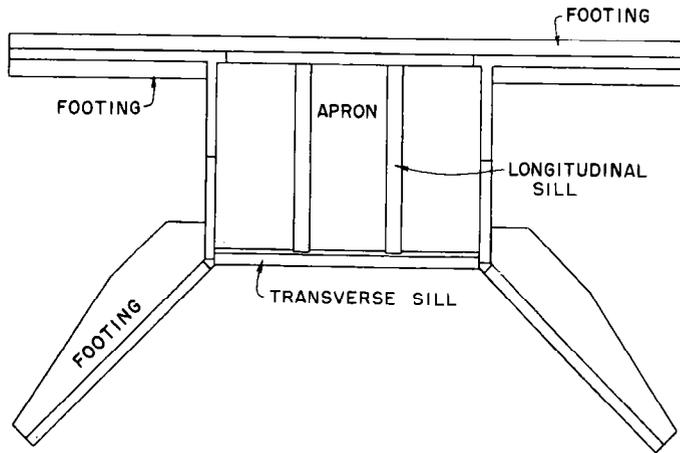
# DROP SPILLWAYS: NOMENCLATURE AND SYMBOLS OF DROP SPILLWAY



DOWNSTREAM ELEVATION



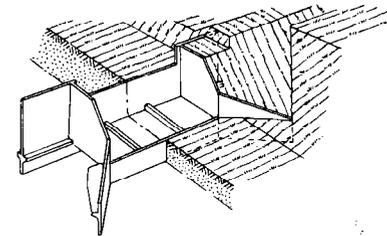
SECTION ON CENTER LINE



PLAN

## SYMBOLS

- $L$  = Length of weir.
- $h$  = Depth of weir.
- $F$  = Drop through spillway from crest of weir to top of transverse sill.
- $s$  = Height of transverse sill.
- $L_B$  = Length of apron.
- $T$  = Depth of toewall below top of apron.
- $C$  = Depth of cutoff wall below top of apron.
- $d_c$  = Critical depth of weir.
- $E$  = Length of headwall extension.
- $J$  = Height of wingwall and sidewall at junction.



PERSPECTIVE VIEW

REFERENCE

Rev. 12-14-53

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE

ENGINEERING STANDARDS UNIT

STANDARD DWG. NO.

ES-63

SHEET 1 OF 1

DATE 1-26-52

## 2. LAYOUT

General. The site selection and proportioning of a structure should be such that it satisfies the objectives and meets the stability requirements at minimum cost.

Site Selection. Proper site selection is dependent upon the availability of adequate field surveys and foundation data on all practicable alternate sites. The extent of field surveys required to prepare the most logical layout will depend upon the complexity of conditions peculiar to the problem. In some cases particular attention must be given to the effect of the proposed work on adjacent highways and their drainage structures, railroads, pipe lines, and other improvements or property that might be affected.

Comparative cost estimates probably will be necessary to determine the best layout. Volumes of earth fill and excavation, the cost of providing adequate protection during the construction period, volumes of concrete as affected by foundation conditions and other factors that vary from site to site, elevation of ground water, and other factors will affect costs. In the final selection, differences in cost at the various sites should be weighed against other advantages and disadvantages.

Channel Alignment. For gradient-control drops with definite approach channels, the site should be selected so that the spillway is located on a reasonably straight section of channel (on tangent), with neither upstream nor downstream curves within 100 to 200 feet of the structure. It often will be desirable to obtain straight alignment above and below the spillway by channel changes that merge smoothly with the existing channel. Modern earth moving equipment has made such channel changes practicable for many locations where, otherwise, poor alignment would have been unavoidable.

Poor upstream alignment, or any other disturbance that produces uneven distribution of velocity and discharge over the weir, is very apt to result in one or more of the following bad effects:

- (1) Reduction in discharge capacity of the weir.
- (2) Excessive scour of the earth embankment and channel banks just above the spillway.
- (3) Uneven distribution of flow across the transverse sill at the end of the apron, and a reduction in energy dissipation by the apron and stilling pool of the structure.
- (4) Excessive scour in the downstream channel just below the spillway apron and wingwalls, and downstream therefrom for a comparatively short distance.

The severity of these effects depends upon the extent of the upstream disturbance. Where the velocity of approach to the weir will be less than 2 feet per second throughout the anticipated life of the spillway, the effect of poor approach-channel alignment may be ignored. Where the approach velocity is apt to be higher, the approach channel must be straight.

Poor downstream alignment is not so serious as poor upstream alignment; however, it also should be avoided. Excessive scour is apt to develop if appreciable channel curvature exists immediately below the spillway. Such scour may be more severe than it would have been otherwise, due to the lack of complete dissipation of the overfall energy by the spillway apron and stilling pool.

The extent of the scour generated as the result of poor alignment will differ considerably from site to site for numerous reasons. It will be affected by the location, amount and rate of curvature, the velocity, depth of flow, duration of discharge, resistance of the channel bottom and banks to erosion, and perhaps other factors. Consequently, it will be difficult to predict the required extent of preventive riprap in advance of the scour. Where such predictions can be made, the riprap should be included in the original construction plans. It will be necessary to inspect such spillways after every significant storm and provide the riprap or other work necessary to protect and preserve them.

If, at a particular site, it is impracticable to avoid curvature, good upstream alignment must take precedence over desirable downstream conditions. In other words, drop spillways should be located so that the center line of a straight approach channel is coincident with the center line of the spillway.

Foundation Conditions. The site selected must provide an adequate foundation for the spillway. The foundation material must have the required supporting strength, resistance to sliding and piping, and be reasonably homogeneous so as to prevent differential or uneven settlement of the structure.

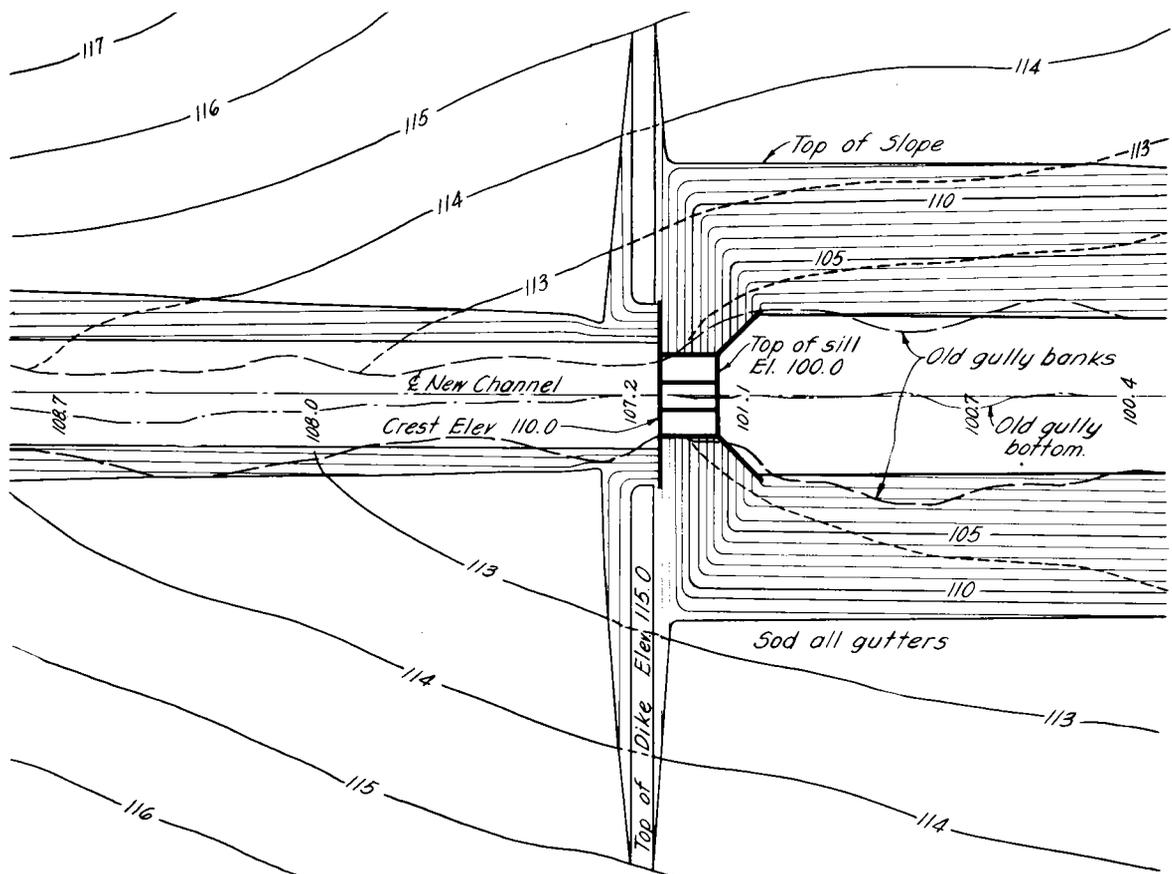
Piping, sliding, and vertical foundation loads are discussed elsewhere. Unequal settlement under various parts of the spillway must be carefully avoided; each part of the foundation must carry its proper share of the load. If the foundation materials vary appreciably as to consolidation under load over the foundation area, the reactions will not be distributed uniformly and cracking and differential settlement are probable.

Articulation of the various component parts of the structure may be desirable if foundation studies indicate only minor differences in foundation profiles. It usually will be wise to search for an alternate site or remove the foundation material and replace it with very carefully compacted homogeneous fill if the foundation profiles indicate appreciable differences that might lead to unequal settlement.

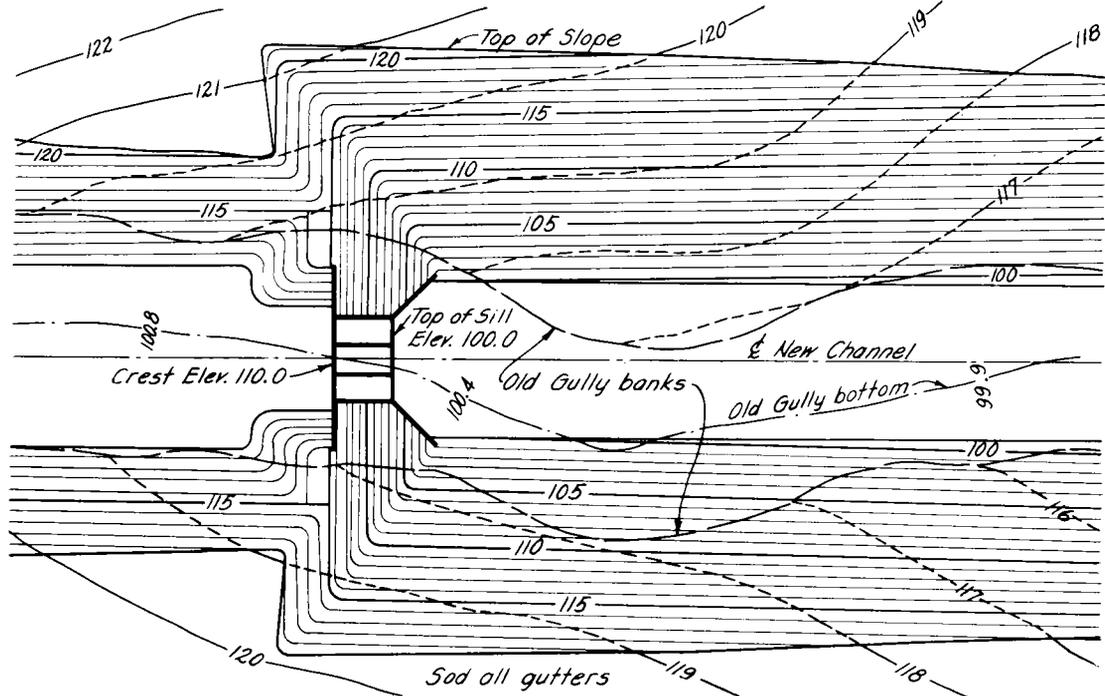
Obviously, foundation investigations must be made at each site. The extent of the investigation should depend upon the size and importance of the structure, known facts concerning the geology of the area, and the findings in the first borings or test pits. The investigations may range from visual classification of soils in one or two test holes, for structures 10 feet or less in height and of low failure hazard, to extensive soil borings, test pits, and soil-mechanics laboratory studies on higher structures where the hazard to life or property is significant should they fail.

Other Considerations. Other factors requiring investigation during the selection of the structure site are: (a) conditions that will affect the type and degree of protection against damage from runoff during

# DROP SPILLWAYS: TYPICAL LAYOUTS



TOP OF HEADWALL EXTENSION ABOVE GULLY BANKS



TOP OF HEADWALL EXTENSION BELOW GULLY BANKS

<p>REFERENCE</p>	<p>U. S. DEPARTMENT OF AGRICULTURE  <b>SOIL CONSERVATION SERVICE</b>          Robert M. Salter, Chief          ENGINEERING STANDARDS UNIT</p>	<p>STANDARD DWG. NO.  <b>ES - 70</b>          SHEET <u>1</u> OF <u>1</u>          DATE <b>4-5-52</b></p>
------------------	---	--



construction (b) material available for earth fill and the required volume of such fill (c) farming operations on the land adjacent to the site (d) roads, railroads, pipelines, and other structures that may affect or be affected by the structure.

Structure Dimensions. The determination of the dimensions of the various parts of the drop spillway is discussed in the subsections dealing with the hydraulic and structural design.

Top Width of Earth Embankment. The top width of the earth embankments should be such that they can be constructed and finished with the standard earth moving equipment. In the majority of cases, this sets the minimum top width at 8 feet. For gradient-control drop spillways without permanent pools of water above them, this minimum top width is sufficient for stability. Where a drop spillway acts as a reservoir spillway, the dikes are usually high for a considerable portion of their length and are in contact with the permanent pool. In such cases, the minimum top width of the dikes is given by the following equation which is applicable for heights of embankment up to 50 feet:

$$W = \frac{H + 35}{5} \quad 2.1$$

where  $W$  = minimum top width in feet

$H$  = maximum height of embankment in feet for  
range in values of 5 to 50 feet

Example: Given a maximum height of fill of 22 feet, find the minimum top width.  $W = (22 + 35) \div 5 = 11.4$ ; hence, use a top width of 12 feet unless some other design consideration requires a larger value.

For practical construction reasons, no attempt should be made to vary the top width of the embankment as the fill height varies; select one top width and use it for the full length of the embankment.

Fill Slopes. The recommended fill slopes are: (a) for fill adjacent to the structure, not steeper than 2 horizontal to 1 vertical; (b) for earth embankment, 3 horizontal to 1 vertical where practical, with a minimum of 2 to 1. These 3-to-1 slopes are recommended not only for stability, but because they will facilitate maintenance operations.

Required Height of Earth Fill Above the Top of the Headwall Extension. For gradient-control drop spillways, the top of the settled earth fill should be at least 1 foot above the top of the headwall extension. Where a reservoir exists above the spillway, the top of the settled earth fill should be higher than the top of the headwall extension by an amount equal to the average depth of frost penetration, taken from fig. 2.1 (page 2.5), but not less than 1 foot. In the western part of United States, local experience on the average of maximum annual frost depths will need to be gathered to supplement the data in fig. 2.1 (page 2.5).

Riprap of Approach Channel. Field experience and observation of laboratory tests indicate that earth backfill, just above the crest of a weir and at the end of embankments adjacent to the ends of the weir opening, will be scoured out and carried away by discharges that approach design values unless it is protected by adequate riprap or vegetation. The depth and duration of discharge, erodability of backfill, alignment of approach

channel, contraction at ends of the weir, sediment being transported, and density, vigor and type of vegetation on the backfill, and probably other factors affect the need for riprap.

Several drop spillways have been observed at which the channel grade line above the weir has been raised by an accumulation of sediment in dense, vigorous vegetation. Such a buildup of the channel results in reduced capacity of the weir.

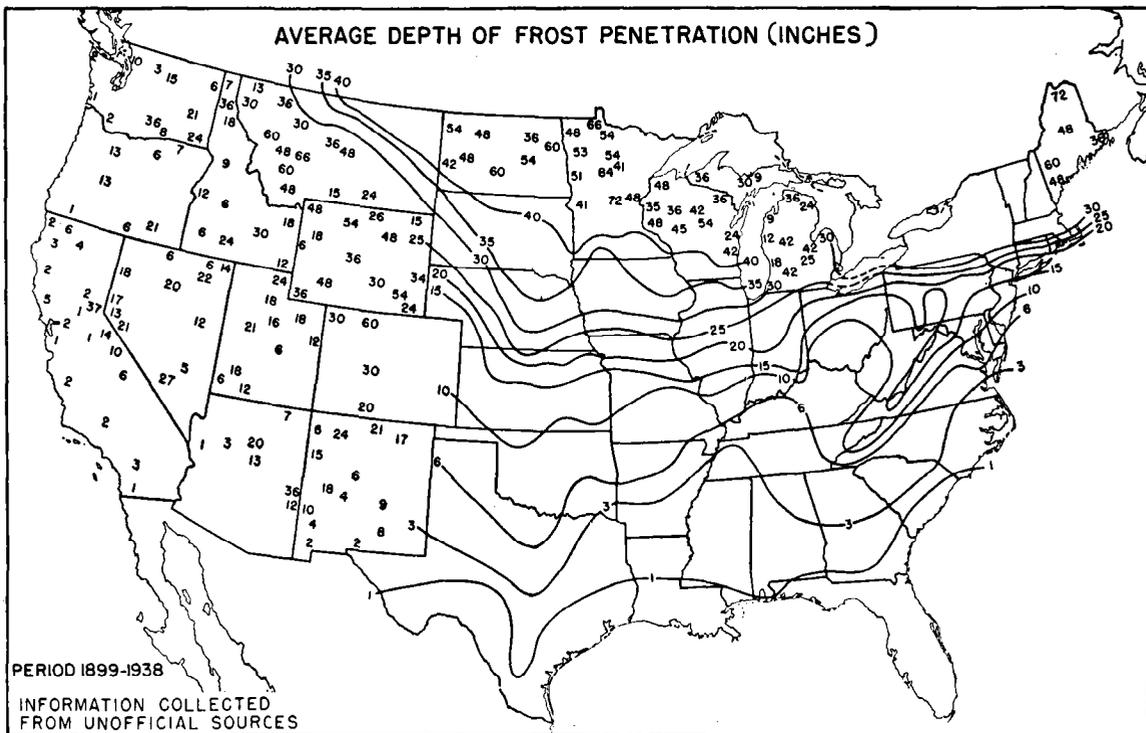
On other spillways, serious deep scour developed just above the weir, and especially at the ends of the weir, where additional turbulence is created by contraction of the flow. The fill just above a weir may be stable for several years, scour under a high discharge, and subsequently build up to original grade on the receding stage or by sedimentation during a series of low discharges.

It is highly desirable to avoid both scour and buildup above the headwall. Properly designed and placed riprap provides very good protection against both of these hazards.

Drop spillways that are located immediately below retarding dams will operate at or near design capacity more often and for longer time intervals than average gradient-control drops. Hence, the hazard from scour will be serious and the need for riprap is apparent. Drop spillways used as gradient-control structures in irrigation canals suffer relatively severe flow conditions and should always be riprapped above the weir.

Recommendations on the layout and requirements of riprap for drop spillways are presented in drawing ES-79 (page 2.6).

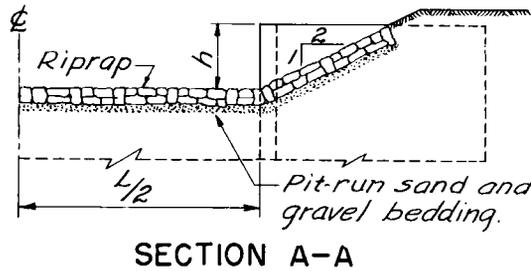
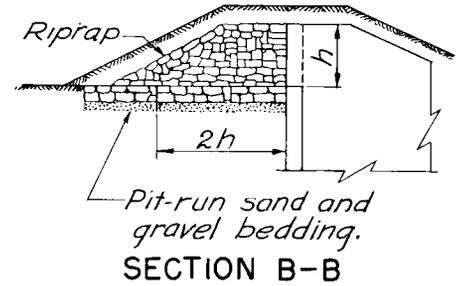
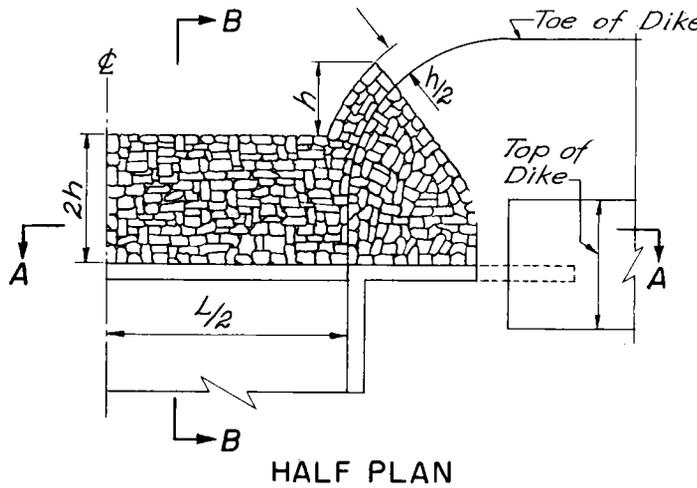
Riprap, placed in accordance with drawing ES-79 (page 2.6), should be considered as "must" protection in all cases where the depth of the weir exceeds 2.5 feet.



From "Climate and Man", Yearbook of Agriculture - 1941, p. 747

FIGURE 2.1

# DROP SPILLWAYS- RIPRAP OF APPROACH CHANNEL- LAYOUT AND REQUIREMENTS



**NOTES:**

The riprap material should be of hard, durable stone or broken concrete with a unit weight equal to or greater than 150 pounds per cubic foot.

Angular, fragmented rock is preferable to rounded stone.

At least 75% of the riprap, by weight, should consist of pieces of rock or concrete, which equal or exceed the weight given in the table opposite the required depth of weir.

The thickness of the layer of riprap should be at least equal to the average diameter of rock, *D*, indicated in the table.

The riprap should be placed on a bed of coarse pit-run sand and gravel. The minimum thickness of the bedding is indicated in the table.

The spaces between the large rock of the riprap should be filled with spalls, smaller rock, and pit-run material.

The dimensions of the area of riprap shown in the above sketches are minimum dimensions.

The surface of the riprap should be as smooth as possible.

Depth of weir, <i>h</i> , in feet.	Average diameter of Rock, <i>D</i> , in inches..	Weight of rock in pounds.	Minimum thickness of bedding in inches.
1.0	3.0	2.0	3.0
1.5	3.0	2.0	3.0
2.0	4.0	3.0	3.0
2.5	5.0	6.0	3.0
3.0	6.0	10.0	3.0
3.5	7.0	16.0	3.5
4.0	8.0	23.0	4.0
4.5	9.0	32.0	4.5
5.0	10.0	44.0	5.0
5.5	11.0	59.0	5.0
6.0	12.0	76.0	6.0

The average diameter, *D*, is defined as the diameter of a spherical rock of equal weight and density.

**REFERENCE**

U. S. DEPARTMENT OF AGRICULTURE  
**SOIL CONSERVATION SERVICE**

ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.

**ES-79**

SHEET 1 OF 1

DATE: 6-30-53



### 3. HYDRAULIC DESIGN

Hydrologic Determinations. Methods of determining peak rates and in-flow hydrographs of runoff are discussed in Hydrology, Section 4 of the Engineering Handbook.

Selection of the frequency of the design flood flow for a particular drop spillway should be based on an evaluation of the following factors:

- (1) Intended life of the structure
- (2) Probable extent of damage, should the spillway fail due to lack of discharge capacity.
- (3) Relative size and cost of the structure.

The discharge characteristics of a weir are such that a relatively large percentage increase in discharge capacity can be provided for a small percentage increase in total cost of the structure. The spillway cost is only a part of the total cost of the structure.

Discharge Capacity Determinations. Two general cases are encountered. They are:

- (1) Those cases where the required discharge capacity and the total drop through the spillway,  $F$ , are known, and the problem is to choose the length and depth of weir to provide the required capacity, maintain an adequate freeboard, and provide economical proportions for the spillway.
- (2) Those cases in which the dimensions of the structure are known, and it is necessary to know the discharge of the spillway operating with adequate freeboard or at maximum capacity.

In either case, the flow may be either free or submerged. Both free and submerged flow are discussed later.

Free Discharge. The discharge capacity of an aerated, rectangular weir without submergence is given by the formula

$$Q = CL \left( H + \frac{v_a^2}{2g} \right)^{3/2} \quad 3.1$$

where  $Q$  = discharge in cfs

$L$  = length of weir in ft

$H$  = head on weir in ft (see fig. 3.1, page 3.4)

$v_a$  = mean velocity of approach in fps

$g$  = acceleration of gravity in ft per sec<sup>2</sup>

$C$  = discharge coefficient

A completely aerated weir is one in which unlimited quantities of air have free access to the space between the nappe and the headwall. Under such conditions, the nappe will be subject to atmospheric pressure on both upper and under surfaces. Ordinarily, complete aeration will not be attained and some small permissible differential in pressure, below atmospheric, will exist under the nappe. Provision must be made in the design for admission of air to the underside of the nappe by the forced development of end contractions or by air vents. Failure to provide aeration will lead to the formation of excessive negative pressures (below atmospheric) under the nappe which in turn cause fluctuation of head, instability of flow, and increased load on the headwall.

Drawing ES-81 (page 3.3) gives the design equations and procedure for estimating the size of air vents required for drop spillways. If the differential pressure,  $p$ , exceeds 0.3 ft. of water, the increased loads on the headwall should be included in the structural analysis and design.

Only a limited investigation of the discharge capacity of drop spillways has been made. The lack of test data, and consideration of the varied conditions under which these structures will operate, lead to a recommended design value for  $C = 3.1$ . Use of this coefficient is based on the assumption that flow approaching the weir is at subcritical velocities, i.e., the depth of flow is greater than critical depth.

Contraction at the ends of the weir has not been treated specifically because of a lack of data applicable to the structures under discussion and because of its small effect on drop spillways of usual proportion.

It is believed that the use of the coefficient  $C = 3.1$  is sufficiently conservative to have made reasonable allowance for possible end contractions and other indeterminate factors that affect the discharge capacity.

Velocity of Approach. The total energy head producing flow over the weir is equal to  $H + (v_a^2 \div 2g)$ . The section in the approach channel at which  $H$  and the approach velocity,  $v_a$ , are estimated should meet the following conditions: (Study fig. 3.1, page 3.4)

- (1) It should be  $3H$  or more upstream from the weir, so as to be above any significant influence of the drop-down curve which results from the increase in velocity as the flow approaches the weir.
- (2) For simplicity in computations, it should be upstream from any constrictions of the approach channel that may be imposed by an earth embankment which diverts the flow to the weir.
- (3) It should not be so far upstream that the energy losses between the chosen section and the weir will affect the design significantly. In other words, the assumption of a level energy line from the chosen section to the weir must be reasonably correct.

The section in question may be chosen at any location which meets conditions 1 and 3, listed above. The actual cross section determined by field measurement may be regular or irregular in shape; if irregular, an equivalent trapezoidal section may often be selected to facilitate computations.

The velocity of approach is equal to the discharge divided by the cross-sectional area of the chosen section.  $v_a = Q \div a_a$

# DROP SPILLWAYS - AERATION OF WEIRS

**EQUATION**

$$\frac{A}{L} = 5.3 \times 10^{-4} \frac{H_e^{3.64}}{p^{1.64}}$$

where:

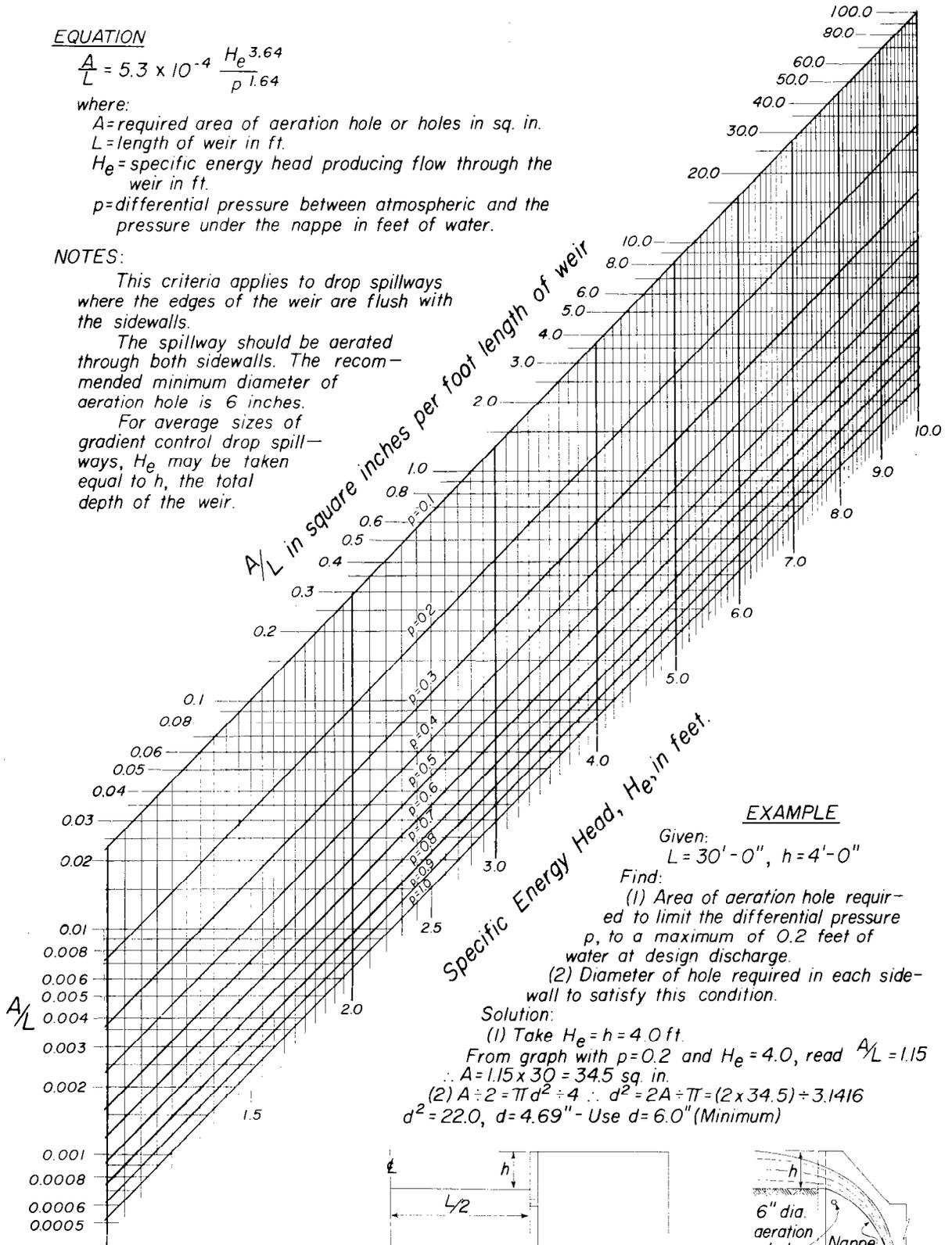
- A = required area of aeration hole or holes in sq. in.
- L = length of weir in ft.
- $H_e$  = specific energy head producing flow through the weir in ft.
- p = differential pressure between atmospheric and the pressure under the nappe in feet of water.

**NOTES:**

This criteria applies to drop spillways where the edges of the weir are flush with the sidewalls.

The spillway should be aerated through both sidewalls. The recommended minimum diameter of aeration hole is 6 inches.

For average sizes of gradient control drop spillways,  $H_e$  may be taken equal to h, the total depth of the weir.



**EXAMPLE**

Given:  
L = 30'-0", h = 4'-0"

Find:

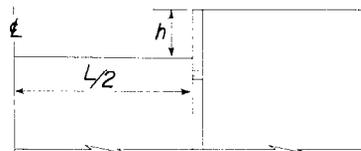
- (1) Area of aeration hole required to limit the differential pressure p, to a maximum of 0.2 feet of water at design discharge.
- (2) Diameter of hole required in each sidewall to satisfy this condition.

Solution:

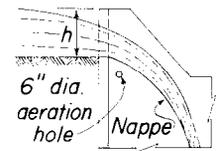
(1) Take  $H_e = h = 4.0$  ft.

From graph with p = 0.2 and  $H_e = 4.0$ , read  $A/L = 1.15$   
 $\therefore A = 1.15 \times 30 = 34.5$  sq. in.

(2)  $A \div 2 = \pi d^2 \div 4 \therefore d^2 = 2A \div \pi = (2 \times 34.5) \div 3.1416$   
 $d^2 = 22.0, d = 4.69"$  - Use d = 6.0" (Minimum)



HALF FRONT ELEVATION



SIDE ELEVATION

**REFERENCE:**

Aeration of Spillways by  
G.H. Hickox. - Trans. ASCE 1944,  
page 537, paper No. 2215

U.S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE

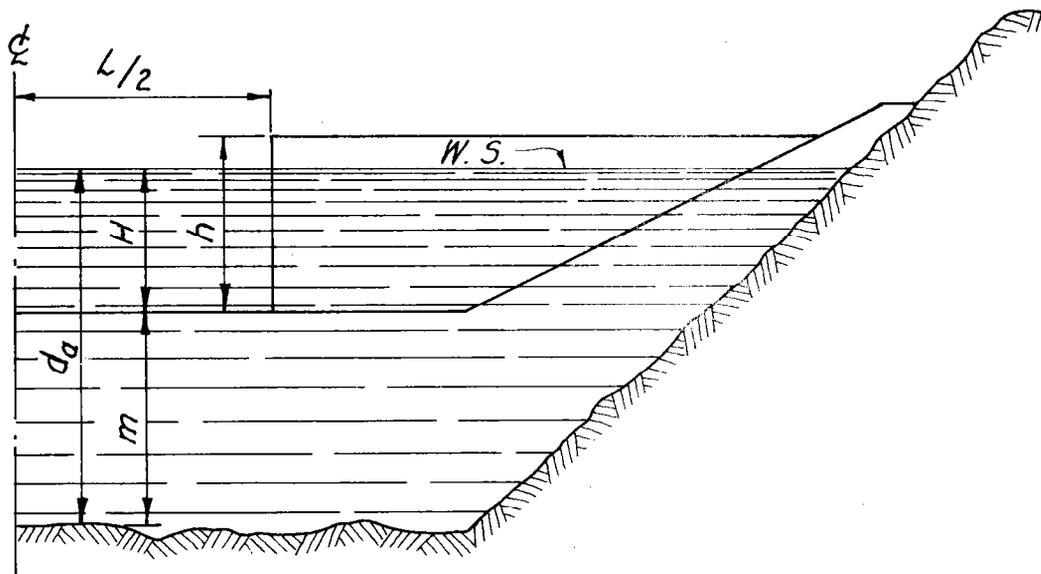
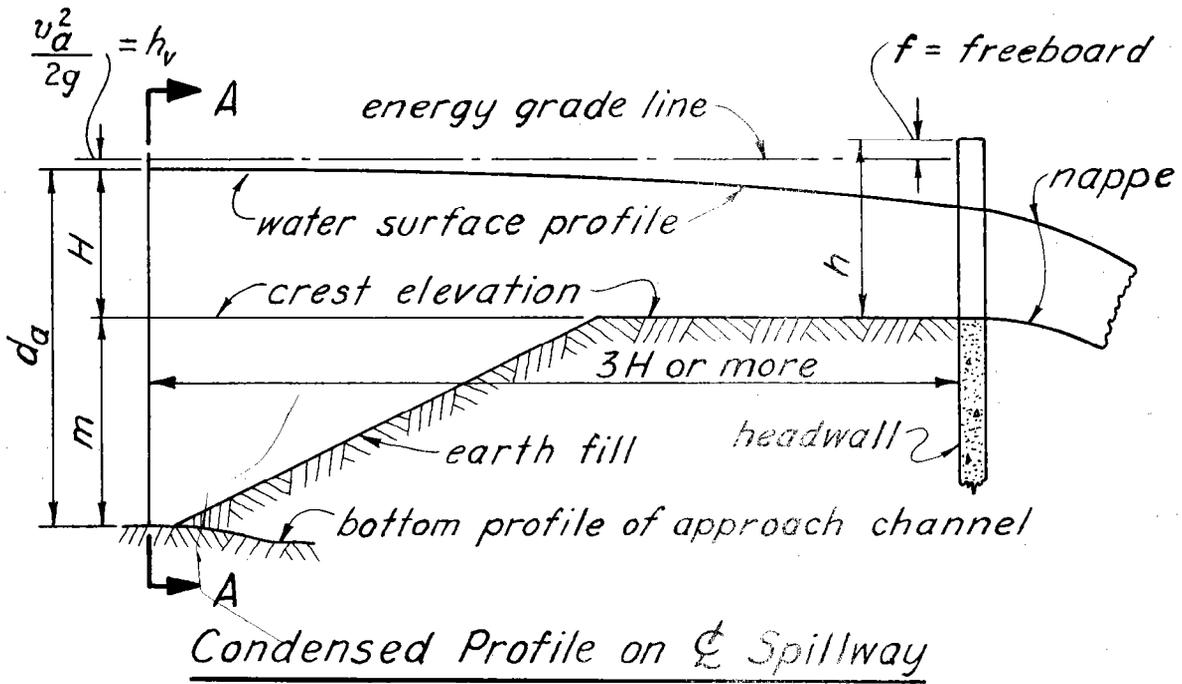
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.

ES-81

SHEET 1 OF 1 SHEETS

DATE 7-10-53



Section AA

Half Cross Section of Approach Channel showing Projected Weir Opening

FIGURE 3.1

From equation 3.1 (page 3.1) the discharge is a function of  $H + (v_a^2 + 2g)$ , so that in the determination of weir dimensions it is necessary only to determine the sum of  $H$  and  $(v_a^2 + 2g)$  since their sum is all that is required to determine discharge.

If, for some reason, it is necessary to know the upstream stage-discharge curve for such a weir, it can be found by the following procedure.

Step 1. Assume various discharges and compute the sum  $H + (v_a^2 + 2g) = [Q + (CL)]^{2/3}$  from equation 3.1 (page 3.1).

Step 2. Determine  $m$  for section AA from physical measurement for the problem at hand and add it to the values of  $H + (v_a^2 + 2g)$  obtained in step 1 to get the specific energy at the section.  $H_e = d_a + (v_a^2 + 2g) = m + H + (v_a^2 + 2g)$  (see fig. 3.1, page 3.4);  $m$  will be positive if channel bottom is below crest elevation at section AA and negative if above.

Step 3. For each value of  $H_e$  determined in step 2, find the velocity head,  $(v_a^2 + 2g)$ ,  $d_a$  and  $H$  at section AA by systematic trial and error. This step is explained best by an example as follows:

### Example 3.1

#### Known:

1. Channel dimensions at section AA: bottom width,  $b = 40.0$  ft; side slopes 2 to 1, or  $z = 2$ ;  $m = -0.10$  ft (i.e., bottom of approach channel is above crest)
2. Weir dimensions:  $L = 30.0$  ft;  $h = 5.0$  ft
3. Discharge,  $Q = 905$  cfs
4. Coefficient of discharge,  $C = 3.1$

#### Find:

1. Velocity head at section AA
2. Velocity of approach,  $v_a$ , at section AA
3.  $H =$  stage of water surface above crest of weir at section AA.

#### Procedure:

Substep 1.  $H + (v_a^2 + 2g) = [Q + (CL)]^{2/3} = [905 + (3.1 \cdot 30)]^{2/3} = 4.56$  ft

Substep 2.  $H_e = m + H + (v_a^2 + 2g) = -0.10 + 4.56 = 4.46$  ft

Substep 3. Prepare a table as follows: Assume trial values of  $d_a$  and for each such assumed value, compute  $a_a$ ,  $v_a$ ,  $(v_a^2 + 2g)$ , and  $H_e$  and compare with actual value of  $H_e$  obtained in step 2. Interpolate to get the value of  $d_a$  which is associated with the actual value of  $H_e$  and then compute the velocity head  $(v_a^2 + 2g) = H_e - d_a$ .

Trial Value of $d_a$	$a_a$	$v_a = \frac{Q}{a_a}$	$\frac{v_a^2}{2g}$	$H_e = d_a + \frac{v_a^2}{2g}$	Remarks
3.80	180.9	5.00	0.39	4.19	too low
4.00	192.0	4.71	0.35	4.35	too low
4.20	203.3	4.45	0.31	4.51	too high
4.14 <sup>1</sup>	199.9	4.53	0.32	4.46	check

Interpolate

$$d_a = 4.00 + \left( \frac{4.46 - 4.35}{4.51 - 4.35} \right) \cdot (4.20 - 4.00) = 4.14$$

This computation can be tabulated easily as follows:

$$\begin{array}{r} 4.51 \text{ -- } 4.20 \\ 4.46 \\ \hline 4.35 \text{ -- } 4.00 \quad 4.00 \\ \frac{11}{16} \cdot 0.20 = \frac{0.14}{4.14} \end{array}$$

<sup>1</sup>Check made after interpolation. Thus  $(v_a^2 \div 2g) = H_e - d_a = 4.46 - 4.14 = 0.32$  ft and  $v_a = 4.53$  fps.

Substep 4. With  $m$  and  $d_a$  known, compute  $H$  from the equation  $H = d_a - m$ . In this example  $H = 4.14 - (-0.10) = 4.24$  ft.

Step 4. Plot values of  $H$  against  $Q$  to give the required stage-discharge curve.

Solution of several examples will demonstrate that where a reservoir full of water exists above the spillway without an excavated approach channel to the weir, the velocity of approach may be ignored. Then  $H$  can be computed directly from equation 3.1 (page 3.1) with  $(v_a^2 \div 2g) = 0$ , or  $H = [Q \div (CL)]^{2/3}$ . In all other cases the velocity of approach should be included in the analysis.

Freeboard. Freeboard is the vertical distance from the maximum water-surface elevation on the upstream side of the headwall extension to the top of the headwall extension for peak design discharge over the weir. It is a safety factor to provide against possible occurrence of conditions, not anticipated during the design, that would decrease the capacity of the spillway or increase the discharge requirements and to provide protection against overtopping by wave action where it can take place.

Most of the velocity head that exists at the section where  $H$  is measured (see fig. 3.1, page 3.4) will be converted to elevation head along the headwall extensions where the stream lines impinge against it. Since the energy grade line may be assumed level between section AA, fig. 3.1 (page 3.4), and the weir, and since most of the approach velocity head is regained at the headwall extension, the total weir depth is given by the equation

$$h = f + H + \frac{v_a^2}{2g} \quad 3.2$$

where  $h$  = total depth of weir in ft

$f$  = freeboard in ft

and other terms are as previously defined.

Where wave action will not occur, it is convenient and logical to consider freeboard in terms of increased weir discharge capacity. It also seems logical to assume that the required freeboard should be some function of the overfall through the drop spillway,  $F$ , since the possible damage due to failure increases with an increase in  $F$ .

Following this line of reasoning, let the maximum discharge capacity of the weir without freeboard be  $Q(1 + \delta)$ . Then from equations 3.1 (page 3.1) and 3.2 (page 3.7)

$$Q(1 + \delta) = CLh^{3/2} = CL \left( f + H + \frac{v_a^2}{2g} \right)^{3/2} \quad 3.3$$

where

$\delta$  = increase in discharge capacity of weir, expressed as a decimal, due to an increase in head on the weir equal to  $f$ .

[A study of various functional relationships between  $\delta$  and  $F$  led to the selection of the following reasonable equation

$$\delta = 0.10 + 0.01 F \quad 3.4$$

Substitution of  $\delta$  from equation 3.4 into equation 3.3 gives

$$Q = \frac{CLh^{3/2}}{1.10 + 0.01 F} \quad 3.5$$

or

$$h = \left[ \frac{Q(1.10 + 0.01 F)}{CL} \right]^{2/3} \quad 3.6$$

or

$$L = \frac{Q(1.10 + 0.01 F)}{Ch^{3/2}} \quad 3.7$$

The use of these equations will be discussed and illustrated later.

Where wave action will occur, the freeboard must be governed by anticipated wave height. Wave freeboard,  $f_w$ , is the difference in elevation between the reservoir water-surface elevation at design discharge and the top of the headwall extension. Wave height is related to wind velocity and the length of water surface subject to wind action, called length of exposure or fetch.

Stephenson's equation for wave freeboard is

$$f_w = 0.0206 D^{1/2} - 0.117 D^{1/4} + 2.5 \quad 3.8$$

where  $f_w$  = wave freeboard in ft

D = length of fetch in ft

This equation requires excessive freeboard for low dams of low failure hazard. Hence, it has been modified to reduce the freeboard requirements for drop spillways with controlled head, F, of less than 20 feet.

The recommended equations are:

(1) For values of D equal to or less than 6000 ft and F equal to or less than 20 ft, use

$$f_w = 0.000095 D + \frac{F^{1/2}}{2} + 0.27 \quad 3.9$$

(2) For values of D greater than 6000 ft and F equal to or less than 20 ft, use

$$f_w = 0.0206 D^{1/2} - 0.117 D^{1/4} + \frac{F^{1/2}}{2} + 0.27 \quad 3.10$$

(3) For values of D equal to or less than 6000 ft and F greater than 20 ft, use

$$f_w = 0.000095 D + 2.50 \quad 3.11$$

(4) For values of D greater than 6000 ft and F greater than 20 ft, use Stephenson's equation, number 3.8.

In the solution of equations 3.8 and 3.10, it is helpful to recognize that  $D^{1/4} = (D^{1/2})^{1/2}$ , i.e., the fourth root of D is equal to the square root of the square root of D.

Equations 3.9 and 3.10 have been plotted in fig. 3.2 (page 3.9).

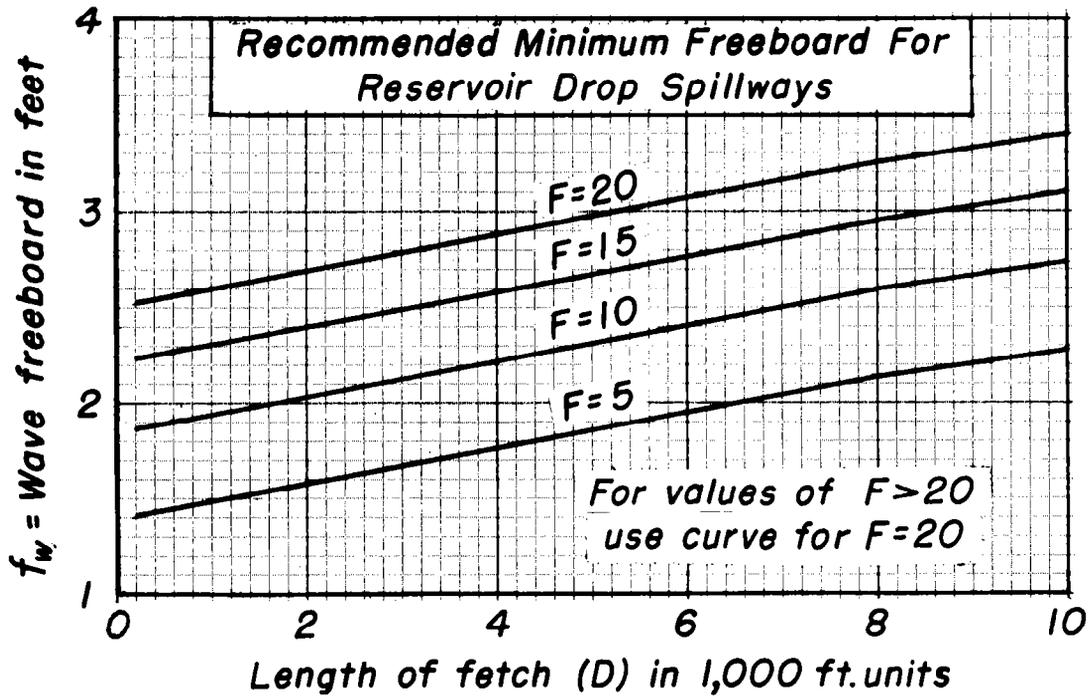


FIGURE 3.2

Example 3.2

Given:  $F = 12 \text{ ft}$ ;  $D = 3600 \text{ ft}$

Find: Required wave freeboard,  $f_w$

Solution: Since  $D$  is less than 6000 ft, substitute given data in equation 3.9 (page 3.8) and solve for  $f_w$  as follows

$$\begin{aligned} f_w &= (0.000095 \cdot 3600) + (12^{1/2} \div 2) + 0.27 \\ &= 0.34 + (3.46 \div 2) + 0.27 = \underline{2.34 \text{ ft}} \text{ Ans.} \end{aligned}$$

Note that this answer can be read directly from fig. 3.2 with sufficient accuracy by eye interpolation.

Working Procedures, Tools, and Examples for Free Flow. The usual design problem of selecting a length and depth of weir to discharge a certain required peak rate of flow is greatly facilitated by the use of drawing ES-65 (page 3.11).

Drawing ES-65 (page 3.11) provides a solution of equation 3.5 (page 3.7) in which the freeboard is a function of the controlled head as defined by equations 3.3 (page 3.7) and 3.4 (page 3.7). It has been prepared to cover the most commonly encountered range in the variables  $F$ ,  $h$ ,  $L$ , and  $Q$ . Where the range of variables on drawing ES-65 (page 3.11) does not cover the situation at hand, equation 3.5 (page 3.7) or one of the equations 3.6 or 3.7 (page 3.7) must be used.

When it is desirable to use a greater freeboard than provided for by equation 3.5 (page 3.7), as for example in a reservoir drop spillway, the required freeboard is determined and added to the value of  $H + (v_a^2 \div 2g)$  (see equation 3.2, page 3.7), which is determined from equation 3.1 (page 3.1), for the required discharge and an assumed trial value of  $L$ . To arrive at reasonable and economical weir proportions, it probably will be necessary to select several trial values of  $L$  and compute the required total weir depth,  $h$ , for each and then select the particular combination of  $L$  and  $h$  that will carry the required discharge with the desired freeboard and produce the lowest-cost structure adaptable to the site under study.

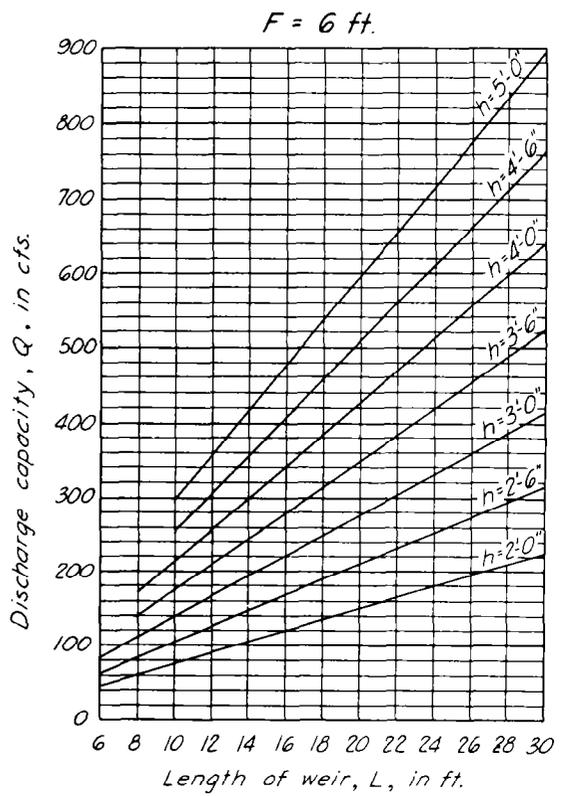
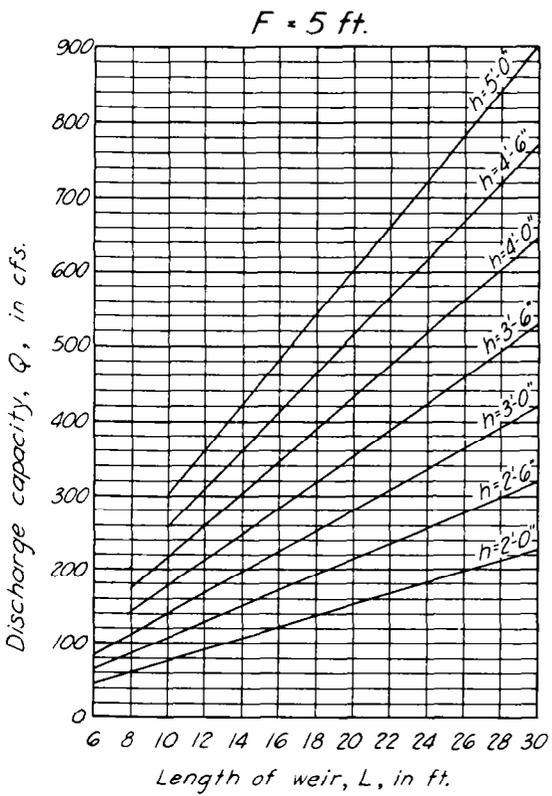
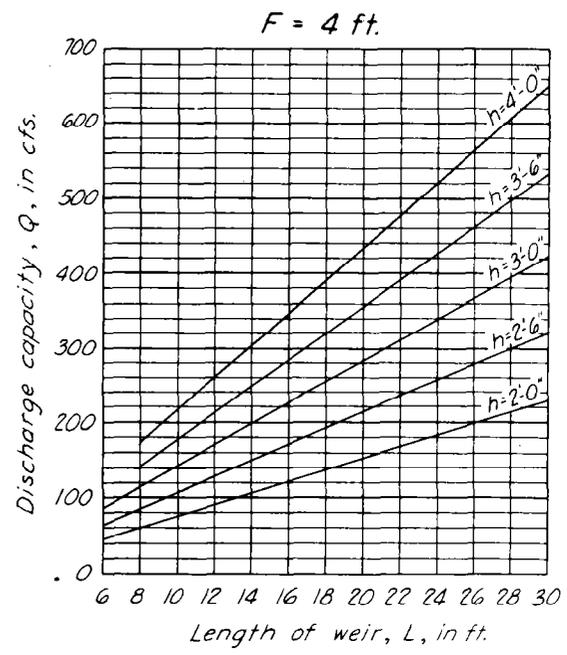
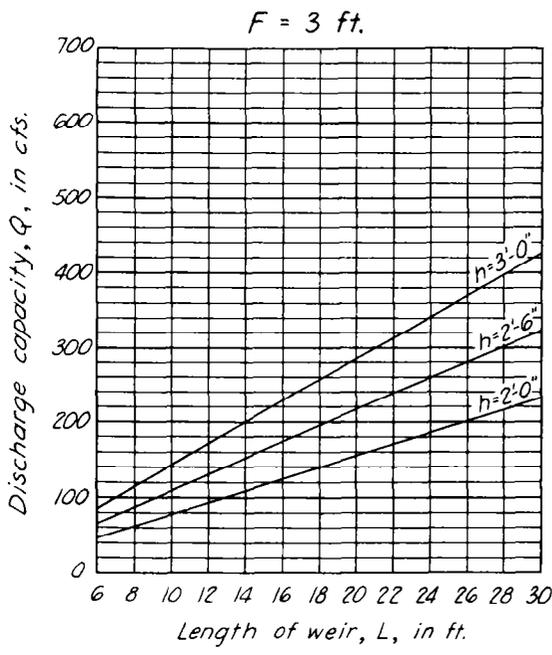
The spillway with the lowest volume of concrete is not necessarily the one which, when combined with the other items of cost, will produce the lowest cost for the entire structure, including excavation, foundation preparation, hand-compacted backfill, earth embankment, and other possible cost items. Carefully prepared cost estimates for the complete structure are necessary for the selection of the best weir proportions. Even after such comparisons have been made, other practical considerations may lead to final selection of a structure other than the one indicated by cost estimates as having the lowest installation cost. In any event, comparative cost estimates are essential as a guide to judgment.

### Example 3.3

- Given:
1. Required discharge capacity,  $Q = 340$  cfs
  2. Net drop,  $F = 8$  ft
  3. Free flow condition (unsubmerged)
  4. Use minimum freeboard as defined by equations 3.3 and 3.4 (page 3.7)
  5. Coefficient of discharge,  $C = 3.1$

- Find:
1. Reasonable combinations of length of weir,  $L$ , and depth of weir,  $h$ , that will carry required discharge capacity and provide minimum freeboard.
  2. Freeboard for one combination of  $L$  and  $h$  (to illustrate how this is done).

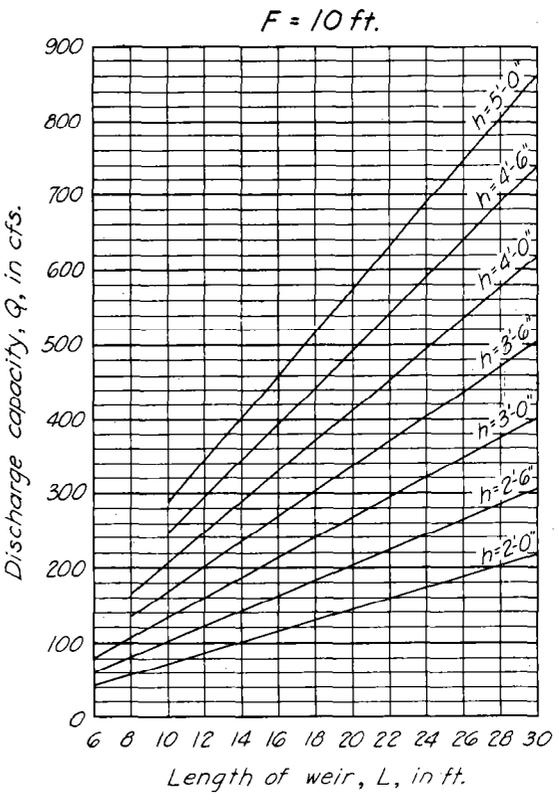
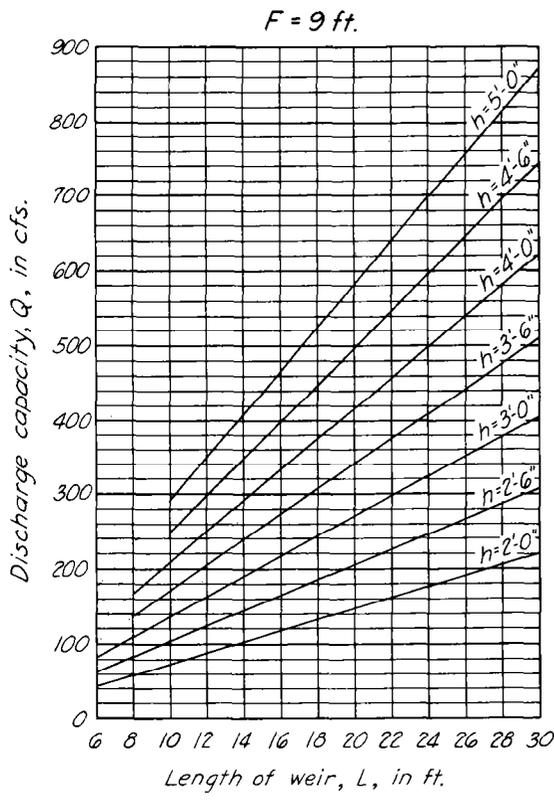
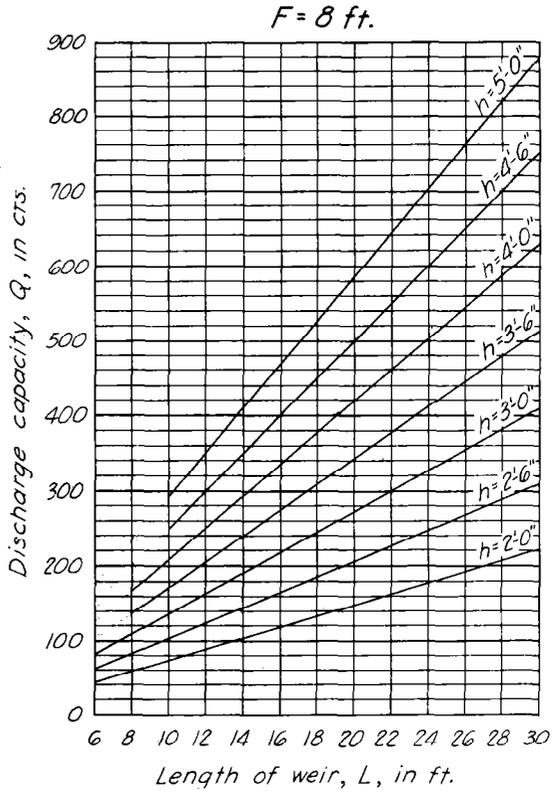
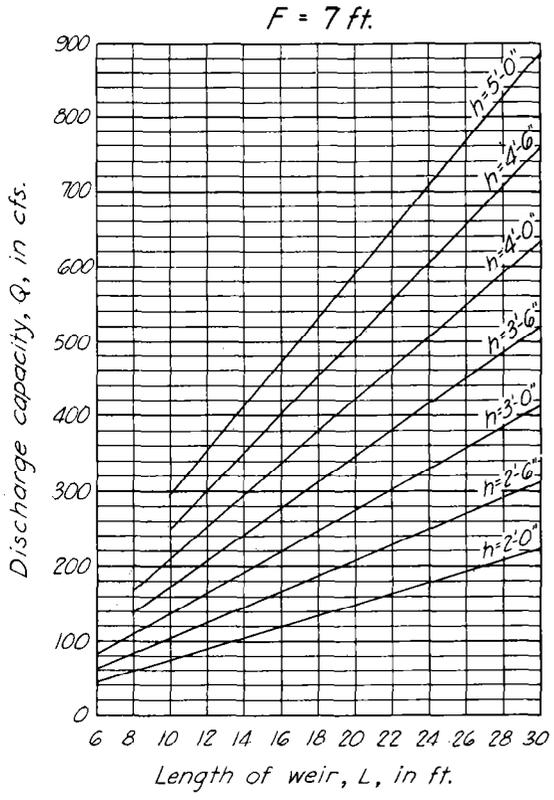
# DROP SPILLWAYS: SOLUTION OF EQUATION $Q = \frac{3.1 L h^{3/2}}{(1.10 + 0.01 F)}$



Note:  $h$  = total depth of weir, in feet (including treeboard)  
 $F$  = net drop from crest to top of transverse sill, in feet  
 (For type B drops keep  $h \div F$  less than 0.75)

<p>REFERENCE</p>	<p>U. S. DEPARTMENT OF AGRICULTURE                  SOIL CONSERVATION SERVICE</p> <p>ENGINEERING STANDARDS UNIT</p>	<p>STANDARD DWG. NO.                  ES-65</p> <p>SHEET <u>1</u> OF <u>2</u></p> <p>DATE <u>2-8-52</u></p>
------------------	---	---

# DROP SPILLWAYS: SOLUTION OF EQUATION $Q = \frac{3.1 L h^{3/2}}{(1.10 + 0.01 F)}$



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE

ENGINEERING STANDARDS UNIT

STANDARD DWG. NO  
**ES-65**

SHEET 2 OF 2

DATE 2-8-52

Solution: Use equation 3.7 (page 3.7) and substitute given values.

$$L = \frac{Q(1.10 + 0.01 F)}{Ch^{3/2}} = \frac{340 [1.10 + (0.01 \cdot 8)]}{3.1 h^{3/2}}$$

$$= 129.4 \div h^{3/2}$$

Next prepare a table as shown below; assume values of  $h$  and complete the computations indicated. Three-halves powers can be obtained from table 38, page 103 of King's "Handbook of Hydraulics," or from drawing ES-37, Engineering Handbook, Section 5 on Hydraulics.

1	2	3	4
$h$	$h^{3/2}$	$L = \frac{129.4}{h^{3/2}}$	Practical Values of $L$
3.0	5.20	24.9	25
3.5	6.55	19.8	20
4.0	8.00	16.2	16
4.5	9.55	13.6	14
5.0	11.18	11.6	12

To illustrate the method of finding the freeboard provided for a specific combination of  $L$  and  $h$  in the above table, choose  $h = 3.00$  ft and the companion  $L = 24.9$  ft. The freeboard is found by computing the head,  $H + (v_a^2 \div 2g)$ , necessary to carry the required discharge and subtracting this value from the depth of the weir,  $h$ .

From equation 3.1 (page 3.1)

$$H + \frac{v_a^2}{2g} = \left(\frac{Q}{CL}\right)^{2/3} = \left(\frac{340}{3.1 \cdot 24.9}\right)^{2/3} = 2.69$$

Then from equation 3.2 (page 3.7)

$$f = h - [H + (v_a^2 \div 2g)] = 3.00 - 2.69 = 0.31 \text{ ft}$$

For the practical value of  $L = 25$  associated with  $h = 3.00$ , the value of  $f$  is found in the same way,  $H + (v_a^2 \div 2g) = [Q \div (CL)]^{2/3} = [3.40 \div (3.1 \cdot 25)]^{2/3} = 2.68$  and  $f = 3.00 - 2.68 = 0.32$  ft

Comment: It should be noted that columns 1 and 4 of the above table can be filled in for this case directly from drawing ES-65 (page 3.11). Of course, if either  $L$  or  $h$  is fixed by site or functional requirements, the other weir size variable may be found directly from equation 3.6 or 3.7 (page 3.7).

Example 3.4

- Given:
1. Net drop,  $F = 15$  ft
  2. Free flow condition (unsubmerged)
  3. Required discharge capacity,  $Q = 2460$  cfs
  4. Reservoir immediately above spillway with length of fetch = 1800 ft
  5. Coefficient of discharge,  $C = 3.1$

- Find:
1. Wave freeboard required
  2. Combinations of  $L$  and  $h$  that will carry required discharge with the required freeboard.

Solution: The required wave freeboard can be computed using equation 3.9 (page 3.8), or it can be read directly from fig. 3.2 (page 3.9). Substituting in equation 3.9 (page 3.8), we have

$$f_w = (0.000095 \cdot 1800) + (15^{1/2} \div 2) + 0.27 = 2.38 \text{ ft}$$

From equation 3.2 (page 3.7),  $H + (v_a^2 \div 2g) = h - f$  and from equation 3.1 (page 3.1),  $H + (v_a^2 \div 2g) = [Q \div (CL)]^{2/3}$ ; hence, for this case

$$h - f_w = \left( \frac{Q}{CL} \right)^{2/3}$$

or

$$L = \frac{Q}{C(h - f_w)^{1.5}} = \frac{2460}{3.1(h - 2.38)^{1.5}} = \frac{794}{(h - 2.38)^{1.5}}$$

With  $f_w$ ,  $C$ , and  $Q$  known, assume values of  $h$  and compute  $L$ . Prepare a table as follows to facilitate the computations.

$h$	$h - 2.38$	$(h - 2.38)^{1.5}$	$L = \frac{794}{(h - 2.38)^{1.5}}$	Practical Value of $L$
7.00	4.62	9.93	80.0	80
7.50	5.12	11.59	68.5	69
8.00	5.62	13.32	59.6	60
8.50	6.12	15.14	52.4	53
9.00	6.62	17.03	46.6	47
9.50	7.12	19.00	41.8	42
10.00	7.62	21.03	37.8	38

Comment: For any selected companion set of weir dimensions, the stage-discharge curve can be determined by methods given in example 3.1 (page 3.5), the paragraph following example 3.1, and as explained in previous discussion.

Those cases where it is necessary to find the discharge capacity of a given weir operating with minimum acceptable freeboard can be solved by a direct application of equation 3.5 (page 3.7). This is illustrated by the following example.

Example 3.5

- Given: 1.  $h = 5.00$  ft;  $L = 18.00$  ft  
 2.  $F = 8$  ft  
 3.  $C = 3.1$   
 4. Free flow conditions (unsubmerged)

Find: 1. Discharge capacity of the weir operating with minimum acceptable freeboard.

Solution: Substitute the given values in equation 3.5 (page 3.7) and compute  $Q$ .

$$Q = \frac{CLh^{3/2}}{1.10 + 0.01 F} = \frac{3.1 \cdot 18 \cdot 5^{3/2}}{1.10 + (0.01 \cdot 8)} = 529 \text{ cfs}$$

Comment: For this case, the above resulting  $Q$  could have been read with sufficient accuracy directly from drawing ES-65 (page 3.11).

It should also be noted that the capacity of such a weir without freeboard =  $Q(1 + \delta) = Q(1.10 + 0.01 F)$ . In this case  $Q(1.10 + 0.01 F) = 1.18 Q = 1.18 \cdot 529 = 624$  cfs, or an 18 percent increase in discharge above that of the same weir operating with minimum freeboard.

The discharge capacity of a given weir operating with a fixed freeboard that is not dependent on  $F$  can be computed from equation 3.1 (page 3.1). This case is illustrated by the following example.

Example 3.6

- Given: 1.  $h = 5.00$  ft;  $L = 18.00$  ft  
 2.  $C = 3.1$   
 3. Free flow conditions (unsubmerged)  
 4. Wave freeboard,  $f_w = 1.80$  ft

Find: Discharge capacity of the weir operating with a freeboard of 1.8 ft

Solution: From equation 3.2 (page 3.7),  $H + (v_a^2 \div 2g) = h - f$   
 $= 5.00 - 1.80 = 3.20$  ft, and from equation 3.1 (page 3.1),  
 $Q = CL [H + (v_a^2 \div 2g)]^{1.5} = 3.1 \cdot 18 \cdot 3.20^{1.5} = 318$  cfs.

Submerged Discharge. No experimental data on submerged flow over drop spillways are available. The following material has been developed from a study of the reported test results of submerged flow over several types of weirs and over earth embankments. The data studied indicates a wide range in the effect of submergence. Hence, precise results should not be expected from submergence computations.

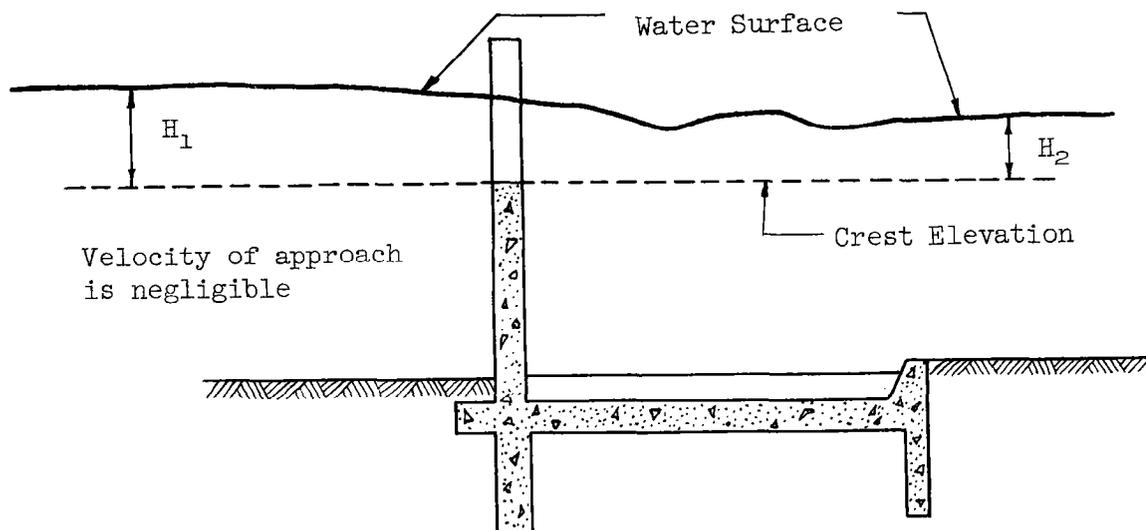
Submerged discharge is related to free discharge by the equations

$$Q_s = RQ_f \quad 3.12$$

$$q_s = Rq_f \quad 3.13$$

where  $Q_s$  = submerged discharge in cfs  
 $Q_f$  = free discharge in cfs  
 $q_s$  = submerged discharge per foot length of weir in cfs =  $Q_s \div L$   
 $q_f$  = free discharge per foot length of weir in cfs =  $Q_f \div L$   
 $R$  = ratio as defined by equations 3.12 and 3.13 (page 3.15)

Analysis of available submergence data resulted in the preparation of fig. 3.4 (page 3.17) which gives the relationship between  $R$  and the ratio ( $H_2 \div H_1$ ) in graphical form. The values  $H_2$  and  $H_1$  are defined below and illustrated in fig. 3.3.



SUBMERGED DROP SPILLWAY

FIGURE 3.3

$H_2$  = submergence = difference in elevation between tail-water and crest of weir in ft

$H_1$  = upstream head on weir with negligible velocity of approach = specific energy of flow at the weir where velocity of approach is significant

From the definition of  $H_1$

$$H_1 = H + (v_a^2 \div 2g) \quad 3.14$$

Then with  $C = 3.1$ , equation 3.1 (page 3.1) becomes

$$Q_f = 3.1 LH_1^{3/2} \quad 3.15$$

and

$$q_f = 3.1 H_1^{3/2} = Q_f \div L \quad 3.16$$

The substitution of  $H_1 = H + (v_a^2 \div 2g)$  from equation 3.14 (page 3.16) into equation 3.2 (page 3.7) gives

$$H_1 = h - f \quad 3.17$$

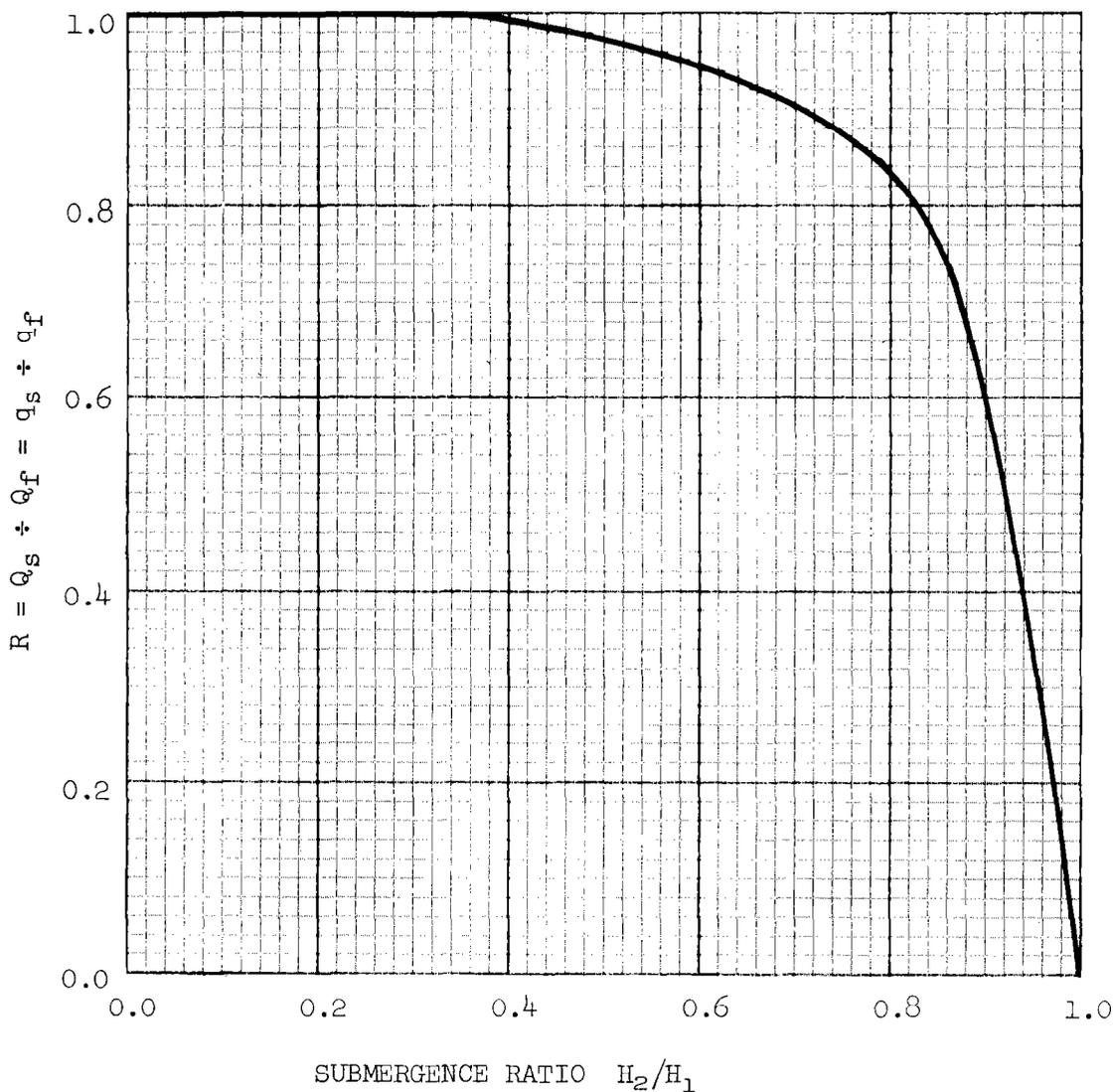


FIGURE 3.4

In a consideration of the effect of submergence, one must first recognize those situations where the effect is apt to exist. Reasonably accurate tailwater elevations will be dependent upon water surface profiles, for various discharges, computed upstream from control points where the stage-discharge relationship is known. See Engineering Handbook, Section 5 on Hydraulics for methods of computing water surface profiles.

Submergence is apt to exist in situations illustrated by the following spillway locations:

1. At the upper end of a drainage ditch where the spillway is designed for a discharge capacity greater than the average bank-full capacity of the drainage ditch below and where the spillway crest elevation is below average ground or bank elevation so as to provide a definite approach channel to the spillway for low flows.

2. Where the spillway is located in a relatively deep gully or channel just upstream from a highway culvert and earth fill, which require and permit a considerable head on the culvert to discharge the spillway design flood.

3. Where the spillway is just upstream from a retarding reservoir in which the maximum flood stage is above the crest of the upstream spillway. Special attention must be given to the element of time in such a problem; will the spillway above the reservoir be required to pass peak or near peak discharge when the reservoir is at or near peak stage? It is quite possible to have a situation in which discharge from other lateral gullies or waterways (runoff from intervening areas) might put the reservoir at or near peak stage at the same time that the spillway under consideration is required to carry maximum discharge.

4. Those in which the channel below the spillway is so flat in grade, so small in cross-sectional area, or so high in resistance to flow that its stage-discharge curve indicates water surface elevations above the spillway crest.

The above possible submergence situations illustrate that the stage just below the spillway may be the primary result of water that has passed over the spillway or of water from some other source. Where the primary source of water producing submergence is above the spillway, remember that the water must first pass through the spillway before it can produce submergence.

Graphs of stage or water-surface elevation just below the spillway, as a function of discharge through the spillway, are valuable and often necessary tools in the solution of submergence problems.

Examples for Submerged Flow. The design of a submerged weir can be accomplished most easily by following a systematic procedure such as outlined and illustrated below. In design, the problem usually resolves itself into one of selecting a certain set of companion values for  $h$  and  $L$  such that the weir will pass the required discharge rate with a predetermined safe freeboard while operating under tailwater conditions that fix the tailwater elevation (and hence the submergence of  $H_2$ ) for the design discharge. It is wise to select a somewhat higher freeboard where submergence is of consequence, because of the possible inaccuracies and uncertainties that exist in the computation of the submergence effect.

#### Example 3.7

- Given:
1.  $Q = 480$  cfs = required discharge capacity
  2.  $H_2 = 2.46$  ft = submergence for  $Q = 480$  cfs
  3.  $f = 0.75$  ft = selected freeboard
  4.  $C = 3.1$  = discharge coefficient

Find: Practical combinations of  $h$  and  $L$  for a weir that will carry the required peak discharge rate with the associated submergence and the chosen freeboard.

Solution: Obviously,  $H_1$  must exceed  $H_2$  for any discharge to take place. The procedure then becomes a matter of selecting values of  $h$  such that  $H_1$  is greater than  $H_2$  and finding companion values of  $L$  as indicated in the tabulation below.

Column 1 lists the assumed values of  $h$ .

Column 2 gives  $H_1$  as computed from equation 3.17 (page 3.17) for each assumed value of  $h$ .

Column 3 gives the values of  $H_1^{3/2}$ , which can be read directly from table 38, page 103 of the third edition of King's "Handbook of Hydraulics," or they can easily be computed by slide rule.

Column 4 gives the solution of equation 3.16 (page 3.17)

Column 5 gives the ratio  $(H_2 \div H_1)$  and is found by dividing the given submergence  $H_2$  (in this case = 2.46) by the values of  $H_1$  given in column 2.

Column 6 lists the values of  $R$  that are taken from fig. 3.4 (page 3.17) for each value of the ratio  $(H_2 \div H_1)$  given in column 5.

Column 7 gives the solution of equation 3.13 (page 3.15) for values of  $R$  and  $q_f$  given in columns 6 and 4.

Column 8 lists the results of dividing the total required discharge capacity,  $Q$  (in this case = 480 cfs) by the submerged discharge per foot of weir,  $q_s$ , from column 7.

Column 9 is merely the result of rounding off the values in column 8 to practical values. It is not practical to detail weir lengths to tenths or any other fraction of a foot.

1	2	3	4	5	6	7	8	9
$h$	$H_1 = h - f$	$H_1^{3/2}$	$q_f = 3.1 H_1^{3/2}$	$\frac{H_2}{H_1}$	$R$	$q_s = Rq_f$	$L = \frac{Q}{q_s}$	$L$
3.50	2.75	4.56	14.1	0.89	0.63	8.9	53.9	54
4.00	3.25	5.86	18.2	0.76	0.87	15.8	30.4	31
4.50	3.75	7.26	22.5	0.66	0.93	20.9	23.0	23
5.00	4.25	8.76	27.2	0.58	0.95	25.8	18.6	19
5.50	4.75	10.35	32.1	0.52	0.97	31.2	15.4	16
6.00	5.25	12.03	37.3	0.47	0.98	36.6	13.1	13

Comment: In the above tabulation, note the increase in efficiency of the weir, as measured by the value of  $R$ , as the value of  $h$  increases and the value of  $(H_2 \div H_1)$  decreases. In other words, for a fixed amount of submergence the effect of submergence is decreased if the depth of the weir is increased.

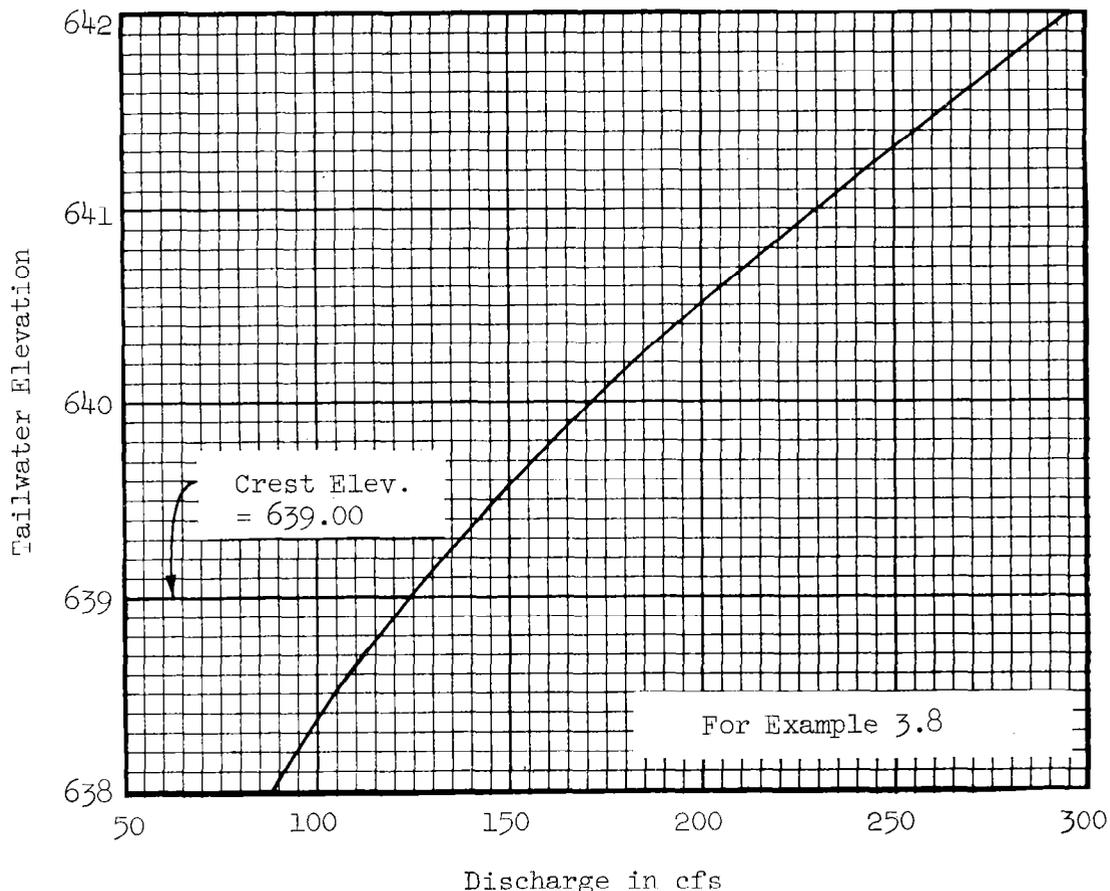
If it is necessary to design a weir with predetermined values of  $h$ ,  $f$ , and  $H_2$ , the procedure is illustrated by the computations for any one of the assumed values of  $h$  in the previous example.

Should the weir length  $L$  be fixed by site conditions or other factors, with predetermined values of  $f$  and  $H_2$ , the problem becomes one of finding the proper value of  $h$ . This problem can be solved by cut-and-try methods, but it is probably easier to prepare a table as illustrated in example 3.7 (page 3.18) for various assumed values of  $h$  and plot the relationship between  $h$  and  $L$ . Then the proper value of  $h$  can be picked from this curve for a given value of  $L$ , or the value of  $h$  can be obtained with sufficient accuracy by interpolation between known companion values of  $h$  and  $L$  that bracket the required set of conditions.

It may be necessary in some cases to compute the discharge capacity of a given structure (both  $L$  and  $h$  fixed by existing structure dimensions) that operates under submerged conditions. The solution of such a case is given in the following example.

### Example 3.8

- Given: 1. Weir dimensions,  $L = 18$  ft;  $h = 3.50$  ft  
 2. Crest elevation = 639.0  
 3. Freeboard,  $f = 0.50$  ft  
 4. Discharge coefficient,  $C = 3.1$   
 5. Stage-discharge curve for tailwater as given below



Find: Discharge capacity of the weir operating under the specified tailwater conditions and with a freeboard of 0.50 ft as specified.

Solution: The solution depends upon trial and error methods; however, a systematic approach will save time.

First compute the free flow capacity of the weir from equation 3.15 (page 3.16). As pointed out before,  $H_1 = H + (v_a^2 \div 2g)$ .

$$\begin{aligned} Q_f &= 3.1 LH_1^{3/2} = 3.1 L (h - f)^{3/2} \\ &= 3.1 \cdot 18 (3.50 - 0.50)^{3/2} = 290 \text{ cfs} \end{aligned}$$

Next prepare a table as shown below. A trial value of  $Q$  is chosen and the value of  $H_2$  for the assumed trial value of  $Q$  is read from the tailwater stage-discharge curve given above. The remaining computations are self-evident. When the trial value of  $Q$  equals the submerged discharge, the solution is complete.

Trial $Q$	$H_2$	$\frac{H_2}{H_1} = \frac{H_2}{3.00}$	$R$	$Q_s = RQ_f$ $= R \cdot 290$	Remarks
260	2.47	0.82	0.82	238	high
250	2.32	0.77	0.86	250	O.K.

Layout and Hydraulic Design Criteria. The apron, sidewalls, and wingwalls must perform functions of both structural and hydraulic character. Structurally, they must provide stability against overturning; the sidewalls and wingwalls must retain the embankment and protect it from scour; the apron protects the stream bed against the force of the overfalling water and changes the direction of the flow. In addition to these and related functions, the outlet portion of the drop, including the apron, sidewalls, wingwalls, and attached devices, should be so designed that erosion of the channel bottom and banks just below the spillway will be reduced to a practical minimum.

A considerable amount of research has been conducted to define the proper proportions of the outlet basin and wingwalls, but as yet a satisfactory set of design criteria has not been found.



#### 4. STRUCTURAL DESIGN

General. The following structural layout and design criteria and methods should not be used when  $F + h$  is greater than 20 ft or when  $F$  is greater than 15 ft. Where these limits are exceeded, a more conservative, complete, and careful analysis is required.

Proportions Required for Stability. The proportions, other than those determined hydraulically, are designed to provide a stable structure. These proportions are often mathematically indeterminate and must be based partially on the designer's experience plus careful consideration of possible mode of failure. Sliding, piping, uplift, undermining, fill slopes, and lateral scour must be considered and analyzed as accurately as possible.

The purposes of the various structural parts are as follows: The headwall extension is to permit a stable fill and to prevent piping around the structure. The cutoff wall is to prevent piping under the structure, to reduce uplift pressures, and to resist sliding. The toewall is to prevent piping under the structure and to prevent undermining of the apron. The sidewall is to hold a stable fill and protect it against erosion due to water passing over the spillway. The wingwall is to hold a stable fill and to prevent serious scour of the fill and gully banks.

The design problem varies greatly with site conditions. In locations where the ground water elevation is a considerable distance below the foundation, the foundation is permeable, and the fill around the structure is normally dry, the problems of piping and uplift are insignificant and the other dangers are greatly reduced.

Horizontal Pressures. Horizontal earth pressures are affected by numerous factors, such as the characteristics of the backfill material against the wall, the relative permeability of the foundation material and the backfill material, the elevation of the water table, and the backfill drainage provided.

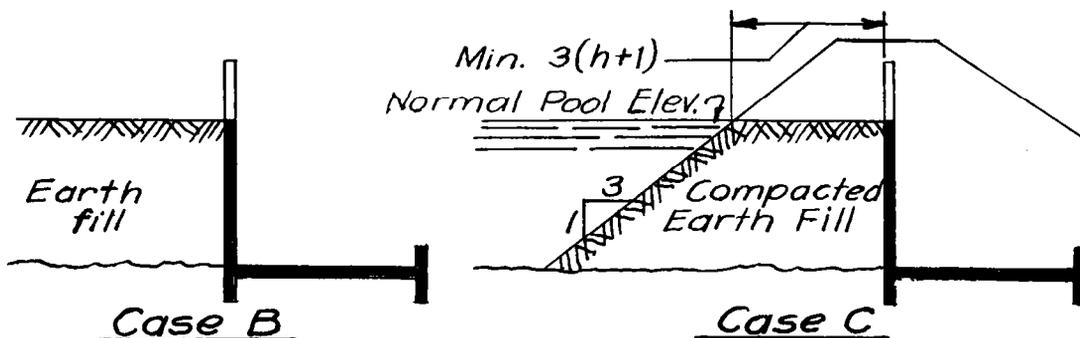
The soil characteristics that affect the horizontal earth pressures are permeability, cohesion, angle of internal friction, weight, void ratio, and moisture content. Refer to Engineering Handbook, Section 6 on Structural Design, part 2.2.2.

Loads on Headwall. The method discussed in the following paragraphs for the determination of the loads on a headwall of a drop spillway is based on judgment and past experience. It is believed that this method results in safe design values and may be applied in the design of drop spillways where  $F$  is 15 feet or less and  $F + h$  is 20 feet or less. In discussing this procedure we will first list and define the variables that affect the equivalent fluid pressures and then cite a numerical example to clarify its use. The following different conditions of backfill must be considered.

Case A. No fill against headwall; therefore, the pressure against the headwall is equal to full hydrostatic pressure.

Case B. Gully graded full to crest elevation.

Case C. An earth fill berm constructed to crest elevation.



Relative Permeability of the Foundation and the Backfill. The following 3 conditions of relative permeability will be considered: the permeability of the foundation is greater than, equal to, and less than the permeability of the backfill.

Effect of Water Table Elevation. The elevation of the water table above and below the spillway, before and after construction, has a significant effect on loads on the headwall and other elements of the design.

If the water table is low and the foundation material is relatively homogeneous and permeable, the flow of water from a reservoir or from percolation through backfill in the channel above the dam tends to pass downward through the foundation in a more or less vertical direction, until it merges with the subsurface flow. The increase in the discharge of subsurface flow will result in a rise in the ground-water elevation at the site; the amount of rise will depend upon the permeability of the foundation, the increase in ground-water discharge, and other factors. In such a case there will be no increase in horizontal pressure on the headwall, due to saturation of an earth backfill, under these conditions: (1) the rise in ground-water elevation does not create hydrostatic pore pressures on the base of the spillway, and (2) either the backfill is homogeneous and more nearly impervious than the foundation, or there is a continuous increase in permeability along the flow lines of the percolating waters.

However, if the water table is high (i.e., close to or above proposed apron elevation) prior to construction, or would be raised to such an elevation by works of improvement downstream, quite a different situation prevails. In such a case, a differential head created by the dam will result in uplift pressures on the base of the spillway and increased pressure on the headwall. The magnitude of this uplift and increased headwall pressure will depend upon the total differential head, type and efficiency of drainage provided above the headwall, relative permeability of the backfill above the headwall and various strata in the foundation, depth of cutoff and toewalls, physical characteristics of backfill and foundation soils, tailwater elevation, and perhaps other factors. With uplift is associated the possibility that the escape gradient of pore pressure below the spillway will be sufficient to cause piping.

Uplift and increased pressures on the headwall are apt to occur, even though the true water table is well below the foundation of the spillway, if a continuous layer of impervious or relatively impervious material exists in the foundation near the surface and this layer or strata is covered with permeable material. Increased pressures on the headwall are certain, and uplift pressures in excess of tailwater will occur unless all flow underneath the spillway is prevented by a watertight cutoff wall which extends well into the impervious strata. This situation is comparable in general to the situation created by a high ground-water elevation, and should be considered so in the estimation of headwall loads.

There are many shades of gray between the picture of black or white presented above. However, thorough studies in soil mechanics to define the flow net with reasonable accuracy are seldom justified in the design of average-size drop spillways. If there is reasonable doubt about the existence of a low-water-table condition, then a high-water-table condition should be assumed for the design.

Drainage of Fill Against Headwall. Two types of drains will be considered. Perforated pipe or porous concrete pipe will be used in both types and should extend a distance equal to  $F$  beyond the edges of the weir opening. The difference in the two types will be in the size and design of the gravel filters.

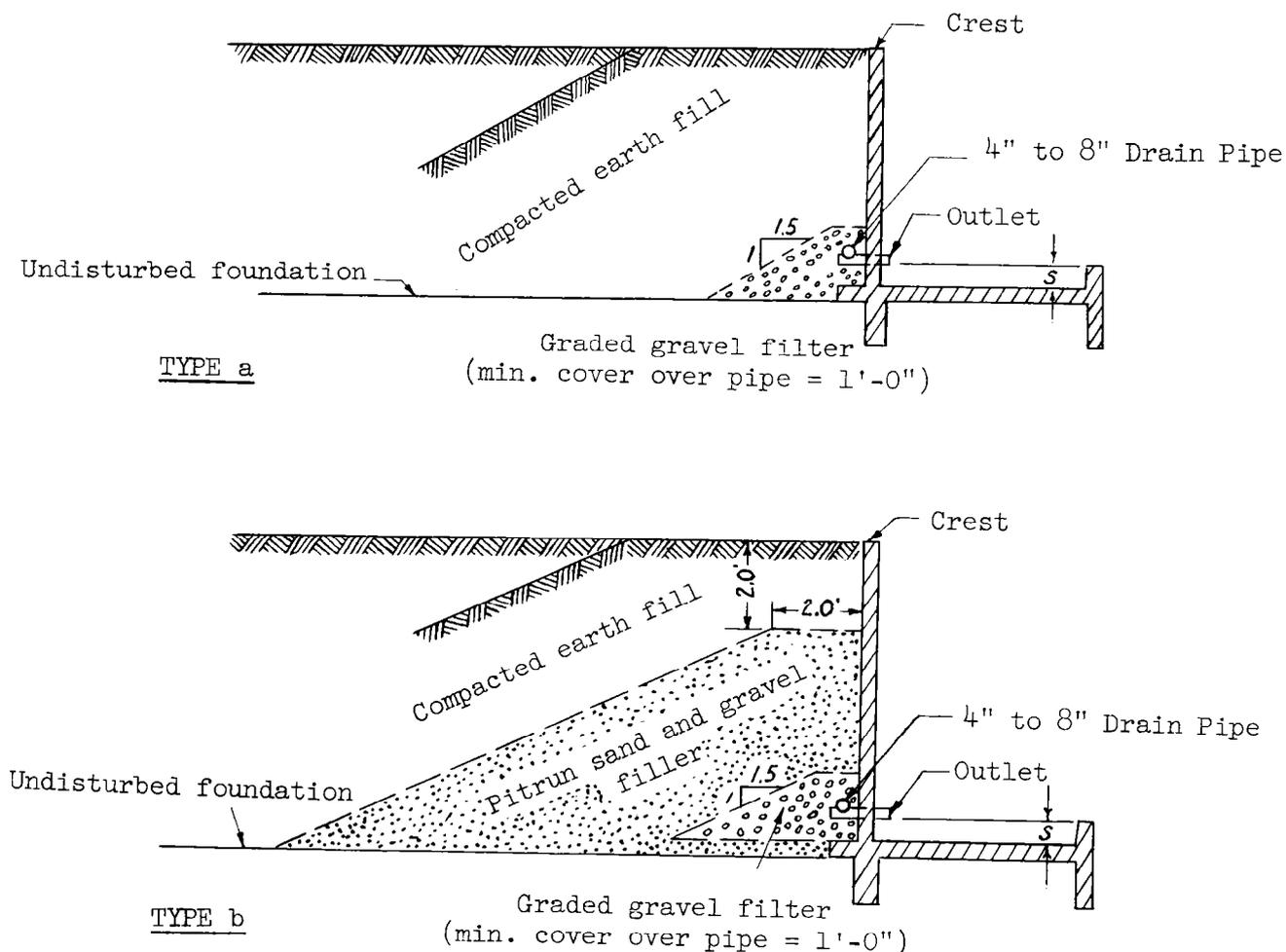


FIGURE 4.1

The criteria for the design of the gradation of the filters are discussed in the drop spillway design example.

The selection of the type of drain will be governed by economics and stability design of the structure. It may be necessary to use the type B drain to insure stability against sliding and piping. In locations where filter materials are readily available at a conservative cost, it is recommended that the type B drain always be considered.

Table 4.1 (page 4.5) furnishes a method of estimating the elevation of the saturation line ( $y_2$ ) above the top of the apron for all combinations of the variables for no-flow and design-discharge-flow conditions. When the table shows that  $y_2$  is greater than  $y_0$ , the backfill will be considered saturated to crest elevation. Such a table represents an obvious over simplification of the problem. However, reasonable care in the interpretation of foundation soil borings and conservative use of the table should give results that are practical and within permissible limits of error. Please note drainage requirements listed in table 4.1 (page 4.5).

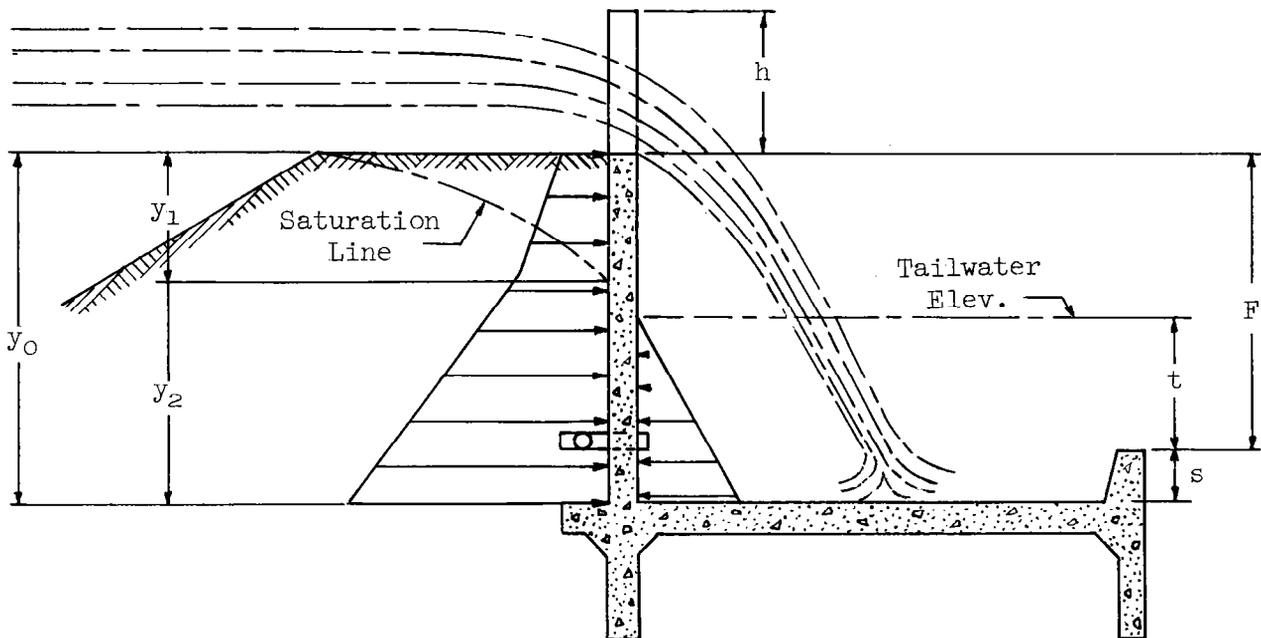


FIGURE 4.2

The headwall is designed as a slab fixed at the bottom and two vertical edges and free at the top. In order to use the moment and shear coefficients of ES-6, Engineering Handbook, Section 6 on Structural Design, the actual load diagram must be resolved into a triangular load diagram. This is done by equating the cantilever moments at the base of the headwall and solving for  $w$ , the equivalent fluid pressure producing the triangular load diagram.

This procedure, along with the use of table 4.1 (page 4.5), can best be explained by examples.

CASE	WATER TABLE	RELATIVE PERMEABILITY OF FOUNDATION TO BACKFILL	DRAINAGE	(See fig. 4.2, page 4.4) $y_2 =$		PIPING A PROBLEM		
				NO FLOW	FULL FLOW			
A	High	---	None	$y_0$	$y_0$	Yes		
	Low	---	None	$y_0$	$y_0$	No		
B	High	greater	a	$s + 0.3F$	$t + s + 0.3F$	Yes		
		greater	b	$s + 0.1F$	$t + s + 0.1F$	Yes		
		equal	a	$s + 0.4F$	$t + s + 0.4F$	Yes		
		equal	b	$s + 0.15F$	$t + s + 0.15F$	Yes		
		less	a	$s + 0.5F$	$t + s + 0.5F$	Yes		
		less	b	$s + 0.2F$	$t + s + 0.2F$	Yes		
		B	Low	greater	None	0	0	No
				equal	None	0	0	No
less	a			$s + 0.3F$	$t + s + 0.3F$	No		
less	b			$s + 0.1F$	$t + s + 0.1F$	No		
C	High	greater	a	$s + 0.4F$	$t + s + 0.4F$	Yes		
		greater	b	$s + 0.1F$	$t + s + 0.1F$	Yes		
		equal	a	$s + 0.5F$	$t + s + 0.5F$	Yes		
		equal	b	$s + 0.15F$	$t + s + 0.15F$	Yes		
		less	a	$s + 0.6F$	$t + s + 0.6F$	Yes		
		less	b	$s + 0.2F$	$t + s + 0.2F$	Yes		
		C	Low	greater	None	0	0	No
				equal	None	0	0	No
less	a			$s + 0.3F$	$t + s + 0.3F$	No		
less	b			$s + 0.1F$	$t + s + 0.1F$	No		

TABLE 4.1

Example 4.1

Given: Drop spillway with  $F = 8.0$  ft,  $h = 3.0$  ft,  $d_c = 1.80$  ft,  
 $s = 1.0$  ft,  $t = 2.5$  ft,  $H = 2.50$  ft

Relative permability; foundation = backfill

Case C (page 4.2)

Backfill properties

	Earth	Pitrun sand and gravel
dry wt. lbs/ft <sup>3</sup>	100	118
e = void ratio	0.65	0.45
percent voids	39.4	31.0
moist wt. lbs/ft <sup>3</sup>	110	125
$\phi$	25°	35°
cohesion	0	0
eff. subm. wt. lbs/ft <sup>3</sup>	62	65

Find:  $w$ , equivalent fluid pressure of triangular load diagram for

- (1) No flow, type (a) drainage
- (2) No flow, type (b) drainage
- (3) With flow, type (a) drainage
- (4) With flow, type (b) drainage

Solutions: (1) No flow, type (a) drainage:

For type (a) drainage consider the backfill as earth for the total height of the headwall.

From table 4.1 (page 4.5)

$$y_2 = s + 0.5F = 1.0 + 4.0 = 5.0 \text{ ft}$$

$p_a$  = unit active lateral earth pressure, psf

$$p_a = W \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)$$

where  $\phi$  = angle of internal friction of backfill

$W$  = vertical weight of material lbs/ft<sup>2</sup>  
 = vertical pressure

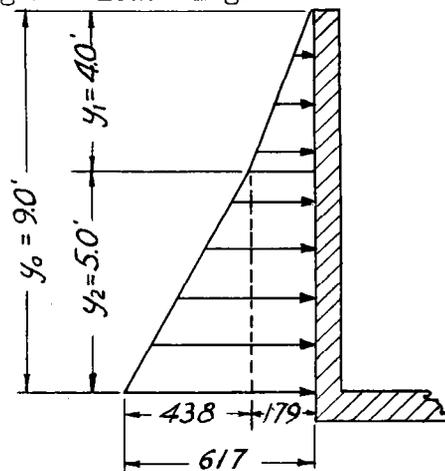
$$\frac{1 - \sin \phi}{1 + \sin \phi} = \text{ratio of lateral pressure to vertical pressure}$$

At crest elevation,  $p_a = 0$

At 4.0 ft below crest

$$\text{vert wt} = 4 \cdot 110 = 440 \text{ lbs}$$

$$p_a = 440 \left( \frac{1 - \sin 25^\circ}{1 + \sin 25^\circ} \right) = 440 \cdot 0.406 = 179 \text{ lbs/ft}^2$$



At 9.0 ft below crest

$$\text{vert intergranular pressure} = 440 + (5 \cdot 62) = 750 \text{ lbs/ft}^2$$

$$p_a = (750 \cdot 0.406) + (62.4 \cdot 5) = 617 \text{ lbs/ft}^2$$

$$\frac{wy_0^3}{6} = (179 \cdot 2.0 \cdot 6.33) + (179 \cdot 5.0 \cdot 2.5) + (438 \cdot 2.5 \cdot 1.67)$$

$$w = 6 \frac{(2270 + 2240 + 1830)}{729} = 52.2 \text{ lbs/ft}^3 = \text{unit weight of equivalent fluid}$$

(2) No flow, type (b) drainage:

For type (b) drainage consider the backfill as pit-run sand and gravel for the total height of the headwall.  
From table 4.1 (page 4.5)

$$y_2 = s + 0.15F = 1.0 + 1.2 = 2.2 \text{ ft}$$

$$p_a = W \left( \frac{1 - \sin 35^\circ}{1 + \sin 35^\circ} \right) = 0.272 W$$

At crest elevation,  $p_a = 0$

At 6.8 ft below crest

$$W = 125 \cdot 6.8 = 850 \text{ lbs/ft}^2$$

$$p_a = 850 \cdot 0.272 = 231 \text{ lbs/ft}^2$$

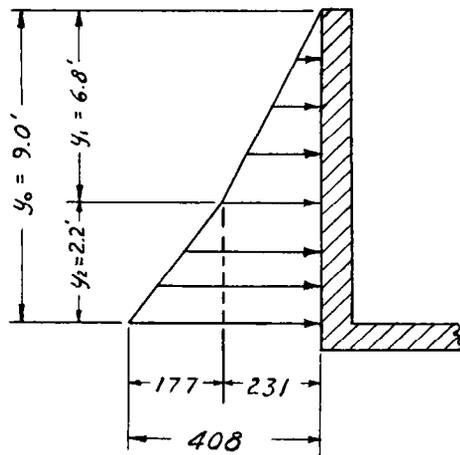
At 9.0 ft below crest

$$W = 850 + (2.2 \cdot 65) = 993 \text{ lbs/ft}^2$$

$$p_a = (993 \cdot 0.272) + (62.4 \cdot 2.2) = 270 + 138 = 408 \text{ lbs/ft}^2$$

$$w = \frac{6}{729} [(231 \cdot 3.4 \cdot 4.47) + (231 \cdot 2.2 \cdot 1.1) + (177 \cdot 1.1 \cdot 0.733)]$$

$$w = \frac{6}{729} (3510 + 560 + 143) = 34.7 \text{ lbs/ft}^3$$



(3) With flow, type (a) drainage

From fig. 5.1 (page 5.2)

$$t = 2.5 \text{ ft for } k = 1.15 \text{ ft}$$

$$t + s = 2.5 + 1.0 = 3.5 \text{ ft}$$

$$y_2 = t + s + 0.5F$$

$$y_2 = 3.5 + 4.0 = 7.5 \text{ ft}$$

At crest

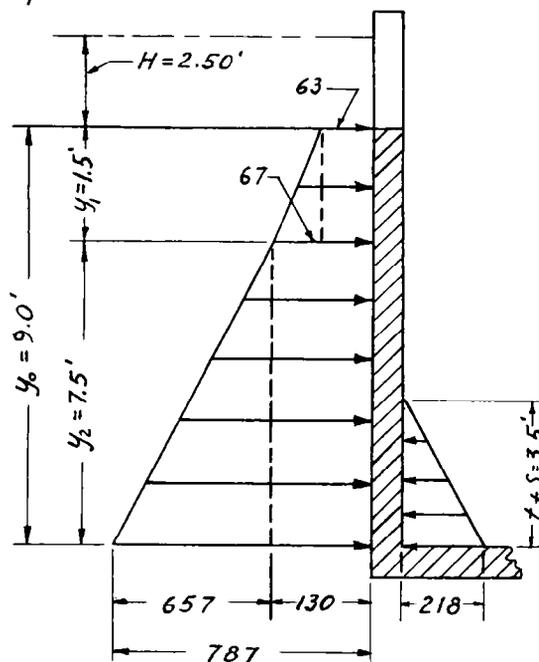
$$W = 62.4 \cdot 2.5 = 156 \text{ lbs/ft}^2$$

$$p = 156 \cdot 0.406 = 63 \text{ lbs/ft}^2$$

At 1.5 ft below crest

$$W = 156 + (110 \cdot 1.5) = 321 \text{ lbs/ft}^2$$

$$p = 321 \cdot 0.406 = 130 \text{ lbs/ft}^2$$



4.8

At 9.0 ft below crest

$$W = 321 + (7.5 \cdot 62) = 786 \text{ lbs/ft}^2$$

$$p = (786 \cdot 0.406) + (7.5 \cdot 62.4) = 319 + 468 = 787 \text{ lbs/ft}^2$$

Tailwater pressure at apron elevation

$$p = 3.5 \cdot 62.4 = 218 \text{ lbs/ft}^2$$

$$w = \frac{6}{729} \left[ (63 \cdot 1.5 \cdot 8.25) + (67 \cdot 0.75 \cdot 8.0) + (130 \cdot 7.5 \cdot 3.75) \right. \\ \left. + (657 \cdot 3.75 \cdot 2.5) - (218 \cdot 1.75 \cdot 1.17) \right]$$

$$w = \frac{6}{729} (780 + 402 + 3660 + 6160 - 445)$$

$$w = \frac{6}{729} (10,557) = 86.9 \text{ lbs/ft}^3$$

(4) With flow, type (b) drainage

$$t + s = 3.5$$

$$y_2 = t + s + 0.15F$$

$$y_2 = 3.5 + 1.2 = 4.7$$

At crest

$$W = 62.4 \cdot 2.5 = 156 \text{ lbs/ft}^2$$

$$p = 156 \cdot 0.272 = 42 \text{ lbs/ft}^2$$

At 4.3 ft below crest

$$W = 156 + (4.3 \cdot 125) = 693 \text{ lbs/ft}^2$$

$$p = 693 \cdot 0.272 = 189 \text{ lbs/ft}^2$$

At 9.0 ft below crest

$$W = 693 + (4.7 \cdot 65) = 998 \text{ lbs/ft}^2$$

$$p = (998 \cdot 0.272) + (62.4 \cdot 4.7) = 565 \text{ lbs/ft}^2$$

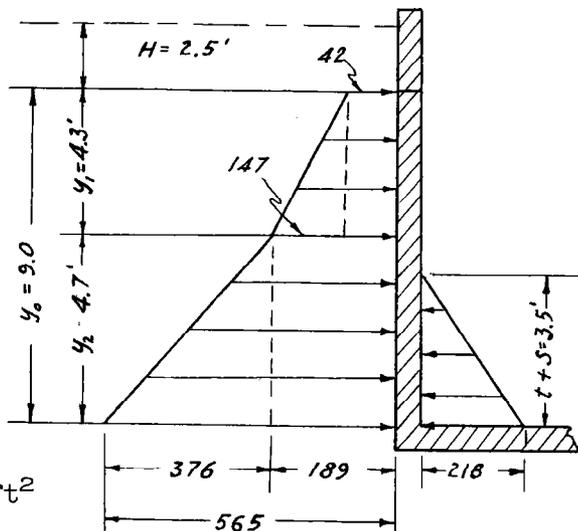
Tailwater pressure

$$p = 3.5 \cdot 62.4 = 218 \text{ lbs/ft}^2$$

$$w = \frac{6}{729} \left[ (42 \cdot 4.3 \cdot 6.85) + (147 \cdot 2.15 \cdot 6.13) + (189 \cdot 4.7 \cdot 2.35) \right. \\ \left. + (376 \cdot 2.35 \cdot 1.57) - (218 \cdot 1.75 \cdot 1.17) \right]$$

$$w = \frac{6}{729} (1237 + 1935 + 2087 + 1387 - 445)$$

$$w = \frac{6}{729} (6201) = 51.1 \text{ lbs/ft}^3$$



Loads on Sidewalls and Wingwalls. For relatively low walls, the equivalent fluid pressures shown in table 6.2-1, Engineering Handbook, Section 6 on Structural Design, may be used as a guide. In the design of large structures, which justify more careful investigations, it is recommended that the graphical method explained in paragraph 2.2.2 of the Structural Design Section be employed to determine the equivalent fluid pressure.

Loads on Headwall Extensions. When the headwall extension is designed monolithically with the rest of the structure, there is a possibility of a differential pressure in the downstream direction or in either direction at different elevations of the wall. If the structure is stable against sliding, without the passive resistance of the earth on the downstream side of the headwall extension coming into play, the differential pressure acting on the wall will be the difference in active earth pressures on both sides of the wall. If this passive resistance is required to stabilize the structure against sliding, the differential pressure will be the difference of the active pressure on the upstream side and the passive pressure on the downstream side. These differential pressures are highly indeterminate. It is, therefore, recommended that the headwall extension be designed for a differential equivalent fluid pressure of 5 to 10 pounds per cubic foot, with the assumption that it may occur in either direction.

For high headwall extension, or where the possibility of differential settlement makes the designer doubtful about using the above assumption, it is suggested that the headwall extension be made articulate from the rest of the structure. The headwall extension will then act as a diaphragm and need be reinforced only to meet the minimum steel requirements. The joint between the headwall extension and the rest of the structure must be made water tight by the use of a continuous rubber water stop or some other equally suitable device.

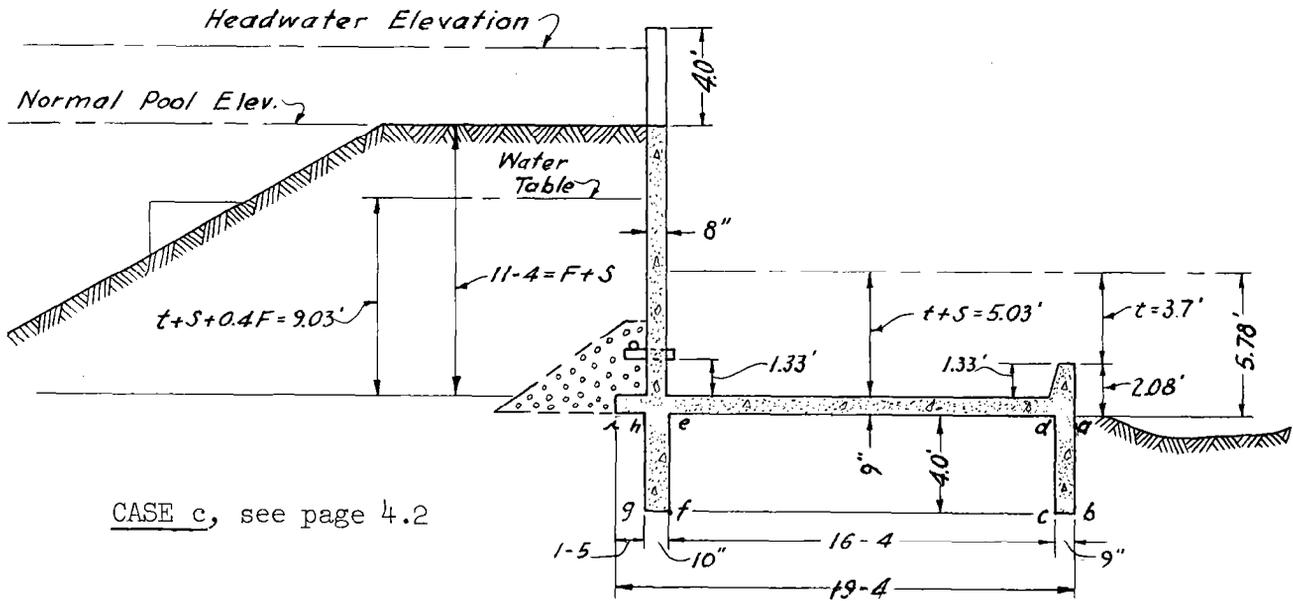
Uplift. Upward hydrostatic pressures may exist on the base of the spillway, as the result of pressure transmitted through the water in a saturated foundation material. If a differential in head exists between the elevation of the water surfaces above and below the spillway, flow or movement of the water will take place and the uplift pressures will vary with the pressure gradient.

For earth foundations, these uplift pressures are assumed to exist over the entire base area of the spillway.

Uplift pressure can be roughly estimated by the "line of creep" theory, see "Piping" page 4.14. The procedure is explained by the following example.

#### Example 4.2

Given: Drop spillway,  $F = 10.0$  ft,  $h = 4.0$  ft,  $d_c = 2.67$  ft,  $s = 1.33$  ft, and  $t = 3.7$  ft (See sketch, page 4.10). Relative permeability of foundation material is greater than fill material. Type (a) drainage used above headwall. (See page 4.3)



Find: Uplift pressures on base of structure and draw uplift diagram for with-flow condition.

Solution: Step 1. Find hydrostatic pressure at point a. Assume downstream channel has eroded to elevation of bottom of apron.

Required depth of tailwater above top of transverse sill = 3.7 ft for  $d_c = 2.67$  ft, see fig. 5.1 (page 5.2) with  $k = 1.0$ .

Depth of tailwater above point a =  $3.70 + 1.33 + 0.75 = 5.78$  ft.

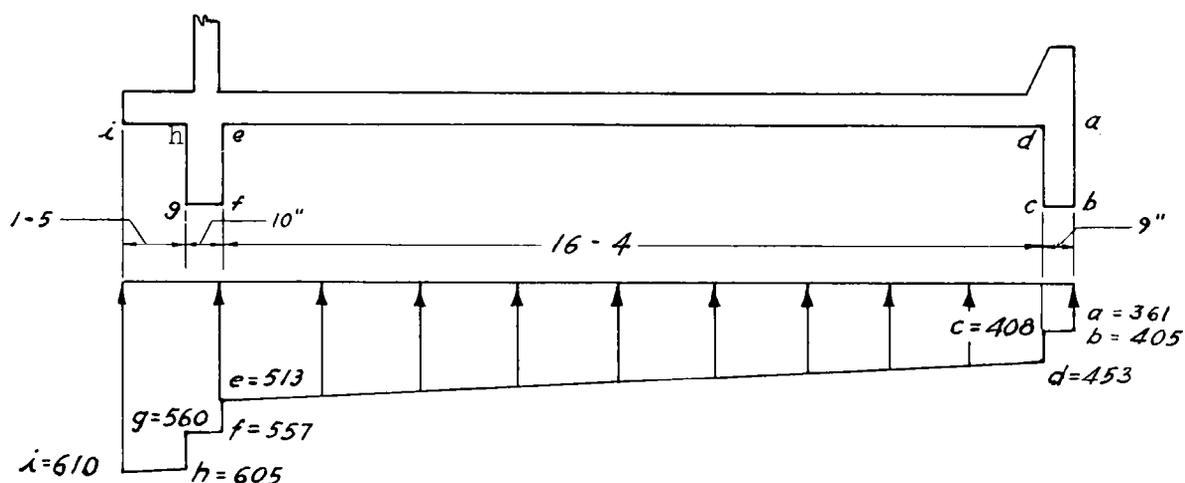
Hydrostatic pressure at point a =  $5.78 \cdot 62.4 = 361$  lbs/ft<sup>2</sup>

Step 2. Find hydrostatic pressure at point i. Estimated elevation of water table above top of apron =  $t + s + 0.4F$ , table 4.1 (page 4.5).  $t + s + 0.4F = 5.03 + 4.0 = 9.03$  ft. Elevation of water table above bottom of apron =  $9.03 + 0.75 = 9.78$  ft. Hydrostatic pressure at point i =  $9.78 \cdot 62.4 = 610$  lbs/ft<sup>2</sup>.

Step 3. Compute the total weighted creep distance and the change in pressure per foot of weighted creep distance. It is assumed that the pressures vary between points a and i in direct proportion to the weighted creep distance. The weighted creep distance =  $(bc + de + fg + hi) \div 3 + ab + cd + ef + gh = (19.33 \div 3) + (4 \cdot 4) = 22.44$  ft. The change in pressure per foot of weighted creep distance =  $(610 - 361) \div 22.44 = 11.07$  lbs/ft<sup>2</sup>.

Step 4. Calculate the pressures at various points and obtain the total uplift on a one foot slice as illustrated by the following tabulation.

Point	Weighted creep distance between points ft	Increase in pressure between points p/ft	Pressure at point psf	Average pressure between points psf	Base area between points ft <sup>2</sup>	Uplift between points lbs
a			361			
b	4.0	44.4	405			
c	0.25	2.8	408	406.5	0.75	305
d	4.0	44.4	453			
e	5.44	60.3	513	483.0	16.33	7,887
f	4.0	44.4	557			
g	0.28	3.1	560	558.5	0.83	464
h	4.0	44.4	605			
i	0.47	5.2	610	607.5	1.42	863
TOTALS	22.44	249.0			19.33	9,519



The sum of the last column in the tabulation gives the total uplift per foot width of the structure as 9,519 lbs. The uplift diagram shows the unit uplift at any point along the base.

For adequate computation of loads on the spillway apron, it is also necessary to compute the uplift pressures for the no-discharge condition. For this condition the pressure at point a is zero. At point i the pressure would be  $62.4 (s + 0.4F + 0.75) = 62.4 (1.33 + 4.0 + 0.75) = 380 \text{ lb/ft}^2$ , see table 4.1 (page 4.5).

Contact Pressures. The total upward load on the base of the spillway can be divided into 2 parts: (1) the uplift described above, and (2) contact pressures. The contact pressures are transmitted into the foundation by direct contact of the foundation material with the spillway. Obviously, the total upward load on the base of the spillway, which consists of both uplift and contact pressures, must equal the sum of all weights and other downward forces.

The distribution of the contact pressures over the base of the structure depends upon the rigidity of the structure, the characteristics of the foundation material, and the magnitude of the resultant overturning moment acting on the structure. This pressure distribution is highly indeterminate.

It is common engineering practice to assume that the vertical foundation contact pressures vary in a straight-line relationship along any longitudinal section parallel to the center line of the spillway, and that these pressures are constant along any section taken at right angles to the center line. The following procedure for computing these contact pressures agrees with these assumptions.

Contact pressures should be computed for the following loading conditions:

1. Before any backfill has been placed around the spillway.
2. After all backfill has been placed, but without flow over the spillway.
3. With the spillway operating at design discharge capacity.

Good design requires that the contact pressures be compressive in nature over the entire base of the structure. Should an analysis, made in accordance with the following procedure, indicate contact pressures that tend to separate the structure from its foundation at any point (tension), the proportions of the spillway must be changed sufficiently to overcome this condition.

Headwall extensions and wingwalls should be ignored in computing contact pressures; in long weirs it is permissible to deal with a typical bay or longitudinal segment of the spillway. In either case, the area over which the contact pressures are assumed to exist will be a rectangle. It is assumed that the spillway is symmetrical about a longitudinal center line parallel to the direction of flow.

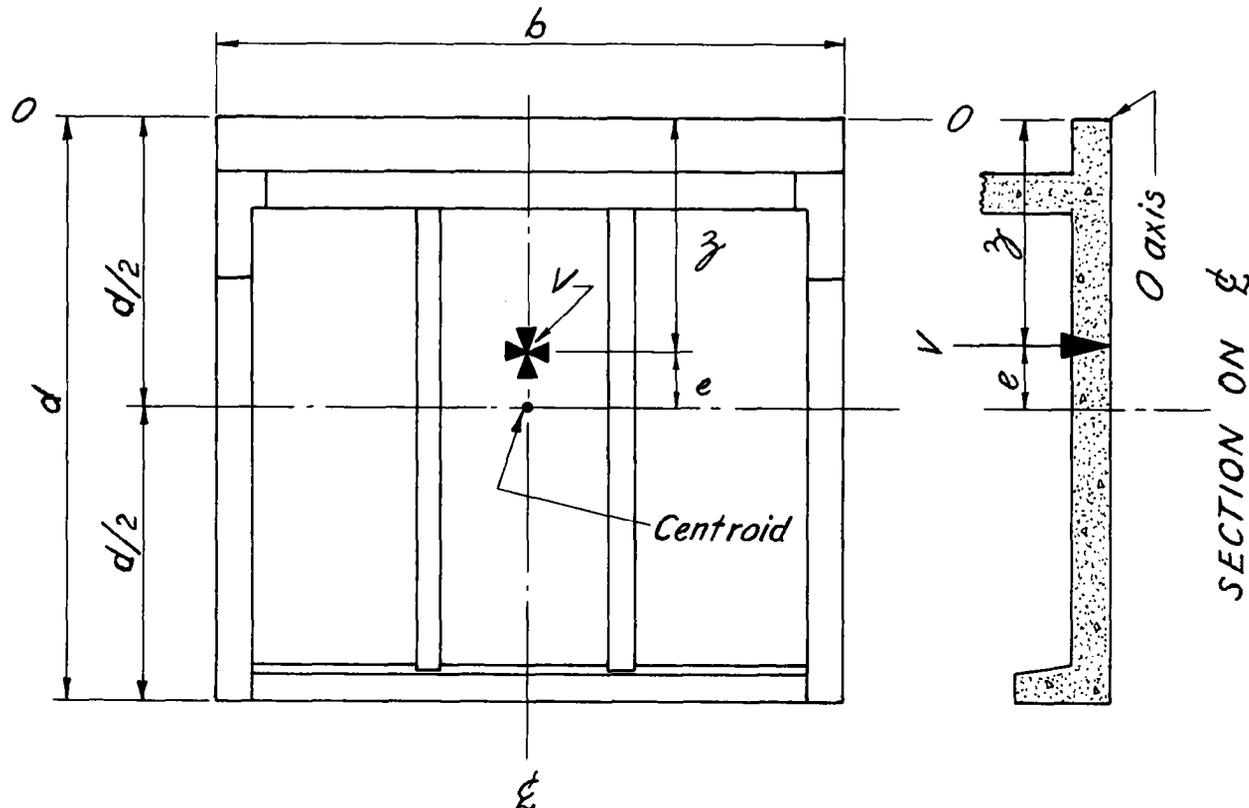
The equation for computing contact pressures for a rectangular base is:

$$p_1 = \frac{V}{A} \left( 1 \pm \frac{6e}{d} \right) \quad 4.1$$

where  $p_1$  = contact pressure at upstream or downstream edge of base (in psf)  
 $V$  = algebraic sum of all vertical loads and weights that act on the structure, including uplift (in lbs)  
 $A$  = area of base on which contact pressures are assumed to act (in ft<sup>2</sup>)  
 $e$  = eccentricity = longitudinal distance between the centroid of the base area and the point of application of the resultant vertical load  $V$  (in ft)  
 $d$  = base length = dimension from upstream edge to downstream edge of base area (in ft)

The centroid of the base area is on the longitudinal center line equidistance from the upstream and downstream edges of the base rectangle.

The area of the base,  $A$ , is equal to  $bd$  where  $b$  is the out-to-out transverse base dimension in feet.



PLAN OF BASE AREA

FIGURE 4.3

The total vertical load,  $V$ , includes the weight of all the concrete, earth above footings, water above any part of the structure under consideration, and uplift. Assume downward weights (loads) to be of positive sign; then uplift forces will be negative.

The location of the resultant  $V$  of all vertical forces including uplift can be found by taking moments about any arbitrarily selected axis. Select an axis  $O-O$  along the upstream edge of the base area at the elevation of the bottom of the apron. Let  $v_1$  be the magnitude of a part of the vertical load or weight and  $l_1$  the perpendicular distance between its line of action and the  $O$ -axis. Then the moment,  $M_V$ , of all such parts of the total vertical force,  $V$ , about the  $O$ -axis is given by

$$M_V = v_1 l_1 + v_2 l_2 + \dots + v_n l_n = \sum v_n l_n \quad 4.2$$

and

$$V = v_1 + v_2 + \dots + v_n = \sum v_n \quad 4.3$$

Next compute the moment of all horizontal loads about the O-axis. Let  $h_1$  be the magnitude of a part of the horizontal load and  $y_1$  the vertical distance from its line of action through its centroid to the O-axis, etc. Then the moment  $M_h$  of all such parts of the total horizontal force  $H$  about the O-axis is given by

$$M_h = h_1 y_1 + h_2 y_2 + \dots + h_n y_n = \sum h_n y_n \quad 4.4$$

$$H = h_1 + h_2 + \dots + h_n = \sum h_n \quad 4.5$$

Then the distance  $z$  from the O-axis to the point of application of the resultant vertical force  $V$  is given by

$$z = \frac{M_h + M_v}{V} = \frac{\sum h_n y_n + \sum v_n l_n}{\sum v_n} \quad 4.6$$

And the eccentricity  $e$  can be figured from relationships indicated in fig. 4.3 (page 4.13). The value of  $z$  may be either greater than or less than  $(d \div 2)$ . If  $z > (d \div 2)$ , the contact pressures at the toe or downstream edge of the base area will be greater than at the upstream edge.

The total resultant contact force acting on the foundation is made up of a vertical component  $V$  and a horizontal component  $H$  as determined by equations 4.3 (page 4.13) and 4.5 respectively. Obviously, the structure will float if the resultant  $V$  acts in an upward direction.

The uplift pressure diagram must be added algebraically to the contact pressure diagram, derived from equation 4.1 (page 4.12), to obtain the diagram of total pressures acting on the base.

The loading to be used in the design of the apron can then be determined by subtracting the weight of the apron and water above it from the total pressure diagram to give net apron load.

Piping. Piping may be defined as the removal of material from the foundation by the action of seepage water as it emerges from the soil below the dam. Failures by piping may result from subsurface erosion or heave. Subsurface erosion starts as a spring or springs near the downstream toe of the dam and progresses upstream along the base of the structure. Failure occurs when the upstream end of the eroded hole nears or reaches the upstream side of the dam. Failure by heave results when a large portion of soil near the downstream toe suddenly rises because the upward pressure of the seepage water is greater than the effective weight of the soil.

Unless the foundation is sealed with a watertight cutoff, water percolates through the foundation and emerges on the downstream side. The characteristics of the flow of this seepage water are similar to those of laminar pipe flow. The length of the path of flow and the frictional resistance to flow govern the outlet velocity of the seepage water and the pressures of the seepage water under the soil at the toe of the dam.

There are two schools of thought regarding the occurrence of seepage through earth foundations. One emphasizes the flow through the foundation material itself. The other believes that the line of least resistance is along the line of contact between the spillway and the foundation.

The "line of creep" theory produces the more usable method of design against failure by piping for structures of the size encountered in our work.

W. G. Bligh was one of the first engineers to advance the "line of creep" theory. It has been revised and refined by E. W. Lane (see "Security from Underseepage--Masonry Dams on Earth Foundations," Trans. of American Society of Civil Engineers, Vol. 100, p. 1235, 1935, and discussion in "Handbook of Applied Hydraulics," Calvin V. Davis, McGraw Hill Book Co.). This theory is based on the conclusion that the "line of creep," i.e., the line of contact between the dam and cutoffs with the foundation, will produce less resistance to percolation than another path through the foundation material. It is believed that the difficulty of securing an intimate contact between the dam and the foundation material, and the danger of unequal settlement which tends to destroy such contact, make this line of contact the one which will provide the least resistance to the flow of water.

After an intensive study of numerous existing dams on earth foundations, Lane was convinced that the majority of failures due to piping occurred along the line of creep. He also found that the majority of failures occurred to dams that had inadequate or no vertical cutoffs. These findings led him to recommend the use of a weighted creep line in which horizontal contacts with the foundation and slopes flatter than  $45^\circ$ , being less liable to have intimate contact, are assigned only one-third the resistance value of steeper contacts. In other words, the weighted creep line is the sum of all the steep contacts, plus one-third of all the contacts flatter than  $45^\circ$ , between the headwater and the tailwater along the contact surface of the dam and foundation. Should the distance between the bottoms of 2 cutoffs be less than one-half the weighted creep distance between, twice the distance between the cutoffs should be used instead of the actual line of creep between them.

Lane's recommended weighted creep ratios ( $C_w$ ), the ratio of the weighted creep distance to head, are given for various foundation materials in the following table.

Material	$C_w$
Very fine sand and silt	8.5
Fine sand	7.0
Medium sand	6.0
Course sand	5.0
Fine gravel	4.0
Medium gravel	3.5
Course gravel including cobbles	3.0
Boulders with some cobbles and gravel	2.5
Soft clay	3.0
Medium clay	2.0
Hard clay	1.8
Very hard clay or hardpan	1.6

TABLE 4.2

Lane's theory resolves itself into the following equation:

$$C_w = \frac{\Sigma L_H + 3 \Sigma L_V}{3H} \quad 4.7$$

where  $C_w$  = weighted creep ratio  
 $L_H$  = horizontal or flat contact distances  
 $L_V$  = vertical or steep contact distances  
 $H$  = head between headwater and tailwater

The foundation materials as listed in Lane's table do not coincide with the descriptions generally used in our work. Therefore, to obtain a working tool more applicable to our problem and to incorporate our past experience with erosion control structures, the following table of weighted creep ratios is recommended for our use.

Material	$C_w$
Clean gravel	5.0
Clean sand or sand and gravel mixture	6.5
Very fine sands and silts	8.5
Well-graded mixture of sand, silt, and less than 15 percent clay	5.5
Well-graded mixture of sand, silt, and more than 15 percent clay	4.0
Firm clay	2.3
Hard clay	1.8

TABLE 4.3

The appurtenances generally used in conjunction with drop spillways to guard against piping are the upstream blanket or fill, the upstream cutoff wall, and the downstream toewall. The upstream blanket should always be used in drop spillway construction when seepage is a problem, as it is an easy and economical means of protection. The upstream blanket also reduces the uplift pressures on the structure. The upstream cutoff wall serves three purposes--it safeguards against piping, reduces uplift pressures, and resists sliding. The downstream toewall serves two purposes--it safeguards against piping and protects the apron from undermining. The toewall has one detrimental effect--it increases the uplift pressures. Therefore, where deep cutoffs are required to safeguard against piping, it may be necessary to increase the depth of the upstream cutoff wall and decrease the depth of the downstream toewall to control the uplift pressures.

Mr. Streiff, in his discussion of Mr. Lane's paper, argues mathematically that weep holes have very little, and only localized effect in reducing pressures. Therefore, weep holes used in the sidewalls and head-wall of drop spillways will be disregarded as far as piping and uplift are concerned.

So many indeterminate variables affect the design for safety against piping that a large factor of safety is mandatory. When one considers that the coefficient of permeability varies from about 10 cm per sec for coarse gravel to  $10^{-9}$  cm per sec for dense clay, the complexity of the problem is apparent. Other factors that affect the problem are methods of construction and the variation of materials in the foundation. If various

materials are encountered in the foundation, the material having the largest weighted creep ratio should be considered as the foundation material and the design made accordingly. It is almost presumptuous to point out that adequate foundation investigations are mandatory for safe dam design.

As pointed out previously, not all structure locations present a danger of piping. If the spillway is located on a deep, permeable foundation, with a low-water table, seepage from above the dam passes downward in a nearly vertical direction until it merges with the water table. Since there is very little, or no, tendency for this seepage to flow under the spillway and emerge in the downstream channel, the problems of uplift and piping do not exist. If soil borings and the geology of the site do not positively indicate the above conditions, the high-water table condition, discussed previously, should be assumed for design purposes.

Where the water table is high, or where the high-water table condition exists because of relatively impervious layers in the foundation near the apron elevation, the data contained in table 4.1 (page 4.5) and the previous discussion on uplift can be used in the solution of the piping problem. Procedure is illustrated in example 4.3.

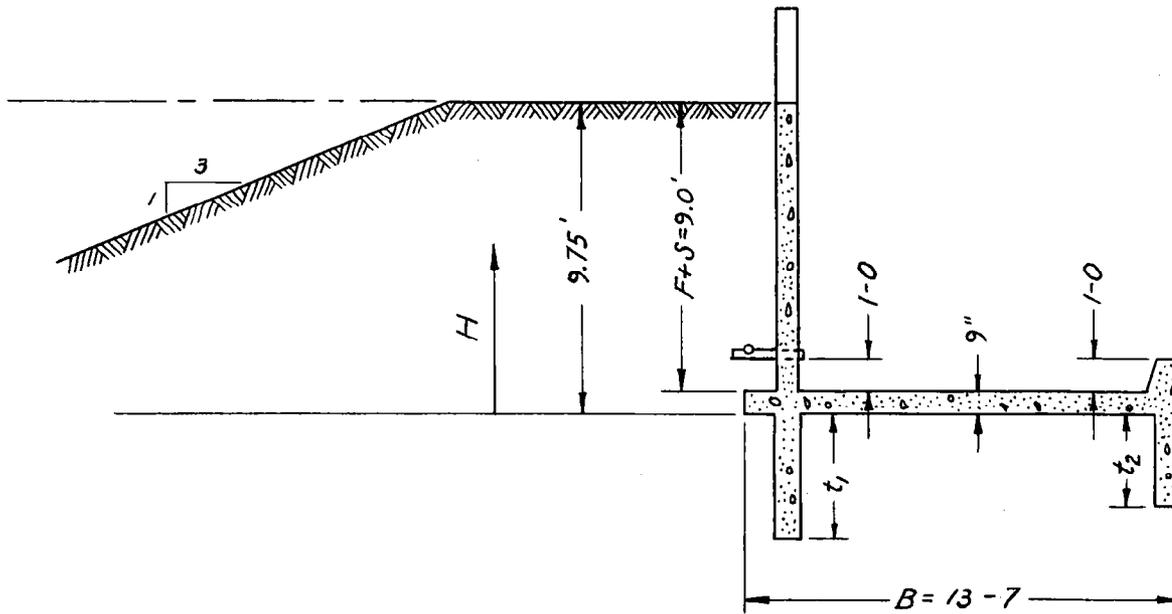
Cutoff walls may be constructed of reinforced concrete, interlocking steel sheet piling, pressure-treated Wakefield timber piling, or dense, well-compacted, impervious earth fill, or combinations of the above. The design of the cutoff will depend upon foundation conditions, availability of materials and construction equipment, cost, and other factors. Wakefield piling over 10 feet long is apt to cause trouble. Steel sheet piling works very well to considerable depth unless large rock and boulders are encountered. If the foundation is dry and the soils are stable at the time of construction, concrete cutoff walls up to 10 feet in depth should not cause undue trouble. If the foundation soils are saturated and the water table cannot be lowered, the construction of concrete cutoff walls to depths of 5 feet can be troublesome and costly. Earth cutoffs, to be impervious and effective, must be made of carefully selected, well-graded materials, placed at proper moisture content, and thoroughly compacted to high density; this is difficult to accomplish, especially if the foundation is wet. Hence, impervious earth cutoffs should normally not be used unless rock and boulders prevent the placement of driven piling; even then, it might be advisable to excavate the rock and boulders to required depth, then backfill with compacted earth and drive sheeting through it to obtain a reasonably watertight cutoff wall.

The cutoff wall must be securely connected to the remainder of the spillway and this connection must be watertight.

### Example 4.3

Given: Drop spillway,  $F = 8.0$  ft,  $h = 3.0$  ft,  $s = 1.0$  ft (see sketch, page 4.18). The foundation material is a well-graded mixture of sand, silt, and clay. Clay content, 20 percent. The relative permeability of the foundation and the fill material is estimated to be equal. A high-water table exists.

Find: The required depth of cutoff wall to insure against piping, if the depth of the toewall ( $t_2$ ) is taken as 3.0 ft, for the following conditions: (1) Pond above structure with no upstream berm against headwall (2) Pond above structure with upstream berm and type (a) drainage, and (3) Pond above structure with upstream berm and type (b) drainage, fig. 4.1 (page 4.3).



$$\text{Solution: } C_w H = B/3 + 2t_1 + 2t_2 = \frac{13.58}{3} + 2t_1 + (2 \cdot 3)$$

$$C_w = 4.0 \text{ for foundation material}$$

$$\therefore 4H = 2t_1 + 4.53 + 6.0$$

$$t_1 = 2H - 5.26$$

1. No upstream berm

$$\text{From sketch } H = 9.75$$

$$\therefore t_1 = 19.50 - 5.26 = 14.24 \text{ ft}$$

2. Upstream berm and type (a) drainage

From table 4.1 (page 4.5), Case C, no flow

$$y_2 = s + 0.5F = 1.0 + 4.0 = 5.0 \text{ ft}$$

$$H = 5.0 + 0.75 = 5.75$$

$$\therefore t_1 = 11.50 - 5.26 = 6.24 \text{ ft}$$

3. Upstream berm and type (b) drainage

From table 4.1 (page 4.5), Case C, no flow

$$y_2 = s + 0.15F = 1.0 + 1.2 = 2.2 \text{ ft}$$

$$H = 2.2 + 0.75 = 2.95$$

$$\therefore t_1 = 5.90 - 5.26 = 0.64 \text{ ft}$$

Use  $t_1 = 2.5 \text{ ft}$  (minimum depth of cutoff wall)

This example indicates the effect of the earth berm and a good drain.

If the tailwater elevation is adequate to provide the desired energy dissipation in the stilling basin of the drop spillway, the maximum head tending to cause piping will occur when there is no flow over the structure.

The danger of piping due to horizontal percolation around the headwall extension must also be considered. If the relative permeability of the abutment material is equal to, or less than, the foundation material, the minimum length of the headwall extension should be 3 times the average depth of the cutoff wall and toewall below the bottom of the apron. If the relative permeability of the abutment material is greater than the foundation material, the minimum length of the headwall extension should be 3 times the average required depth of the cutoff wall and toewall, assuming that the foundation is made of the abutment material. In the second case, in lieu of extending the headwall extension, a core trench could be excavated into the abutment to the elevation of the bottom of the cutoff wall and backfilled with a material that is considerably more impervious than the foundation material. The core trench should extend into the abutment, measured from the end of the headwall extension, a distance equal to twice the length of the headwall extension. The minimum bottom width of the core trench should be 4.0 feet and the side slopes should not be steeper than one-half horizontal to one vertical.

Overturning. The structure is safe against overturning if positive contact pressures exist over the entire base area.

Uplift. As pointed out previously, the total weight of the structure plus all vertical downward forces acting on it must be greater than the uplift forces. Should the uplift be greater than the downward force, the structure will tend to float--a situation which, obviously, cannot be tolerated.

Sliding. The horizontal forces acting on the structure in the downstream direction have a tendency to slide the structure. The horizontal resisting forces must be sufficient to withstand this tendency with a margin of safety.

In the case of a drop spillway designed and constructed as a monolithic unit, the forces resisting sliding are the frictional resistance of the foundation, the friction resistance between the sidewalls and the earth fill, the passive resistance of the earth downstream from the toewall and headwall extensions, and, during times of flow, the hydrostatic pressure of the tailwater against the headwall and wingwalls.

Past field experience indicates that drop spillways, with  $F$  equal to 10 feet or less and with headwall extensions poured monolithically with the remainder of the spillway, are safe against failure by sliding if the minimum requirements of the depth of cutoff walls and length of headwall extensions are met.

In the design of large drop spillways, however, sliding must be considered and the design made with a liberal safety factor. It is possible to make reasonable estimates of the total possible resisting forces, but it is impossible to ascertain the distribution of the actual required forces to maintain the structure in equilibrium. The designer, therefore, does not know what the design loads should be for various parts of the structure. It is wise, therefore, in large structures to design the headwall extensions and wingwalls articulate from the rest of the structure, and provide watertight joints at the junctions.

The following procedure is recommended for computing stability against sliding. The plane of sliding is assumed to be on a plane between the bottom of the cutoff wall and the bottom of the toewall. The passive resistance of the earth downstream from the toewall is neglected. A safety factor of 1.5 is recommended. Therefore, the ratio of horizontal resisting forces to the total downstream forces should be equal to, or greater than, 1.5.

Figure 4.4 shows a cross section of the headwall and apron of a drop spillway and the forces that act on a longitudinal slice of the spillway. For any loading condition the horizontal force acting above the bottom of the cutoff wall in the downstream direction is  $H$ . The total downward vertical force,  $V$ , is the weight of the structure, minus uplift, plus the effective weight of the soil between the cutoff wall and the toewall above the plane of sliding. For equilibrium to exist, the vertical component of the resultant reaction,  $R_V$ , must equal  $V$  and the horizontal component,  $R_H$ , must equal  $H$ . The force resisting sliding,  $R_H$ , is made up of two parts, the friction force,  $fV$ , and the cohesion force,  $cA$ , so that

$$H = R_H = fV + cA \quad 4.8$$

where  $R_H$  = horizontal resisting force in lbs  
 $f = \tan \phi$ , the coefficient of friction  
 $\phi$  = angle of internal friction of foundation material  
 $V$  = total vertical load in lbs  
 $c$  = cohesion resistance of foundation material in lbs/ft<sup>2</sup>  
 $A$  = area of plane of sliding in ft<sup>2</sup>

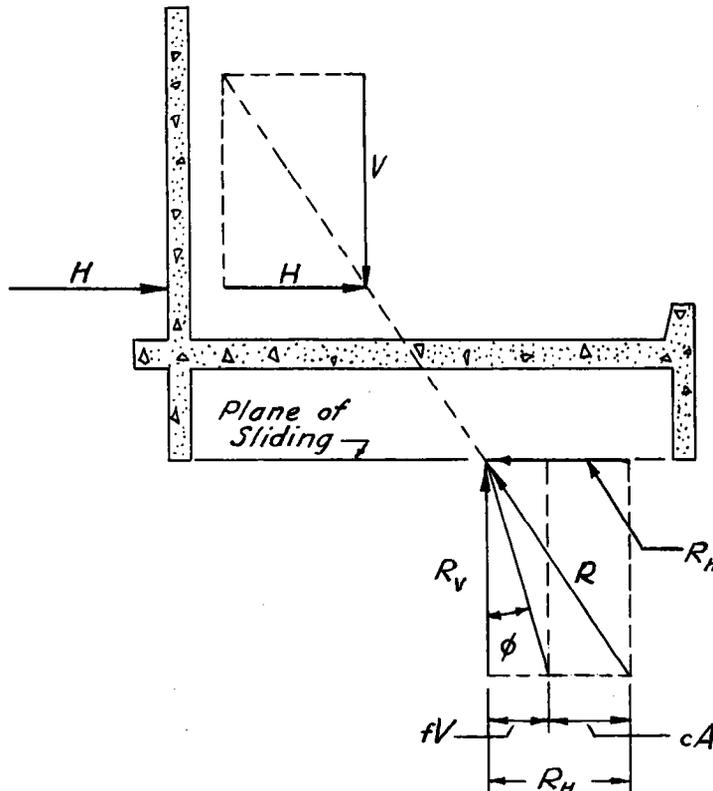


FIGURE 4.4

To provide a safety factor of 1.5, it is obvious that  $fV + cA$  must equal  $1.5H$ . If it is not possible to meet this criteria with a cutoff wall and toewall of reasonable depth, it will be necessary to provide an anchor whose pull or resistance to sliding,  $T$ , will satisfy the equation

$$T = 1.5H - fV - cA \quad 4.9$$

If an anchor is provided, it must be placed on a level with the apron and upstream from the headwall a distance equal to or greater than that given by the formula

$$X = (F + s) \cot \left( 45^\circ - \frac{\phi}{2} \right) = \frac{F + s}{\tan \left( 45^\circ - \frac{\phi}{2} \right)} \quad 4.10$$

where  $X$  = minimum distance from headwall to anchor in ft  
 $\phi$  = coefficient of internal friction of saturated backfill above the spillway  
 $F + s$  = vertical distance from crest of spillway to top of apron in ft

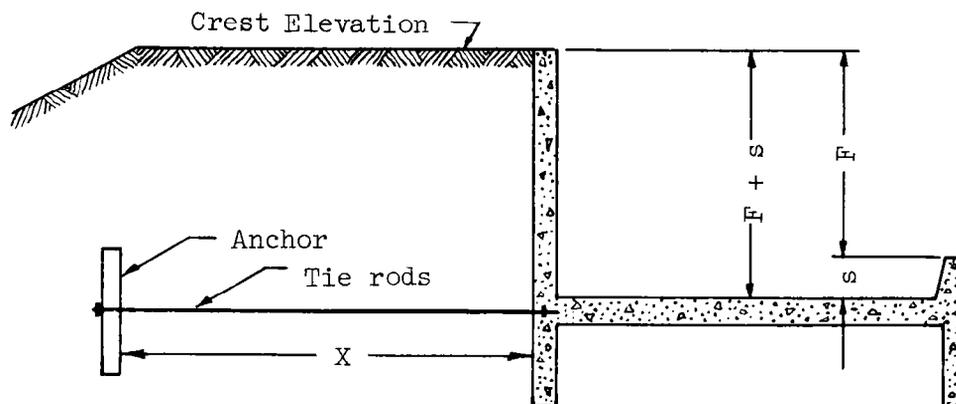


FIGURE 4.5

Passive pressures on the anchor can be computed from equations in paragraph 2.2.3, Engineering Handbook, Section 6 on Structural Design. In applying these equations, submerged weight of the backfill must be used for  $w$ .

Codes and Criteria. The design codes and criteria to be followed are given in the Engineering Handbook, Section 6 on Structural Design.

Headwall Analysis. The headwall may be designed as a slab considered fixed on three edges and free at the top in accordance with the Portland Cement Association Publication, "Rectangular Concrete Tanks," (ST-63). Drawing ES-6, Engineering Handbook, Section 6 on Structural Design is a plot of the moment and shear coefficients taken from this reference.

Sidewall Analysis. The procedure for design of the sidewall depends on the angle between it and the wingwall and on whether the sidewall and wingwall are monolithic or not. Three cases are cited below.

1. Monolithic with straight wingwall.

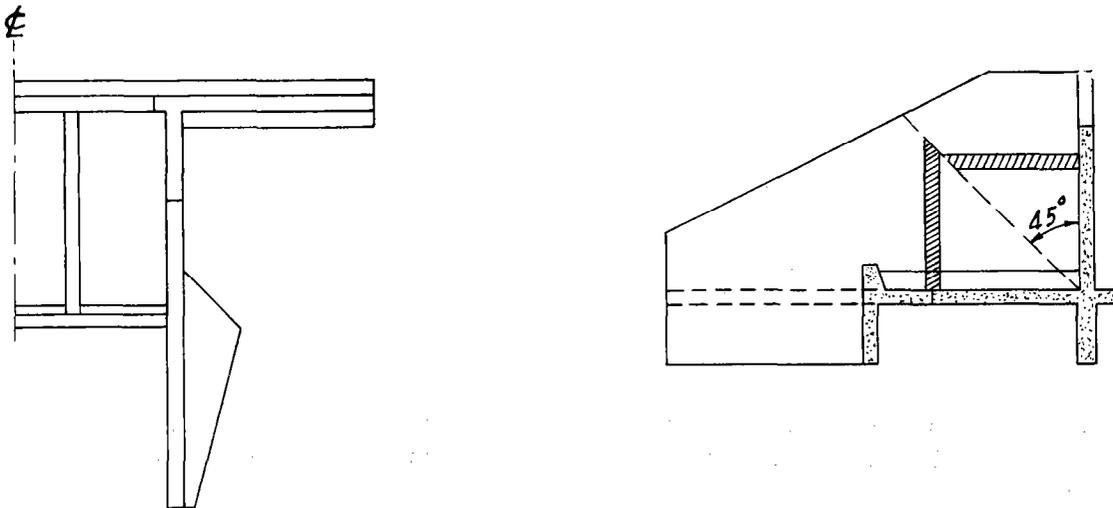


FIGURE 4.6

This sidewall may be assumed to act both as a horizontal and vertical cantilever, with the load between these two structural elements divided by a  $45^\circ$  line from the lower upstream corner of the wall as indicated in the sketch.

2. Sidewall and wingwall not monolithic, with angle between the two walls ( $\beta$ ) between  $0^\circ$  and  $45^\circ$ .

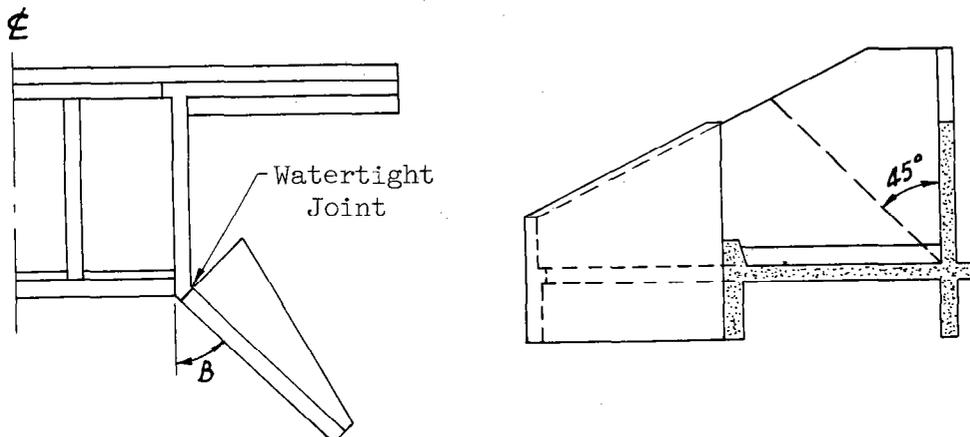


FIGURE 4.7

This sidewall may be designed in the same manner as the previous example.

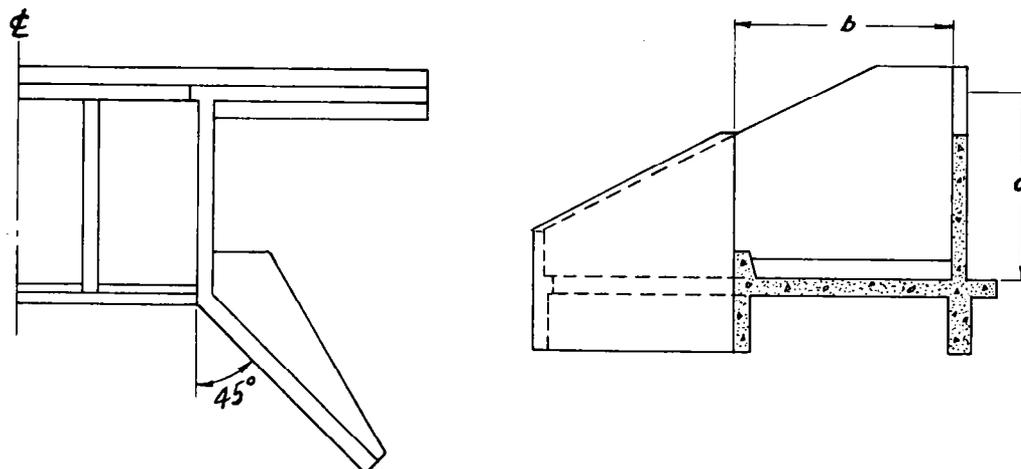
3. Sidewall and wingwall monolithic, with  $45^\circ$  angle between the walls.

FIGURE 4.8

The following procedure provides an economical design, and results in the placement of reinforcing steel where past experience and good judgment indicate that it should be.

The basic assumptions for the design of the sidewall and wingwall are as follows:

(a) The sidewall is a slab fixed along its boundaries with the headwall and the apron. The downstream vertical edge is assumed to be supported and partially restrained by the wingwall. The top edge is free.

(b) The wingwall acts both as a vertical and horizontal cantilever, with the load between these two structural elements divided by the  $45^\circ$  line indicated in the sketch below.

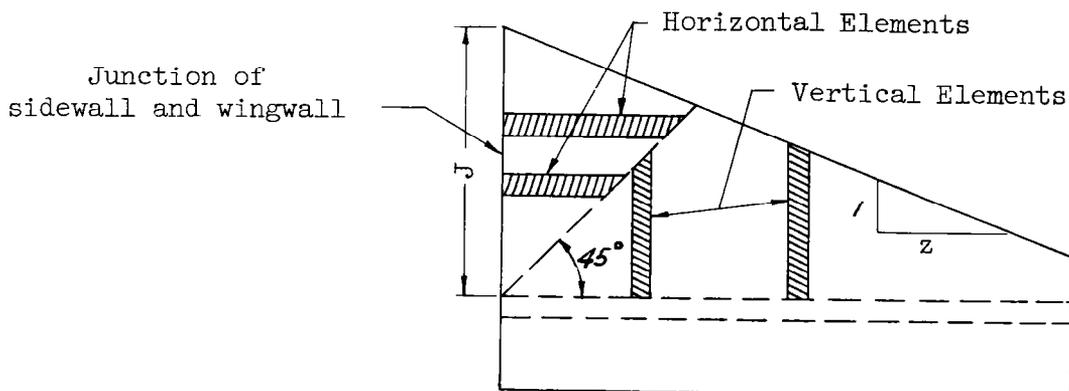


FIGURE 4.9

(c) The load distribution on both walls is triangular in the vertical plane normal to the wall.

The design procedure is outlined below by steps.

Step 1. Consider the vertical joint between the sidewall and wingwall as fixed against rotation and, from coefficients given in drawing ES-6, Engineering Handbook, Section 6 on Structural Design, determine the moments and shears in the sidewall for a slab fixed on three edges and free at the top. Take the average height of the sidewall as "a" for computing the b/a ratio.

Step 2. Compute maximum horizontal moment and maximum shear along the vertical edge of the sidewall in accordance with assumption (a), page 4.23.

$$M_S = C_m w a^3, \quad V_S = C_S w a^2 \quad 4.11$$

where  $M_S$  = maximum horizontal moment in ft lbs  
 $V_S$  = maximum shear in lbs  
 $C_m$  = moment coefficient from drawing ES-6  
 $C_S$  = shear coefficient from drawing ES-6  
 $w$  = equivalent fluid weight in lbs per ft<sup>3</sup>  
 $a$  = average height of sidewall in ft

Step 3. Compute maximum horizontal moment and maximum shear along the junction of sidewall and wingwall from the wingwall in accordance with assumption (b), page 4.23.

$$M_w \text{ is maximum when } y = J \left( \frac{z + 2}{3z + 2} \right)$$

$$M_w = \frac{w(J - y)^2}{6z} [3zy - 2(J - y)] \quad 4.12$$

$$V_w \text{ is maximum when } y = J \left( \frac{z + 1}{2z + 1} \right)$$

$$V_w = w \left[ y(J - y) - \frac{1}{2z} (J - y)^2 \right] \quad 4.13$$

where  $M_w$  = maximum horizontal moment in ft lbs  
 $V_w$  = maximum shear in lbs  
 $w$  = equivalent fluid weight in lbs per ft<sup>3</sup>  
 $J$  = height of wall from top of apron at junction of sidewall and wingwall in ft  
 $y$  = distance from top of wall in ft  
 $z$  = ratio of horizontal to vertical of the slope of the top of the wingwall

The above equations are derived from the load distribution assumptions previously described.

Step 4. Assume that the maximum moments and shears from the sidewall and the wingwall occur on the same horizontal slice, and find the centroidal tension thrust in the wingwall and sidewall necessary to counteract the shearing forces from the wingwall and sidewall for the assumed one-foot horizontal slice.

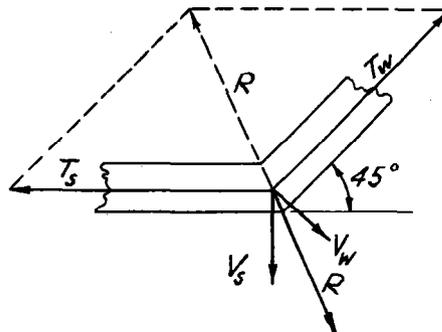


FIGURE 4.10

$$T_W = 1.414 V_S + V_W \quad 4.14$$

$$T_S = V_S + 1.414 V_W \quad 4.15$$

$T_W$  = centroidal tension thrust in wingwall in lbs

$T_S$  = centroidal tension thrust in sidewall in lbs

The following free body diagram shows all of the external forces acting on the joint of the above hypothetical slice.

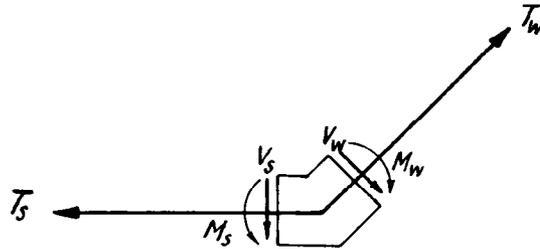


FIGURE 4.11

This joint is in equilibrium except for the moments which are unbalanced.

It is reasonable to assume that this and the other unbalanced moments along the junction of the sidewall and the wingwall will be distributed mainly into the apron, because of the rigidity of this general stress path. With this assumption, the release of the downstream vertical joint of the sidewall would have no effect on the moments along its other vertical joint at the headwall. This line of reasoning leads to the assumption that the maximum horizontal moment at the center of the sidewall should be increased by  $0.5 (M_S - M_W)$  as indicated in the sketch below.

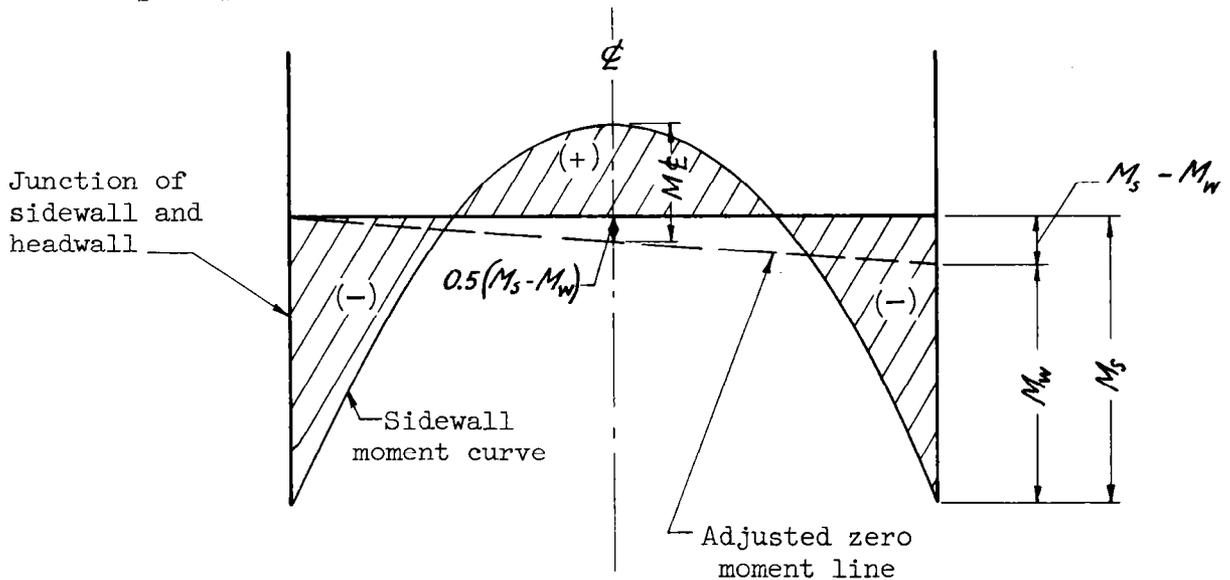


FIGURE 4.12

For conservative design, the sidewall and wingwall should be designed for the maximum moments and shears found from the above assumptions, whether they occur before or after the adjustment.

The horizontal steel in the exposed face of the sidewall will be designed for a moment,  $M = C_m w a^3 + 0.5 (M_S - M_W)$  for the full height of the wall.

The horizontal steel in the unexposed face of the sidewall at the upstream end of the wall will be designed for the maximum moment and shear as determined from drawing ES-6, Engineering Handbook, Section 6.

The horizontal steel in the unexposed face of the sidewall at the downstream end of the wall (junction with wingwall) will be designed to take the moment,  $M_S$ , plus the axial tension force,  $T_S$ . The steel required for  $M_S$  may be cut at the quarter point of the span, but the steel required by  $T_S$  should be extended to lap with the steel at the other end of the wall.

The principal vertical steel in both faces of the sidewall will be designed for the vertical moments determined from drawing ES-6.

The horizontal cantilever steel in the wingwall will be designed for the moment,  $M_S$ , plus the axial tension force,  $T_W$ .

The vertical steel in the wingwall will be determined from cantilever moments.

Wingwall Analysis. Refer to the three cases cited under sidewall analysis. The wingwall in the first two cases may be designed as a vertical cantilever. The wingwall analysis for case (3) is explained along with the sidewall design for this case. The required wingwall footing for all three cases is determined by considering the wingwall as an independent wall and making it stable against overturning.

In some cases it may be impracticable to provide sufficient resistance to sliding of the wingwall by frictional resistance on the bottom of the footing. Passive resistance on the toewall extension under the wingwall should be neglected because the fill in front of the toewall is apt to be wet and of low shearing strength when maximum loads are against the wingwall and because this fill may scour and be washed away. Where frictional resistance to sliding is not adequate the toewall, footings, and an upstream extension of the footing should all be poured monolithically with the apron and its toewall. Then the wingwall footing can be designed as a horizontal cantilever to transfer a part of the horizontal loads on the wingwall into the apron slab. This design procedure is illustrated in the structural design example.

Apron Analysis. The apron may be designed as a series of beams perpendicular to the sidewalls. The beams are considered as supported at the sidewalls and continuous with them, and continuous over the longitudinal sills or buttresses. Refer to pages 4.9 to 4.14 for the method of determining the apron loading. See drawing ES-56 (page 4.27) for moment and shear determinations.

Buttress Analysis. Buttresses for the headwalls of drop spillways of average size can usually be designed as cantilever beams. They should have a minimum width of 12 inches and a depth sufficient to carry the overturning moment which results from shears from the headwall. Vertical compressive stresses computed on the assumption of a rectangular beam should be corrected to give the maximum compressive stress parallel to the downstream face of the buttress. This method of analysis is illustrated in the structural design example.

The load to be used for the design of the buttress is the sum of the shears along the fixed vertical edges of the adjacent headwall slabs. For values of  $(b \div a)$  equal to or less than 2, the distribution and magnitude

# DROP SPILLWAY APRON DESIGN: MOMENTS AND SHEARS

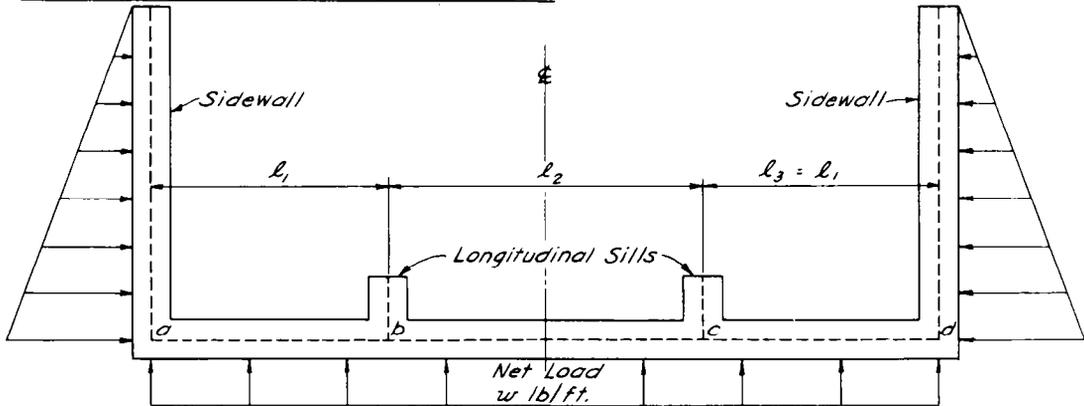
## Assumptions:

Loads and structure are symmetrical about  $\epsilon$  of structure.  
The analysis is based on  $\epsilon$  dimensions of members.

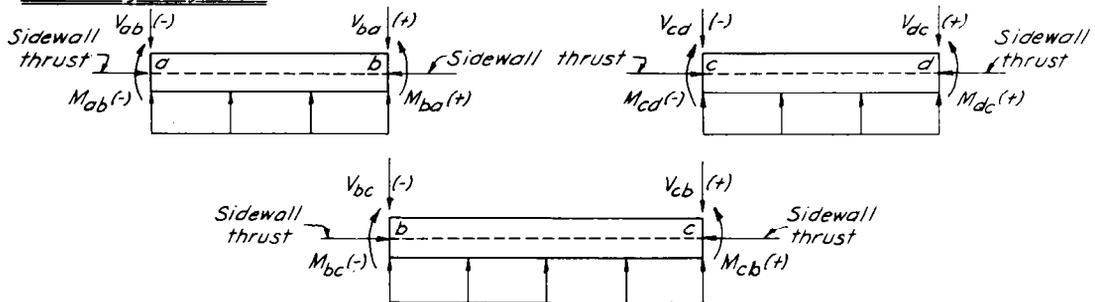
## Sign Convention:

A moment acting in a clockwise direction on a joint is positive.  
Shear acting down on the left side of a section is positive.

## Transverse Slice through Apron and Sidewall



## Free-body Diagrams:



**NOTE:**  $M_{ab}$  and  $M_{dc}$  are equal to the corresponding vertical sidewall moments but are of opposite sign.

## Moment Equations:

$$-M_{ab} = M_{dc}, M_{ba} = -M_{bc}, M_{cb} = -M_{cd}$$

When  $l_1 = l_2 = l_3$

$$M_{ba} = \frac{1}{5} M_{ab} + \frac{1}{10} w l_1^2$$

When  $l_1 = l_3$  and  $l_2$  is less or greater

$$M_{ba} = \frac{M_{ab} l_1 + \frac{1}{4} w (l_1^3 + l_2^3)}{2l_1 + 3l_2}$$

**Caution:** Use proper sign of moments and shears when substituting them in the given equations.

## Shear Equations:

$$V_{ab} = -\frac{1}{2} w l_1 + \left( \frac{M_{ab} + M_{ba}}{l_1} \right) = -V_{dc}$$

$$V_{ba} = w l_1 + V_{ab} = -V_{cd}$$

$$V_{bc} = -\frac{1}{2} w l_2 = -V_{cb}$$

## Example:

**Given** -  $l_1 = 6.0$  ft,  $l_2 = 8.0$  ft,  $l_3 = 6.0$  ft, Vertical sidewall moment = 1600 ft. lb.

**Problem** - Find moments and shears by equations. Check moments by moment distribution procedure. Plot moment and shear diagrams.

**Solution by equations**

$$M_{ab} = -1600 \text{ ft. lb.}$$

$$M_{ba} = \frac{(-1600)(6) + \frac{1}{4}(300)(6^3 + 8^3)}{(2)(6) + (3)(8)}$$

$$M_{ba} = \frac{-9600 + 54,600}{36} = +1250 \text{ ft. lb.}$$

$$M_{bc} = -M_{ba} = -1250 \text{ ft. lb.}$$

$$V_{ab} = -\frac{1}{2}(300)(6) + \left( \frac{-1600 + 1250}{6} \right)$$

$$V_{ab} = -900 - 58 = -958 \text{ lb.}$$

$$V_{ba} = (300)(6) - 958 = 1800 - 958 = 842 \text{ lb.}$$

$$V_{bc} = -\frac{1}{2}(300)(8) = -1200 \text{ lb.}$$

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE

ENGINEERING STANDARDS UNIT

STANDARD DWG. NO.

ES-56

SHEET 1 OF 2

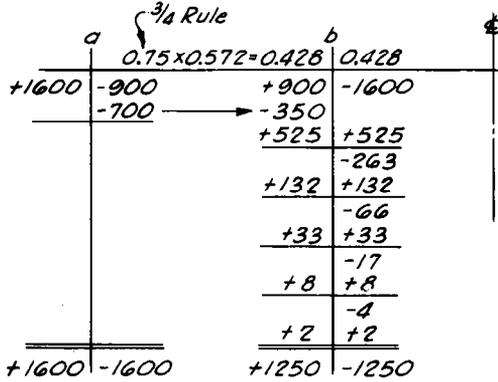
DATE 8-6-51

Revised 3-11-53

# DROP SPILLWAY APRON DESIGN: MOMENTS AND SHEARS

## Checking Moments by Moment Distribution

References: Theory of Simple Structures, Second Edition, by Shedd & Vawter.  
 No. ST40, Moment Distribution applied to Continuous Concrete Structures by  
 Portland Cement Association.



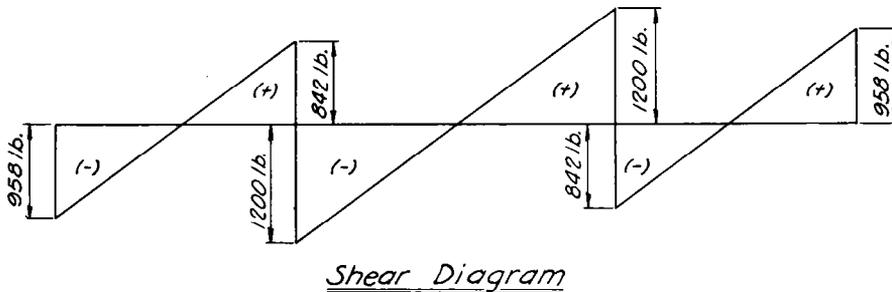
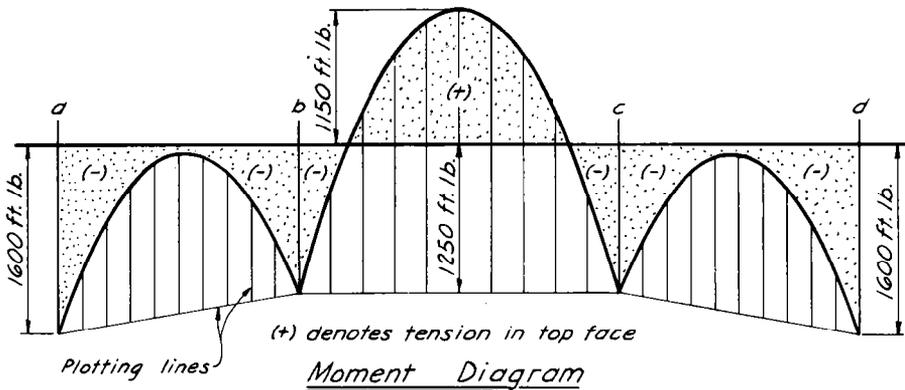
Stiffness of members,  $K = 4EI \div l$   
 $4EI$  is equal for all members  
 Let  $4EI = 8$  then  
 $K_1 = 8 \div 6 = 1.333, K_2 = 8 \div 8 = 1.0$   
 Relative Stiffness  
 of  $L_1 = \frac{K_1}{K_1 + K_2} = \frac{1.333}{1.333 + 1.0} = 0.572$   
 of  $L_2 = \frac{K_2}{K_1 + K_2} = \frac{1.0}{1.333 + 1.0} = 0.428$   
 Fixed End Moments =  $\frac{1}{2} w l^2$   
 $M_{ab} = -M_{ba} = -\frac{1}{12} (300)(6)^2 = -900 \text{ ft. lb.}$   
 $M_{bc} = -M_{cb} = -\frac{1}{12} (300)(8)^2 = -1600 \text{ ft. lb.}$

## Plotting Moment and Shear Diagrams

Simple bending moment =  $\frac{1}{8} w l^2$   
 End span,  $M^s = \frac{1}{8} w l^2$   
 $= \frac{1}{8} (300)(6)^2 = 1350 \text{ ft. lb.}$   
 Center span,  $M^s = \frac{1}{8} w l^2$   
 $= \frac{1}{8} (300)(8)^2 = 2400 \text{ ft. lb.}$

$\frac{1}{2} l$ k	$M_x$ M@€	Simple bending moments	
		In end spans	In center spans
0.1	0.36	486	864
0.2	0.64	864	1,536
0.3	0.84	1,134	2,016
0.4	0.96	1,296	2,304
0.5	1.00	1,350	2,400

See ES-1 in Structural Design  
 Section (Section 6)



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE  
 SOIL CONSERVATION SERVICE  
 H. H. Bennett, Chief  
 ENGINEERING STANDARDS UNIT

STANDARD DWG. NO.

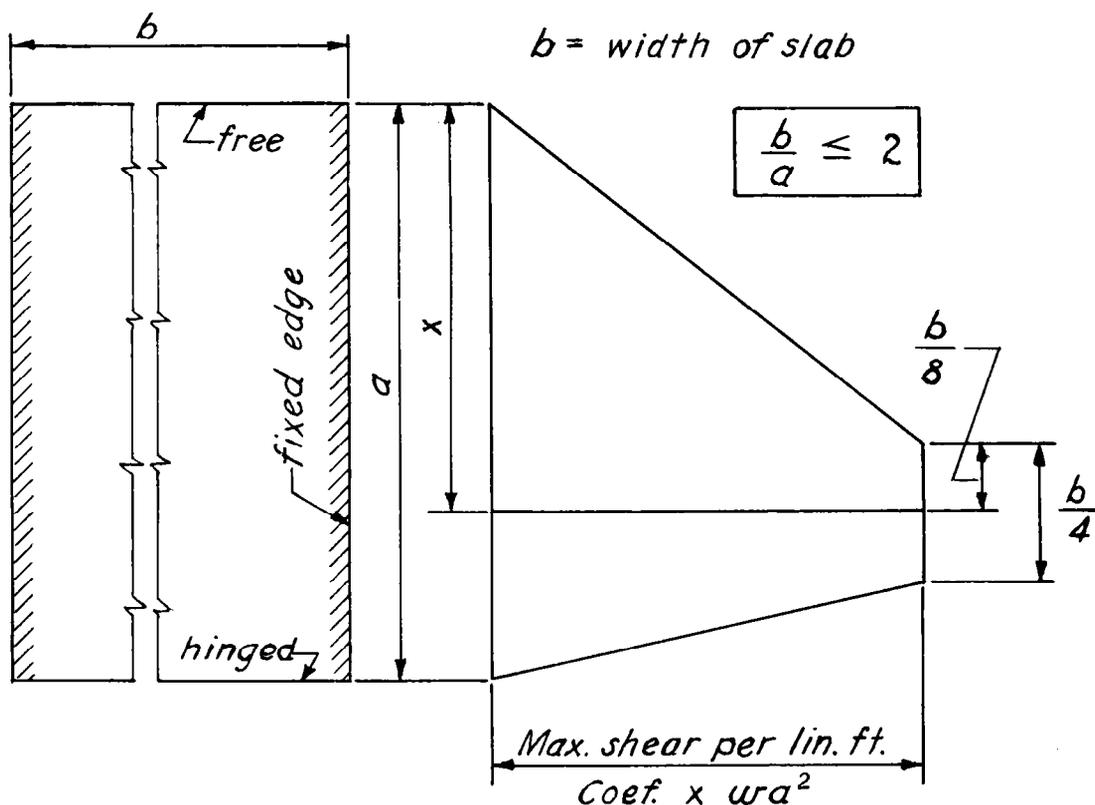
ES-56

SHEET 2 OF 2

DATE 8 - 6 - 51

Revised 11-8-51

of the shear along the vertical edge of a headwall slab are given with reasonable accuracy by the following shear diagram, fig. 4.13.



DISTRIBUTION OF UNIT SHEARING STRESS ALONG FIXED EDGE

FIGURE 4.13

In fig. 4.13, the distribution of unit shearing stress is represented by a trapezoid, the area of which represents the total shear along the edge of the slab. The coefficient used to determine the maximum shear per linear foot and the value of  $x$ , which determines the location of the center of maximum shearing-stress intensity, are found from drawing ES-6, sheet 9 of 10, Engineering Handbook, Section 6 on Structural Design. The shearing-stress distribution shown in fig. 4.13 is good only for values of  $(b \div a)$  equal to, or less than, 2.

The total load is equal to the sum of the shears from the two adjacent spans. If the spans adjacent to the buttress are equal (the usual case), then the total load on the buttress is equal to twice the load indicated by the diagram in fig. 4.13.

When the overturning moments on the drop spillway are high and result in high toe pressures under the spillway, or when the weir length is relatively great, it may be necessary to devise special methods of analysis for the buttress, longitudinal sill, and transverse sill. Experienced structural designers should be consulted in such cases.

Longitudinal Sill Analysis. Longitudinal sills may be used with or without buttresses. In either case, the procedure of analysis is the same.

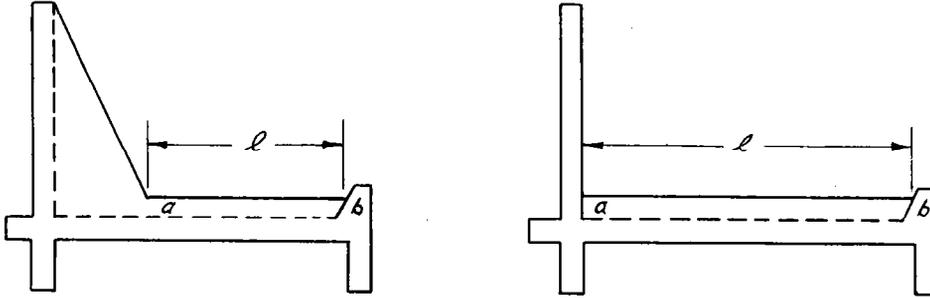


FIGURE 4.14

The longitudinal sill may be considered as a beam fixed against rotation at the toe of the buttress or at the headwall, point a, fig. 4.14, as the case may be, and as both partially restrained and freely supported at the transverse sill, point b, fig. 4.14. For the partially restrained condition at point b, the moment at b is taken as one-half the fixed end moment. The load on the longitudinal sill is taken as the maximum net reaction from the apron slab at the longitudinal sill less the weight of the sill.

Transverse Sill Analysis. When the longitudinal sills are designed as outlined in the preceding paragraph, the transverse sill acts as a support for the longitudinal sills. The analysis following these assumptions may be handled as follows: The transverse sill and toewall may be considered as a beam supported at the sidewalls. This beam should be designed for both fixed and half-fixed end moments; the actual degree of restraint at the end is unknown. The loads on the beam will be the reactions from the longitudinal sills as concentrated loads, plus a uniform load equal to the difference between the overturning pressure acting on the toewall and the weight of the beam.

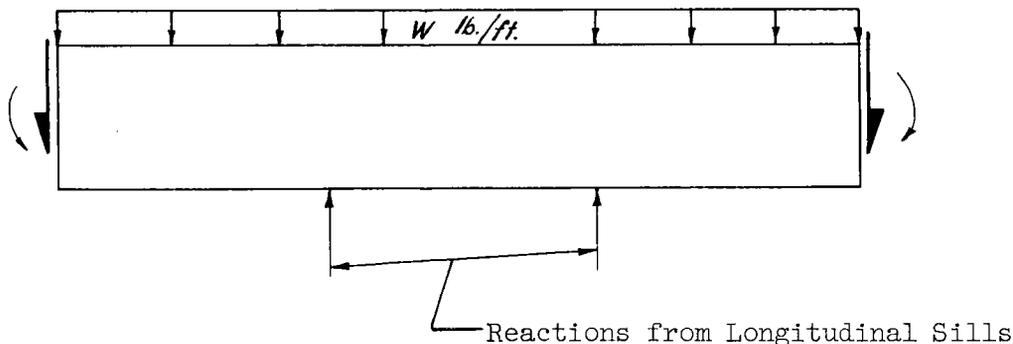


FIGURE 4.15

The resultant uniform load may act either in an upward or downward direction.

Headwall Extension Analysis. If the headwall extension is not joined monolithically with the rest of the structure, it acts merely as a diaphragm. The differential in earth loads on the two sides of the wall at any point will be very small. Therefore, the stresses in the wall will be small and the wall need only be reinforced to meet minimum steel requirements. This type of design for the headwall extension is used in the design example.

If the headwall extension is designed to be monolithic with the rest of the structure, it may be designed as a vertical and horizontal cantilever. See page 4.9 for load recommendations. In the vertical direction, the wall may be designed as a series of cantilever beams. In the horizontal direction, the wall and footings will be designed as a unit. The steel in the footings will be designed to carry the remainder of the total moment not carried by the horizontal steel in the wall.



## 5. TYPE B DROP SPILLWAY

General. Minimum layout and hydraulic design criteria for a type B drop spillway are given on drawing ES-67 (page 5.3) and in the following discussion. These criteria are patterned after those suggested by Messrs. B. T. Morris and D. C. Johnson in a paper entitled "Hydraulic Design of Drop Structures for Gully Control" which was published in the Trans. of the American Society of Civil Engineers, Vol. 108 (1943). Study of this paper will disclose the differences between the criteria proposed by Morris and Johnson and those proposed herein.

The type B drop spillway is not recommended for sites where easily eroded soils exist in channel bottoms or banks, or at sites where prolonged flow will occur as in irrigation channels. The scour in these sites is apt to be so severe that it will endanger the stability of the structure. Drop structures in such locations require a longer apron and more effective energy dissipation. Pending availability of results of research now underway on drop spillways for such conditions, it is suggested that Field Engineers present such problems to the Washington Engineering Division for assistance.

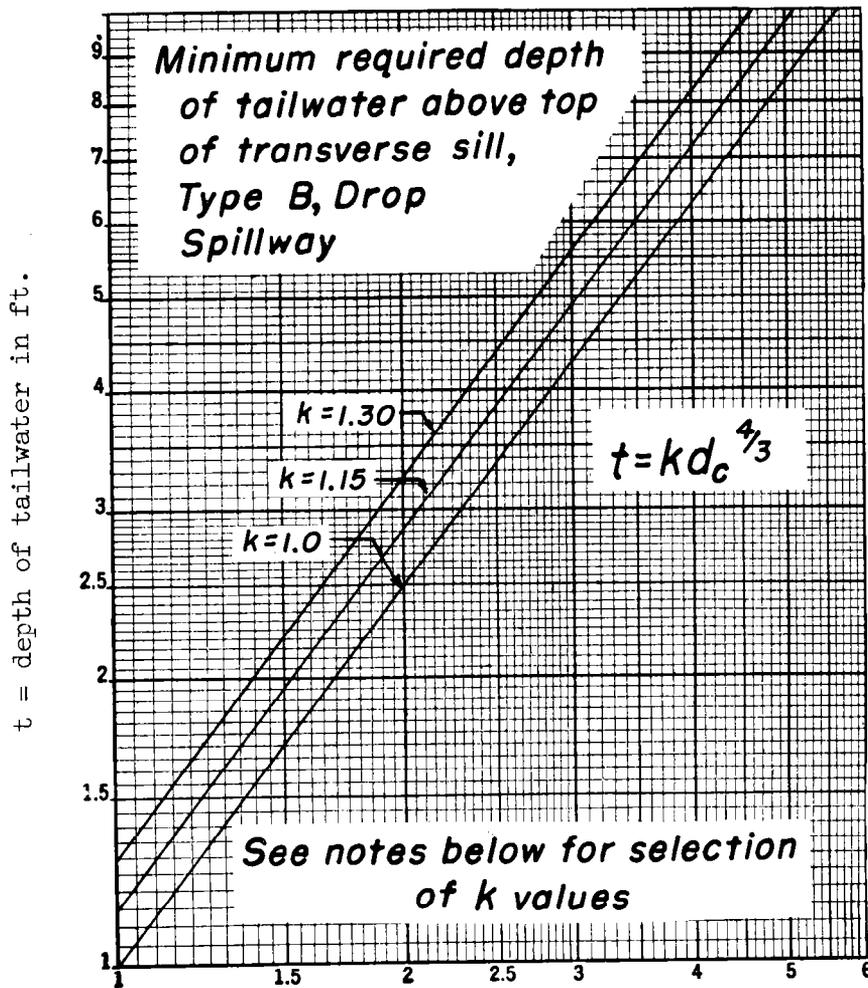
On dense firm clays, dense well-graded and compacted glacial tills, and dense well-graded and compacted mixtures of silt, sand and clay, the type B drop spillway should give satisfactory performance. On silts and sandy silts, riprap will probably be required to protect the channel bottom and banks just below the spillway. Where the need of riprap is anticipated, it should be placed during the original construction, especially if systematic maintenance of the structure is doubtful. The type B drop spillway should not be used where the channel bottom and banks below it are composed of loose or easily eroded materials such as sand.

As the ratio  $h + F$  increases, the tendency for scour to occur also increases. For this reason, and because the most economical spillways for a given discharge tend toward low values of the  $h + F$  ratio, it is recommended that this ratio be kept lower than 0.50 with an absolute maximum of 0.75.

The ratio of  $L + h$  should always be equal to, or greater than, 2. This criterion applies to all rectangular weirs.

As will be seen later, the longitudinal sills become an important element in the structural design of the apron. In long weirs, where the value of  $F + h$  is approximately 12 feet or more, it is structurally advantageous to shorten the horizontal span in the headwall by the use of buttresses; such buttresses should be placed so that the horizontal length of headwall is divided into equal spans. For practical construction and design reasons, the location of buttresses and longitudinal sills should coincide. Hence, it is recommended that the longitudinal sills be located so that the distance between center lines of the sidewalls will be divided into approximately equal spans of practical length, and that the location of longitudinal sills and buttresses be made to coincide where buttresses are used.

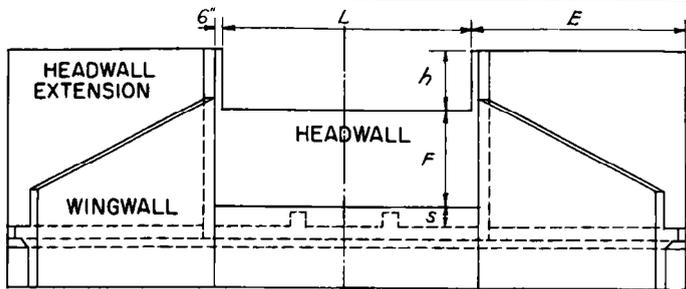
**Tailwater.** A minimum tailwater elevation is required to reduce the scour of the channel bottom and banks just below the spillway to tolerable limits. A significant amount of model testing and field observation show that a low tailwater permits the jet of water, cast upward by the transverse sill, to strike the channel bottom with serious scour effects and that strong side eddies which attack the channel banks are created. With a high tailwater, the velocity of the jet off the transverse sill is reduced and the jet merges with the downstream flow with less serious results. The proper amount of tailwater for a type B drop spillway has not been definitely established. Pending additional research, the recommended minimum required tailwater depth above the top of the transverse sill,  $t$  in feet, is given in fig. 5.1. A tailwater depth  $t = 2d_c$  is desirable and should be obtained where practicable.



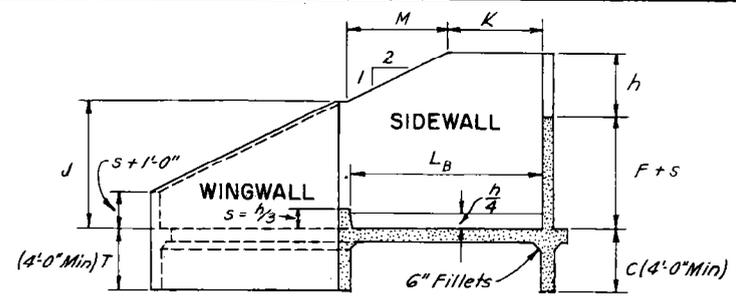
$d_c = \text{critical depth at weir, ft.}$

**FIGURE 5.1**

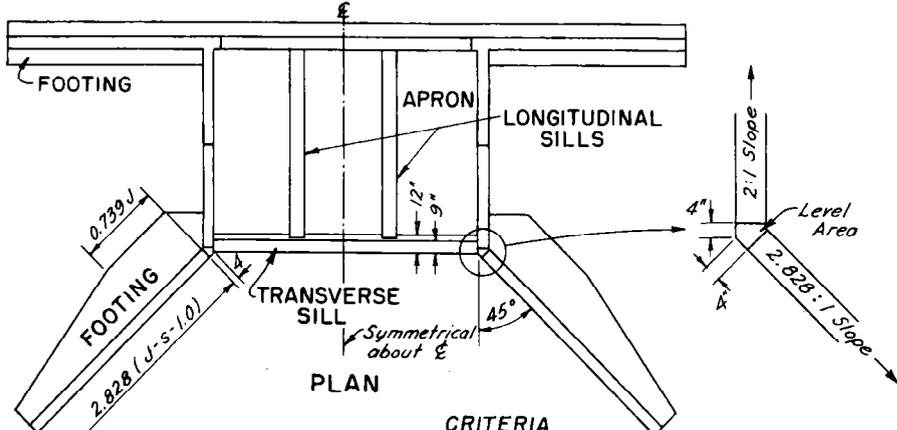
# DROP SPILLWAYS: LAYOUT AND HYDRAULIC DESIGN CRITERIA - TYPE B



DOWNSTREAM ELEVATION



SECTION ON CENTER LINE



PLAN  
CRITERIA

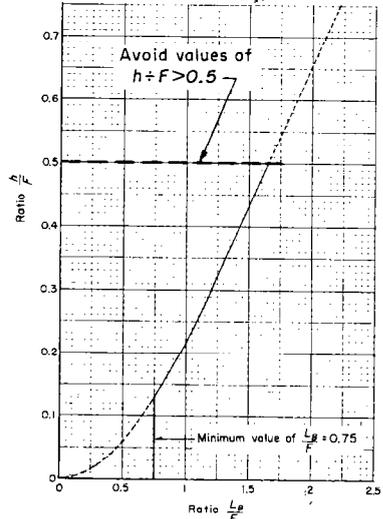
- $E =$  Minimum length of headwall extension (in feet) =  $[3h + 2]$  or  $[1.5F]$  whichever is greater.
- $J =$  Height of wingwall and sidewall at junction (in feet) =  $[2h]$  or  $[F + h + s - (\frac{L_B + 0.42}{2})]$  or  $[t + 1]$  whichever is greater.
- $L_B =$  Length of basin (in feet) =  $[F(2.28 \frac{h}{F} + 0.52)]$
- $M = [2(F + \frac{4}{3}h - J)]$
- $K = [(L_B + 0.42) - M]$

NOTE: 1.— See Engineering Handbook, Section 11, Drop Spillways, for criteria on placement of longitudinal sills, tailwater requirements, hydraulic capacity, structural design, etc.  
2.— All dimensions in foot units.

Minimum Length of Apron

$$\frac{L_B}{F} = [2.041 + 0.599(\frac{h}{F}) + 0.169(\frac{h}{F})^3] (\frac{h}{F})^{\frac{1}{2}}$$

for range of values  
 $\frac{L_B}{F} \geq 0.75 \sim 2.22$   
 or use  
 $\frac{L_B}{F} = 2.28 \frac{h}{F} + 0.52$   
 for same range



REFERENCE

Revised 12-16-53

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE

ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.

ES - 67

SHEET 1 OF 1  
DATE 2-27-52

Use  $k = 1.0$ , fig. 5.1 (page 5.2)

Where flow is intermittent, the periods of high discharges are of short duration, and the channel below the structure is highly resistant to scour, such as a tight clay or glacial till.

Use  $k = 1.15$

- (a) Where flow is intermittent, the periods of high discharge are of short duration, and the channel below the structure is resistant to scour. (This covers most average conditions.)
- (b) Where flow is intermittent, but the periods of high discharge are relatively long and the channel below the structure is highly resistant to scour.

Use  $k = 1.30$

Where flow is intermittent, periods of high discharge are relatively long, and the channel below the structure is resistant to scour.

It may be necessary to set the elevation of the top of the transverse sill below the elevation of the stable grade line of the downstream channel in order to provide the required tailwater.

In addition to the above requirements for minimum tailwater, there are limitations of maximum permissible tailwater for the type B drop spillway. Tailwater depths above maximum permissible values cause the jet of water coming through the weir to be deflected upward and outward to such an extent that a considerable part of the discharge may not hit the apron but falls on the unprotected stream bed in front of the apron. To avoid this situation and prevent the excessive scour associated with it, the tailwater depth above the top of the transverse sill,  $t$ , should not be greater than  $0.5 (F + h)$ .

Then the range in value of tailwater depth,  $t$ , is given by the following formula.

$$kd_c^{4/3} \leq t \leq \frac{F + h}{2} \quad 5.1$$

Volumes of Concrete and Steel. The volume of reinforced concrete required to construct certain sizes of type B drop spillways with minimum dimensions is given in drawing ES-66 (page 5.7). Drawing ES-74 (page 5.9) gives the weir dimensions and concrete volume of the type B drop spillway that requires the minimum volume of concrete for a given design discharge,  $Q$ , and net drop,  $F$ . These quantities are based on the use of class B concrete as defined in the Engineering Handbook, Section 6 on Structural Design, and a working stress for reinforcing steel,  $f_s = 20,000$  psi. Load assumptions were:

1. Weight of concrete = 150 lbs/ft<sup>3</sup>
2. Weight of earth fill = 100 lbs/ft<sup>3</sup>
3. Weight of equivalent fluid against headwall = 62.4 lbs/ft<sup>3</sup>
4. Weight of equivalent fluid against sidewalls = 35 lbs/ft<sup>3</sup>
5. Weight of equivalent fluid against wingwalls = 35 lbs/ft<sup>3</sup>
6. Weight of equivalent fluid against headwall extensions = 5 lbs/ft<sup>3</sup>
7. Allowable soil bearing pressure = 2000 lbs/ft<sup>2</sup>

In some cases, for stability reasons, it will be necessary to increase the length of the headwall extensions and increase the depth of the cutoff wall beyond the minimum values indicated on drawing ES-67 (page 5.3). Such changes will require additional concrete above the amount indicated in the tables of drawing ES-66 (page 5.7). The amount of additional concrete can be estimated with reasonable accuracy from drawing ES-48 (page 5.10) with  $b = 8$  in.

The amount of reinforcing steel required, as measured in terms of pounds of reinforcing steel per cubic yard of concrete, will vary considerably with the height of the structure and to a smaller extent with the length of the weir. Approximate amounts of reinforcing steel to be used for estimating purposes only are given in fig. 5.2.

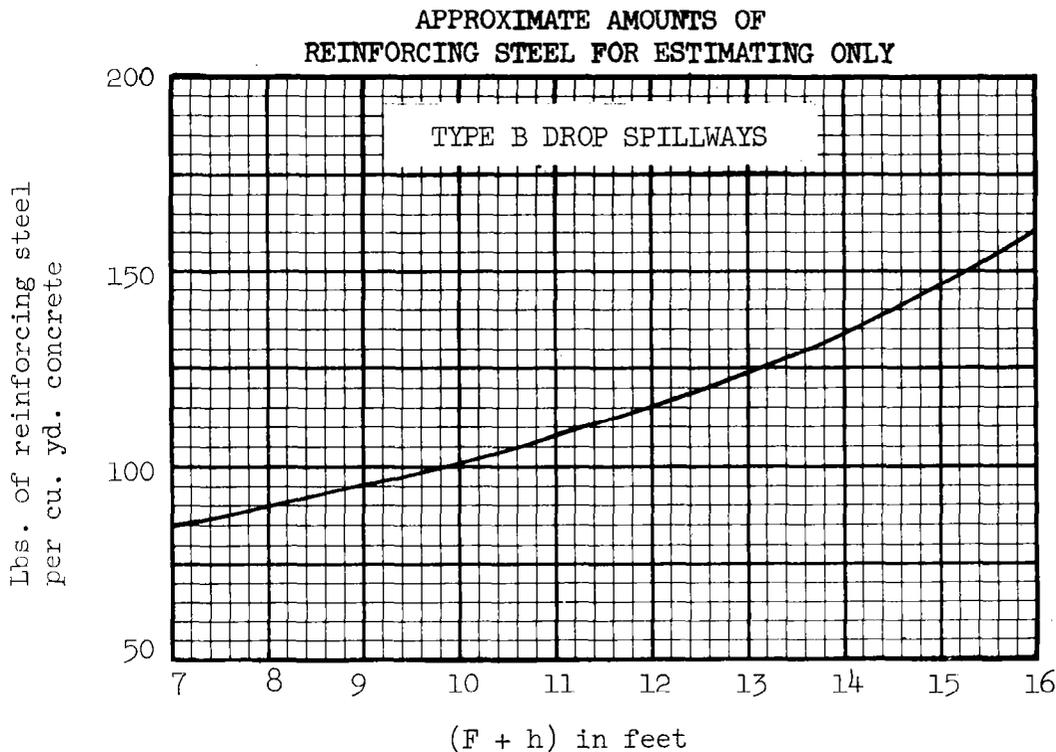


FIGURE 5.2

Example 5.1

- Given:
1. Required discharge capacity,  $Q = 200$  cfs
  2. Net drop,  $F = 7$  ft
  3. Free flow condition (no submergence)
  4. Use drawing ES-74 (page 5.9)

- Find:
1. The length  $L$  and depth  $h$  of a weir of a type B drop spillway that will carry the required discharge and contain a minimum amount of concrete.
  2. The estimated amount of reinforcing steel required for each structure considered.

Solution: Drawing ES-74 (page 5.9) shows that for a Q of 200 cfs and F of 7.0 ft, a drop spillway having weir dimensions of  $L = 20'-0''$  and  $h = 2'-6''$  requires the minimum volume of concrete, namely 34.9 cubic yards.

Figure 5.2 (page 5.5) shows that for  $F + h = 7 + 2.5 = 9.5$  ft the estimated reinforcing steel requirement is 98 pounds per cubic yard of concrete. Therefore, the total estimated steel requirement is  $34.9 \cdot 98 = 3420$  pounds.

Comment: Drawing ES-74 (page 5.9) also shows there is only 0.5 cubic yard difference in concrete volume required by the 2.5 x 20 weir and the 3 x 16 weir. As pointed out previously, final selection of weir proportions should be based on site requirements and comparative cost estimates that include costs of excavation, fill, drainage, etc.

### Example 5.2

Given: A type B drop spillway with  $F = 7$  ft,  $h = 3$  ft, and  $L = 16$  ft of minimum dimensions as indicated on drawing ES-67 (page 5.3) and a wall thickness,  $b = 8$  inches.

Find: The increase in concrete yardage if the length of each headwall extension is increased 4.0 ft and the depth of cutoff wall is increased 2.0 ft.

Solution: Reference to drawings ES-48 (page 5.10) and ES-67 (page 5.3) should make the following computations self-evident:  $s = h \div 3 = 3 \div 3 = 1$ ; then  $Y = F + h + s = 7 + 3 + 1 = 11$  ft. For  $b = 8$  in, and  $Y = 11$  ft,  $X$  as taken from the curve = 0.8 ft, but use  $X = 1.0$  ft;  $b + 1 = 9$  in = 0.75 ft;  $b + 2 = 10$  in = 0.833 ft. Now compute added volume in headwall extension without increase in cutoff wall depth.

$$\begin{aligned} 11 \cdot 0.667 &= 7.34 \\ 2.667 \cdot 0.75 &= 2.00 \\ 3.25 \cdot 0.833 &= 2.71 \\ 2 \cdot 0.5 \cdot 0.5 \cdot 0.5 &= \underline{0.25} \\ 12.30 \cdot 2 \cdot 4 &= 98.40 \text{ ft}^3 \end{aligned}$$

Now compute added volume due to increase in depth.

Original length of cutoff wall =  $L + 2(3h + 2)$  or  $L + 2(1.5F)$ , whichever is greater.

$$L + 2(3h + 2) = 16 + 2(9 + 2) = 38 \text{ ft} \quad (\text{Use})$$

$$L + 2(1.5F) = 16 + 2(1.5 \cdot 7) = 37 \text{ ft}$$

Final length of cutoff wall =  $38 + 8 = 46$  ft; increase in volume of cutoff wall =  $46 \cdot 2 \cdot 0.833 = 76.64 \text{ ft}^3$ ; total increase in volume =  $(98.40 + 76.64) \div 27 = 6.5$  cu yd.

Comment: In actual practice, a sketch should be made to facilitate such computations.



**DROP SPILLWAYS: APPROXIMATE VOLUMES OF REINFORCED CONCRETE IN CUBIC YARDS - TYPE B**

F	h \ L													
		6	8	10	12	14	16	18	20	22	24	26	28	30
3	2-0	14	15	16	18	19	20	21	22	23	24	25	26	27
4	2-0	15	16	18	19	20	21	22	24	25	26	27	28	29
	2-6	19	20	22	23	24	26	27	28	29	31	32	33	34
	3-0	23	25	26	28	29	30	32	33	34	36	37	38	40
5	2-0	16	17	19	20	21	23	24	25	26	28	29	30	31
	2-6	20	22	23	25	26	27	29	30	31	33	34	35	37
	3-0	25	26	28	29	31	32	33	35	36	38	39	41	42
	3-6	30	31	33	34	36	37	39	40	42	43	45	46	48
6	2-0	19	20	22	23	24	26	27	28	30	31	32	34	35
	2-6	21	23	24	26	27	29	30	32	33	34	36	37	39
	3-0	26	28	29	31	32	34	35	37	38	40	41	43	44
	3-6	32	33	35	37	38	40	41	43	45	46	48	49	51
	4-0	37	39	40	42	44	46	47	49	51	52	54	56	57
	4-6	43	45	47	49	51	52	54	56	58	59	61	63	65
7	2-0	23	24	26	27	29	30	31	33	34	36	37	38	40
	2-6	25	26	28	29	31	32	34	35	37	38	40	41	43
	3-0	28	29	31	33	34	36	37	39	41	42	44	45	47
	3-6	33	35	36	38	40	42	43	45	47	48	50	52	53
	4-0	39	41	43	45	47	48	50	52	54	55	57	59	61
	4-6	45	47	49	51	53	55	57	59	61	62	65	67	69
	5-0	52	54	56	58	60	62	64	66	68	70	72	74	76
8	2-0	27	29	30	32	33	35	36	38	39	41	43	45	46
	2-6	29	31	32	34	35	37	39	40	42	43	46	47	49
	3-0	30	32	34	36	37	39	41	42	44	46	47	50	52
	3-6	35	36	38	40	42	44	46	47	49	51	53	55	57
	4-0	41	43	44	47	48	50	52	54	56	58	60	62	64
	4-6	47	49	51	53	55	57	59	61	63	66	68	70	72
	5-0	54	56	58	61	63	65	67	69	72	74	76	78	80

## DROP SPILLWAYS: APPROXIMATE VOLUMES OF REINFORCED CONCRETE IN CUBIC YARDS - TYPE B

F	h \ L		6	8	10	12	14	16	18	20	22	24	26	28	30
	h	L													
9	2-0		32	33	35	37	38	40	41	43	45	47	49	50	52
	2-6		34	36	37	39	41	43	44	46	49	50	52	54	55
	3-0		35	37	39	41	43	44	46	48	51	53	54	56	58
	3-6		37	39	41	43	45	47	49	51	53	55	57	59	61
	4-0		42	44	46	48	50	52	54	57	59	61	63	65	67
	4-6		49	51	53	56	58	60	62	65	67	69	71	73	75
	5-0		56	58	61	63	65	67	69	73	75	77	79	81	83
10	2-0		37	39	41	42	44	46	47	50	52	54	55	57	59
	2-6		39	41	43	44	46	48	50	53	54	56	58	60	61
	3-0		40	42	44	46	48	50	53	55	57	59	60	62	64
	3-6		43	45	47	49	51	53	56	58	60	62	64	66	68
	4-0		45	47	50	52	54	56	59	61	63	65	67	69	71
	4-6		51	53	55	58	60	62	65	68	70	72	74	76	78
	5-0		59	61	63	66	68	70	74	76	78	80	83	85	87

- Note: (1) These volumes apply only to drop spillways designed in accordance with criteria set forth in drawing ES-67, page 5.3, and on page 5.4 of the Engineering Handbook, Section 11, Drop Spillways.
- (2) F = net drop from crest of weir to top of transverse sill in feet.  
 h = total depth of weir in feet.  
 L = length of weir in feet.

REFERENCE

*Revised 7-9-53*

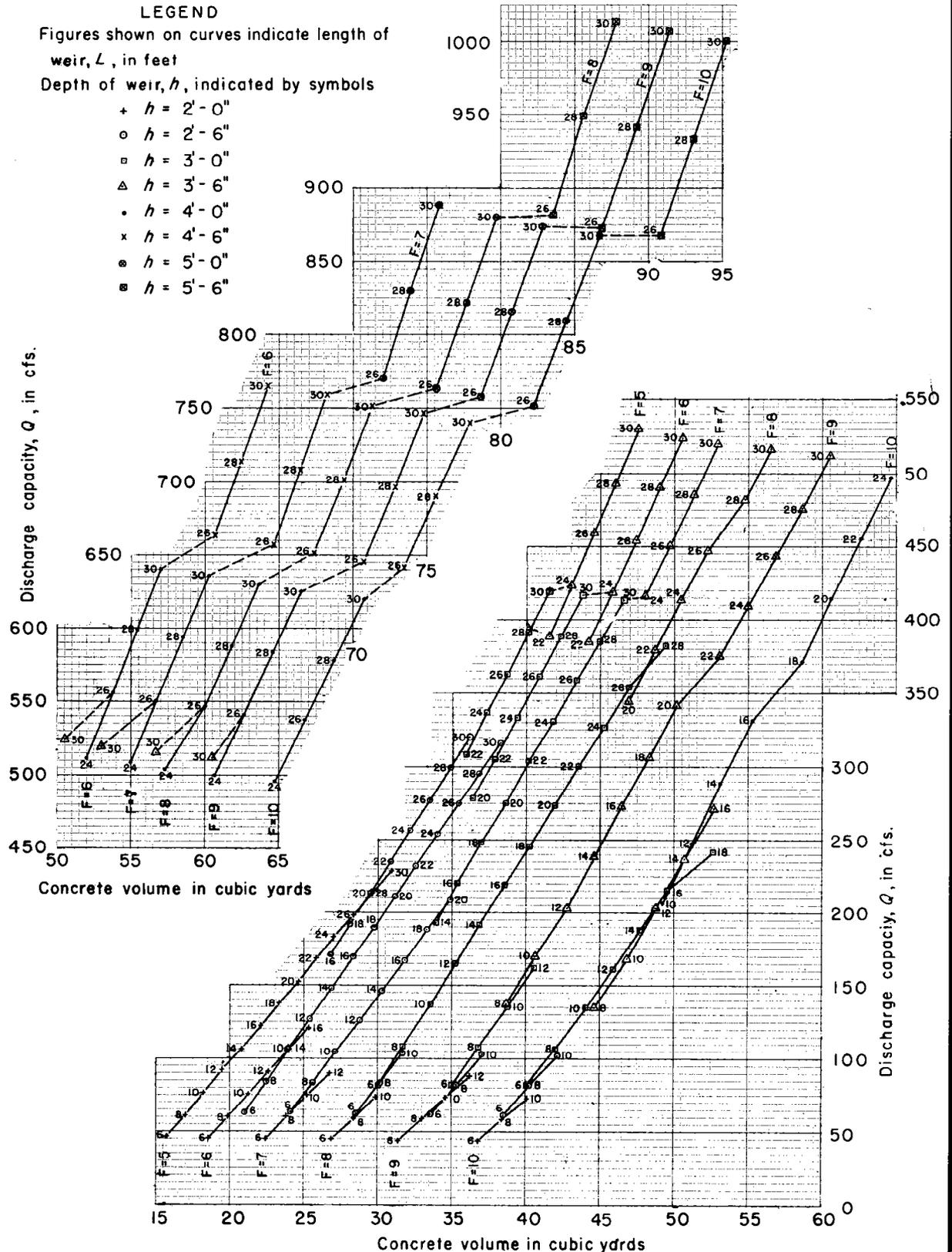
U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE  
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.

ES-66

SHEET 2 OF 2DATE 2-13-52

# DROP SPILLWAYS - TYPE B - MINIMUM CONCRETE VOLUME FOR VARIOUS DISCHARGES FOR NET DROPS, F, OF 5 TO 10 FEET AND WEIR LENGTHS, L, UP TO 30 FEET.



REFERENCE

U.S. DEPARTMENT OF AGRICULTURE  
 SOIL CONSERVATION SERVICE

ENGINEERING DIVISION - DESIGN SECTION

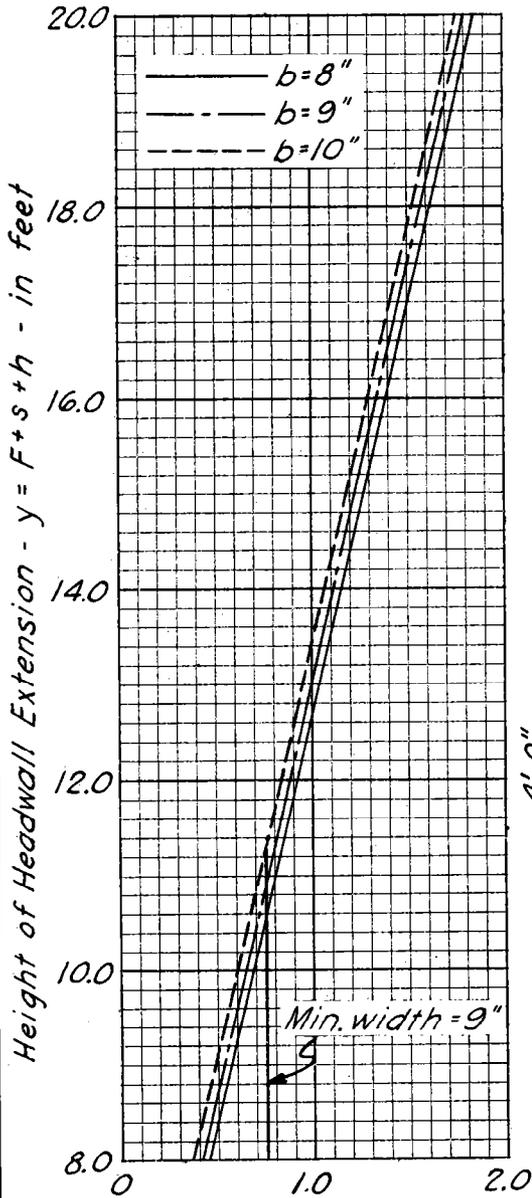
STANDARD DRAWING NO.

ES 74

SHEET 1 OF 1

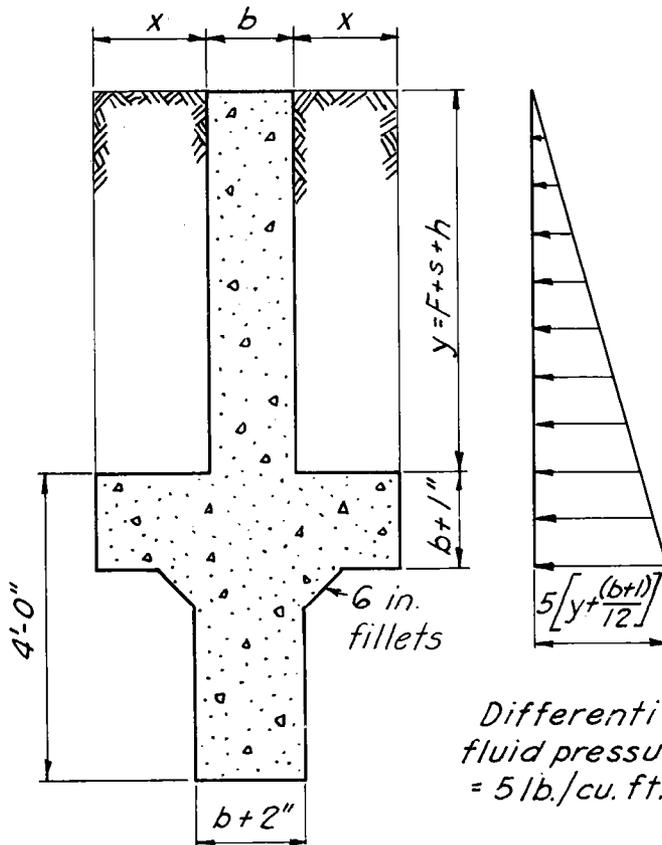
DATE 3-19-53

# DROP SPILLWAYS: REQUIRED WIDTH OF HEADWALL EXTENSION FOOTINGS FOR TYPE B



Minimum width of footing -  $x$  - in feet.

Note: Use values of  $x$  of 0.75, 1.00, 1.25 etc. (i.e. in even increments of 0.25 ft.)



Differential fluid pressure = 5 lb./cu. ft.

## HEADWALL EXTENSION

REFERENCE

Revised 7-7-53

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE

STANDARD DWG. NO.

ES-48

SHEET 1 OF 1

DATE 12-21-50

## Hydraulic Design of Type C Drop Spillways

The nomenclature associated with the type C drop spillway is given in ES-111, page 5-18. The hydraulic design is illustrated by examples beginning on page 5-23.

The type C drop spillway was developed for use in locations where type B drop spillways are considered inadequate. These locations include the following situations:

1. Continuous flow;
2. Long durations of flow at discharges nearly equal to design discharge;
3. High tailwater; and
4. Values of  $h/F$  greater than 0.5.

### Limitations of Type C Drop Spillways

Type C drop spillways were developed by model studies.\* These model studies were limited to a range of values of

1.  $0.1 \leq \frac{h}{F} \leq 1.43$
2.  $L \geq 1.5 d_c$
3.  $t \geq 1.75 d_c$

where  $h$  is the design head over the crest, ft

$F$  is the vertical distance from the crest of the spillway to the top of the end sill, ft

$L$  is the length of spillway crest, ft

$t$  is the tailwater depth above the transverse sill, ft

$d_c$  is defined in the next paragraph

### Values of $d_c$ , ft

The parameter  $d_c$  is used in the determination of various dimensions of the drop spillway which are set by the hydraulic design. The critical depth  $d_c$  is that critical depth corresponding to the capacity-without-freeboard  $Q_s$  in the weir notch of the drop spillway. The value of  $d_c$  corresponding to various discharges for rectangular sections are given on the line charts of ES-111, page 5-19.

### Design Discharge $Q_r$ , cfs

Design discharge  $Q_r$  is that discharge the structure is required to convey with a freeboard. It is determined from hydrologic data, reservoir routing, and economic considerations. The hydrologic aspect of this determination is given in the National Engineering Handbook, Section 4, Hydrology.

---

\* St. Anthony Falls Hydraulic Laboratory, Technical Paper No. 15, Series B, Straight Drop Spillway Stilling Basin, by Charles A. Donnelley and Fred W. Blaisdell.

Required-Capacity-Without-Freeboard  $Q_{fr}$ , cfs

The required-capacity-without-freeboard  $Q_{fr}$  is that discharge the structure must convey without freeboard. Knowing this required capacity  $Q_{fr}$  will enable the designer to select the structure with sufficient capacity-without-freeboard  $Q_s$ .

$$Q_{fr} = (1.10 + 0.01 F)Q_r$$

where  $Q_r$  = design discharge, cfs

F = vertical distance between the crest of the spillway and the top of transverse sill, ft

Capacity-Without-Freeboard  $Q_s$ , cfs

The capacity-without-freeboard  $Q_s$  of any hydraulic structure is equal to the maximum discharge the structure is capable of conveying without overtopping. It is determined solely by the size of the structure and its operating conditions. The capacity-without-freeboard  $Q_s$  must be equal to or greater than the required-capacity-without-freeboard  $Q_{fr}$ .

$$Q_s \geq Q_{fr}$$

The capacity-without-freeboard  $Q_s = q_s L$  with and without high tailwater is given on ES-111, page 5-20. The effect of submergence of the crest on the capacity-without-freeboard is reflected in the graph and is in accordance with Figure 3.4, page 3.17. Thus, the capacity-without-freeboard  $Q_s$  sometimes depends on the tailwater depth.

Required Tailwater Depth  $t \geq 1.75 d_c$ 

Tailwater depths over the transverse sill are determined by computing water surface profiles.

The tailwater depth over the transverse sill  $t$  must be greater than or equal to  $1.75d_c$  to prevent excess scour in the downstream bed and banks. Sufficient tailwater depth over the transverse end sill can always be obtained by increasing the value of F; that is, by lowering the apron and transverse sill.

Length of Stilling Basin Apron  $L_B$ 

The tailwater depth  $t$ , the value of F, and the values of  $h$  are the parameters required in the determination of apron length  $L_B$ . The minimum length of the stilling basin  $L_B$  is given graphically by ES-111, page 5-21.

Values along the right side of the graph represent submergence of the crest of  $0.7d_c$ . For submergence of the crest greater than  $0.7d_c$ , use a value of  $L_B$  equal to that  $L_B$  obtained for a submergence of the crest of  $0.7d_c$ . Thus, increasing the tailwater depth over the crest of the spillway greater than  $0.7d_c$  does not require that  $L_B$  be increased more than that computed for  $0.7d_c$ .

Values of  $\frac{h}{t} > 0.857$  (Note that  $\frac{h}{t} = \frac{1.5d_c}{1.75d_c} = 0.857$ ) are impermissible because they represent tailwater depths which are smaller than that which is required.

It is required that the value of  $\frac{h}{F} < 1.43$ . The value of  $\frac{h}{F} = 1.43$  is a maximum value of  $\frac{h}{F}$  tested by models. When  $\frac{h}{F} = 1.43$ , the minimum tail-water depth  $1.75d_c$  causes a submergence of the crest of  $0.7d_c$ . Because of this, the minimum value of  $F$  is  $1.05d_c$ .

The length of the stilling basin may be increased from the minimum.

#### Location of Floor Blocks $L_f$

The distance  $L_f$  between the headwall and the floor blocks is given graphically by ES-111, page 5-22. The distance  $L_f$  is required to assure that the trajet of the nappe will be upstream from the floor blocks. When this distance is too small, a high boil occurs because of the floor blocks and the floor blocks are ineffective in the dissipation of kinetic energy.

If, for some reason, the length of the apron  $L_B$  is increased above the minimum amount, the distance  $L_f$  from the headwall to the floor block should not be increased.

The minimum distance between the floor block and the transverse sill,  $L_B - L_f = 1.75d_c$ , may be increased. This distance permits the reduction of turbulence downstream from the floor blocks.

#### Height of Floor Blocks $0.8d_c$

The heights of the floor blocks and the end sill are significant in the performance of the stilling basin. The primary function of the floor blocks is to control bank or lateral erosion of the channel downstream from the spillway. The recommended height of floor blocks is  $0.8d_c$ . This may be varied slightly to permit the use of even dimensions.

#### Floor Block Width $(0.4 \pm 0.15)d_c$

The floor block width and the spacing of the floor blocks are important parts of the design. Floor blocks which are too wide do not function properly in dissipating the kinetic energy and require high sidewalls. The recommended width of floor blocks (in a direction transverse to the flow) is  $0.4d_c$ . This may be varied slightly to permit the use of even dimensions but the floor block width should be within the interval  $(0.4 \pm 0.15)d_c$ .

#### Floor Block Length $(0.4 \pm 0.15)d_c$

The recommended length (in the direction of flow) of floor blocks is  $0.4d_c$ . This dimension effects the required dimension between the floor blocks and the end sill. This distance is required for energy dissipation of the flow which has been divided by the floor blocks. The length may be varied slightly to permit the use of even dimensions.

Floor Block Spacing

Floor blocks which occupy over 60 percent of the transverse length of the stilling basin tend to function like a solid sill. If they occupy less than 50 percent of the transverse length of the stilling basin, they function less efficiently. A half space ( $0.2d_c$ ) shall be allowed adjacent to the sidewalls, thus, no floor block will be placed adjacent to the sidewalls.

Longitudinal Sills

Longitudinal sills may be used for structural purposes. Their width will be equal to or less than the floor block width, and their height is determined from structural requirements. They are not to be spaced between the floor blocks. Longitudinal sills are neither beneficial nor harmful hydraulically.

Transverse Sill Height  $0.4d_c$ 

The transverse sill prevents erosion in the channel bed immediately downstream from the drop spillway. The lowest height of the transverse sill was selected from model study to reduce the tailwater requirement. The recommended height of the transverse sill is  $0.4d_c$ . This height may be increased slightly to permit the use of even dimensions.

Sidewall Height ( $t + 0.85d_c$ )

The sidewall must extend above the tailwater to prevent overtopping of the sidewalls. The water surface in the stilling basin fluctuates considerably. The floor blocks and end sills cause boils and standing waves. The highest boils are  $0.60d_c$  above the tailwater. The recommended minimum height of the sidewall at the end sill is  $t + 0.85d_c$ , but not greater than  $F + h$ . From the standpoint of hydraulics, the top of the sidewalls may be level and have the recommended height.

Wingwalls

Wingwalls are set at an angle of  $45^\circ$  with the centerline of the basin. The top of the wingwall should have a slope not steeper than 1 to 1. The length of the wingwall is usually controlled by the backfill slope and should be sufficient to intersect the backfill slope in the horizontal plane at the top of the transverse sill.

Approach Channel

Certain approach channel conditions are necessary for this type of drop spillway to function properly. These conditions are

1. The bottom of the approach channel must be level and have the same elevation as the crest of the drop spillway for a minimum distance of  $6d_c$  upstream from the crest. When the bottom of the approach channel is below the crest of the nappe, it will not have the same trajectory and trajet as that used in the model study. This could cause the nappe to strike the floor too close to the floor blocks. Lowering the approach channel bottom a distance of  $0.1d_c$  will cause a significant and unsatisfactory change in the position of the nappe trajectory.

2. The dikes covering the upstream face of the headwalls are essential for the proper functioning of this structure. See ES-111, page 5-18. It is preferable hydraulically, that the slope of the dike along the face of the headwall be steeper than a 2:1 slope. When this slope is flatter than 2:1, the discharge over the weir is concentrated in the central portion of the stilling basin.

When dikes are omitted in wide channels or when the toe of dike at the upstream face of the headwall is not at the weir notch corner, a significant end contraction of flow occurs in the weir section. This causes an unfavorable distribution of discharge in the stilling basin and poor stilling basin performance.

If, for some reason, the bottom width of the approach channel is equal to the length of the weir notch, no dikes will be required, provided the side slope of the channel at the structure is not flatter than 2:1.

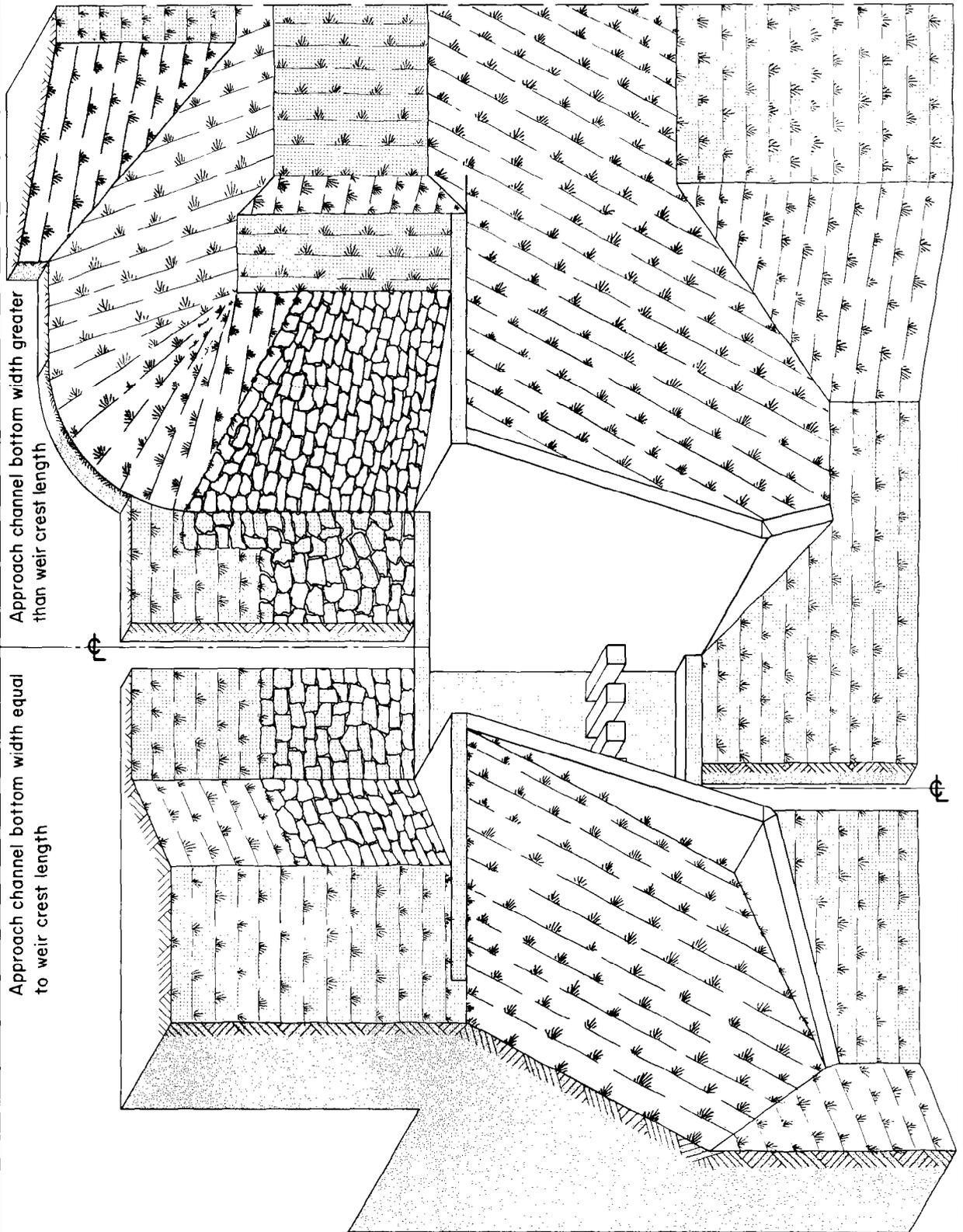
3. The channel bottom and the dikes covering the upstream face of the headwall require riprap to prevent their erosion. The recommended use of riprap and the specifications for riprap size and weight are given on pages 2.4, 2.5 and ES-79, page 2.6. Of course, concrete paving may be used in place of riprap.
4. The general channel alignment, both upstream and downstream from the drop spillway, is prescribed on pages 2.1 and 2.2.

#### Aeration Under Nappe

No provision for aeration of the nappe is required unless two or more headwall buttresses are used. The recommended approach channel conditions insure sufficient end contraction of the flow to permit ample aeration for ordinary weir lengths. The nappe over that portion of the weir sections supported by buttresses require provisions for aeration. Proper aeration can be provided by the construction of holes in the top of the buttresses. The determination of the size of these openings is given in ES-81, page 3.3. It is recommended that the top of buttresses be placed six inches below the crest of the weir.



# DROP SPILLWAYS: HYDRAULIC DESIGN, STRAIGHT DROP SPILLWAY—Type C



Approach channel bottom width greater than weir crest length

Approach channel bottom width equal to weir crest length

REFERENCE

St. Anthony Falls hydraulic laboratory technical paper No. 5, series B Straight Drop Spillway Stilling Basin, by Charles A. Donnelly and Fred. W. Blaisdell.

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE  
ENGINEERING DIVISION - DESIGN SECTION

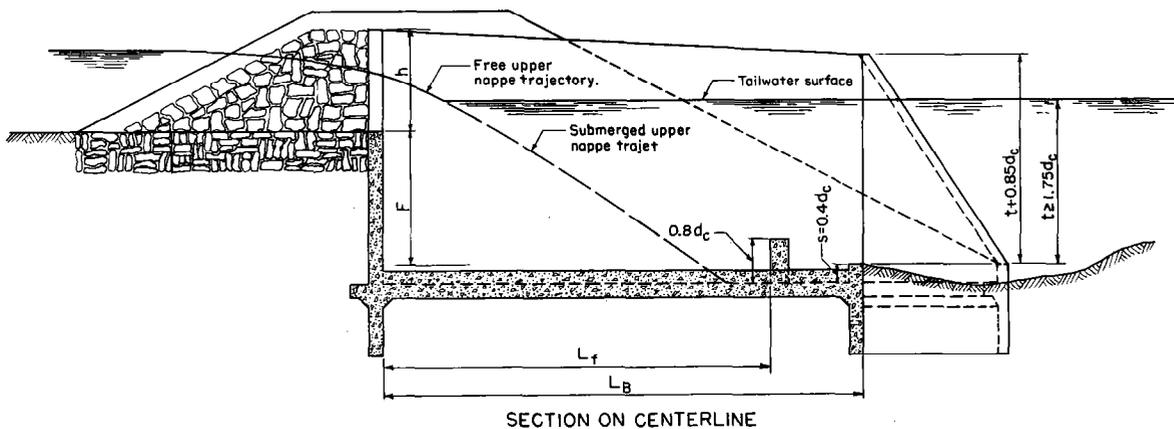
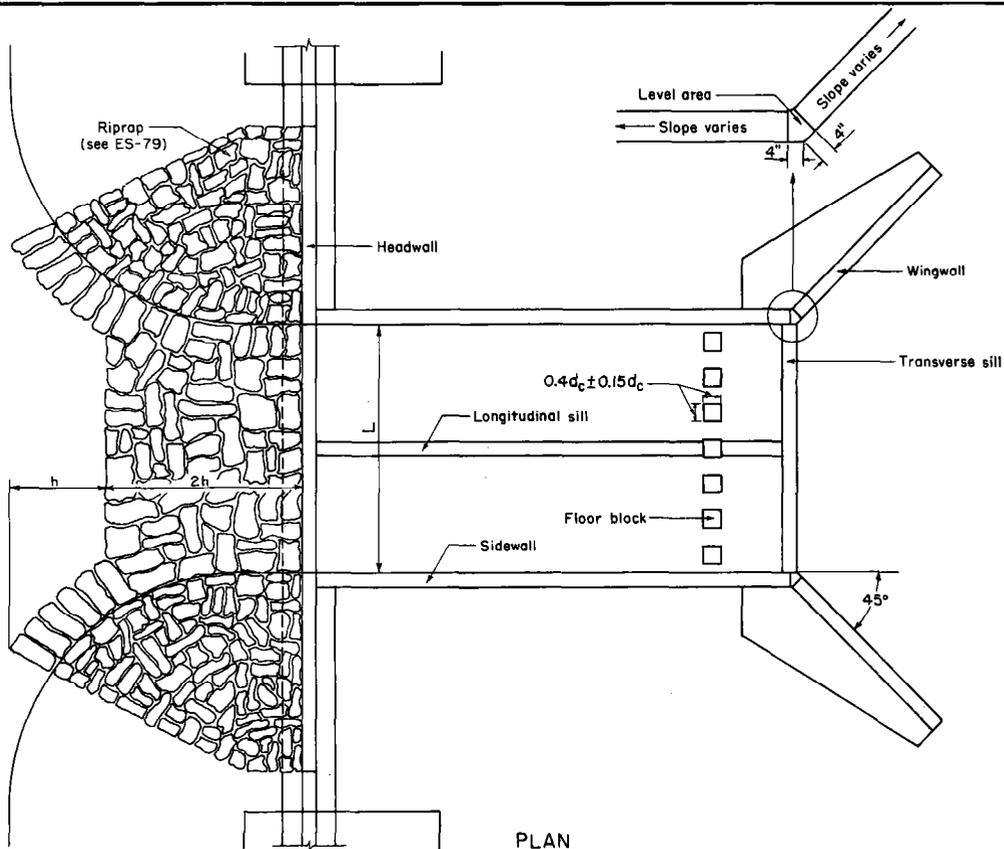
STANDARD DWG. NO.

ES-111

SHEET 1 OF 6

DATE 2-9-59

# DROP SPILLWAYS: HYDRAULIC DESIGN, STRAIGHT DROP SPILLWAY STILLING BASIN—Type C.



**Symbols :**

- $d_c$  = Critical depth for the weir section of the spillway
- $F$  = Vertical distance from top of transverse sill to spillway crest
- $h$  = Depth of weir
- $L$  = Crest length = stilling basin width
- $L_B$  = Minimum stilling basin length
- $L_f$  = Distance from downstream face of headwall to upstream face of floor blocks
- $Q_S$  = Maximum discharge structure is capable of conveying without overtopping:  $Q_S = LQ_S$
- $s$  = Height of transverse sill
- $t$  = Vertical distance from tailwater surface to top of transverse sill

**REFERENCE**

St. Anthony Falls hydraulic laboratory technical paper No. 5, series B Straight Drop Spillway Stilling Basin, by Charles A. Donnelly and Fred W. Blaisdell.

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE  
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.

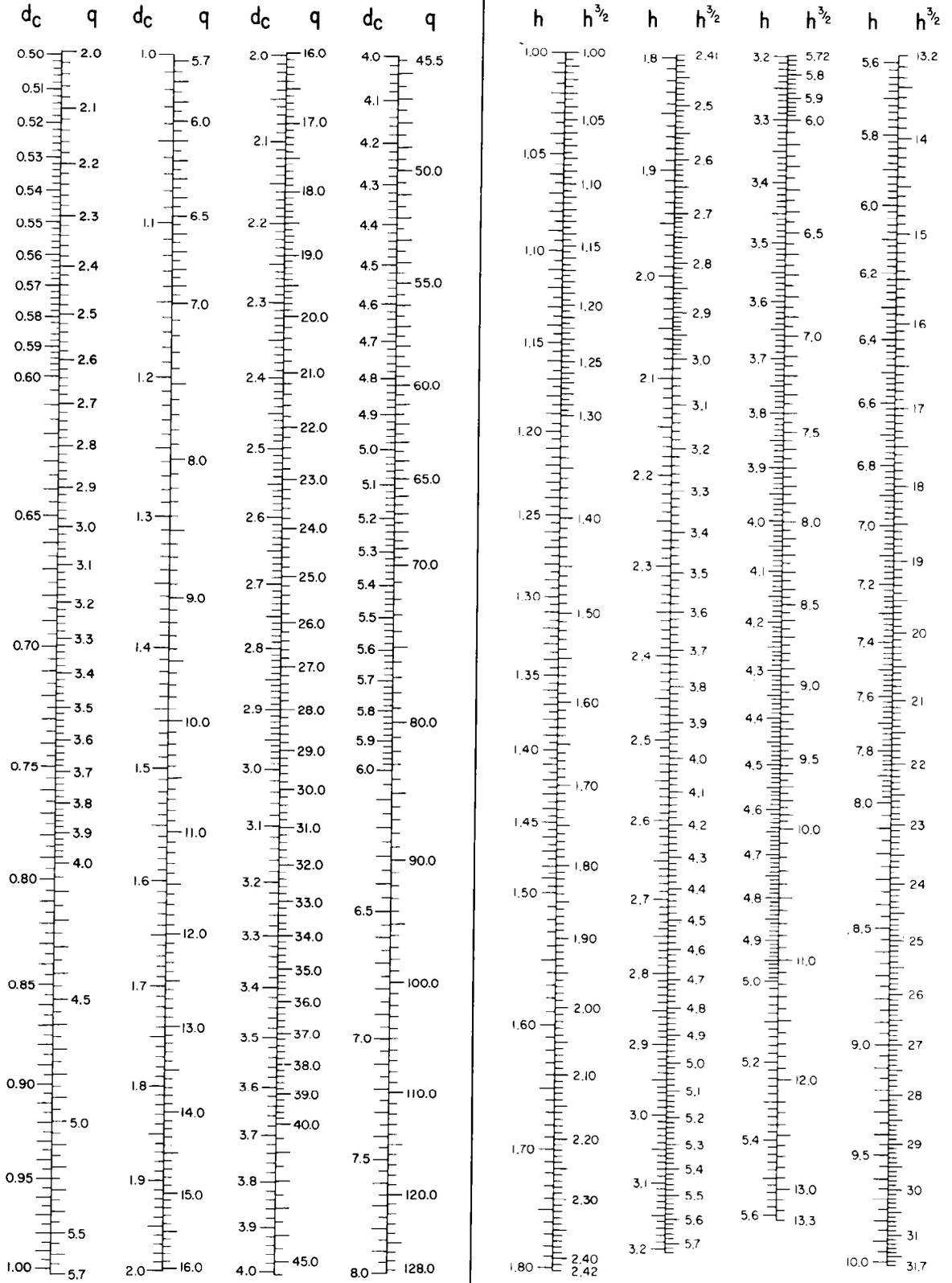
ES-111

SHEET 2 OF 6

DATE 1-28-58

**DROP SPILLWAYS:**

**TYPE C**  
 $d_c$  vs.  $q$  and  $h$  vs.  $h^{3/2}$



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE  
 SOIL CONSERVATION SERVICE  
 ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.

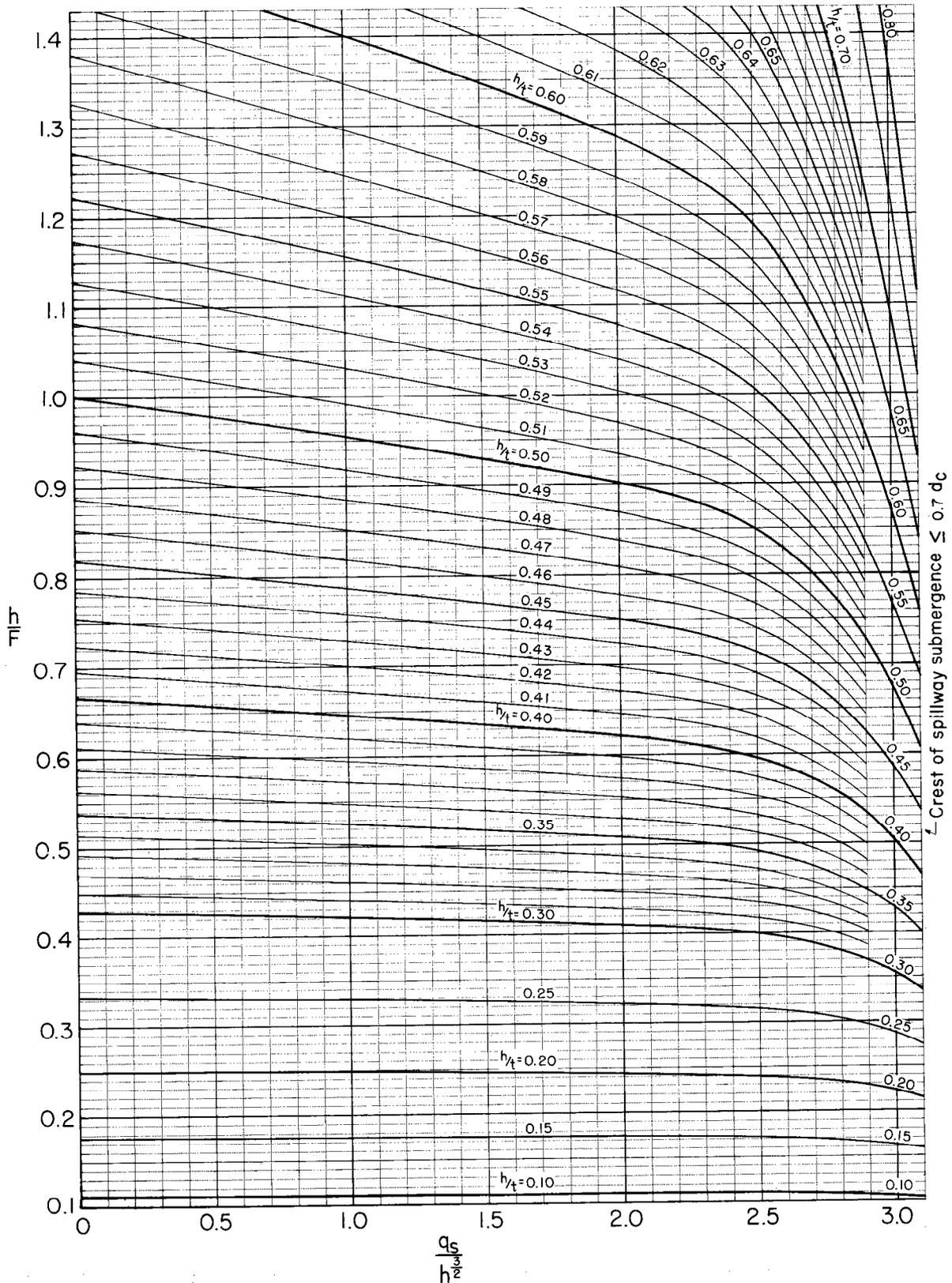
ES- 111

SHEET 3 OF 6

DATE 5-28-58

# DROP SPILLWAYS: Type C

Capacity without freeboard per foot length of weir  $q_s$



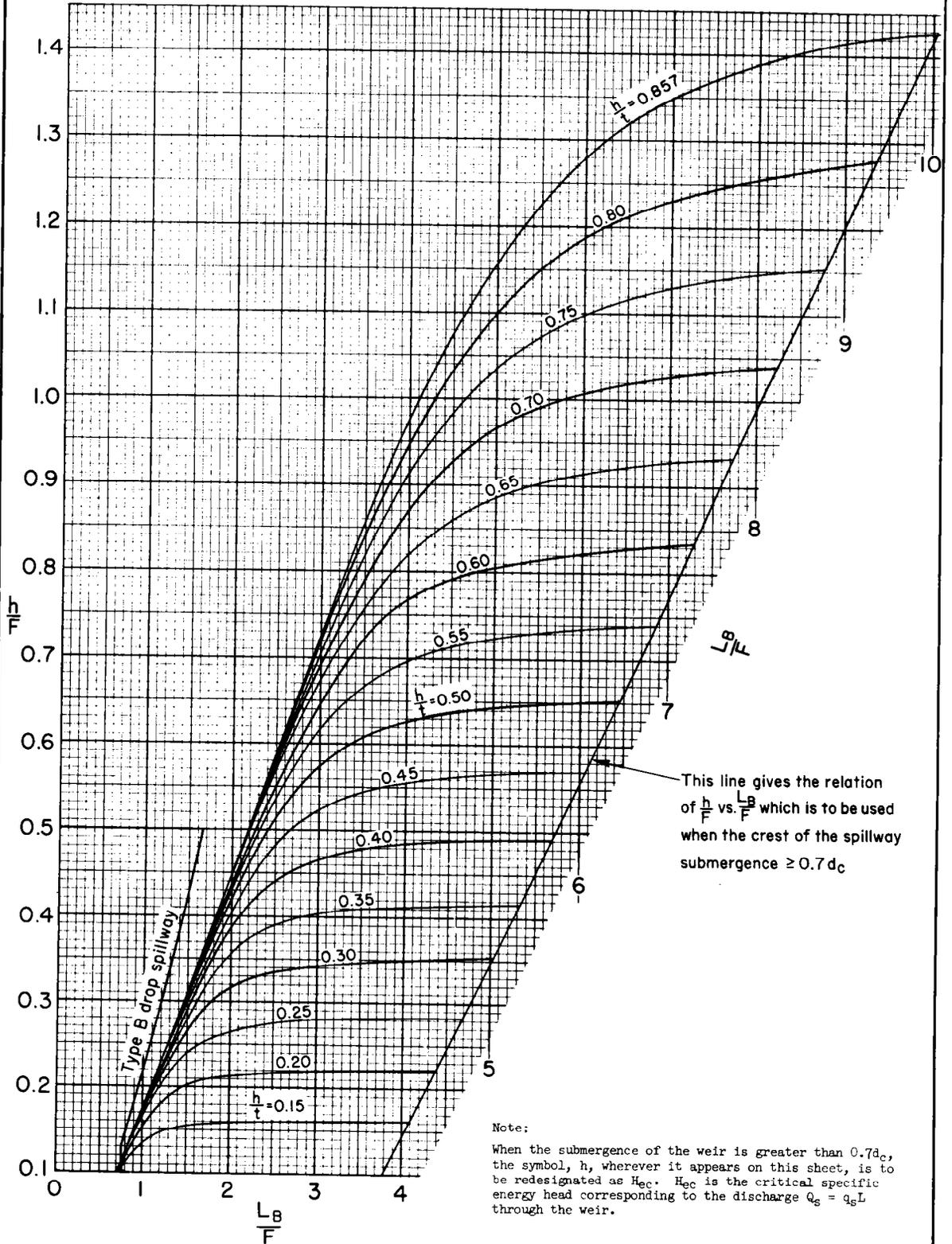
z - Crest of spillway submergence  $\leq 0.7$  dc

REFERENCE  
Data taken from page 3.17  
figure 3.4

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE  
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.  
ES-111  
SHEET 4 OF 6  
DATE 5-28-58

# DROP SPILLWAYS: Type C—Apron length $L_B$



Note:  
 When the submergence of the weir is greater than  $0.7d_c$ , the symbol,  $h$ , wherever it appears on this sheet, is to be redesignated as  $H_{ec}$ .  $H_{ec}$  is the critical specific energy head corresponding to the discharge  $Q_c = q_c L$  through the weir.

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE  
 SOIL CONSERVATION SERVICE  
 ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.

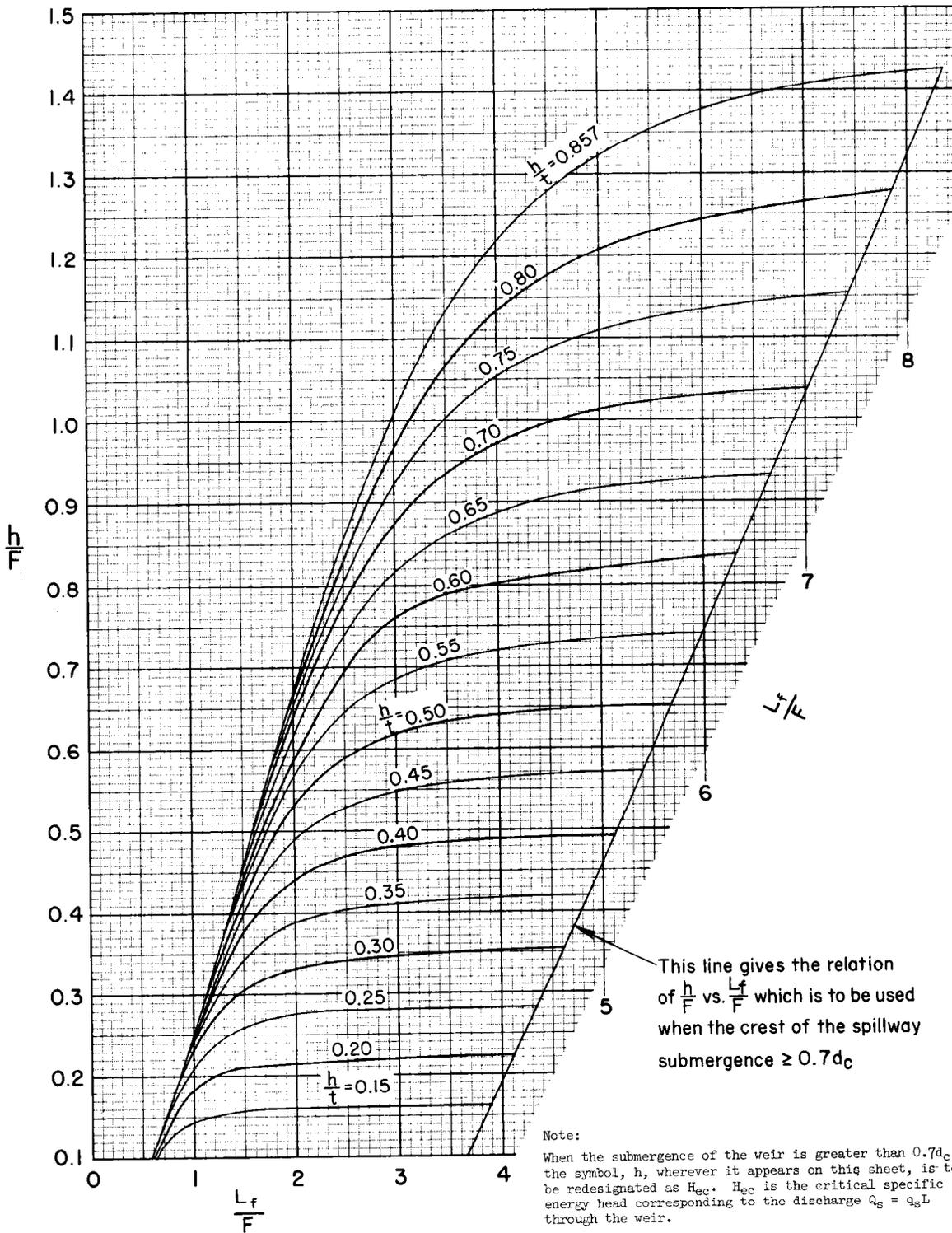
ES-111

SHEET 5 OF 6

DATE 5-28-58

Revised 4-68

# DROP SPILLWAYS: Type C— Distance to floor blocks $L_f$



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE  
SOIL CONSERVATION SERVICE  
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.

ES-111

SHEET 6 OF 6

DATE 5-28-58

Design Examples

These are strictly academic examples and are only complete insofar as they illustrate the hydraulic design of the Straight Drop Spillway - Type C.

Given: The design discharge  $Q_r = 745$  cfs. The approach channel is flat and has a bottom width of 44.0 ft. The maximum total energy head in the approach channel at the weir notch is limited to  $H_e = 4.0$  ft. The drop to be controlled in the channel grade is 8.0 ft. The depth of flow in the downstream channel is 4.0 ft. Two buttresses and longitudinal sills are used in the design.

- Determine:
1. Weir notch depth.
  2. Vertical distance  $t$  from the top of the transverse sill to the tailwater surface.
  3. The vertical distance  $F$  from the top of the transverse sill to the crest.
  4. The required-capacity-without-freeboard  $Q_{fr}$ , the crest length  $L$ , and the capacity-without-freeboard  $Q_s$ .
  5. Approach channel hydraulic requirements.
  6. Minimum transverse sill height  $s$ .
  7. Stilling basin length  $L_B$ .
  8. Location, width, spacing, and height of floor blocks.
  9. Minimum sidewall height.
  10. Wingwall length.
  11. Size and location of aeration holes in buttresses.

- Solution:
1. Set the weir notch depth equal to the maximum allowable total energy head in the approach channel, or  $h = 4.0$  ft. Then,  $d_c = 2/3h = (0.667)(4.0) = 2.67$  ft.
  2.  $t \geq 1.75 d_c = (1.75)(2.67) = 4.67$  ft
  3. If the controlled drop is 8.0 ft and the tailwater depth is 4.0 ft, the vertical distance from the tailwater to the crest is 4.0 ft. Then,  $F = 4.0 + t = 4.0 + 4.67 = 8.67$  ft. The required tailwater depth places the top of the transverse sill 0.67 ft below the downstream channel grade.
  4.  $Q_{fr} = (1.10 + 0.01F)Q_r$   
 $= 1.10 + (0.01)(8.67)745 = 884$  cfs  
 Since the weir is submerged less than  $0.7 d_c$ , the value of  $\frac{q_s}{h^{3/2}} = 3.1$  (from ES-111, page 5-20).  
 $q_s = 24.8$   
 $L = \frac{884}{24.8} = 35.6$  ft -- use 36.0 ft  
 $Q_s = (36.0)(24.8) = 893$  cfs

5. The bottom width of the approach channel must be reduced to 36.0 ft at the spillway crest. The channel side slopes at the headwall must be 2:1. This is accomplished by the addition of a conical shaped fill between the upstream face of the headwall and the side slope of the 44.0 ft approach channel. The approach channel is then riprapped in accordance with ES-79, page 2.6.

6. The minimum transverse sill height.

$$s = 0.4 d_c = (0.4)(2.67) = 1.07 \text{ ft} \quad \text{-- use } 1' - 0''$$

7. Stilling basin length  $L_B$  is determined from ES-111, page 5-21.

$$\frac{h}{F} = \frac{4.0}{8.67} = 0.461$$

$$\frac{h}{t} = \frac{4.0}{4.67} = 0.857$$

$$\frac{L_B}{F} = 2.04 ; L_B = (8.67)(2.04) = 17.76 \quad \text{-- use } 17' - 9''$$

8. The distance from the downstream face of the headwall to the upstream face of the floor blocks  $L_F$  is determined from ES-111, page 5-22.

$$\frac{h}{F} = 0.461$$

$$\frac{h}{t} = 0.857$$

$$\frac{L_F}{F} = 1.50 ; L_F = (8.67)(1.50) = 13.00 \text{ ft}$$

The blocks are square in plan. The width  $w = 0.4 d_c = (0.4)(2.67) = 1.07 \text{ ft}$  (use 1' - 0"). Twenty blocks would occupy 20/36 or 55 percent of the basin width. The blocks are spaced 0' - 9 1/2" apart with the face of the outside blocks 0' - 5 3/4" from the sidewall.

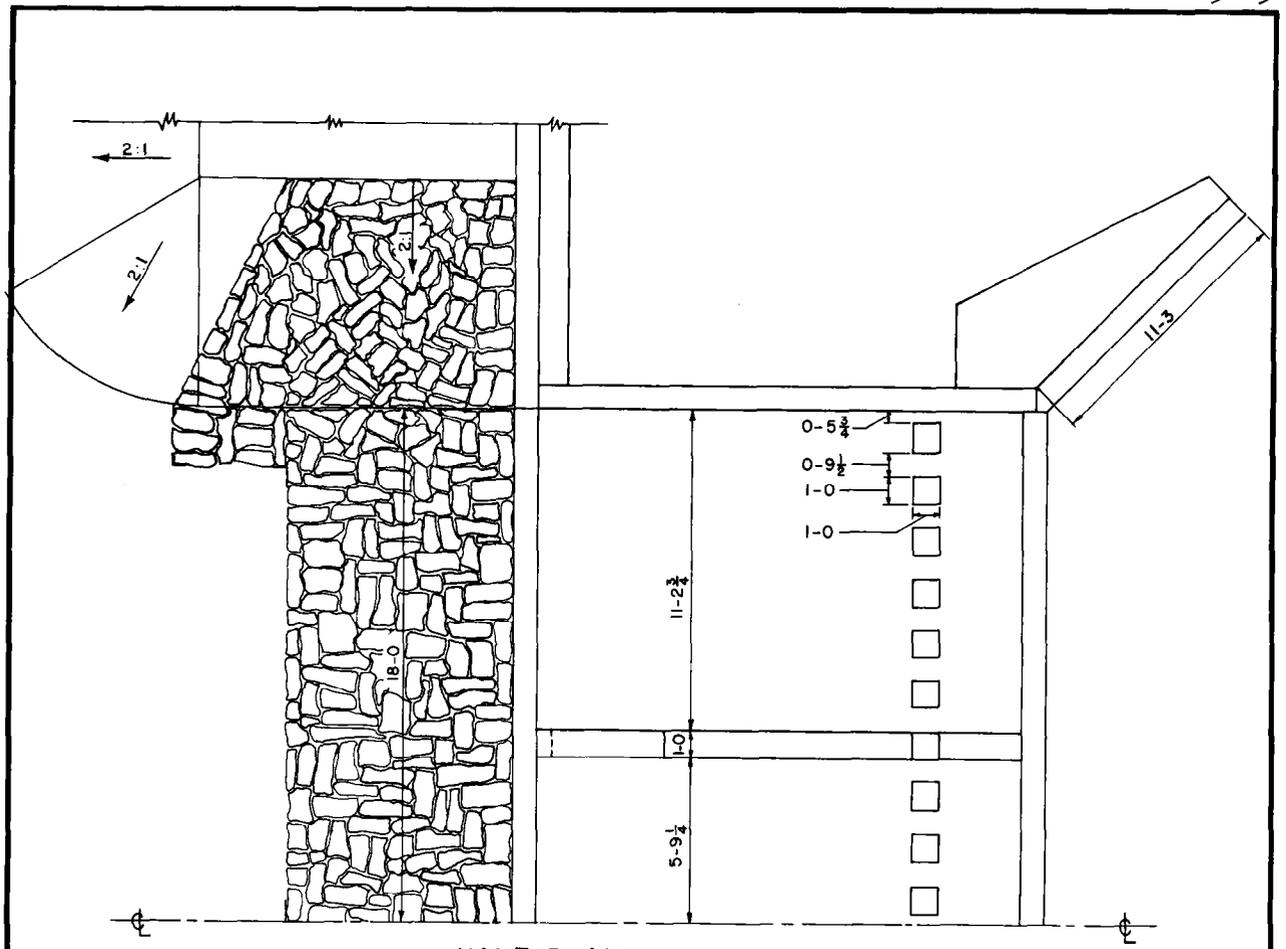
$$\text{Height of blocks} = 0.8 d_c = 2.14 \text{ ft.} \quad \text{-- use } 2' - 0''$$

9. The sidewall height above the top of the transverse sill is  $t + 0.85 d_c = 4.67 + (0.85)(2.67) = 6.94 \text{ ft}$  (use 7' - 0").

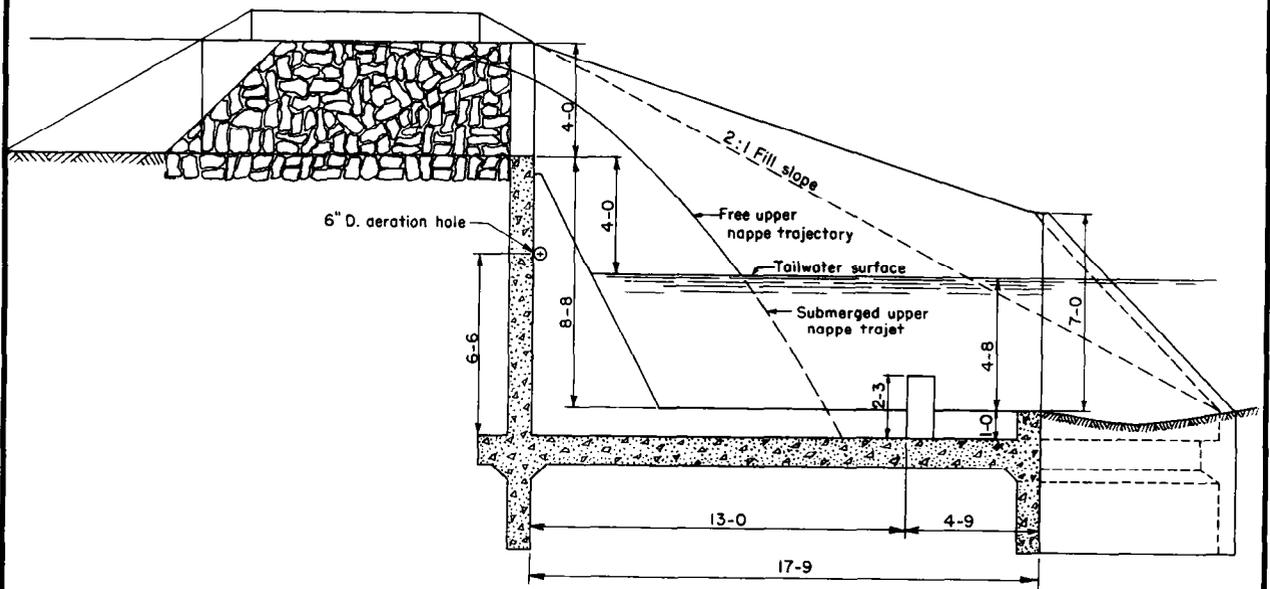
10. The 2:1 fill slope or the dike is 3.96 ft above the top of the transverse sill (end elevation of the top of the wingwall) at the junction of the wingwall and sidewall. The wingwall length is

$$\frac{(2)(3.96)}{0.707} = 11.20 \text{ ft (use } 11' - 3'')$$

11. The 12' - 6 1/2" weir length between the centers of the two buttresses is aerated by 6-inch diameter holes in each buttress. The 6-inch diameter holes will provide a differential pressure between atmospheric and pressure under the nappe of 0.17 ft (determined by ES-81, page 3.3). This does not influence the headwall design. The aeration holes are placed above the tailwater. (Use 6' - 6" above the floor of the stilling basin.)



HALF PLAN



SECTION ON CENTERLINE

Figure 1. — Design example: Straight drop spillway—Type C

Given: The required capacity-without-freeboard is  $Q_{fr} = 1450$  cfs.  
 The maximum total energy head in the approach channel at the weir, measured from the weir crest, cannot be greater than 5.0 ft. The controlled drop is  $F = 5.0$  ft. and tailwater depth is  $t = 9.0$  ft.

Determine:

1. Weir notch depth,  $h$ .
2. Crest length,  $L$ , and the capacity-without-freeboard  $Q_s$ .
3. The approach channel hydraulic requirements.
4. Minimum transverse sill height  $s$ .
5. Stilling basin length  $L_B$ .
6. Location, width, spacing and height of floor blocks.
7. Minimum sidewall height.

Solution: 1. Set the weir notch depth,  $h$ , equal to the maximum allowable total energy head in the approach channel at the weir, measured from the weir crest, thus,  $h = 5.0$  ft.

2. a. Determine the crest length of weir,  $L$ , and critical depth,  $d_c$ .

$$\frac{h}{F} = \frac{5.0}{5.0} = 1.0 \text{ and } \frac{h}{t} = \frac{5.0}{9.0} = 0.556$$

From ES-111, page 5-20

$$\frac{q_s}{h^{3/2}} = 2.6$$

$$q_s = (5^{3/2})(2.6) = 29.07 \text{ cfs.}$$

From ES-111, page 5-19

$$d_c = 2.97 \text{ ft.}$$

$$L = \frac{Q_{fr}}{q_s} = \frac{1450}{29.07} = 49.88 \text{ ft. -- Use 50 ft.}$$

b. The capacity-without-freeboard,  $Q_s$ , when  $L = 50$  ft. is

$$Q_s = q_s L = (29.07)(50) = 1453.5 \text{ cfs.}$$

3. The approach channel bottom width may be set at 50.0 ft. with side slope  $z = 2$ . (See Approach Channel, page 5-14).

4. Determine the minimum transverse sill height,  $s$ .

$$s = 0.4d_c$$

$$s = (0.4)(2.97) = 1.19 \text{ ft. -- Use 1' - 3"}$$

5. The stilling basin length  $L_B$  is determined from ES-111, page 5-21.

a. Ascertain if the submergence of the weir crest is greater than  $0.7d_c$ .

The tailwater which causes submergence of the weir of  $0.7d_c$  is

$$F + 0.7d_c = 5 + 0.7(2.97) = 7.08 \text{ ft.}$$

Since the tailwater is 9.0 ft., submergence is greater than  $0.7d_c$ .

When designing Type C drop spillways having greater than  $0.7d_c$  submergence, the symbol,  $h$ , wherever it appears on sheets 5 and 6 of ES-111, should be redesignated as  $H_{ec}$ .  $H_{ec}$  is the critical specific energy head corresponding to the unit discharge,  $q_s$ , through the weir.

- b. Determine stilling basin length,  $L_B$ .

$$H_{ec} = \frac{3}{2}d_c = 4.46 \text{ ft.}$$

$$\frac{H_{ec}}{F} = \frac{4.46}{5.0} = 0.892$$

From ES-111, page 5-21, read at the  $0.7d_c$  submergence curve, the values

$$\frac{L_B}{F} = 7.54 \text{ when } \frac{H_{ec}}{F} = 0.892$$

Since  $\frac{H_{ec}}{t} = \frac{4.46}{9.0} = 0.496$  represents submergence greater than  $0.7d_c$ , the value of  $\frac{L_B}{F} = 7.54$  is satisfactory,

(see Length of Stilling Basin Apron  $L_B$ , page 5-12).

$$L_B = 7.54(5) = 37.7 \text{ ft. -- Use } 37' - 9''$$

6. The distance from the downstream face of the headwall to the upstream face of the floor blocks  $L_f$  is determined from ES-111, page 5-22.

$$\frac{L_f}{F} = 6.52 \text{ when } \frac{H_{ec}}{F} = 0.892 \text{ at the } 0.7d_c \text{ submergence curve.}$$

$$L_f = 32.6 \text{ ft. -- use } 32' - 8''$$

The floor blocks are square in plan. The side width  $w = 0.4d_c = (0.4)(2.97) = 1.19 \text{ ft. -- use } 1' - 3''$

Twenty-two blocks will occupy 55 percent of the basin width. The blocks are spaced  $1' - 0''$  apart with the face of the outside blocks  $0' - 9''$  from the sidewall. Height of blocks =  $0.8d_c = 2.38 \text{ ft. -- use } 2' - 6''$

7. The sidewall height above the top of the transverse sill is

$$t + 0.85d_c = (9.0) + (0.85)(2.97) = 11.52 \text{ ft. -- use } 10' - 0'' \text{ (the maximum sidewall height is } F + h = 5 + 5 = 10' - 0'').$$



## 6. STRUCTURAL DESIGN EXAMPLE

General. The following example deals with the stability and structural design only. It was assumed that the site location had been selected and that the hydrologic data indicated that the structure should be designed for a peak discharge of 610 cfs. The explanation of the methods and procedures of design are contained in the example or referenced to the corresponding sections of the Engineering Handbook.

Hydraulic Design (Design Q = 610 cfs)(F = 12.0 ft)

Weir dimensions: Try h = 4'-0" and L = 30'-0" and compute capacity by equation 3.5 (page 3.7).

$$Q = \frac{3.1 Lh^{3/2}}{1.10 + 0.01F} = \frac{3.1 \cdot 30 \cdot 8}{1.22} = 610 \text{ cfs} \quad \text{OK}$$

Height of Transverse Sill (s):  $s = h \div 3 = 4.0 \div 3 = 1.33' = 1'-4"$

Apron Length ( $L_B$ ) and Height of Sidewall (J): (See ES-67, page 5.3)

$$J_{(\min)} = 2h = 2 \cdot 4 = 8.0', \quad L_{B(\min)} = 1.28 \cdot 12 = 15.36'$$

Use sidewall layout shown in the following layout drawing and determine combinations that will maintain 2 to 1 fill slope with 1.0 ft. of fill over upstream edge of headwall extension.

$$J = F + h + s + 1.0 - \left( \frac{L_B + 5''}{2} \right) - \frac{10''}{2}$$

$$L_B = 2 (F + h + s + 1.0 - J) - 0.417 - 0.833$$

$$L_B = 2 (12 + 4.0 + 1.33 + 1 - J) - 1.25$$

$$L_B = 35.42 - 2J$$

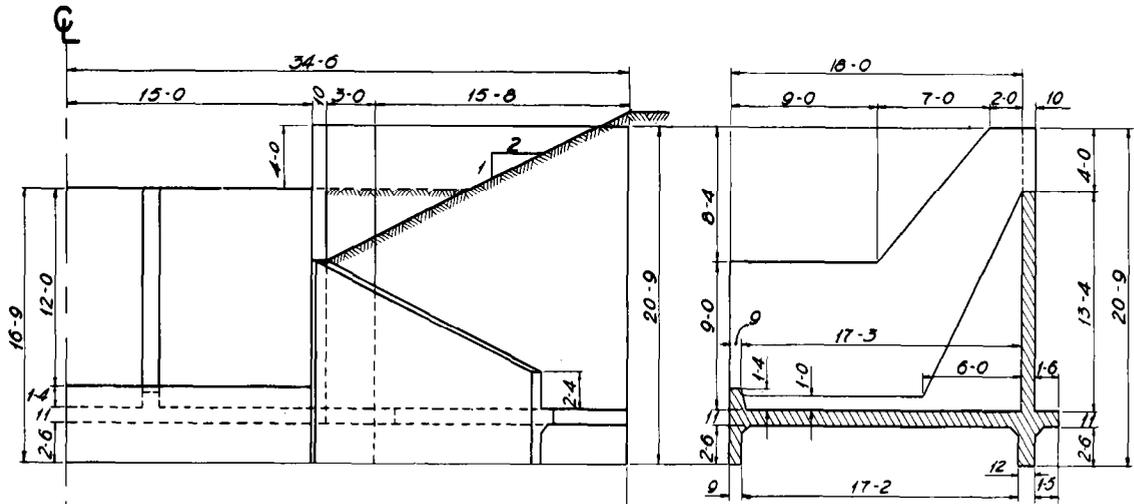
$$\text{When } J = 8.0 \text{ ft, } L_B = 19.42 \text{ ft}$$

$$\text{When } J = 9.0 \text{ ft, } L_B = 17.42 \text{ ft}$$

$$\text{When } J = 10.0 \text{ ft, } L_B = 15.42 \text{ ft}$$

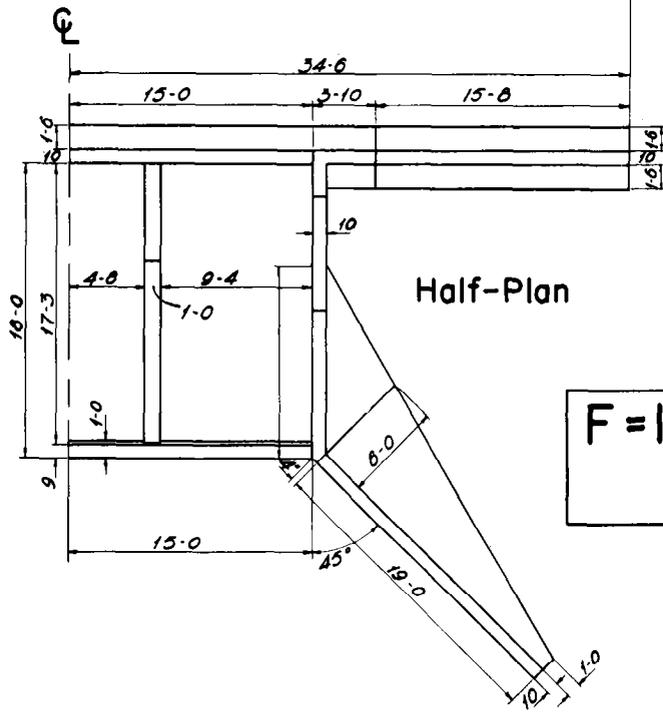
$$\text{Select and use } J = 9'-0", \quad L_B = 17'-3"$$

Shape of Sidewall: The sidewall will have a level top except near the headwall, where it will be shaped to confine the nappe. Plot the path of the upper nappe (See ES-68, page 6.3) from the crest elevation to the level portion of sidewall. Take  $d_c = 2/3 h = 2.67 \text{ ft}$ .



Half-Downstream Elevation

Cross-section along C-C



Half-Plan

$F = 12-0, h = 4-0, L = 30-0$   
Handbook Example

# DROP SPILLWAYS: DIMENSIONLESS COORDINATES OF WATER SURFACES FOR AERATED NAPPE OVER WEIR WITH LEVEL, FLUSH APPROACH CHANNEL

## EQUATIONS

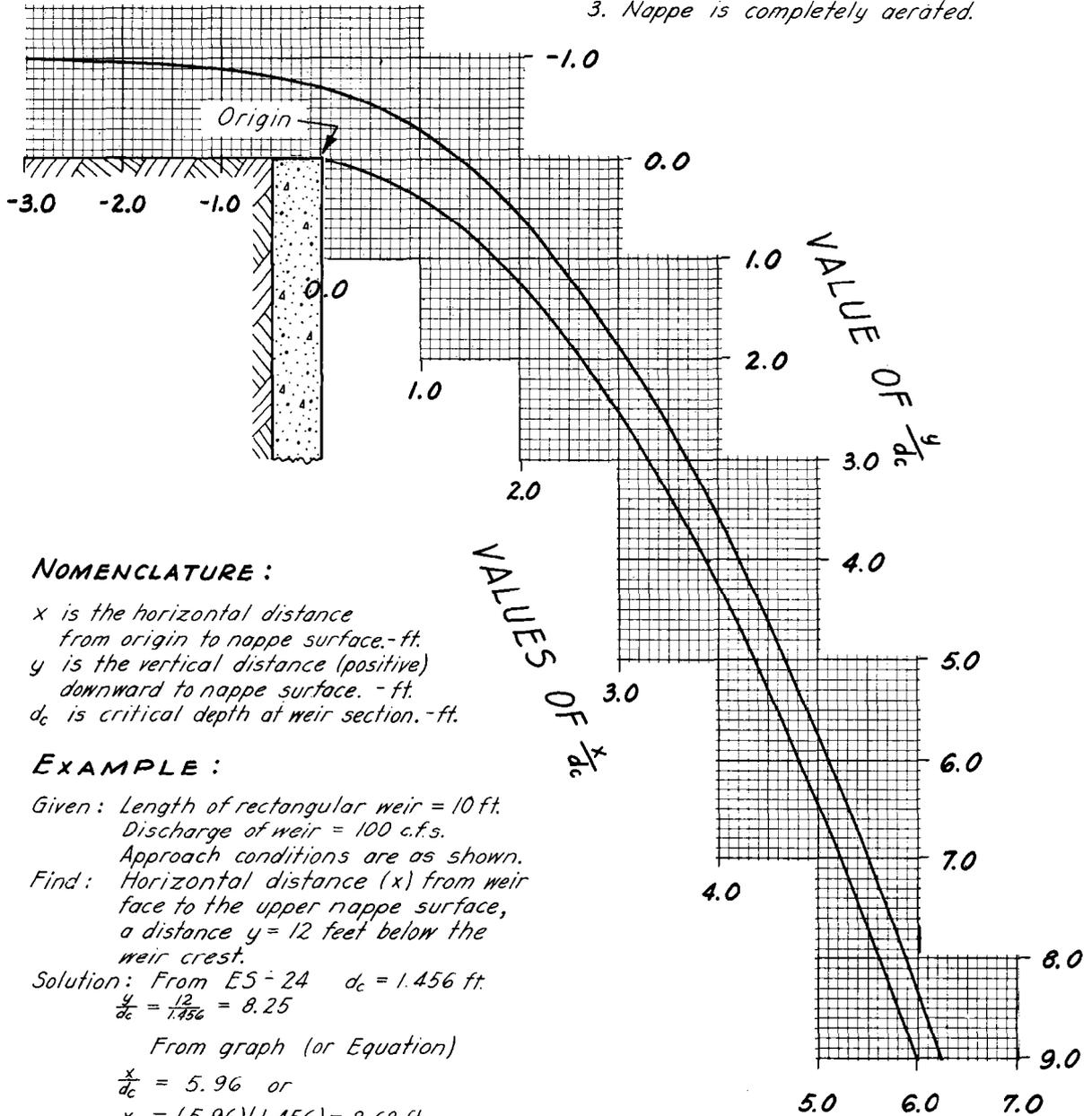
Upper Surface of nappe  
 $\frac{x}{d_c} = 2.12\sqrt{\frac{y}{d_c}} + 0.69 - 0.38$  when  $\frac{y}{d_c} > 0.3$

Lower Surface of nappe  
 $\frac{x}{d_c} = 2.12\sqrt{\frac{y}{d_c}} - 0.38$  when  $\frac{y}{d_c} > 1.0$

## NOTE

The given equations define the nappe surfaces only when the flow conditions are as follows:

1. Approach Channel is level for a distance of  $3.0 d_c$  upstream from the weir.
2. Approach Channel is flush with the weir crest.
3. Nappe is completely aerated.



## NOMENCLATURE:

- $x$  is the horizontal distance from origin to nappe surface, -ft.
- $y$  is the vertical distance (positive) downward to nappe surface, -ft.
- $d_c$  is critical depth at weir section, -ft.

## EXAMPLE:

Given: Length of rectangular weir = 10 ft.  
 Discharge of weir = 100 c.f.s.  
 Approach conditions are as shown.  
 Find: Horizontal distance ( $x$ ) from weir face to the upper nappe surface, a distance  $y = 12$  feet below the weir crest.

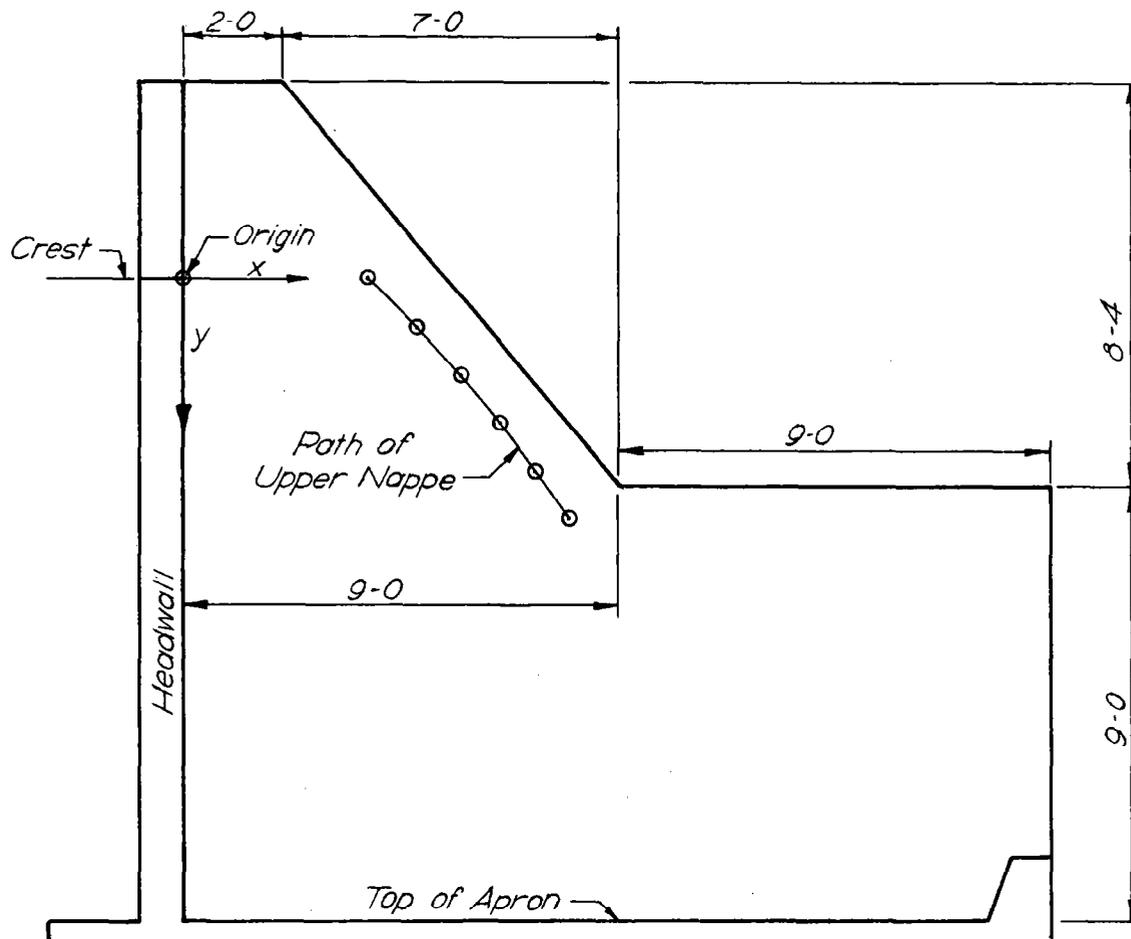
Solution: From ES - 24  $d_c = 1.456$  ft.  
 $\frac{y}{d_c} = \frac{12}{1.456} = 8.25$

From graph (or Equation)

$\frac{x}{d_c} = 5.96$  or

$x = (5.96)(1.456) = 8.68$  ft.

y	y/d <sub>c</sub>	x/d <sub>c</sub>	x
0	0	1.4	3.74
1	0.375	1.8	4.81
2	0.750	2.15	5.73
3	1.125	2.45	6.55
4	1.500	2.73	7.29
5	1.875	3.00	8.00



Downstream flow: (Tailwater depth above transverse sill, t)

Required depth,  $t = 4.6$  ft (See fig. 5.1, page 5.2)

( $d_c = 2.67$ ,  $k = 1.25$ )

Allowable velocity (max) = 3.0 fps

$s = 0.00095$  ft/ft,  $n = 0.035$ , sideslopes 3 to 1 or  $z = 3$

Try channel bottom width,  $b = 30$  ft (See ES-33, Section 5, Hydraulics)

$$\text{Hydraulic radius, } r = \frac{bd + zd^2}{b + 2d \sqrt{z^2 + 1}} = \frac{30 \cdot 4.6 + 3 \cdot 4.6^2}{30 + (2 \cdot 4.6 \sqrt{9 + 1})}$$

$$r = \frac{201.5}{59.1} = 3.41$$

See ES-34, Section 5, Hydraulics

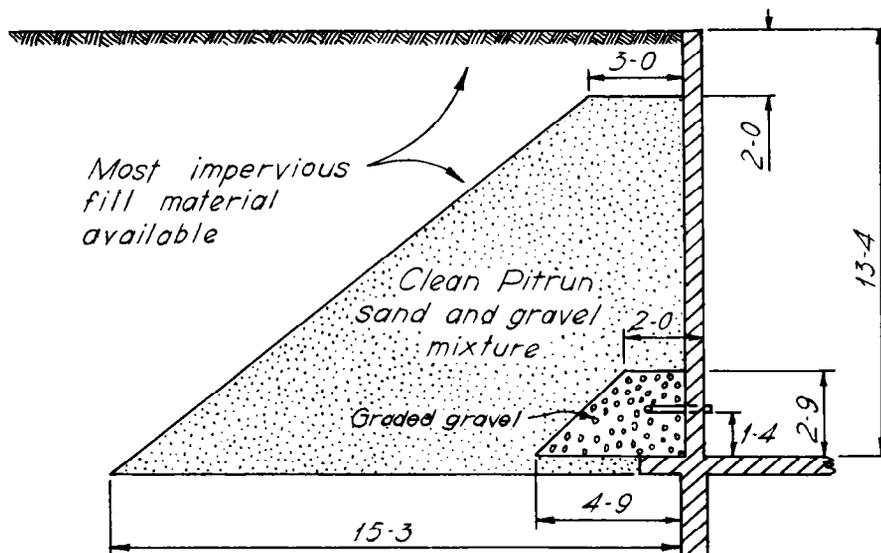
With  $r = 3.41$ ,  $s = 0.00095$ , and  $n = 0.035$ ,  $v = 2.97$  fps OK

$t + s =$  tailwater depth above apron  $= 4.60 + 1.33 = 5.93$  ft

Use 6.0 ft for design purposes

Stability Design: The stability of the structure will be checked for a representative bay. The representative bay will extend  $30.83 \div 6 = 5.14$  ft on both sides of the center line of a buttress.

Design of drain and drain filter above structure: Preliminary investigations indicated it would be advantageous to design a very good drain and filter upstream from the headwall to reduce uplift and to minimize the forces tending to slide the structure. Therefore, a drain similar to the one discussed on pages 6.2-13 and 6.2-14 of the Structural Design Section will be used. Use 4 in. Perforated Clay Pipe, with 1/4 in. diameter perforation, four rows, 3 in. spacing. A.S.T.M. Designation C-211.



Criteria for Filter Gradation

Graded Gravel

$$\frac{85 \text{ percent size of gravel}}{\text{diameter of perforation}} \geq 1.0$$

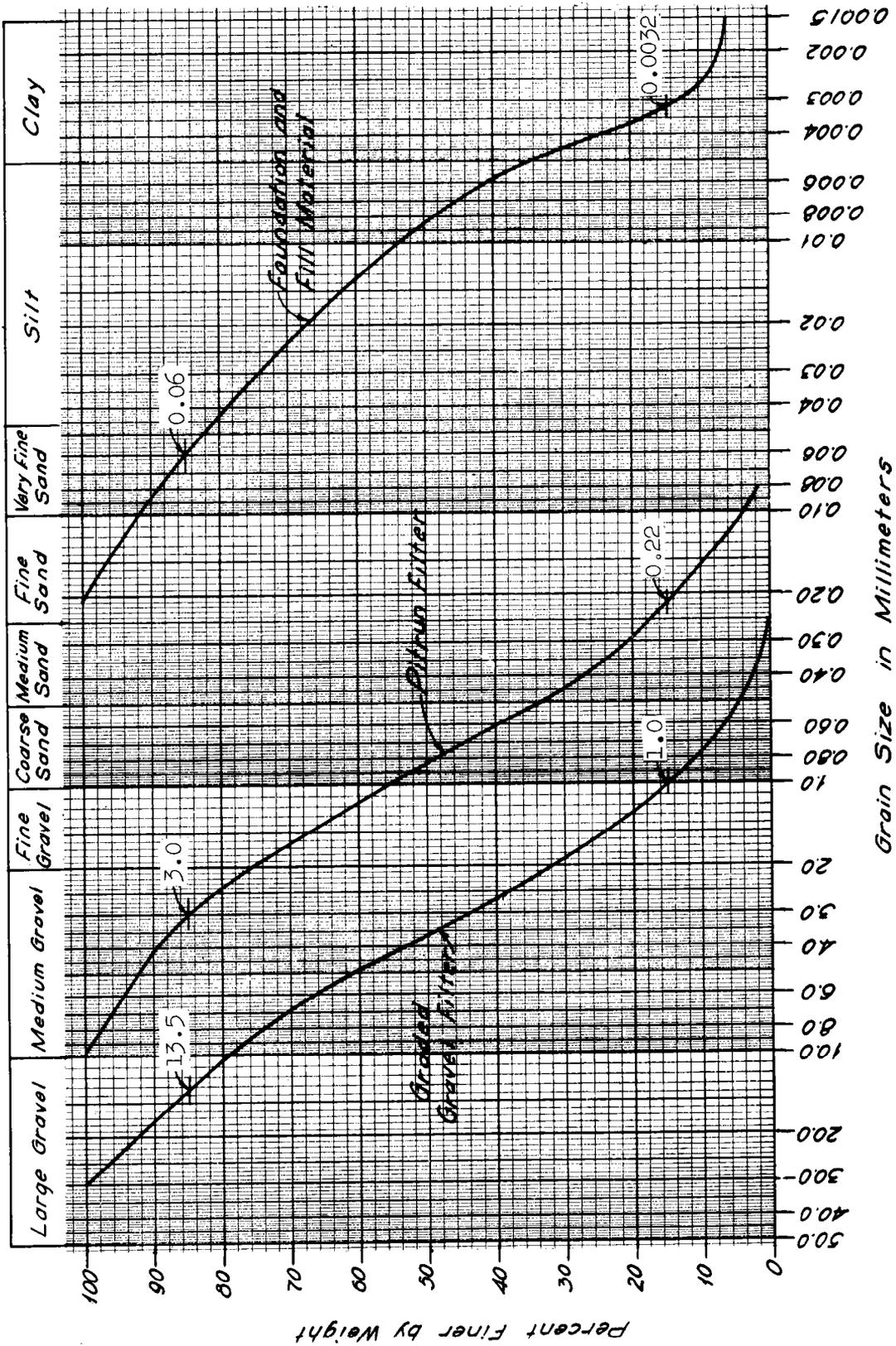
Pitrun Sand and Gravel

$$\frac{15 \text{ percent size of gravel}}{85 \text{ percent size of pitrun}} \leq 4 \text{ to } 5 \leq \frac{15 \text{ percent size of gravel}}{15 \text{ percent size of pitrun}}$$

and

$$\frac{15 \text{ percent size of pitrun}}{85 \text{ percent size of foundation}} \leq 4 \text{ to } 5 \leq \frac{15 \text{ percent size of pitrun}}{15 \text{ percent size of foundation}}$$

# GRAIN SIZE CURVES (U.S. Bureau of Soils Classification)



Check Filter Gradation: (See preceding page for grain sizes)

	15 percent size	85 percent size
Graded Gravel	1.0 mm	13.5 mm
Pitrun Sand and Gravel	0.22 mm	3.0 mm
Foundation and Fill Material	0.0032 mm	0.06 mm

Pipe perforations =  $1/4$  in. diam. =  $0.25 \cdot 25.4 = 6.35$  mm

$$\frac{\text{85 percent size of gravel}}{\text{diameter of perforations}} = \frac{13.5}{6.35} = 2.13 > 1.0 \quad \text{OK}$$

$$\frac{\text{15 percent size of gravel}}{\text{85 percent size of pitrun}} = \frac{1.0}{3.0} = 0.33 < 4.0 \quad \text{OK}$$

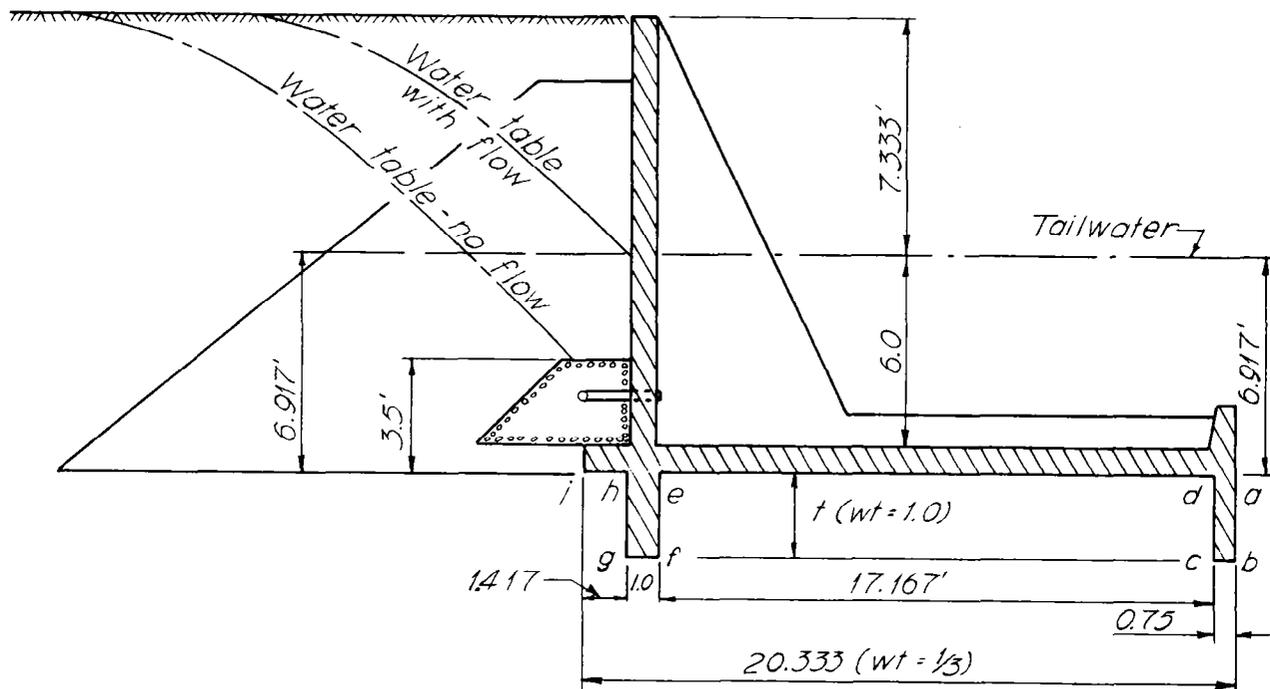
$$\frac{\text{15 percent size of gravel}}{\text{15 percent size of pitrun}} = \frac{1.0}{0.22} = 4.55 > 4.0 \quad \text{OK}$$

$$\frac{\text{15 percent size of pitrun}}{\text{85 percent size of foundation}} = \frac{0.22}{0.06} = 3.67 < 4.0 \quad \text{OK}$$

$$\frac{\text{15 percent size of pitrun}}{\text{15 percent size of foundation}} = \frac{0.22}{0.0032} = 68.8 > 4.0 \quad \text{OK}$$

The 15 percent size of any given material is a unique size for that material such that 15 percent by weight of the material is finer than it. Likewise, the 85 percent size is the size for which 85 percent by weight is finer.

Determine required depth of cutoff wall and toewall: (See Piping, page 4.14)



Foundation Material (Firm clay)--20 percent sand, 45 percent silt, 35 percent clay

$$\text{Wt. Creep Ratio} = C_w = 2.3 \text{ (See page 4.16)}$$

It is estimated that the drain will lower the water table at the structure, as shown in the above sketch. The maximum differential head occurs during periods of no flow.

$$\text{Required Wt. Creep Dist.} = 2.3 \cdot 3.5 = 8.05 = 4t + (20.33 \div 3)$$

$$4t = 8.05 - 6.78 = 1.27, \quad t = 0.32, \quad \text{Use minimum of 2'-6"} \underline{\hspace{1cm}}$$

$$\text{Actual Wt. Creep Dist.} = 4(2.5) + 6.78 = 16.78$$

$$\text{Safe differential head} = H = 16.78 \div 2.3 = 7.30 \text{ ft}$$

Compute Uplift (Refer to sketch in foregoing paragraph)

With flow

$$\text{Uplift} = 6.92 \cdot 62.4 = 432 \text{ lbs/ft}^2$$

$$\text{Total Uplift on bay} = 432 \cdot 20.33 \cdot 10.28 = 90,300 \text{ lbs}$$

$$\text{Wt. of bay} = 163,009 \text{ lbs (See base pressures, with flow)}$$

$$\text{Ratio} = \text{Wt.} \div \text{Uplift} = 163,009 \div 90,300 = 1.80 \quad \text{OK}$$

No flow

$$\text{Pressure at point (a)} = 0, \quad \text{Pressure at point (i)} = 3.5 \cdot 62.4 = 218 \text{ lbs/ft}^2$$

$$\text{Wt. Creep Dist.} = 16.78 \text{ ft, Increment of pressure} = \text{loss per unit of weighted creep dist.} = 218 \div 16.78 = 13.0 \text{ lbs/ft}^2$$

Point	Wt. Creep Dist. ft	Inc. of Pres. lbs/ft <sup>2</sup>	Pres. at Point lbs/ft <sup>2</sup>	Ave. Pres. between points lbs/ft <sup>2</sup>	Base Length ft	Uplift per ft width lbs	Base Width ft	Total Uplift lbs
a			0					
b	2.5	32.5	32.5					
c	0.25	3.2	35.7	34	0.75	26	10.28	267
d	2.5	32.5	68.2					
e	5.72	74.4	142.6	105	17.17	1802	10.28	18,500
f	2.5	32.5	175.1					
g	0.333	4.3	179.4	177	1.0	177	10.28	1,820
h	2.5	32.5	211.9					
i	0.472	6.1	218.0	215	1.42	304	10.28	3,120
Total uplift on bay =								23,707

$$\text{Wt. of bay} = 88,959 \text{ lbs}$$

$$\text{Ratio} = \text{Wt.} \div \text{Uplift} = 88,959 \div 23,707 = 3.75$$

OK

Determine contact and total pressures on base: Take moments about downstream edge at elevation of bottom of apron of representative bay.

Before fill (No uplift)

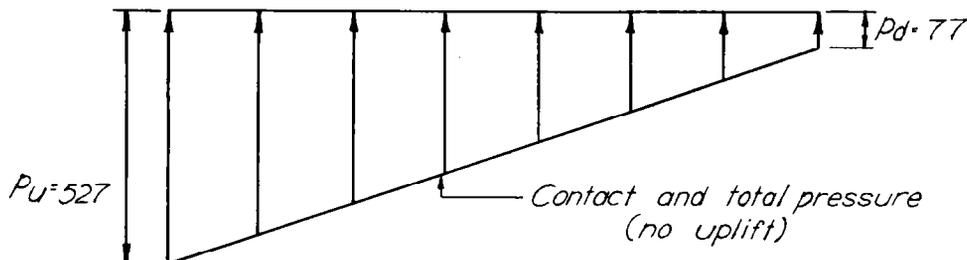
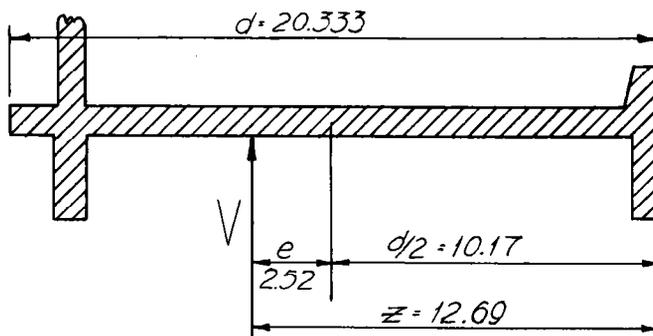
Part		Wt.	Arm	Moment
Apron	$10.28 \cdot 20.33 \cdot 0.917 \cdot 150$	28,700	10.17	292,000
Headwall	$10.28 \cdot 13.33 \cdot 0.833 \cdot 150$	17,100	18.42	315,000
C. O. Wall	$10.28 \cdot 2.5 \cdot 1.0 \cdot 150$	3,860	18.42	71,100
C. O. Fillets	$10.28 \cdot 0.25 \cdot 150$	386	18.42	7,100
Toewall	$10.28 \cdot 2.5 \cdot 0.75 \cdot 150$	2,890	0.375	1,086
Twl. Fillets	$10.28 \cdot 0.125 \cdot 150$	193	0.917	177
Trans. Sill	$10.28 \cdot 1.33 \cdot 0.875 \cdot 150$	1,800	0.440	792
Long. Sill	$17.09 \cdot 1.0 \cdot 1.0 \cdot 150$	2,560	9.44	24,200
Buttress	$12.33 \cdot 3.0 \cdot 1.0 \cdot 150$	5,550	16.00	88,800
		63,039	12.69	800,255

$$z = 800,255 \div 63,039 = 12.69 \text{ ft}, e = 12.69 - 10.17 = 2.52 \text{ ft}$$

$$\text{Area of base} = 10.28 \cdot 20.33 = 209.0 \text{ ft}^2$$

$$p = (V \div A) [1 \pm (6e \div d)] = (63,039 \div 209) [1 \pm (6 \cdot 2.52 \div 20.33)]$$

$$p_u = 302 (1 + 0.744) = \underline{527 \text{ lbs/ft}^2}, p_d = 302 (1 - 0.744) = \underline{77 \text{ lbs/ft}^2}$$



After fill (No flow)

Backfill Properties (upstream from headwall) (compacted, clean pitrun sand and gravel)

Dry wt.	115 lbs/ft <sup>3</sup>
Void ratio, e	0.35
Percent voids	$\left(\frac{e}{1+e} \cdot 100\right)$ 25.9
Moist wt.	125 lbs/ft <sup>3</sup>
$\phi$	35° $\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) = 0.271$
Cohesion	0
Eff. submerged wt. =	$[115 - (1 - 0.259) 62.4] = 69 \text{ lbs/ft}^3$
Saturated wt. =	$115 + (0.259 \cdot 62.4) = 131.2 \text{ lbs/ft}^3$

For design purposes assume pitrun material to crest elevation.

Resolve Uplift into Trapezoidal Diagram, to simplify calculations.

Rate of change per ft of horizontal contact =  $13.0 \div 3 = 4.33$   
(See uplift, no flow)

Let x = unit uplift at downstream edge, y = unit uplift at upstream edge

$$\text{then } y = x + (4.33 \cdot 20.33) = x + 88$$

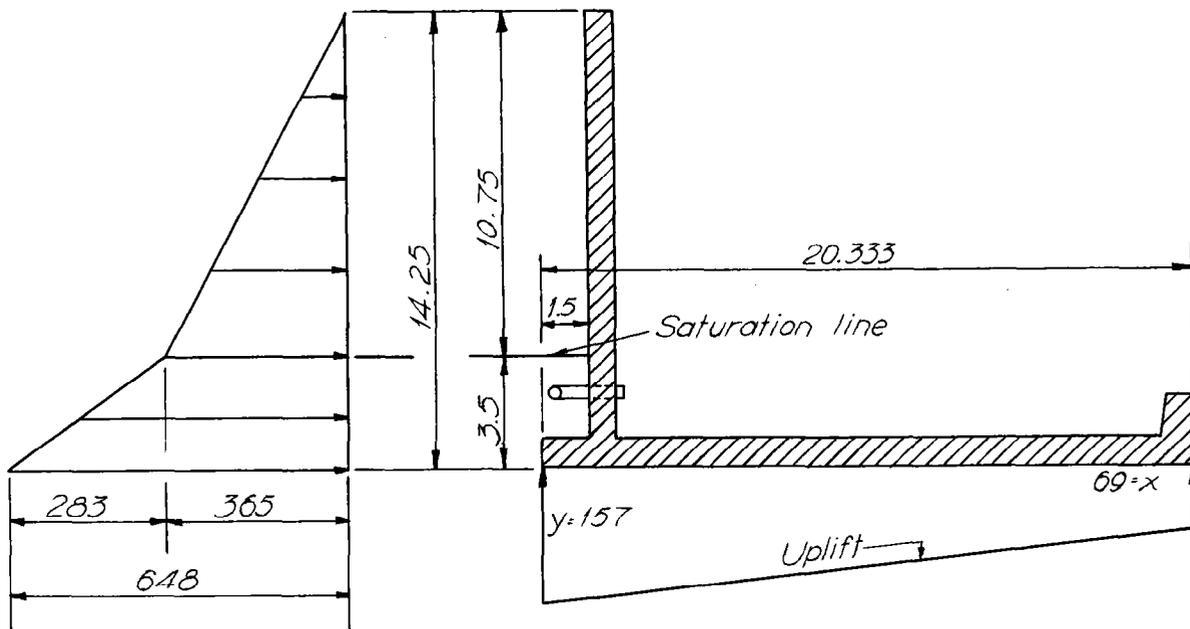
$$\text{Total uplift per ft width} = 23,707 \div 10.28 = 2305 \text{ lbs}$$

$$\left[\frac{(x + y) \div 2}{d}\right] = 2305 \text{ lbs}$$

$$(x + x + 88) \div 2 = 2305 \div 20.33 = 113 \text{ lbs/ft}^2$$

$$x = 113 - 44 = 69 \text{ lbs/ft}^2$$

$$y = 69 + 88 = 157 \text{ lbs/ft}^2$$



Determine horizontal pressuresAt 10.75 ft depth

$$\text{Vert. wt.} = 10.75 \cdot 125 = 1345 \text{ lbs}$$

$$\text{Unit equiv. fluid pres.} = \text{vert. wt.} (1 - \sin \phi \div 1 + \sin \phi)$$

$$p = 1345 (0.271) = 365 \text{ lbs/ft}^2$$

At 14.25 ft depth

$$\text{Vert. wt. (earth)} = 1345 + (3.5 \cdot 69) = 1345 + 242 = 1587 \text{ lbs}$$

$$\text{Unit equiv. fluid pres.} = 1587 (0.271) = 430 \text{ lbs/ft}^2$$

$$\text{Water pres.} = 3.5 \cdot 62.4 = 218 \text{ lbs/ft}^2$$

$$p = 430 + 218 = 648 \text{ lbs/ft}^2$$

Part	Force	Wt.	Arm	Moment
Concrete		63,039	12.69	800,255
Moist sand	$10.28 \cdot 10.75 \cdot 1.5 \cdot 125 =$	20,700	19.58	405,500
Sat. sand	$10.28 \cdot 2.58 \cdot 1.5 \cdot 131 =$	5,220	19.58	102,300
Hor. Pres.	$10.28 \cdot 0.5 \cdot 365 \cdot 10.75 =$	-20,200	7.08	-143,000
Hor. Pres.	$10.28 \cdot 365 \cdot 3.5 =$	-13,130	1.75	-23,000
Hor. Pres.	$10.28 \cdot 0.5 \cdot 283 \cdot 3.5 =$	-5,100	1.17	-5,950
	-38,430	88,959		1,136,105
Uplift 	$10.28 \cdot 69 \cdot 20.33$	-14,400	10.17	-146,300
Uplift 	$10.28 \cdot 0.5 \cdot 88 \cdot 20.33$	-9,200	13.55	-124,700
		-23,600		-271,000
		+65,359	13.23	+865,105

$$z = 865,105 \div 65,359 = 13.23 \text{ ft}$$

$$e = 13.23 - 10.17 = 3.06 \text{ ft}$$

Contact Pressures

$$P_u = (65,359 \div 209) [1 + (6 \cdot 3.06 \div 20.33)]$$

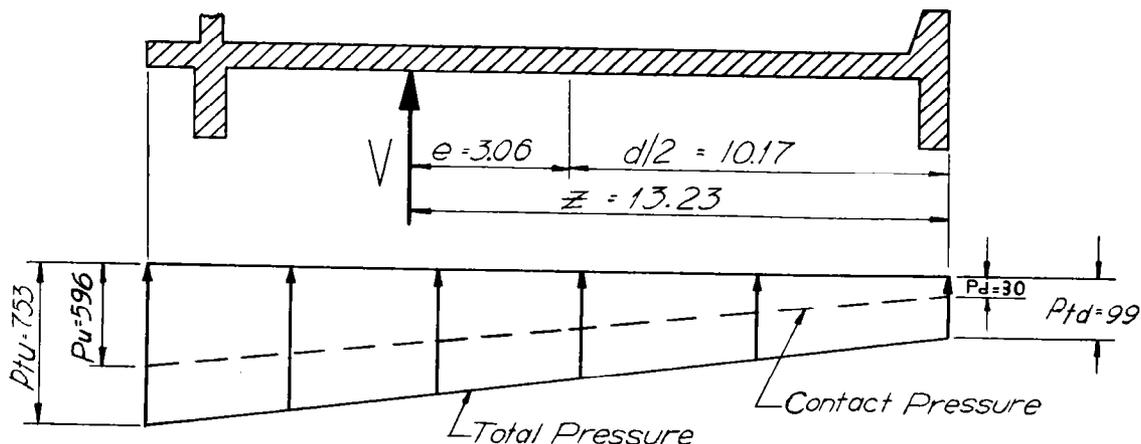
$$P_u = 313 (1 + 0.905) = 596 \text{ lbs/ft}^2$$

$$P_d = 313 (1 - 0.905) = 30 \text{ lbs/ft}^2 \quad \text{OK Positive pressures}$$

Total Pressures

$$P_{tu} = 596 + 157 = 753 \text{ lbs/ft}^2$$

$$P_{td} = 30 + 69 = 99 \text{ lbs/ft}^2$$

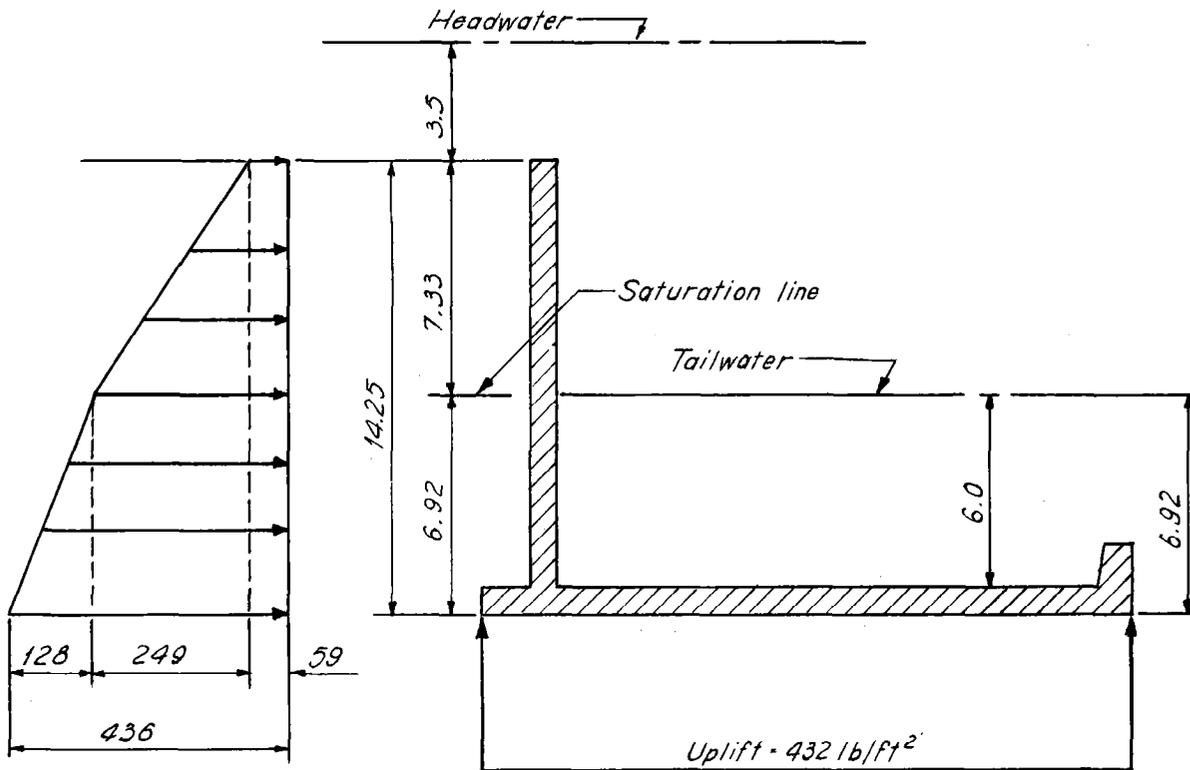


With flow (Design Discharge)

Depth of headwater above crest, use 3.5 ft

Depth of tailwater above bottom of apron = 6.92 ft

Uplift is uniform = 432 lbs/ft<sup>2</sup>

Determine Horizontal Pressures

At Crest (surcharge of headwater)

$$\text{Vert. Wt.} = 3.5 \cdot 62.4 = 218 \text{ lbs, } p = 218 \cdot 0.271 = 59 \text{ lbs/ft}^2$$

At 7.33 ft depth

$$\text{Vert. Wt.} = 218 + 125 \cdot 7.33 = 218 + 916 = 1134 \text{ lbs}$$

$$p = 1134 \cdot 0.271 = 308 \text{ lbs/ft}^2$$

At 14.25 ft depth

$$\text{Vert. Wt.} = 1134 + 6.92 \cdot 69 = 1134 + 477 = 1611 \text{ lbs}$$

$$p = 1611 \cdot 0.271 = 436 \text{ lbs/ft}^2$$

NOTE: Water pressures in both directions are equal and cancel one another.

Part		Force	Wt.	Arm	Moment
Concrete			63,039		800,255
Moist Sand	$10.28 \cdot 7.33 \cdot 1.5 \cdot 125$		14,150	19.58	277,000
Satur. Sand	$10.28 \cdot 6.92 \cdot 1.5 \cdot 131$		13,980	19.58	274,000
H <sub>2</sub> O on Foot.	$10.28 \cdot 3.5 \cdot 1.5 \cdot 62.4$		3,370	19.58	66,000
H <sub>2</sub> O on Apron	$9.28 \cdot 6.0 \cdot 17.25 \cdot 62.4$		59,900	9.38	562,000
H <sub>2</sub> O on T. S.	$10.28 \cdot 4.67 \cdot 0.75 \cdot 62.4$		2,250	0.375	843
H <sub>2</sub> O on Butt.	$1.0 \cdot 2.5 \cdot 2.43 \cdot 62.4$		379	12.81	4,850
H <sub>2</sub> O on L. S.	$1.0 \cdot 11.25 \cdot 5.0 \cdot 62.4$		3,510	6.38	22,400
Hor. Pres.	$10.28 \cdot 59 \cdot 14.25$	8,650	—	7.13	— 61,600
Hor. Pres.	$10.28 \cdot 0.5 \cdot 249 \cdot 7.33$	9,390	—	9.36	— 87,900
Hor. Pres.	$10.28 \cdot 249 \cdot 6.92$	17,700	—	3.46	— 61,300
Hor. Pres.	$10.28 \cdot 0.5 \cdot 128 \cdot 6.92$	4,550	—	2.31	— 10,500
		40,290	+160,578		+1,786,048
Uplift	$10.28 \cdot 432 \cdot 20.33$		- 90,300	10.17	- 918,000
			+ 70,278	12.36	+ 868,048

$$z = 868,048 \div 70,278 = 12.36 \text{ ft}, e = 12.36 - 10.17 = 2.19 \text{ ft}$$

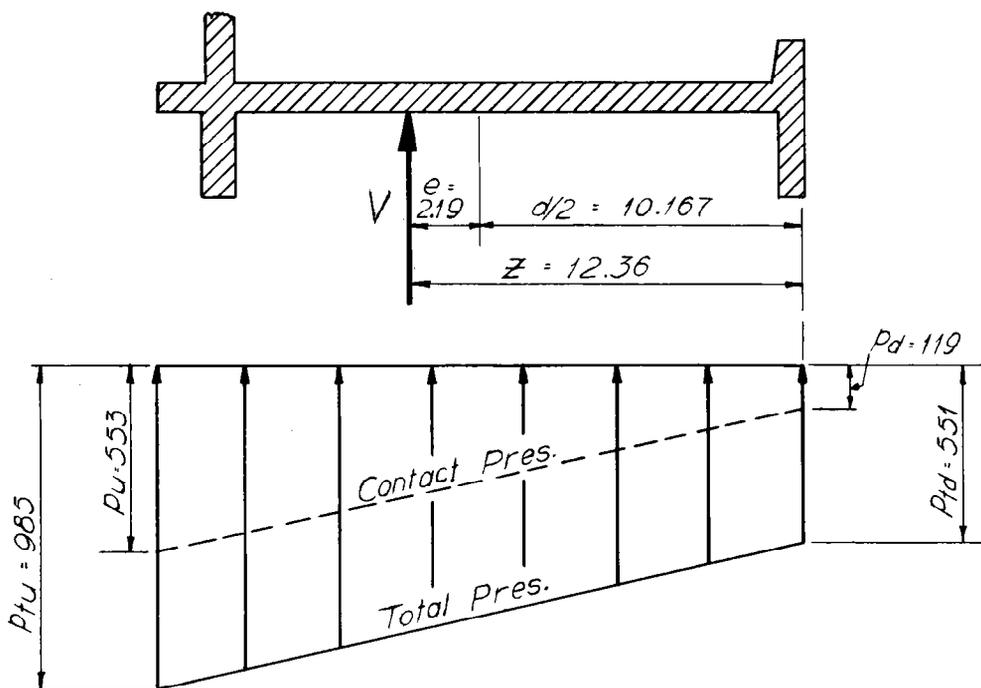
$$p_u = (70,278 \div 209) [1 + (6 \cdot 2.19 \div 20.33)]$$

$$p_u = 336 (1 + 0.646) = 553 \text{ lbs/ft}^2$$

$$p_d = 336 (1 - 0.646) = 119 \text{ lbs/ft}^2$$

$$p_{tu} = 553 + 432 = 985 \text{ lbs/ft}^2$$

$$p_{td} = 119 + 432 = 551 \text{ lbs/ft}^2$$



Check Sliding: Assume plane of sliding at elevation of bottom of cutoffs. Neglect passive resistance in front of toewall.

Properties of foundation material (Undisturbed and saturated)

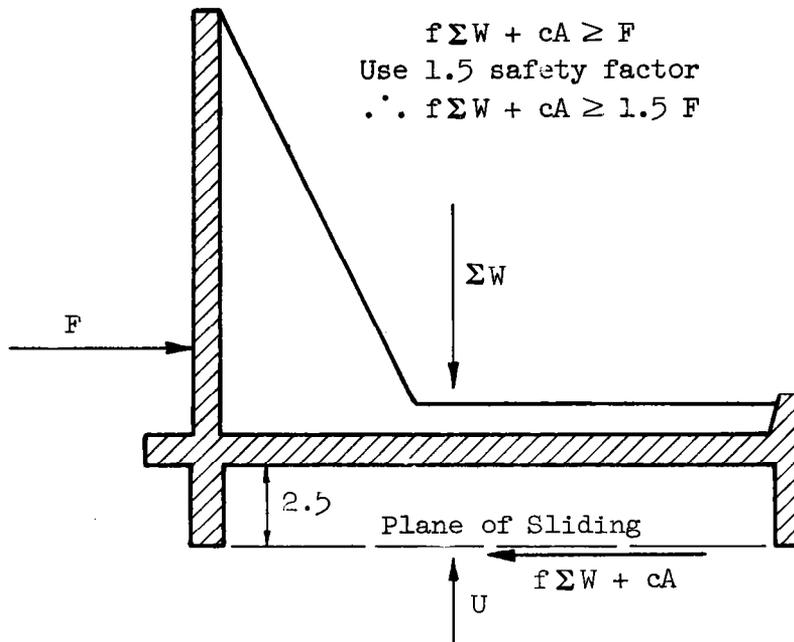
$$\text{Angle of internal friction, } \phi = 12^\circ, \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right) = 0.655$$

$$\text{Friction coefficient} = \tan \phi = 0.213 = f$$

$$\text{Void ratio} = 0.65$$

$$\text{Effective submerged wt.} = 62.2 \text{ lbs/ft}^3$$

$$\text{Cohesion, } c = 500 \text{ lbs/ft}^2 \text{ for sliding}$$



No flow

$$\text{Eff. wt. of soil between cutoffs} = 2.5 \cdot 10.28 \cdot 17.17 \cdot 62.2 = 27,500 \text{ lbs}$$

$$\text{Wt. of bay minus uplift (See base pressures)} = 65,359 \text{ lbs}$$

$$\Sigma W = 92,859 \text{ lbs}$$

$$\text{Area of sliding plane, } A = 17.17 \cdot 10.28 = 176.5 \text{ ft}^2$$

$$\therefore f \Sigma W + cA = (0.213 \cdot 92,859) + (500 \cdot 176.5) = 108,000 \text{ lbs}$$

Determine F (See contact pressure calculation for horizontal forces from crest to bottom of apron, pages 6.10 and 6.11.)

At 14.25 ft depth (just below apron)

$$\text{Vert. wt.} = 1587 \text{ lbs}$$

$$p = 1587 \cdot 0.655 = 1040 \text{ lbs/ft}^2$$

At 16.75 ft depth

$$\text{Vert. wt.} = 1587 + (2.5 \cdot 62.2) = 1587 + 155 = 1742 \text{ lbs}$$

$$p = 1742 \cdot 0.655 = 1140 \text{ lbs/ft}^2$$

$$F = 10.28 \left[ (365 \cdot 0.5 \cdot 10.75) + (1013 \cdot 0.5 \cdot 3.5) + (2180 \cdot 0.5 \cdot 2.5) \right]$$

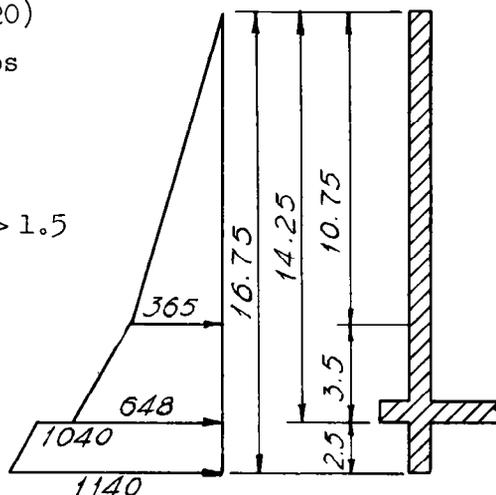
$$F = 10.28 (1960 + 1770 + 2720)$$

$$F = 10.28 \cdot 6450 = 66,300 \text{ lbs}$$

$$1.5F = 99,450 \text{ lbs}$$

OK Safe from sliding

$$\frac{f \Sigma W + cA}{F} = \frac{108,000}{66,300} = 1.63 > 1.5$$



With flow (Design discharge)

$$\text{Eff. wt. of soil} = 27,500 \text{ lbs}$$

$$\text{Wt. of bay - uplift} = \underline{70,278} \text{ lbs}$$

$$\Sigma W = 97,778 \text{ lbs}$$

$$f \Sigma W + cA = 0.212 (97,778) + 500 (1765) = 20,720 + 88,250 = 108,270$$

Determine F (See contact pressure calculations, page 6.12 and 6.13)

At 14.25 ft depth (just below apron)

$$\text{Vert wt.} = 1611 \text{ lbs, } p = 1611 \cdot 0.655 = 1055 \text{ lbs/ft}^2$$

At 16.75 ft depth

$$\text{Vert. wt.} = 1611 + 155 = 1766 \text{ lbs, } p = 1766 \cdot 0.655 = 1157 \text{ lbs/ft}^2$$

$$F = 10.28 \left[ (367 \cdot 0.5 \cdot 7.33) + (744 \cdot 0.5 \cdot 6.92) + (2212 \cdot 0.5 \cdot 2.5) \right]$$

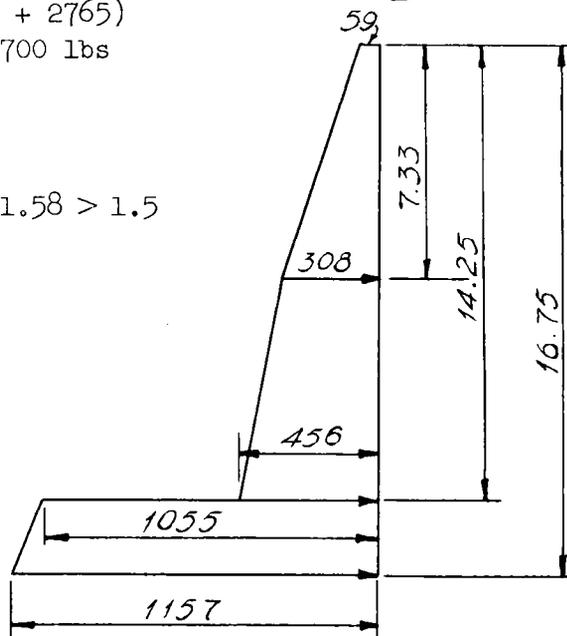
$$F = 10.28 (1345 + 2575 + 2765)$$

$$F = 10.28 \cdot 6685 = 68,700 \text{ lbs}$$

$$1.5F = 103,050 \text{ lbs}$$

OK Safe from sliding

$$\frac{f \Sigma W + cA}{F} = \frac{108,270}{68,700} = 1.58 > 1.5$$



Headwall Design: The headwall will be designed as a slab with fixed sides and bottom and a free top (see ES-6, Engineering Handbook, Section 6). To use this reference, it is necessary to resolve the actual load diagram into a triangular load diagram. This is done by making the cantilever moment at the base of the headwall equal for both load diagrams. The maximum load will occur with the design flow over the structure. (See load computed to determine contact pressures.)

Determine Equivalent Hydrostatic Pressure (w) for Triangular Load Diagram. (See actual load computed to determine contact pressures)

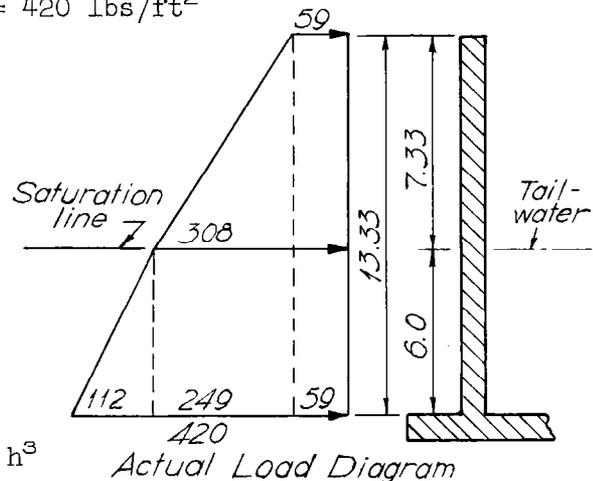
At 13.33 ft depth

$$\text{Vert. wt.} = 1134 + 6(69) = 1134 + 414 = 1548 \text{ lbs}$$

$$p = 1548(0.271) = 420 \text{ lbs/ft}^2$$

Taking Moments about base of headwall

Force	Arm	M
$59 \cdot 13.33 = 787$	6.67	5,250
$249 \cdot 0.5 \cdot 7.33 = 912$	8.44	7,700
$249 \cdot 6.0 = 1494$	3.0	4,482
$112 \cdot 0.5 \cdot 6.0 = 336$	2.0	672
M =		18,104



For Triangular Load Diagram

$$M = wh^3 \div 6 \text{ or } w = 6M \div h^3$$

$$w = 6(18,104) \div (13.33)^3$$

$$w = 45.9 \text{ lbs/ft}^3 \quad \text{Use } 46.0 \text{ lbs/ft}^3$$

Slab Design (with two buttresses) (Class B Concrete)

$$b = 30.83 \div 3 = 10.28 \text{ ft, } a = 13.33 \text{ ft, } b/a = 0.77$$

Check minimum wall thickness of 10 in (d = 7.5 in)

$$M_{\max} = -0.025 \cdot 46 \cdot (13.33)^3 = 2730 \text{ ft lbs (see ES-6, sheet 1)}$$

$$\text{Req'd. } d = 3.8 \text{ in OK (See ES-45, Engr. Hbk., Sec. 6)}$$

$$V_{\max} = 0.27 \cdot 46 \cdot (13.33)^2 = 2210 \text{ lbs}$$

$$\text{req'd. } d < 3.0 \text{ in OK (See ES-51, Engr. Hbk., Sec. 6)}$$

Vertical Steel (Unexposed face) d = 10 - 2.5 = 7.5 in

$$-M_x = 0.025 \cdot 109,300 = 2730 \text{ ft lbs, } A_s = 0.235 \text{ (from ES-45)}$$

$$V = 0.197 \cdot 8190 = 1610 \text{ lbs, } \Sigma O < 1.0 \text{ (from ES-44, Sec. 6)}$$

Use No. 5 at 15 ( $A_s = 0.25$ ,  $\Sigma O = 1.57$ ) (See ES-46, Sec. 6)

Find where No. 4 at 15 ( $A_s = 0.16$ ) can be used

Allowable M = 1900 ft lbs (from ES-45)

Allowable  $-C_m = 1900 \div 109,300 = 0.0174$  which corresponds to a point  $(2.3 \div 40) 13.33 = 0.77 \text{ ft}$  above top of apron.

(See  $M_x$  moment curves)

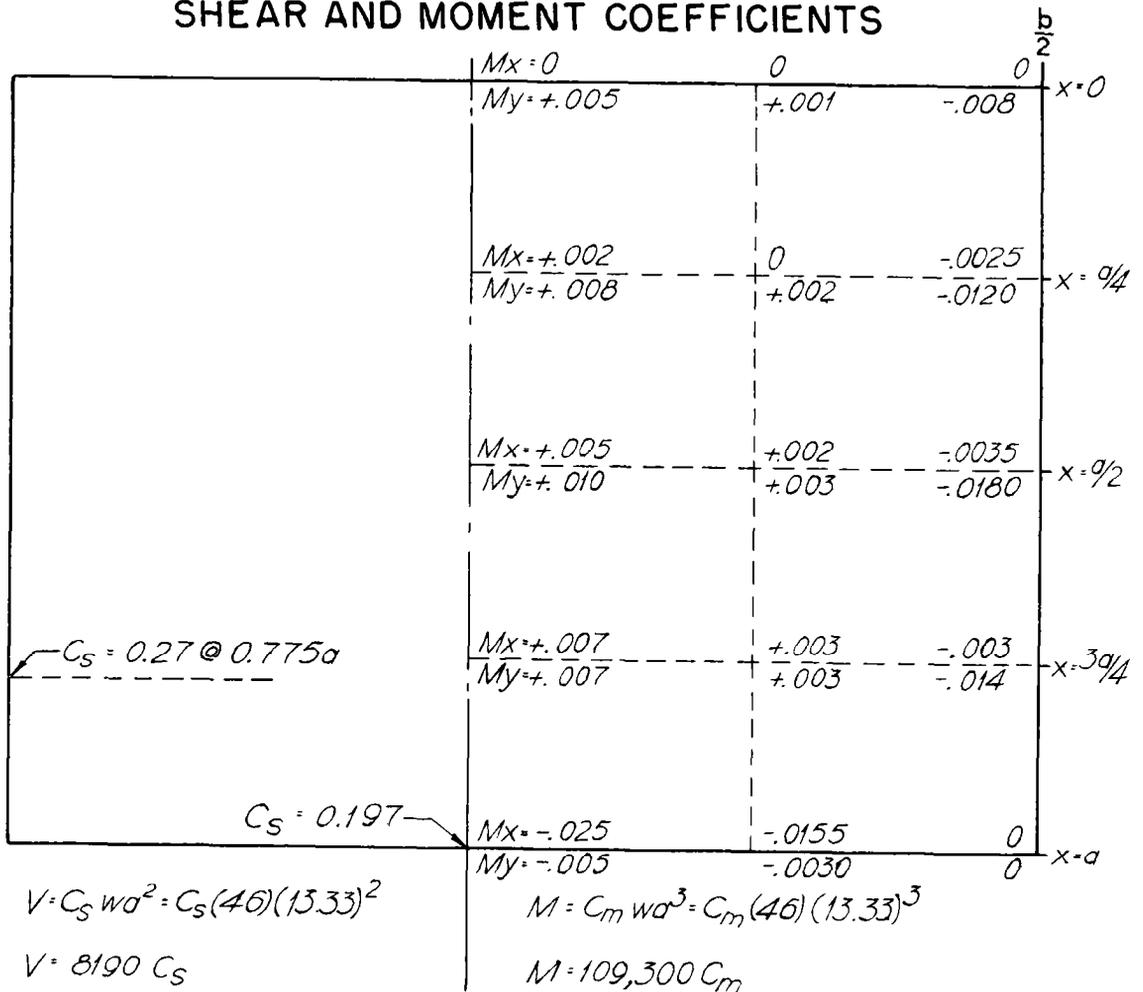
∴ No. 5 bars may be cut  $0.77 + 1.0 = 1.77$  ft. above apron and will extend into cutoff wall. Length of No. 5 bars  $1.77 + 0.92 + 2.5 - 0.25 = 4.94$  ft.

Use 5'-0" and extend bars 1'-10" above apron

Set No. 4 bars on construction joint

Length =  $(13'-4") - (0'-4") - (0'-3") = 12'-9"$

### SHEAR AND MOMENT COEFFICIENTS



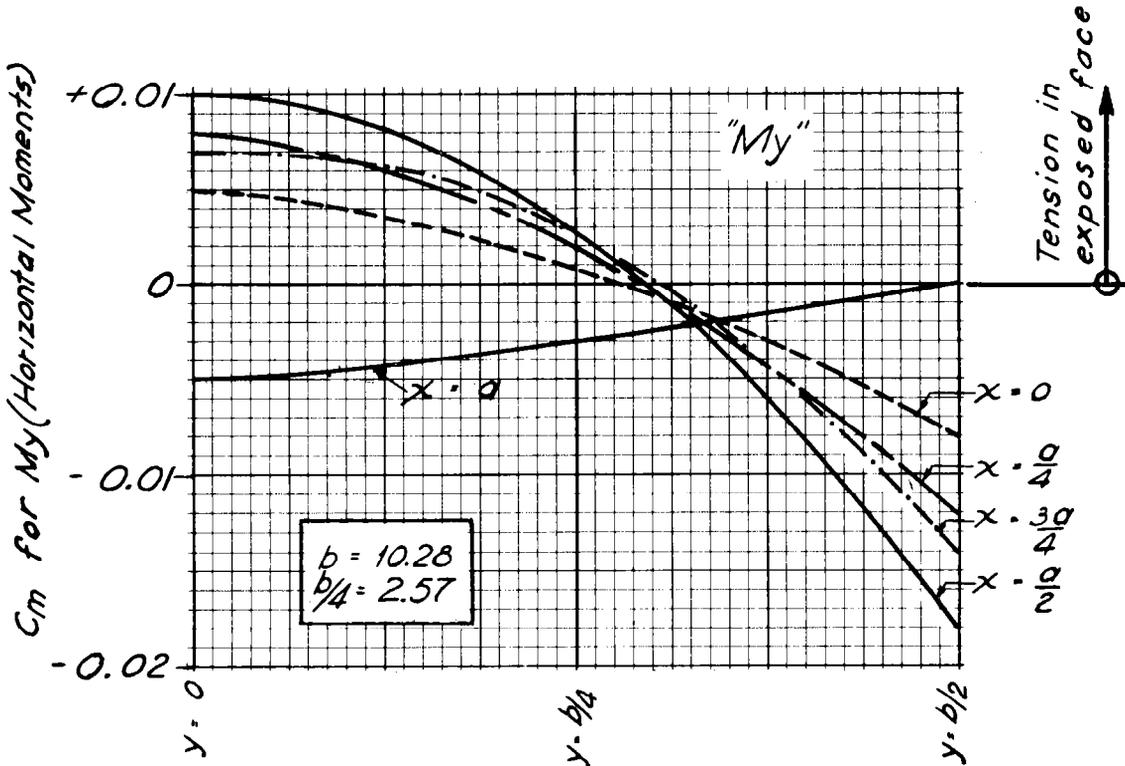
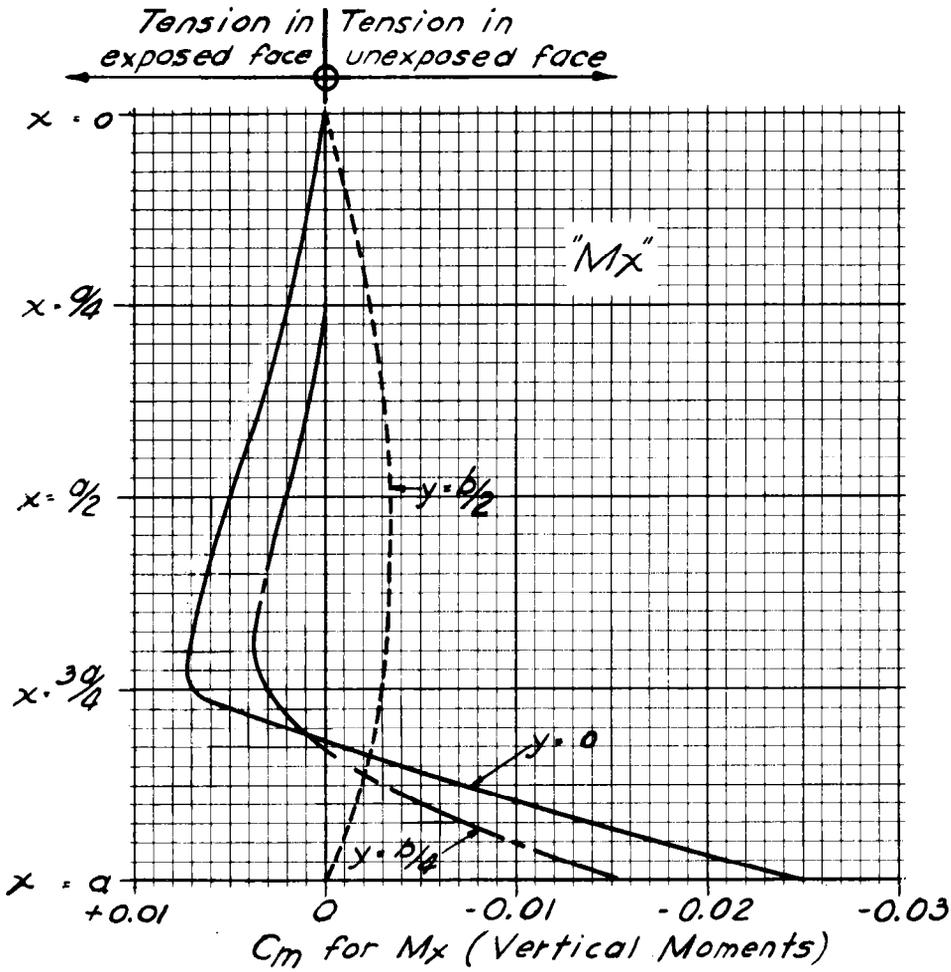
Horizontal Steel (Unexposed face)  $d = 10 - 2.0 - 5/8 - 1/4 = 7 \frac{1}{8}"$

$-M_y = 0.018 \cdot 109,300 = 1970$  ft lbs, Req'd.  $A_s = 0.18$  in<sup>2</sup>

$V = 0.27 \cdot 8190 = 2210$  lbs, Req'd.  $\Sigma O = 1.2$  in

Use No. 4 at 12 Splice at center line of structure and extend into headwall extension stub.

# HEADWALL MOMENT COEFFICIENT CURVES



Horizontal Steel (Exposed face)  $d = 7 \frac{1}{2}$  in

$$+ M_y = 0.01 \cdot 109,300 = 1093 \text{ ft lbs, Req'd. } A_s < 0.10 \text{ in}^2$$

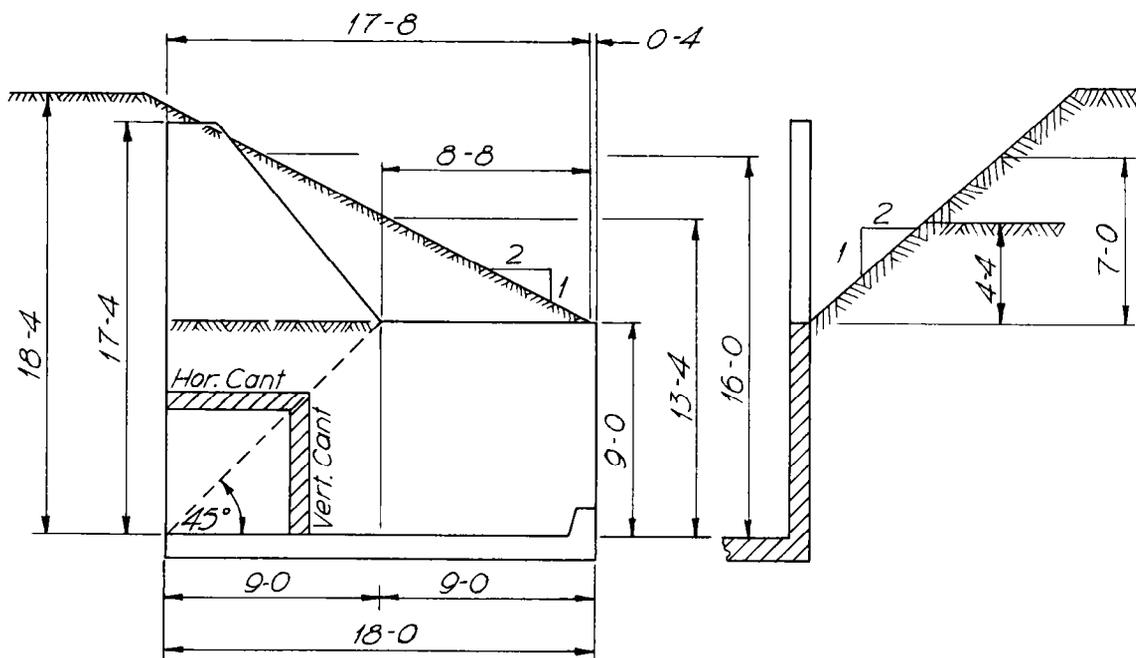
Use No. 5 at 15 ( $A_s = 0.25$ ) to meet temp. requirements.  
Splice at center line of structure and extend into headwall extension stub.

Vertical Steel (Exposed face)  $d = 10 - 2 - \frac{1}{2} - \frac{1}{4} = 7 \frac{1}{4}$  in

$$+ M_x = 0.0075 \cdot 109,300 = 820 \text{ ft lbs, Req'd. } A_s < 0.10 \text{ in}^2$$

Use No. 4 at 15 ( $A_s = 0.16$ ) Min. for 10 in. wall. Extend into cutoff wall and cut 1'-7" above top of apron. Length of bars = 4'-9". Lap on No. 4 set on construction joint. Length = 12'-9"

Sidewall Design: The sidewall is designed as vertical and horizontal beams divided as shown in the sketch below by the  $45^\circ$  line.



Backfill Properties (Compacted)

Dry wt.	= 100 lbs/ft <sup>3</sup>	specific gravity of solids	= 2.65
Void ratio (e)	= 0.65		
Percent voids	= 39.4		
Moist wt.	= 110 lbs/ft <sup>3</sup>		
Percent moisture	= 10		
$\phi$ (moist)	= $25^\circ$	$\left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)$	= 0.406
Cohesion (c)	= consider as 0		

Equivalent fluid pressure on vertical cantilever section

Maximum ht. of surcharge on 2 to 1 slope = 4.33 ft  
 $w = 70 \text{ lbs/ft}^3$  (See graphical solution)

Equivalent fluid pressures on horizontal cantilever section

Assumed average ht. of surcharge on 2 to 1 slope = 7.0 ft  
 $w = 78 \text{ lbs/ft}^3$  (See graphical solution)

Vertical Steel (Unexposed face)  $d = 10 - 2.5 = 7.5 \text{ in}$ 

Use ES-4  $M = 121.5 \cdot 70 = 8505 \text{ ft lbs}$ , Req'd.  $A_s = 0.77 \text{ in}^2$   
 $V = 40.5 \cdot 70 = 2835 \text{ lbs}$ , Req'd.  $\Sigma_o = 1.46 \text{ in}$

Use No. 7 at 9 ( $A_s = 0.80$ ,  $\Sigma_o = 3.67$ )

Find where No. 4 at 9 ( $A_s = 0.26$ ) can be used

Allowable  $M = 3010 \text{ ft lbs}$  (from ES-45)  
 $3010 \div 70 = 43.2$ ; (from ES-4, EH, Sec. 6) distance from top of wall to point where No. 4 at 9 will carry moment = 6'-3" which corresponds to point 2'-9" above top of apron. Length of vertical leg of No. 7 at 9 =  $2.75 + 1.00 + 0.67 = 4.42 \text{ ft}$  (Use 4'-6")

Horizontal Steel (Unexposed face)  $d = 10 - 2 - 7/8 - 1/2 = 6 \text{ } 5/8 \text{ in}$ 

Find points of maximum shear and moments

$$V = w (H - x) x ; \frac{dV}{dx} = H - 2x = 0$$

$$\therefore x = \frac{H}{2}$$

$$V_{\max} = 78 (4.5) 4.5 = 1580 \text{ lbs}$$

$$M = w (H - x) \frac{x^2}{2} ; \frac{dM}{dx} = Hx - \frac{3}{2} x^2 = 0$$

$$\therefore x = \frac{2}{3} H$$

$$M_{\max} = 78 (3) \frac{36}{2} = 4210 \text{ ft lbs}$$

Req'd.  $A_s = 0.42 \text{ in}^2$ , Req'd.  $\Sigma_o < 1.0 \text{ in}$

Use No. 7 at 15 ( $A_s = 0.48$ ,  $\Sigma_o = 2.20$ )

Find where No. 4 at 15 ( $A_s = 0.16$ ) is sufficient

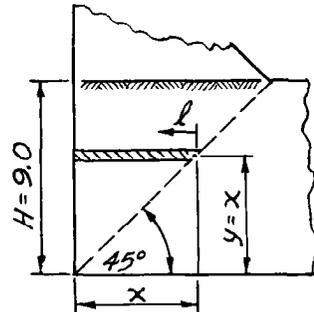
$M = 1660 \text{ ft lbs}$  (from ES-45, EH, Sec. 6)

$$M = w (H - x) \frac{l^2}{2} ; 1660 = 78 \cdot 3 (l^2 \div 2);$$

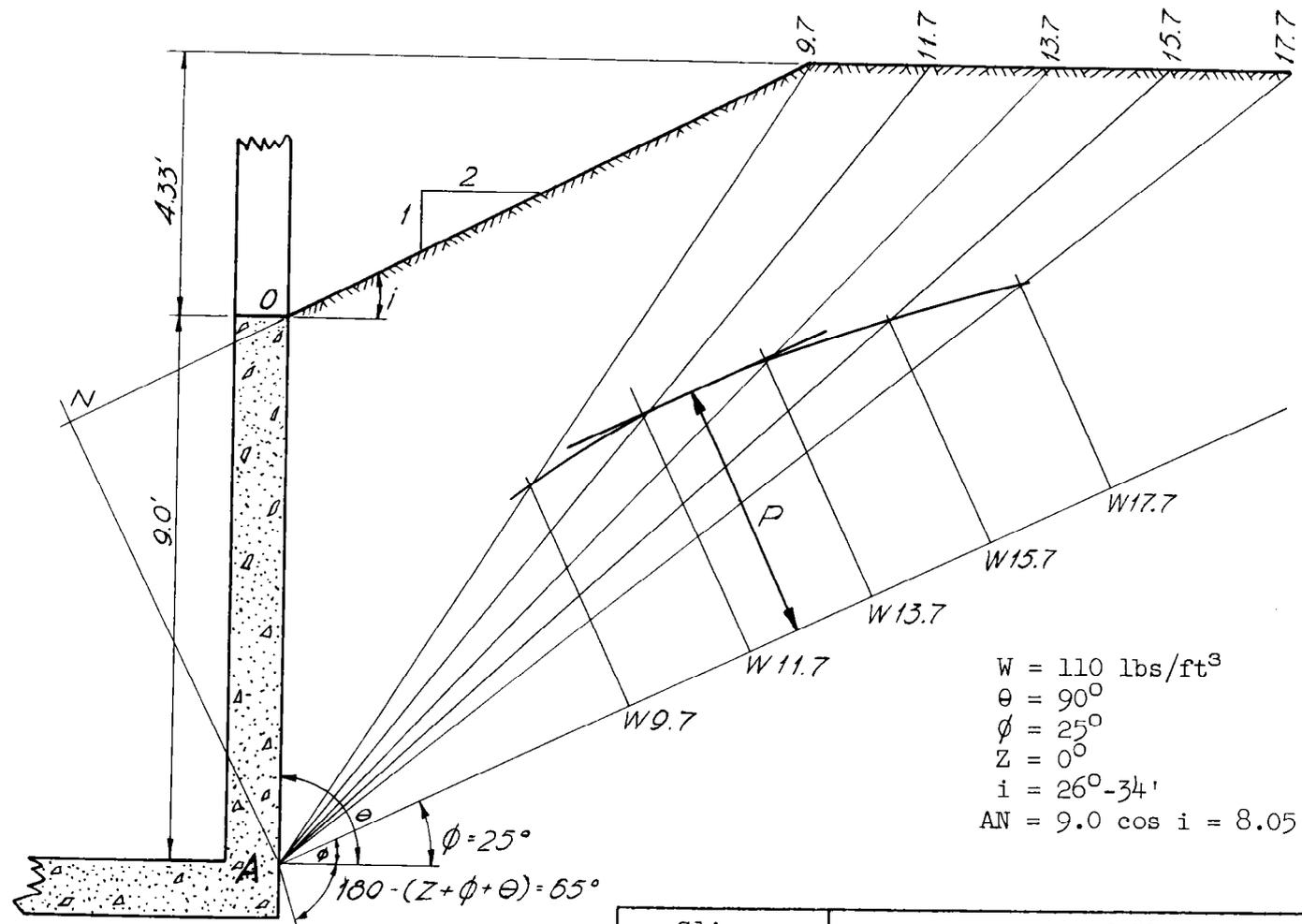
$$l^2 = \frac{1660 \cdot 2}{78 \cdot 3} = 14.2 \text{ ft}^2$$

$l = 3.77 \text{ ft}$  which corresponds to a point  $6 - 3.77 = 2.23 \text{ ft}$  from downstream face of headwall.

$$\begin{aligned} \text{Length of leg of bar} &= (2'-3") + (1'-0") + (0'-7 \text{ } 1/2") \\ &= 3'-10 \text{ } 1/2 \text{ in. (Use 4'-0")} \end{aligned}$$

Steel in Exposed Face

Horizontal Steel--Use No. 5 at 15 (temp. steel) place nearer face.  
 Vertical Steel--Use No. 4 at 15 (tie steel)



$W = 110 \text{ lbs/ft}^3$   
 $\theta = 90^\circ$   
 $\phi = 25^\circ$   
 $Z = 0^\circ$   
 $i = 26^\circ-34'$   
 $AN = 9.0 \cos i = 8.05$

$P = 2870 \text{ lbs} = \frac{wH^2}{2}$   
 $w = \frac{5740}{(9)^2} = 70.9 \text{ lbs/ft}^3$   
Use 70.0 lbs/ft<sup>3</sup>

Slice	Weight of Slice	Acc Wt
A-0-9.7	$8.05 \cdot 0.5 \cdot 110 = 4,290$	4,290
A-9.7-11.7	$13.33 \cdot 1.0 \cdot 110 = 1,467$	5,757
A-11.7-13.7	$13.33 \cdot 1.0 \cdot 110 = 1,467$	7,224
A-13.7-15.7	$13.33 \cdot 1.0 \cdot 110 = 1,467$	8,691
A-15.7-17.7	$13.33 \cdot 1.0 \cdot 110 = 1,467$	10,158

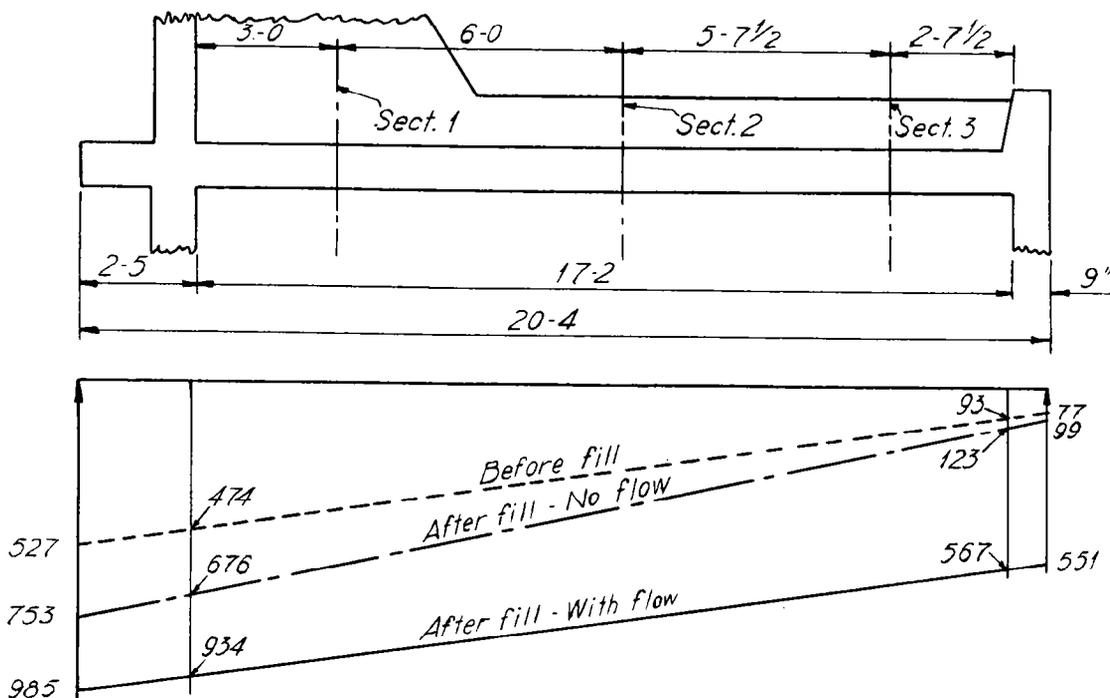
See Paragraph 2.2.2 of Structural Design



Apron Design: The apron will be designed as a series of beams in the transverse direction, considered restrained at the sidewalls and continuous over the longitudinal sills. The apron will be designed for the following loading conditions: (1) before backfill has been placed, (2) after backfill has been placed with no flow, (3) after backfill has been placed with design flow.

Upward Total Pressures on Base

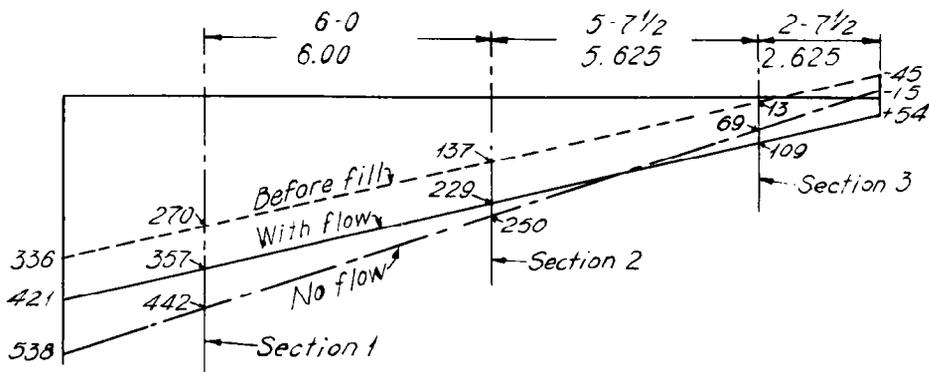
(See determination of contact and total pressures)



Net Loads on Apron (Total Pressure less wt. of concrete and H<sub>2</sub>O)

Wt. of concrete/ft<sup>2</sup> of apron =  $11/12 \cdot 150 = 138$  lbs

Wt. of H<sub>2</sub>O on apron/ft<sup>2</sup> =  $62.4 \cdot 6.0 = 375$  lbs



Due to the wide variation in loads across the apron, the apron was divided into three approximately equal parts. The stress analysis will be made at the mid-sections of these parts at Sections 1, 2, and 3.

At Section 1

Before fill  $l = 30.83 \div 3 = 10.28 \text{ ft}$ ,  $l^2 = 105.7 \text{ ft}^2$

Sidewall Moment,  $M_{ab} = 0$

Load on apron,  $w = 270 \text{ lbs/ft}^2$

$M_{ba} = \frac{1}{10} w l^2 = 27 \cdot 105.7 = 2850 \text{ ft lbs}$  (See ES-56, page 4.27)

$V_{ab} = -\frac{1}{2} w l + \frac{1}{l} (M_{ab} + M_{ba}) = - (135 \cdot 10.28) + (2850 \div 10.28)$

$V_{ab} = -1388 + 277 = -1111 \text{ lbs}$

$V_{ba} = w l + V_{ab} = 2776 - 1111 = +1665 \text{ lbs}$

$V_{bc} = -\frac{1}{2} w l = -1388 \text{ lbs}$

$R_a = 1111 \text{ lbs (down)}$

$R_b = 1665 + 1388 = 3053 \text{ lbs (down)}$

After fill--No flow  $w = 442 \text{ lbs/ft}^2$

Sidewall Moment,  $= -M_{ab}$  (at Q of apron)

$M_{ab} = -\frac{6 \cdot 70 \cdot (3.42)^2}{2} - \frac{3.42 \cdot 70 (3.42)^2}{6}$

$M_{ab} = -2460 - 467 = -2927 \text{ ft lbs}$

$M_{ba} = \frac{1}{5} M_{ab} + \frac{1}{10} w l^2$

$M_{ba} = -585 + 44.2 (105.7) = -585 + 4670 = +4085 \text{ ft lbs}$

$V_{ab} = -\frac{1}{2} w l + \frac{1}{l} (M_{ab} + M_{ba}) = - (221 \cdot 10.28) + \frac{1}{10.28} (-2927 + 4085)$

$V_{ab} = -2270 + 113 = -2157 \text{ lbs}$

$V_{ba} = w l + V_{ab} = 4540 - 2157 = +2383 \text{ lbs}$

$V_{bc} = -\frac{1}{2} w l = -2270 \text{ lbs}$

$R_a = 2157 \text{ lbs (down)}$

$R_b = 2383 + 2270 = 4653 \text{ lbs (down)}$

After fill--With flow  $w = 357 \text{ lbs/ft}^2$

$M_{ab} = -2927 + (3 \cdot 62.4 \cdot 3 \cdot 1.92) + (3 \cdot 62.4 \cdot 1.5 \cdot 1.42)$

$M_{ab} = -2927 + 1078 + 399 = -1450 \text{ ft lbs}$

$M_{ba} = (-1450 \div 5) + (35.7 \cdot 105.7)$

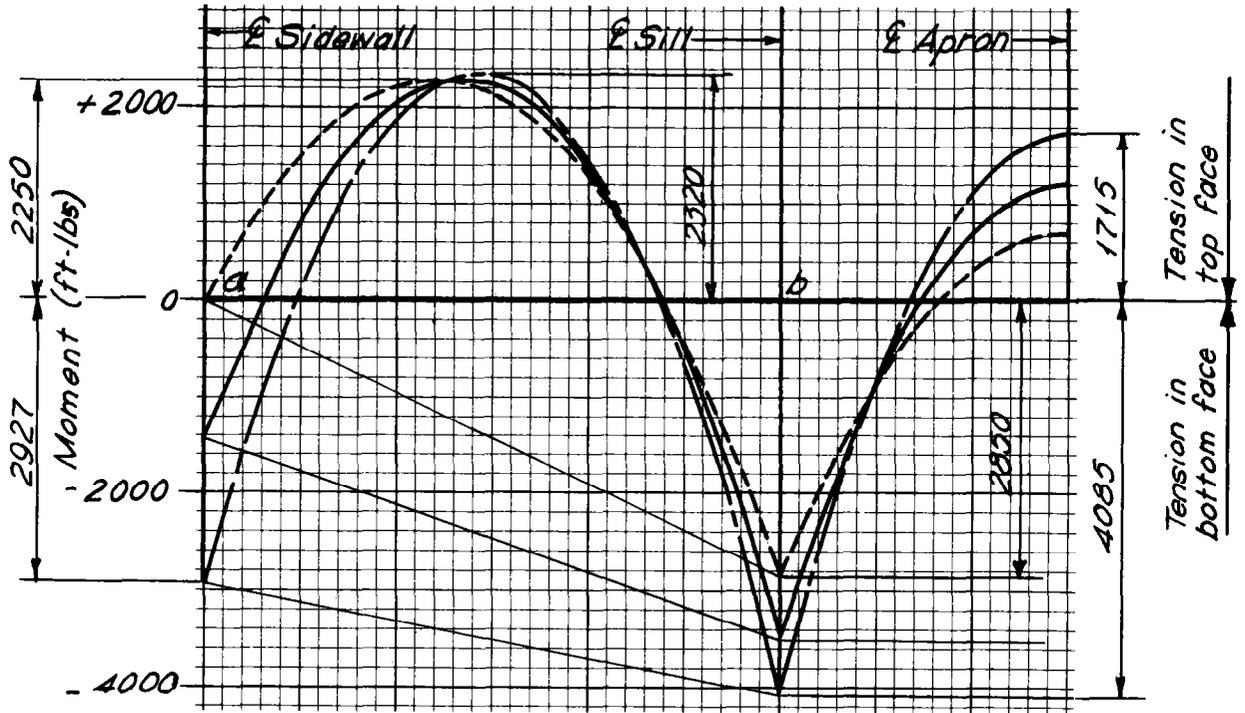
$M_{ba} = -290 + 3770 = +3480 \text{ ft lbs}$

$V_{ab} = (-178 \cdot 10.28) + \frac{1}{10.28} (-1450 + 3470)$

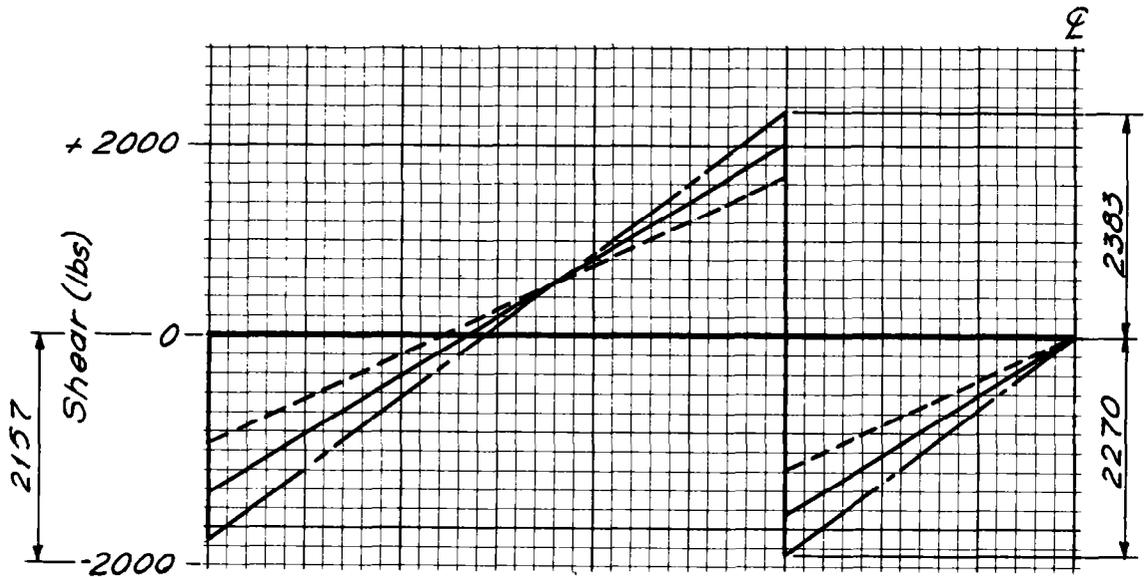
$V_{ab} = -1830 + 196 = -1634 \text{ lbs}$

# APRON DESIGN (Section 1)

Before fill -----  
 After fill - no flow -----  
 After fill - with flow \_\_\_\_\_



**MOMENT DIAGRAMS**



**SHEAR DIAGRAMS**

$$V_{ba} = w\ell + V_{ab} = 3670 - 1634 = 2036 \text{ lbs}$$

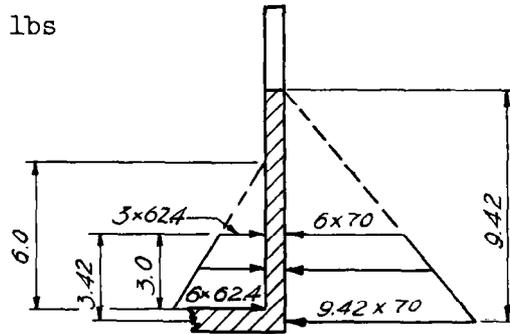
$$V_{bc} = -\frac{1}{2} w\ell = -1830 \text{ lbs}$$

$$R_a = 1634 \text{ lbs (down)}$$

$$R_b = 2036 + 1830 = 3866 \text{ lbs (down)}$$

Simple Moments

$$M_Q = \frac{1}{8} w\ell^2 = \frac{1}{8} w (10.28)^2 = 13.2 w$$



	$\frac{M_x}{M_Q}$	Before fill $w = 270$	No flow $w = 442$	With flow $w = 357$
at Q	1.0	3560	5830	4710
at 0.4ℓ	0.96	3420	5600	4520
at 0.3ℓ	0.84	2990	4900	3960
at 0.2ℓ	0.64	2280	3730	3020
at 0.1ℓ	0.36	1280	2100	1700

In determining the steel requirements, the horizontal compression thrust in the apron due to the load on the sidewall will be taken into account. (See ES-45, EH, Sec. 6)

#### Magnitudes of Horizontal Compression Thrust (N)

Before fill,  $N = 0$

$$\text{After fill--No flow, } N = \left( \frac{(6 \cdot 70) + (9.42 \cdot 70)}{2} \right) 3.42 = 1850 \text{ lbs}$$

$$\text{After fill--With flow, } N = 1850 - (4.5 \cdot 62.4 \cdot 3.0)$$

$$N = 1850 - 842 = 1008 \text{ lbs}$$

#### At Section 2

$$\text{Before fill } \ell = 10.28 \text{ ft, } \ell^2 = 105.7 \text{ ft}^2$$

$$\text{Sidewall Moment, } M_{ab} = 0$$

$$\text{Load on apron} = 137 \text{ lbs/ft}^2$$

$$M_{ba} = \frac{1}{10} w\ell^2 = 13.7 \cdot 105.7 = 1450 \text{ ft lbs}$$

$$V_{ab} = -\frac{1}{2} w\ell + \frac{1}{\ell} (M_{ab} + M_{ba}) = -\frac{1}{2} \cdot 137 \cdot 10.28 + (1450 \div 10.28)$$

$$V_{ab} = -704 + 141 = -563 \text{ lbs}$$

$$V_{ba} = w\ell + V_{ab} = 1408 - 563 = +845 \text{ lbs}$$

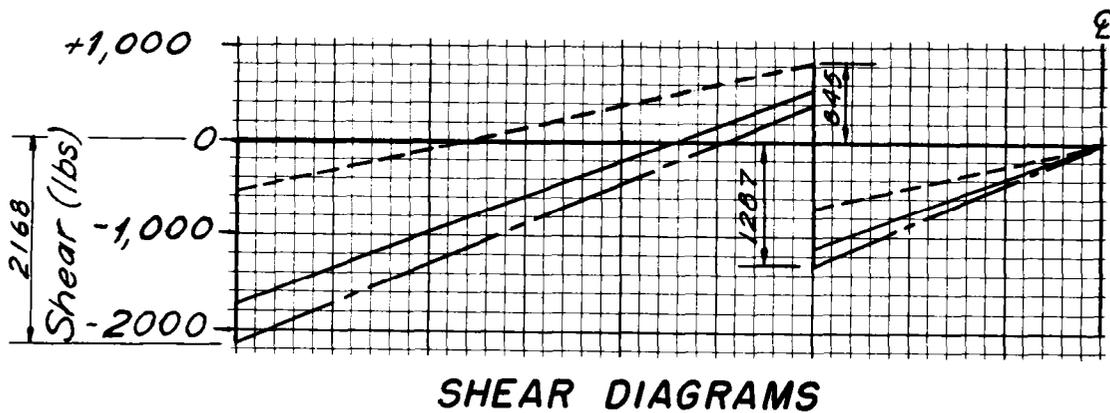
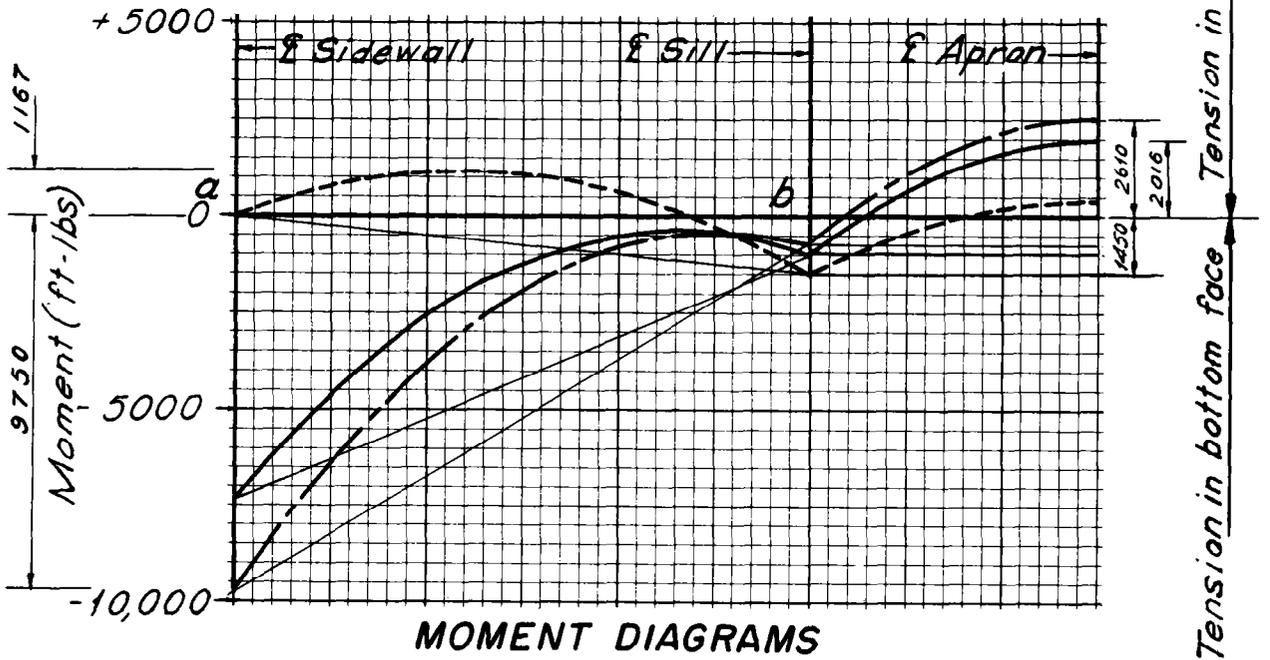
$$V_{bc} = -\frac{1}{2} w\ell = -704 \text{ lbs}$$

$$R_a = 563 \text{ lbs (down)}$$

$$R_b = 704 + 845 = 1549 \text{ lbs (down)}$$

# APRON DESIGN (Section 2)

Before fill \_\_\_\_\_  
 After fill - no flow \_\_\_\_\_  
 After fill - with flow \_\_\_\_\_



After fill--Now flow  $w = 250 \text{ lbs/ft}^2$

$$M_{ab} = -\frac{wl^3}{6} = -\frac{70(9.42)^3}{6} = -9750 \text{ ft lbs (at } Q \text{ apron)}$$

$$M_{ba} = \frac{1}{5} M_{ab} + \frac{1}{10} wl^2$$

$$M_{ba} = -1950 + (25 \cdot 105.7) = -1950 + 2640 = +690 \text{ ft lbs}$$

$$V_{ab} = -\frac{1}{2} wl + \frac{1}{l} (M_{ab} + M_{ba}) = (-125 \cdot 10.28) + \frac{1}{10.28} (-9750 + 690)$$

$$V_{ab} = -1287 - 881 = -2168 \text{ lbs}$$

$$V_{ba} = wl + V_{ab} = 2573 - 2168 = +405 \text{ lbs}$$

$$V_{bc} = -\frac{1}{2} wl = -1287 \text{ lbs}$$

$$R_a = 2168 \text{ lbs (down)}$$

$$R_b = 405 + 1287 = 1692 \text{ lbs (down)}$$

After fill-- With flow  $w = 229 \text{ lbs/ft}^2$

$$M_{ab} = -9750 + \frac{62.4(6)^2}{2} \cdot 2.42 = -9750 + 2720 = -7030 \text{ ft lbs}$$

$$M_{ba} = \frac{1}{5} (-7030) + \frac{1}{10} (229)(105.7) = -1406 + 2420 = +1014 \text{ ft lbs}$$

$$V_{ab} = -\frac{1}{2} wl + \frac{1}{l} (M_{ab} + M_{ba}) = -\frac{1}{2} (229)(10.28) + \frac{1}{10.28} (-7030 + 1014)$$

$$V_{ab} = -1177 - 585 = -1762 \text{ lbs}$$

$$V_{ba} = wl + V_{ab} = 2354 - 1762 = +592 \text{ lbs}$$

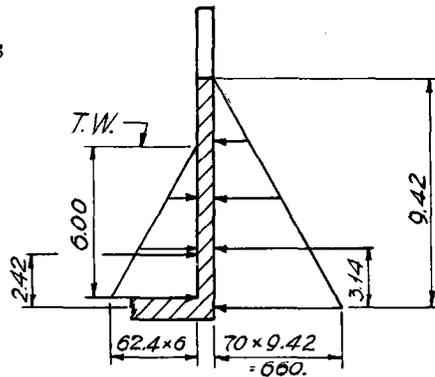
$$V_{bc} = -\frac{1}{2} wl = -1177 \text{ lbs}$$

$$R_a = 1762 \text{ lbs (down)}$$

$$R_b = 592 + 1177 = 1769 \text{ lbs (down)}$$

Simple Moments

$$M_Q = \frac{1}{8} wl^2 = 13.2 w$$



	$\frac{M_x}{M_Q}$	Before fill $w = 137$	No flow $w = 250$	With flow $w = 229$
at $Q$	1.0	1820	3300	3030
at $0.4l$	0.96	1747	3165	2910
at $0.3l$	0.84	1530	2770	2540
at $0.2l$	0.64	1166	2110	1940
at $0.1l$	0.36	655	1190	1090

Magnitude of Horizontal Compression Thrust (N)

Before fill,  $N = 0$

$$\text{After fill--No flow, } N = \frac{70 \cdot (9.42)^2}{2} = 3100 \text{ lbs}$$

$$\text{After fill--With flow, } N = 3100 - \frac{62.4(6)^2}{2} = 3100 - 1120$$

$$N = 1980 \text{ lbs}$$

At Section 3  $l = 10.28$  ft,  $l^2 = 105.7$  ft<sup>2</sup>

Before fill

$$\text{Sidewall Moment} = M_{ab} = 0$$

$$\text{Load on apron} = 13 \text{ lbs/ft}^2$$

$$M_{ba} = \frac{1}{10} w l^2 = 1.3 (105.7) = 138 \text{ ft lbs}$$

$$V_{ab} = -\frac{1}{2} w l + \frac{1}{l} (M_{ab} + M_{ba}) = -\frac{1}{2} (13)(10.28) + \frac{1}{10.28} (138)$$

$$V_{ab} = -67 + 13 = -54 \text{ lbs}$$

$$V_{ba} = w l + V_{ab} = 134 - 54 = +80 \text{ lbs}$$

$$V_{bc} = -\frac{1}{2} w l = -67 \text{ lbs}$$

$$R_a = 54 \text{ lbs (down)}$$

$$R_b = 80 + 67 = 147 \text{ lbs (down)}$$

After fill--No flow  $w = 69$  lbs/ft<sup>2</sup>

$$M_{ab} = -9750 \text{ ft lbs (Same as Section 2)}$$

$$M_{ba} = \frac{1}{5} M_{ab} + \frac{1}{10} w l^2 = -1950 + 730 = -1220 \text{ ft lbs}$$

$$V_{ab} = -\frac{1}{2} w l + \frac{1}{l} (M_{ab} + M_{ba}) = -\frac{1}{2} (69)(10.28) + \frac{1}{10.28} (-9750 - 1220)$$

$$V_{ab} = -354 - 1068 = -1422 \text{ lbs}$$

$$V_{ba} = w l + V_{ab} = 708 - 1422 = -714 \text{ lbs}$$

$$V_{bc} = -\frac{1}{2} w l = -354 \text{ lbs}$$

$$R_a = 1422 \text{ lbs (down)}$$

$$R_b = 714 - 354 = 360 \text{ lbs (up)}$$

After fill--With flow  $w = 109$  lbs/ft<sup>2</sup>

$$M_{ab} = -7030 \text{ ft lbs (Same as Section 2)}$$

$$M_{ba} = \frac{1}{5} M_{ab} + \frac{1}{10} w l^2 = -1406 + 1152 = -254 \text{ ft lbs}$$

$$V_{ab} = -\frac{1}{2} w l + \frac{1}{l} (M_{ab} + M_{ba}) = -\frac{1}{2} (109)(10.28) + \frac{1}{10.28} (-7030 - 254)$$

$$V_{ab} = -560 - 709 = -1269 \text{ lbs}$$

$$V_{ba} = w l + V_{ab} = 1120 - 1269 = -149 \text{ lbs}$$

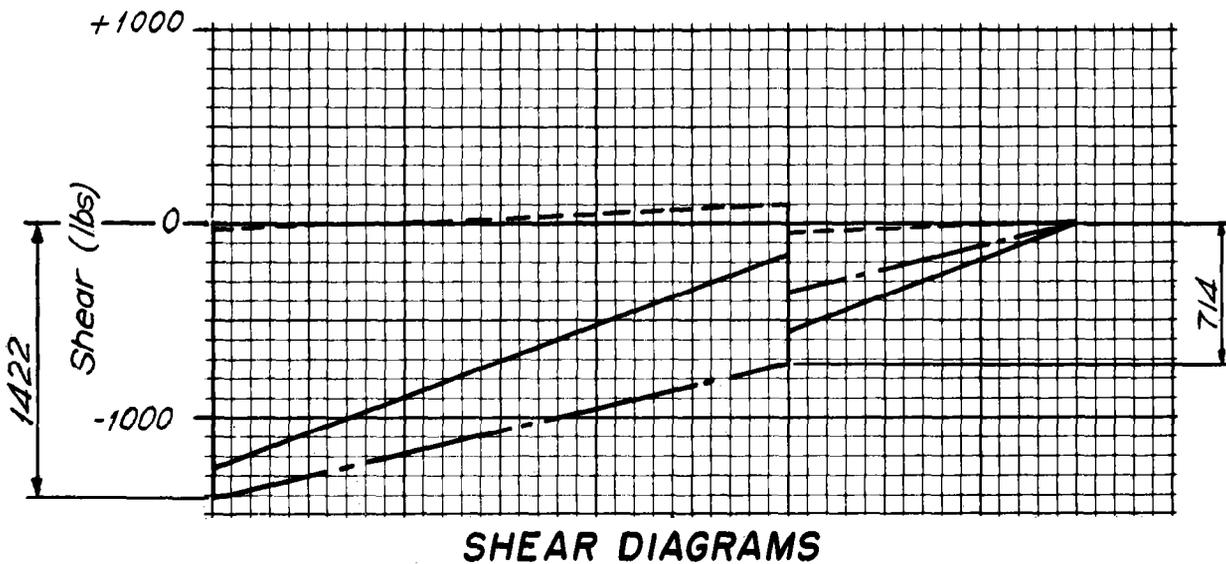
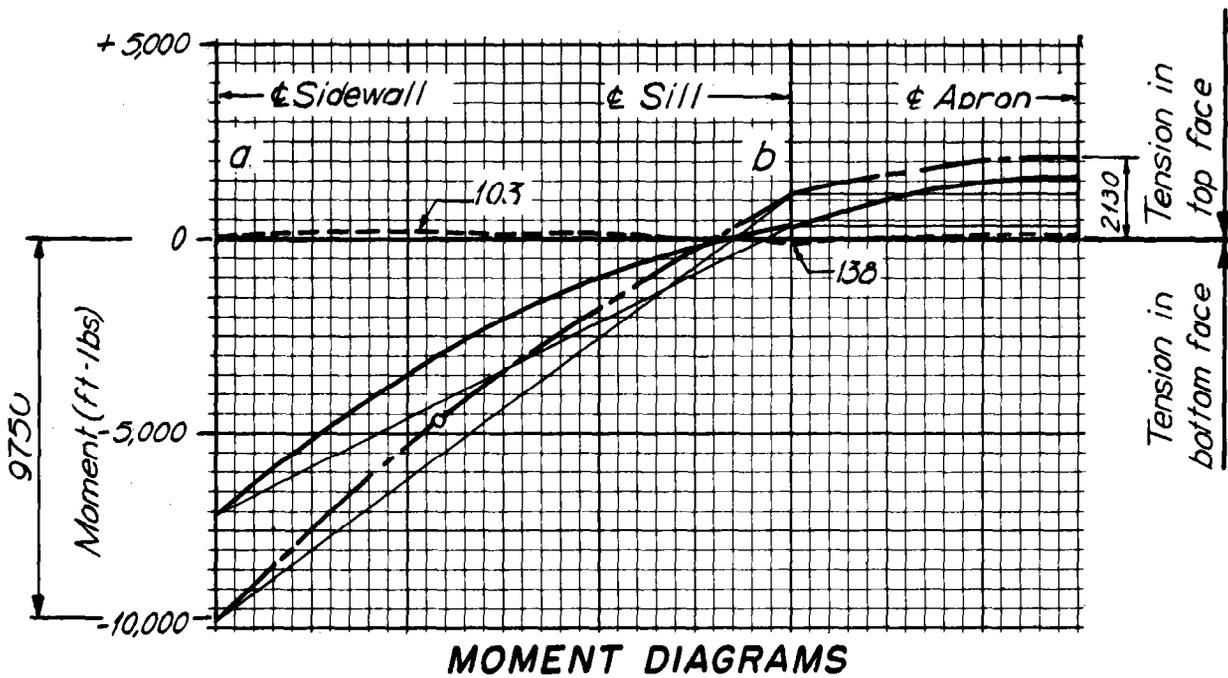
$$V_{bc} = -\frac{1}{2} w l = -560 \text{ lbs}$$

$$R_a = 1269 \text{ lbs (down)}$$

$$R_b = 560 - 149 = 411 \text{ lbs (down)}$$

# APRON DESIGN (Section 3)

Before fill -----  
 After fill - no flow -----  
 After fill - with flow -----



## Simple Moments

$$M_Q = \frac{1}{8} w l^2 = 13.2 w$$

	$\frac{M_x}{M_Q}$	Before fill w = 13	No flow w = 69	With flow w = 109
at Q	1.0	172	910	1440
at 0.4l	0.96	165	873	1380
at 0.3l	0.84	145	765	1210
at 0.2l	0.64	110	582	920
at 0.1l	0.36	62	328	518

Values of N are the same for Sections 2 and 3

Steel Requirements: Because of nature of design assumptions and to facilitate the placing of the steel during construction, the apron will be reinforced the same at all sections and will be designed to meet the maximum stress requirement in any section.

Bottom Steel (Transverse)  $d = 11 - 3 \frac{1}{2} = 7 \frac{1}{2}$  in

At Sidewall

$M = 9750$  ft lbs (Sections 2 and 3, No flow governs)

$V = 2168$  lbs,  $N = 3100$  lbs

$M_S = M + (Nd'' \div 12)$  (See ES-45, EH, Sec. 6)  $d'' = 6 - 3.5 = 2.5$  in

$M_S = 9750 + (3100 \cdot 2.5 \div 12) = 9750 + 646 = 10,396$  ft lbs

$A = 0.94$  in<sup>2</sup> (See ES-45)

Req'd.  $A_S = A - (N \div f_s) = 0.94 - (3.1 \div 20) = 0.94 - 0.16 = 0.78$  in<sup>2</sup>

Req'd.  $\Sigma_O = 1.11$  in (See ES-44, EH, Sec. 6)

Use steel from sidewall--No. 7 at 9 ( $A_S = 0.80$ ,  $\Sigma_O = 3.67$ )

At Sill

$M = 4085$  ft lbs--(Section 1, No flow governs)

$V = 2383$  lbs,  $N = 1850$  lbs

$M_S = M + (Nd'' \div 12) = 4085 + (1850 \cdot 2.5 \div 12)$

$M_S = 4085 + 385 = 4470$  ft lbs

$A = 0.39$  in<sup>2</sup> (ES-45)

Req'd.  $A_S = A - (N \div f_s) = 0.39 - (1.85 \div 20) = 0.39 - 0.09 = 0.30$  in<sup>2</sup>

Req'd.  $\Sigma_O = 1.21$  in (ES-44)

Use No. 5 at 12 ( $A_S = 0.31$ ,  $\Sigma_O = 1.96$ )

Find where No. 5 at 12 ( $A_S = 0.31$ ) can be used in end spans.

(Section 3, No flow governs)  $N = 3100$  lbs

$A = A_S + (N \div f_s) = 0.31 + (3.1 \div 20) = 0.31 \div 0.16 = 0.47$  in<sup>2</sup>

$M_S = 5330$  ft lbs (ES-45)

$M = M_S - (Nd'' \div 12) = 5330 - (3100 \cdot 2.5 \div 12)$

$M = 5330 - 662 = 4668$  ft lbs which corresponds to a point

$11.5/30 \cdot 10.28 = 3.94$  ft from the  $Q$  of sidewall

Length of horizontal legs of No. 7 bar

$3.94 =$  distance from  $Q$  sidewall to theoretical cutoff point

$+0.25 =$  distance from  $Q$  sidewall to back of bar

$+1.00 =$  for contingencies

$5.19'$  total (Use  $5'-3''$ ) (lap)

Length of No. 5 bars =  $(31'-4'') - 2(5'-3'') + 2(1'-7'') = 24'-0''$

Top Steel (Transverse)  $d = 10 - 2.5 = 7.5$  in

End Span

$M_s = M = 2250$  ft lbs (Section 1, Before fill governs as  $N = 0$ )

$A_s = A = 0.192$  in<sup>2</sup>

$V = 1450$  lbs (Section 1, No flow governs)

$\Sigma_0 < 1.0$  in

Center Span

$M = 2610$  ft lbs (Section 2, No flow governs)  $N = 3100$

$M_s = M + (Nd'' \div 12) = 2610 + 646 = 3256$  ft lbs

$A = 0.281$ ,  $A_s = A - (N \div f_s) = 0.281 - 0.15 = 0.13$  in<sup>2</sup>

Use No. 4 at 12 ( $A_s = 0.20$ ,  $\Sigma_0 = 1.57$ ) Length =  $31'-0''$

Longitudinal Steel

Bottom Face

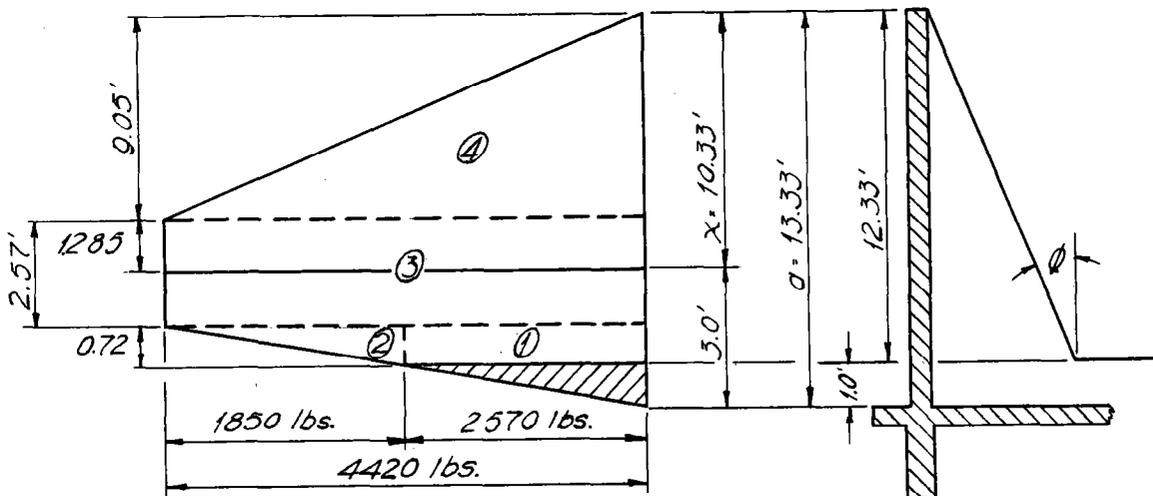
Temp. steel--Use No. 4 at 15, Length =  $21'-0''$ , horizontal leg =  $20'-0''$ , vertical leg =  $1'-0''$  in downstream face of toewall

Top Face

Temp. steel--Use No. 5 at 12, Length =  $20'-6''$ , horizontal leg =  $19'-6''$ , vertical leg =  $1'-0''$  in upstream face of toewall

Buttress Design: The buttress will be designed as a vertical cantilever beam for a load equal to the sum of the shears from the adjacent headwall slabs as explained on page 4.26.

Determine Load (See Headwall Design, page 6.16)



From Headwall Design

$$V_{\max} = 0.27 \cdot 8190 = 2210 \text{ lbs/ft (from one side only)}$$

$$\text{Max. load on buttress} = 2 \cdot 2210 = 4420 \text{ lbs/ft (both sides)}$$

$$x = 0.775 a = 0.775 \cdot 13.33 = 10.33 \text{ ft}$$

$$b/4 = 10.28 \div 4 = 2.57 \text{ ft, } b/8 = 1.285 \text{ ft}$$

$$\text{Load at top of sill elevation} = (1.0 + 1.72) 4420 = 2570 \text{ lbs}$$

Determine cantilever M and V at top of sill elevation

	Force	Arm	Moment
1	$0.72 \cdot 2570 = 1,850$	0.36	665
2	$0.72 \cdot 0.5 \cdot 1850 = 666$	0.48	319
3	$2.57 \cdot 4420 = 11,360$	2.0	22,720
4	$9.05 \cdot 0.5 \cdot 4420 = 20,000$	7.30	146,000
	33,876		169,704

$$V = 33,876 \text{ lbs}$$

$$M = 169,704 \text{ ft lbs}$$

Determine base width of buttress

$$\text{Shear } V = 33,876 \text{ lbs, } b = 12$$

$$v = V \div bjd \quad \therefore d = V \div bjv$$

For Class B Concrete

$$v = 90 \text{ lbs/in}^2, j = 0.875, k = 0.375, f_c = 1200 \text{ lbs/in}^2$$

$$\text{From ES-51, EH, Sec. 6, } d = 36 \text{ in}$$

$$\text{Moment } M = 169,704 \text{ ft lbs} = 2.37 \cdot 10^6 \text{ in lbs}$$

$$M_c = f_c b k j d^2 / 2$$

$$d = \sqrt{\frac{2M_c}{f_c b k j}} = \sqrt{\frac{2 \cdot 2.37 \cdot 10^6}{1.2 \cdot 10^3 \cdot 12 \cdot 0.375 \cdot 0.875}} = 29.4 \text{ in}$$

Shear governs

$$\text{Required base width} = 36 - 7.5 = 28.5 \text{ in}$$

A 6'-0" base width will be used for the following reasons: (1) to reduce the span length of the longitudinal sills, (2) to allow room for an aeration hole through the buttress at a higher elevation, (3) to reduce steel area requirement and thus facilitate placement of tension steel for buttress.

Steel RequirementsFor cantilever moment

$$A_s = M \text{ (in ft kips)} \div ad, a = 1.44 \text{ (for Class B)}$$

$$d = (6 \cdot 12) + 7.5 = 79.5 \text{ in}$$

$$A_s = 169.7 \div (1.44 \cdot 79.5) = 1.48 \text{ in}^2$$

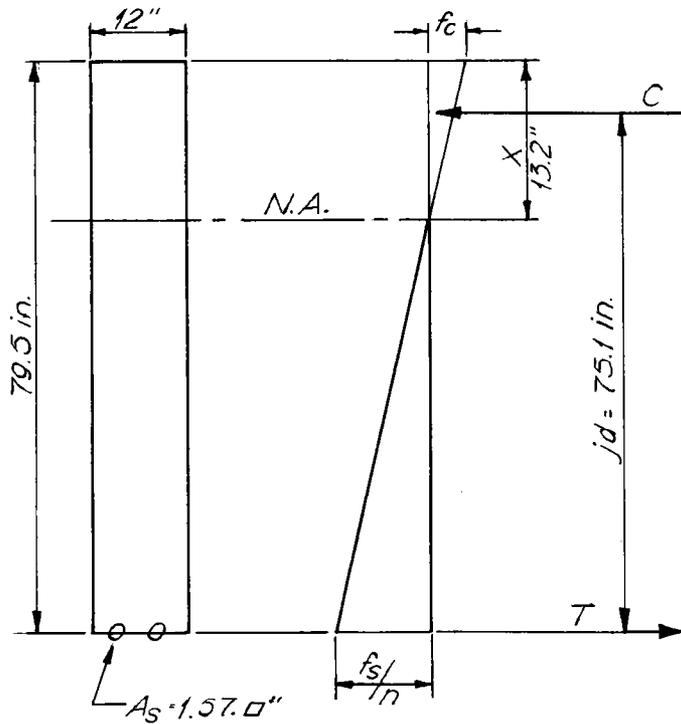
Use 2 No. 8 bars ( $A_s = 1.57$ ) in unexposed face of headwall.

In both faces of buttress

Use Temp. steel requirement

No. 5 at 12 (both faces, both directions)

Compute compressive stresses in buttress



Locate neutral axis--Try  $x = 10$

$$\begin{array}{rcl}
 10 \cdot 12 & = & 120 \quad \cdot \quad 5 & = & 600 \\
 1.57 \cdot 10 & = & \underline{15.7} & \cdot & 79.5 & = & \underline{1250} \\
 & & 135.7 & & & & )1850(13.64 \\
 3.64 \cdot 12 & = & \underline{43.7} & \cdot & 11.82 & & \underline{516} \\
 & & 179.4 & & & & )2366(13.2 \\
 - 0.44 \cdot 12 & = & \underline{-5.3} & \cdot & 13.42 & & \underline{-71} \\
 & & 174.1 & & & & 2295(13.2 = x
 \end{array}$$

$$jd = d - x/3 = 79.5 - 4.4 = 75.1$$

Compute vertical stresses

$$C = T = \frac{M}{jd} = \frac{2.37 \cdot 10^6}{75.1} = 31,600 \text{ lbs}$$

$$C = f_c/2 \cdot 13.2 \cdot 12 = 31,600$$

$$f_c = \frac{31,600 \cdot 2}{13.2 \cdot 12} = 399 \text{ psi}, \quad f_s = \frac{31,600}{1.57} = 20,100 \text{ psi}$$

Correct vertical compressive stress to stress parallel to face of buttress.  $\tan \phi = 6 \div 12.33 = 0.487$ ;  $\phi = 25^{\circ}-58'$ ;  $\cos \phi = 0.899$ .  
 Max.  $f_c$  parallel to face =  $399 \div \cos^2 \phi = 399 \div 0.81 = 493$  psi OK  
 Allowable stress =  $0.225 f_c' (1.3 - 0.03 \frac{h}{d})$   
 $h = (12.33^2 + 6^2)^{1/2} = 13.71$   
 Allowable stress =  $0.225 \cdot 3000 [1.3 - (0.03)(13.71 \div 1)] = 600$  psi  
 (See Section 1107, ACI Code)

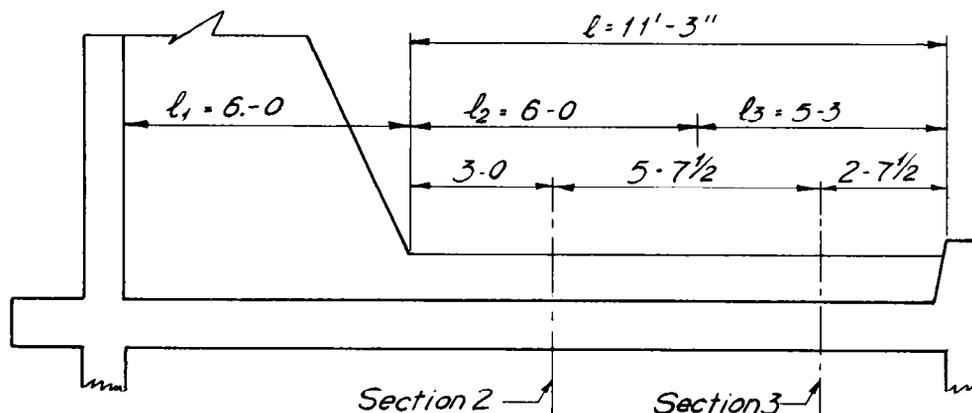
Longitudinal Sill Design: The longitudinal sills will be designed as beams considered fixed at the toe of the buttress and both fixed and simply supported at the transverse sill. The loads on the longitudinal sills will be taken as a uniform load, equal in magnitude and opposite in direction to the reactions at the sills as determined in the apron design at Section 2 and 3, plus or minus the weight of the longitudinal sill.

Reaction at Sill from Apron Design

	Section 2	Section 3
Before fill	1549 (down)	147 (down)
No flow	1692 (down)	360 (up)
With flow	1769 (down)	411 (down)

From inspection of the reactions, it is evident that both the no-flow and with-flow conditions must be considered because of the reversal of direction at Section 3.

Determination of Loads



Wt. of longitudinal sill/ft length =  $1.0 \cdot 1.0 \cdot 1.0 \cdot 150 = 150$  lbs

Loads at Section 2

No flow, load =  $1692 - 150 = 1542$  lbs/ft (up)  
 With flow, load =  $1769 - 150 = 1619$  lbs/ft (up)

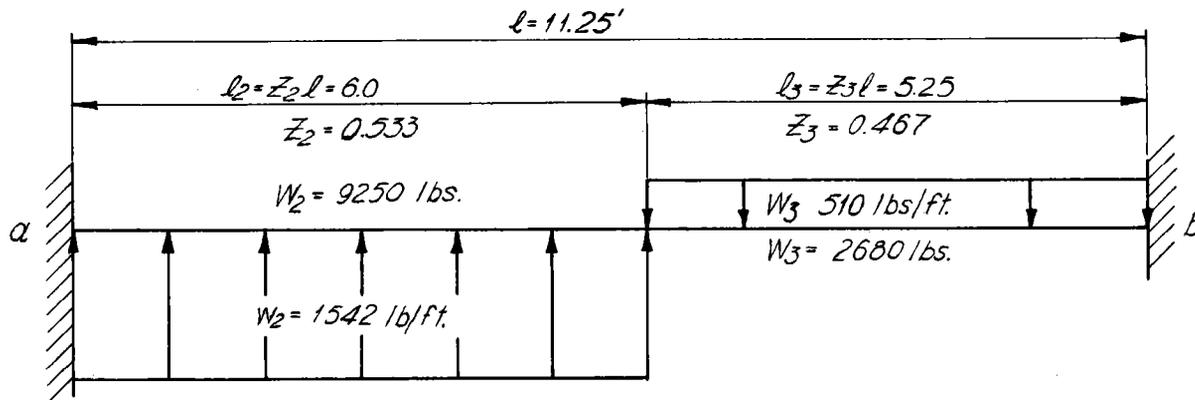
Loads at Section 3

No flow, load = 360 + 150 = 510 lbs/ft (down)

With flow, load = 411 - 150 = 261 lbs/ft (up)

Moments and Shears

No flow--both ends fixed



(See ES-32, EH, Sec. 6)

$$M_{ab}^F = -\frac{1}{12} l z_2 (6 - 8z_2 + 3z_2^2) W_2 + \frac{1}{12} l z_3^2 (4 - 3z_3) W_3$$

$$M_{ab}^F = -\frac{1}{12} (11.25)(0.533)(6 - 4.26 + 0.85)(9250) + \frac{1}{12} (11.25)(0.218)(2.60)(2680)$$

$$M_{ab}^F = -11,970 + 1425 = \underline{-10,545 \text{ ft lbs}}$$

$$M_{ba}^F = -\frac{1}{12} l z_3 (6 - 8z_3 + 3z_3^2) W_3 + \frac{1}{12} l z_2^2 (4 - 3z_2) W_2$$

$$M_{ba}^F = -\frac{1}{12} (11.25)(0.467)(2.91)(2680) + \frac{1}{12} (11.25)(0.284)(2.40)(9250)$$

$$M_{ba}^F = -3420 + 5920 = \underline{+2500 \text{ ft lbs}}$$

$$V_{ab}^S = R_a^S = \frac{1}{l} \left[ W_3 z_3 l \cdot \frac{1}{2} - W_2 (z_3 l + z_2 l \cdot \frac{1}{2}) \right]$$

$$V_{ab}^S = \frac{1}{11.25} \left[ (2680 \cdot 2.62) - (9250 \cdot 8.25) \right]$$

$$V_{ab}^S = \frac{1}{11.25} (7040 - 76,300) = \underline{-6150 \text{ lbs}}$$

$$V_{ba}^S = R_b^S = W_2 - W_3 - V_{ab}^S = 9250 - 2680 - 6150 = \underline{+420 \text{ lbs}}$$

$$V_{ab}^F = V_{ab}^S + \frac{1}{l} (M_{ab}^F + M_{ba}^F) = -6150 + \frac{1}{11.25} (-10,545 + 2500)$$

$$V_{ab}^F = -6150 - 715 = \underline{-6865 \text{ lbs}}$$

$$V_{ba}^F = V_{ba}^S + \frac{1}{l} (M_{ab}^F + M_{ba}^F) = + 420 - 715 = \underline{-295 \text{ lbs}}$$

$$M_c^S = R_b^S z_3 l + \frac{1}{2} W_3 z_3 l = (420 \cdot 5.25) + (1340 \cdot 5.25) = \underline{9240 \text{ lbs}}$$

Simple Moments (See ES-1, EH, Sec. 6)

$$\text{At Section 2, } M_c = \frac{1}{8} W_2 z_2 l = + 6940 \text{ ft lbs}$$

$$\text{At Section 3, } M_c = \frac{1}{8} W_3 z_3 l = - 1760 \text{ ft lbs}$$

	$M_x/M_c$	for span $z_2 l$	for span $z_3 l$
at $0.5z l$	1.0	+6940	-1760
at $0.4z l$	0.96	+6660	-1690
at $0.3z l$	0.84	+5830	-1480
at $0.2z l$	0.64	+4440	-1127
at $0.1z l$	0.36	+2500	- 634

Plot moments and shears as explained in ES-32, EH, Sec. 6.

No flow--Simply supported at b

Satisfy this condition by correcting moments and shears when both ends are fixed.

With both ends fixed

$$M_{ab}^F = - 10,545 \text{ ft lbs, } M_{ba}^F = + 2500 \text{ ft lbs}$$

$$V_{ab}^F = - 6865 \text{ lbs, } V_{ba}^F = - 295 \text{ lbs}$$

Simply supported at b

$$M_{ba} = 0$$

$\therefore M_{ab} = M_{ab}^F + 0.5 (-M_{ba}^F)$ , for 0.5 carryover factor and reason for minus sign, refer to any good reference on structural mechanics.

$$M_{ab} = - 10,545 - 1250 = - 11,795 \text{ ft lbs}$$

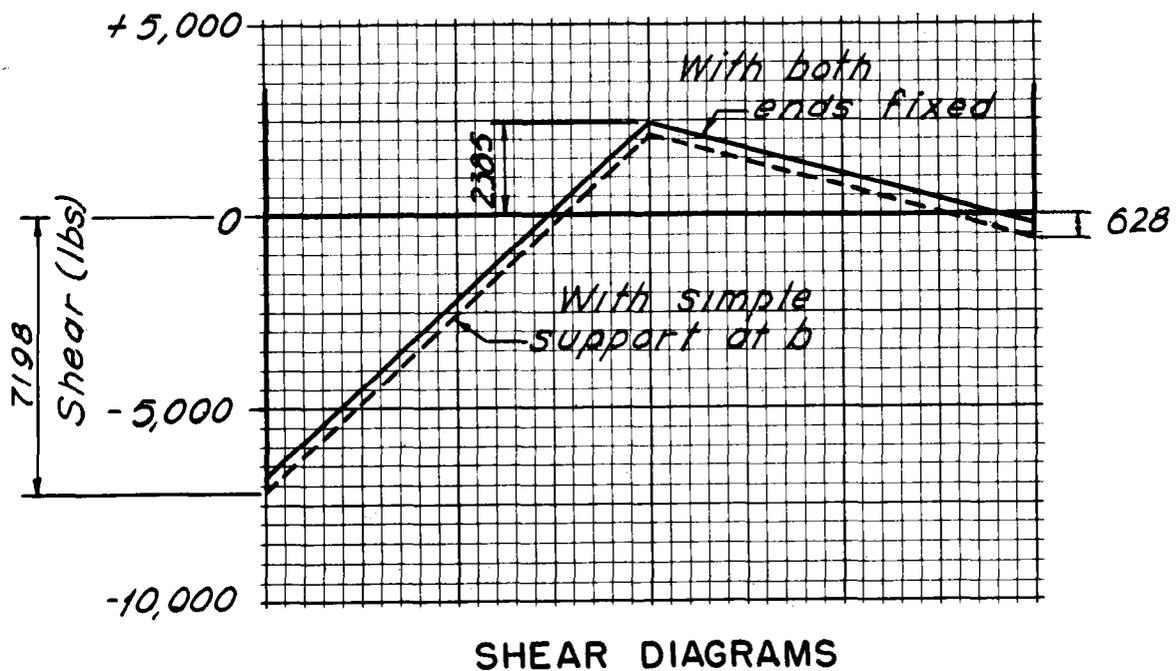
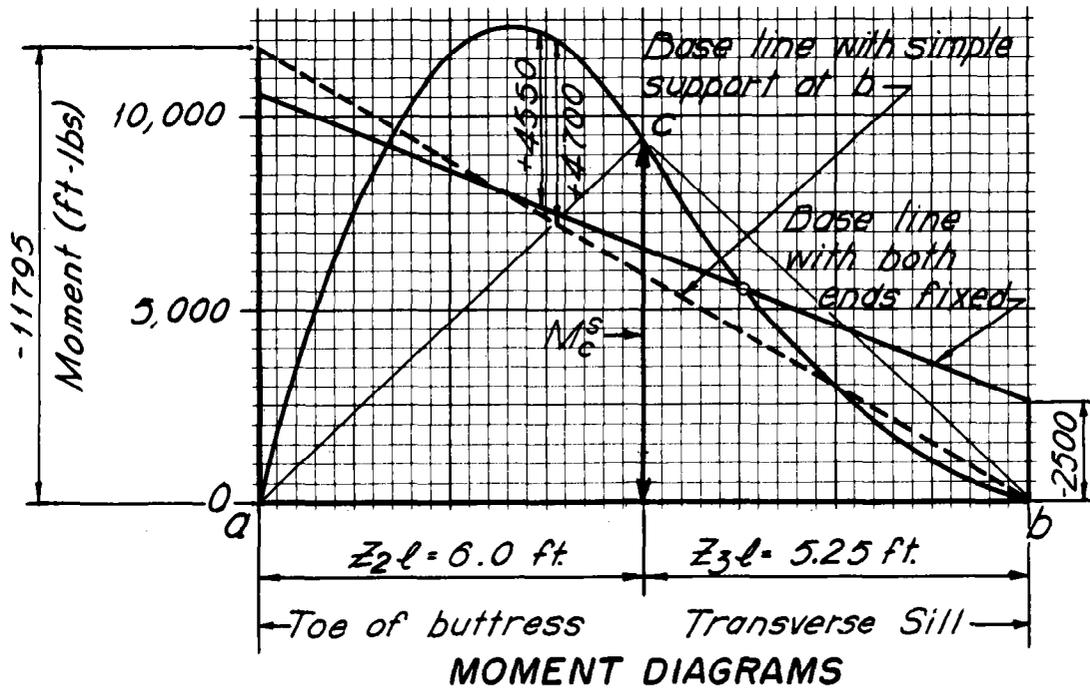
$$V_{ab} = V_{ab}^F + \frac{1}{l} (-1.5 M_{ba}^F) = - 6865 + \frac{1}{11.25} (- 3750)$$

$$V_{ab} = - 6865 - 333 = \underline{- 7198 \text{ lbs}}$$

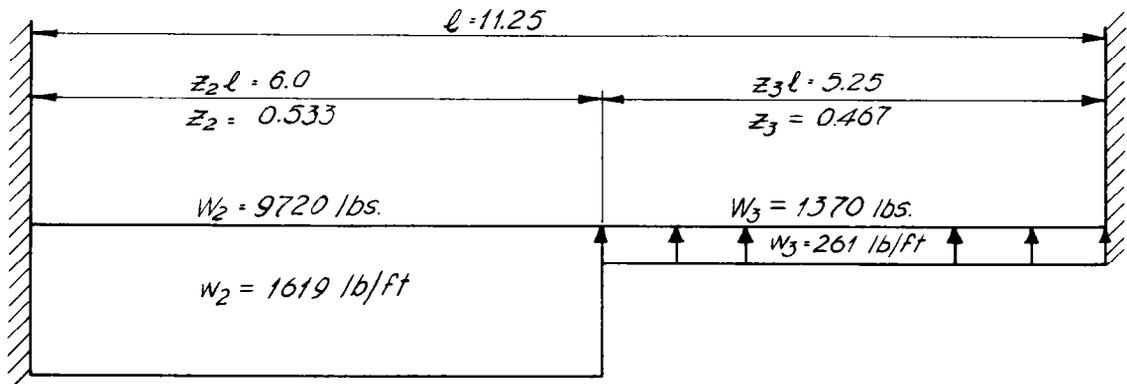
$$V_{ba} = V_{ba}^F - 333 = - 295 - 333 = - 628 \text{ lbs}$$

# LONGITUDINAL SILL (No flow)

+ indicates tension in top face  
- indicates tension in bottom face



With flow--both ends fixed



$$M_{ab}^F = -\frac{1}{12} l z_2 (6 - 8z_2 + 3z_2^2) W_2 - \frac{1}{12} l z_3^2 (4 - 3z_3) W_3$$

$$M_{ab}^F = -\frac{1}{12} (11.25)(0.533)(2.59)(9720) - \frac{1}{12} (11.25)(0.218)(2.60)(1370)$$

$$M_{ab}^F = -12,600 - 728 = \underline{-13,328 \text{ ft lbs}}$$

$$M_{ba}^F = +\frac{1}{12} l z_3 (6 - 8z_3 + 3z_3^2) W_3 + \frac{1}{12} l z_2^2 (4 - 3z_2) W_2$$

$$M_{ba}^F = +\frac{1}{12} (11.25)(0.467)(2.91)(1370) + \frac{1}{12} (11.25)(0.284)(2.40)(9720)$$

$$M_{ba}^F = 1746 + 6220 = \underline{+7966 \text{ ft lbs}}$$

$$V_{ab}^S = \frac{1}{l} \left[ (-W_3 z_3 l \cdot \frac{1}{2}) - W_2 (z_3 l + \frac{1}{2} z_2 l) \right]$$

$$V_{ab}^S = \frac{1}{11.25} \left[ (-1370 \cdot 2.62) - (9720 \cdot 8.25) \right]$$

$$V_{ab}^S = \frac{1}{11.25} (-3600 - 80,100) = \underline{-7440 \text{ lbs}}$$

$$V_{ba}^S = W_2 + W_3 - V_{ab}^S = 9720 + 1370 - 7440 = \underline{+3650 \text{ lbs}}$$

$$V_{ab}^F = V_{ab}^S + \frac{1}{l} (M_{ab}^F + M_{ba}^F) = -7440 + \frac{1}{11.25} (-13,328 + 7966)$$

$$V_{ab}^F = -7440 - 476 = \underline{-7916 \text{ lbs}}$$

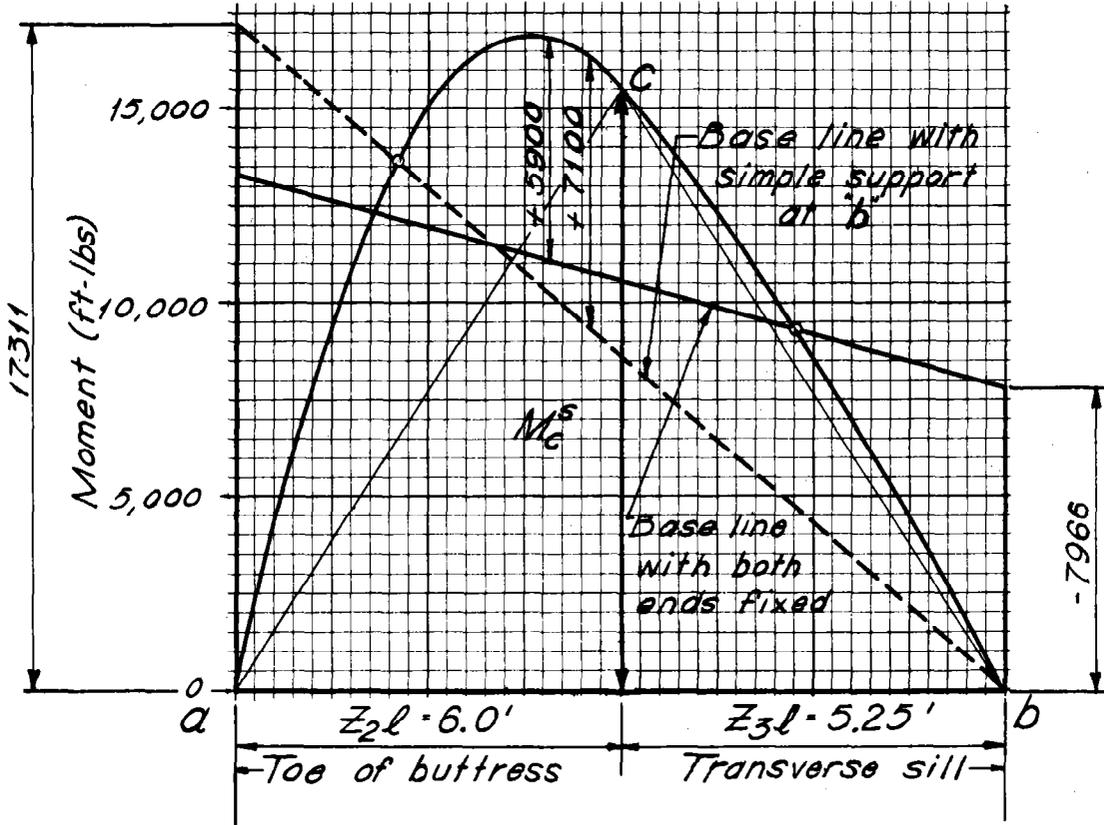
$$V_{ba}^F = V_{ba}^S - 480 = 3650 - 476 = \underline{+3174 \text{ lbs}}$$

$$M_c^S = V_{ba}^S z_3 l - \frac{1}{2} W_3 z_3 l = (3650 \cdot 5.25) - (\frac{1}{2} \cdot 1370 \cdot 5.25)$$

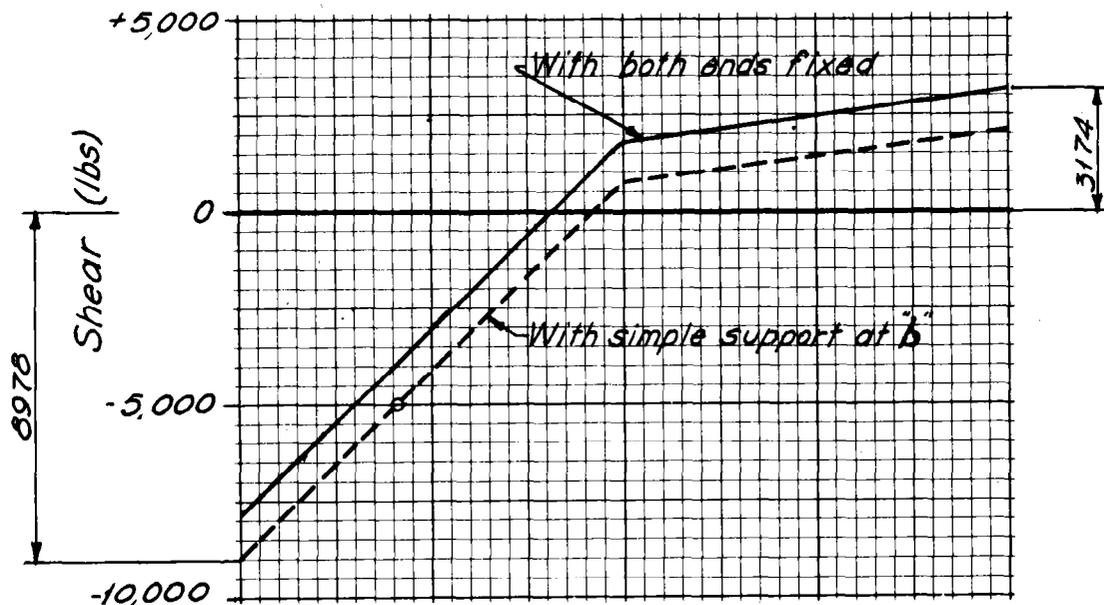
$$M_c^S = 19,160 - 3600 = \underline{15,560 \text{ ft lbs}}$$

# LONGITUDINAL SILL (With flow)

+ indicates tension in top face  
- indicates tension in bottom face



**MOMENT DIAGRAMS**



**SHEAR DIAGRAMS**

Simple Moments

$$\text{At Section 2, } M_Q = \frac{1}{8} W_2 z_2 \ell = 7290 \text{ ft lbs}$$

$$\text{At Section 3, } M_Q = \frac{1}{8} W_3 z_3 \ell = 900 \text{ ft lbs}$$

	$M_x/M_Q$	At Section 2	At Section 3
at $0.5z\ell$	1.0	7290	900
at $0.4z\ell$	0.96	6990	863
at $0.3z\ell$	0.84	6120	755
at $0.2z\ell$	0.64	4660	576
at $0.1z\ell$	0.36	2620	324

With flow--Simply supported at b

Correcting fixed end moments and shears

$$M_{ba} = 0$$

$$M_{ab} = M_{ab}^F + 0.5 (-M_{ba}^F) = -13,328 - 3983 = \underline{-17,311 \text{ ft lbs}}$$

$$V_{ab} = V_{ab}^F + \frac{1}{\ell} (-1.5 M_{ba}^F) = -7916 + \frac{1}{11.25} (-1.5 \cdot 7966)$$

$$V_{ab} = -7916 - 1062 = -8978 \text{ lbs}$$

$$V_{ba} = V_{ba}^F - 1062 = 3174 - 1062 = \underline{+2112 \text{ lbs}}$$

Steel in bottom face  $d = 11 + 12 - 3 - \frac{3}{8} - \frac{3}{4} = 18 \frac{7}{8} \text{ in}$  (Use 19 in)

At Buttress

$$M_{ab} = -17,311, V_{ab} = 8978 \text{ (With flow, simple support at b)}$$

$$\text{Req'd. } A_s = 0.58 \text{ in}^2 \text{ (ES-45, EH, Sec. 6) Req'd. } \Sigma_o = 1.79 \text{ in (ES-44)}$$

$$\text{Use 1 No. 7 bar (} A_s = 0.60, \Sigma_o = 2.75)$$

$$\text{Extend bar } \frac{8.3}{20} (6.0) + 1.0 = 3.49 \text{ ft Say 3'-6" into sill and 2'-0" under buttress. Length = 5'-6".}$$

At Sill

$$M_{ba}^F = 7966, V_{ab}^F = 3174, \text{ With flow, both ends fixed.}$$

$$\text{Req'd. } A_s = 0.273 \text{ in}^2, \text{ Req'd. } \Sigma_o < 1.0 \text{ in}$$

$$\text{Use 1 No. 5 bar (} A_s = 0.31, \Sigma_o = 1.96) \text{ Length of horizontal}$$

$$\text{leg} = \left(\frac{15}{20} \cdot 5.25\right) + 1.0 + 0.583 \text{ ft (No flow, both ends fixed}$$

$$\text{governs) Use 5'-6" Vertical leg} = 1'-0" \text{ in downstream face of toewall. Total length} = 6'-6".$$

Steel in top face Use  $d = 19$  in

$M = 7100$ ,  $V = 5000$  (With flow, simple support at b governs)

Req'd.  $A_s = 0.235$  in<sup>2</sup>, Req'd.  $\Sigma O = 1.45$  in ( $u = 210$  lbs/in<sup>2</sup>)

Use 1 No. 5 bar ( $A_s = 0.31$ ,  $\Sigma O = 1.96$ ) Start bar 2 in. from downstream edge of transverse sill and extend it about 1'-0" under buttress. Length = (0'-7") + (11'-3") + (1'-0") = 12'-10"  
 Use 13'-0". Use L bars No. 4 at 18 for tie bars. Both legs 1'-6", Length = 3'-0"

Transverse Sill Design: The transverse sill, together with the toewall, will be designed as a rectangular beam with its thickness equal to that of the toewall. The design moment at the ends of the beam will be determined, considering the beam as fixed at both ends. The design moment at mid-span will be determined, considering the beam to be partially restrained at the ends and assuming the end moments to be equal to one-half the fixed end moments. The loads will be the total base pressure minus the weight of water and/or concrete and the reactions from the longitudinal sills.

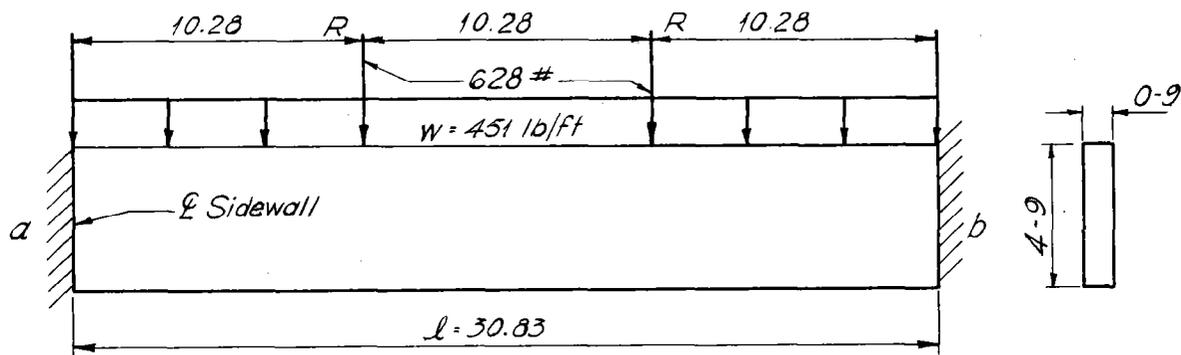
After fill--No flow

Total base pressures/ft length =  $0.75 (99 + 123) \div 2 = 83$  lbs/ft (up)  
 (See apron design)

Wt. of concrete =  $4.75 \cdot 0.75 \cdot 1.0 \cdot 150 = 534$  lbs/ft (down)

Uniform load per ft =  $534 - 83 = 451$  lbs/ft (down)

Concentrated load =  $R_b$  from longitudinal sill =  $R = 628$  lbs (down)



Moments and Shears

$$V_{ab} = -V_{ba} = \frac{wl}{2} + R = (451)(30.83) \frac{1}{2} + 628 = 6950 + 628$$

$$V_{ab} = 7578 \text{ lbs}$$

$$M_{ab} = -M_{ba} = \frac{1}{12} wl^2 + \frac{2}{9} Rl = \frac{1}{12} (451)(30.83)^2 + \frac{2}{9} (628)(30.83)$$

$$M_{ab} = -M_{ba} = 35,700 + 4300 = 40,000 \text{ ft lbs}$$

Simple Moments

From Concentrated load at point of load

$$M = 628 \cdot 10.28 = 6460 \text{ ft lbs}$$

From Uniform Load

$$M_Q = \frac{1}{8} w l^2 = \frac{1}{8} \cdot 451 (30.83)^2 = 53,600 \text{ ft lbs}$$

$$\begin{aligned} \text{at } 0.4l, M &= 0.96 \cdot 53,600 = 51,400 \text{ ft lbs} \\ 0.3l, M &= 0.84 \cdot 53,600 = 45,000 \text{ ft lbs} \\ 0.2l, M &= 0.64 \cdot 53,600 = 34,300 \text{ ft lbs} \\ 0.1l, M &= 0.36 \cdot 53,600 = 19,300 \text{ ft lbs} \end{aligned}$$

After fill--With flow

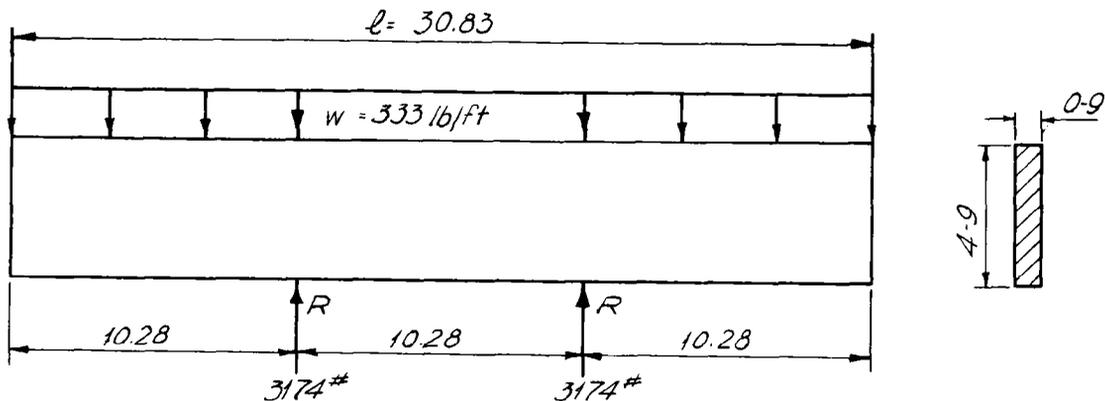
$$\text{Total base pressure} = 0.75 (551 + 567) \frac{1}{2} = 419 \text{ lbs/ft (up)}$$

$$\text{Wt. of concrete} = 534 \text{ lbs (down)}$$

$$\text{Wt. of H}_2\text{O on sill} = 62.4 \cdot 4.67 \cdot 0.75 = 218 \text{ lbs (down)}$$

$$\text{Uniform load/ft} = 534 + 218 - 419 = 333 \text{ lbs/ft (down)}$$

$$\text{Concentrated load} = R_p = R = 3174 \text{ lbs (up)}$$

Moments and Shears

$$V_{ab} = -V_{ba} = \frac{w l}{2} - R = (333 \cdot 15.41) - 3174 = 1966 \text{ lbs}$$

$$M_{ab} = -M_{ba} = \frac{1}{12} w l^2 - \frac{2}{9} R l = \left[ \frac{1}{12} \cdot 333 \cdot (30.83)^2 \right] - \left[ \frac{2}{9} (3176) (30.83) \right]$$

$$M_{ab} = -M_{ba} = 26,400 - 21,800 = 4,600 \text{ ft lbs}$$

Simple MomentsFrom Concentrated load at point of load

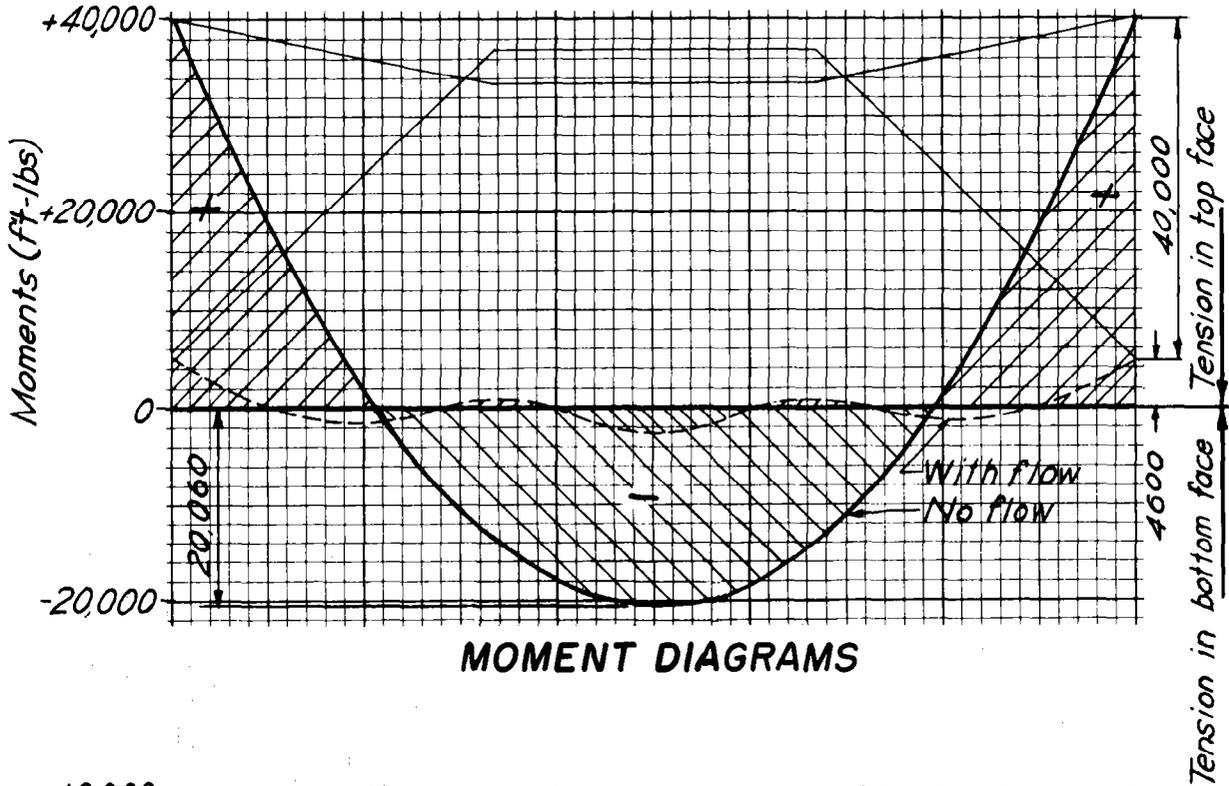
$$M = 3176 \cdot 10.28 = 32,600 \text{ ft lbs}$$

From Uniform load

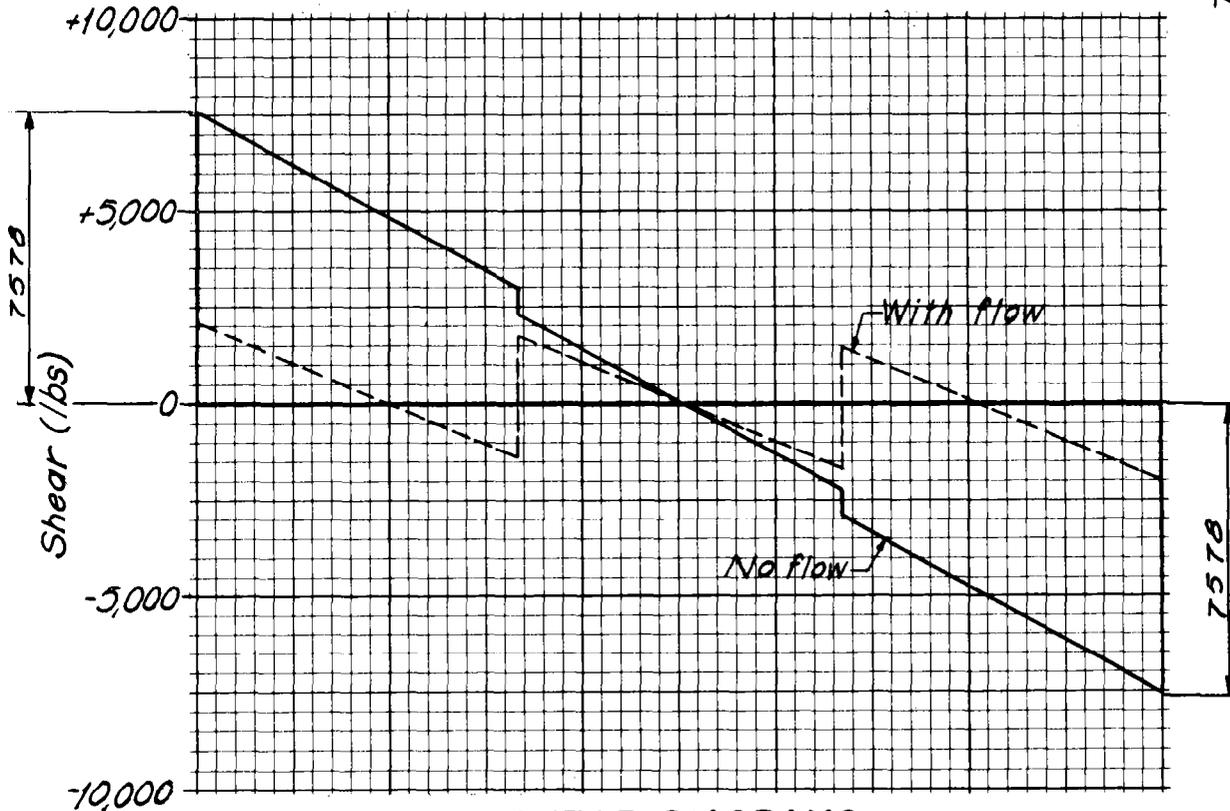
$$M_Q = \frac{1}{8} w l^2 = \frac{1}{8} \cdot 333 (30.83)^2 = 39,600 \text{ ft lbs}$$

$$\begin{aligned} \text{at } 0.4l, M &= 0.96 \cdot 39,600 = 38,000 \text{ ft lbs} \\ 0.3l, M &= 0.84 \cdot 39,600 = 33,300 \text{ ft lbs} \\ 0.2l, M &= 0.64 \cdot 39,600 = 25,300 \text{ ft lbs} \\ 0.1l, M &= 0.36 \cdot 39,600 = 14,260 \text{ ft lbs} \end{aligned}$$

# TRANSVERSE SILL



MOMENT DIAGRAMS



SHEAR DIAGRAMS

Steel in top face  $d = 57'' - 4'' = 53$  in

$M = 40,000$  ft lbs,  $V = 7578$  lbs (No flow governs)

Req'd.  $A_s = M \div ad = 40.0 \div (1.44 \cdot 53) = 0.524$  in<sup>2</sup>

Req'd.  $\Sigma_o < 1.0$  in. Use 2 No. 5 bars ( $A_s = 0.61$ ,  $\Sigma_o = 3.93$ )

Find where 2 No. 3 bars ( $A_s = 0.22$ ) can be used

$M = ad A_s = 1.44 \cdot 53 \cdot 0.22 = 16.8$  ft kips which corresponds to a point  $5.7/50 \cdot 30.83 = 3.52$  ft from  $\zeta$  of sidewall.

Length of horizontal leg of No. 5 bars =  $3.52 + 0.25 + 1.0 = 4.77$  ft

Use 4'-9". Make vertical leg 2'-0". Total length = 6'-9"

Length of No. 3 bars =  $(30'-0'') + 2 (0'-8'') + 2 (1'-0'') - 2 (4'-9'')$   
 $= 23'-10''$ , Use 24'-0" with 1'-1" lap

Steel in bottom face Increase moment 50 percent of fixed end moment

Moment =  $20.06 + (0.5 \cdot 40) = 40.06$  ft kips; shear = 4600 lbs (max)

$A_s = M \div ad = 40.06 \div (1.44 \cdot 53) = 0.532$  sq in

$\Sigma_o = 8 V \div 7 u d = (8 \cdot 4600) \div (7 \cdot 300 \cdot 53) = 36,800 \div 111,300$   
 $= 0.33$  in

Use 2 No. 5 bars ( $A_s = 0.61$ ,  $\Sigma_o = 3.93$ ). Extend bars 0'-6" into sidewall on both ends then length = 31'-0"

Other Steel

Vertical Steel

Use No. 3 at 12 in downstream face (*Apron bar*)

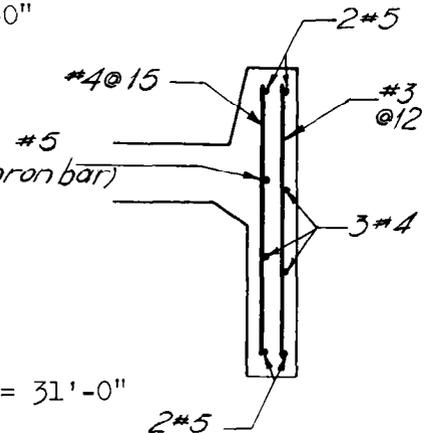
Use No. 4 at 15 in upstream face

Length =  $(4'-9'') - 2 (0'-3'') = 4'-3''$

Horizontal Steel

Use No. 4 in both faces

Length =  $(30'-0'') + (0'-6'') + (0'-6'') = 31'-0''$



Wingwall Design: The wingwall will be designed independently of the rest of the structure and as a series of cantilever beams. The wingwall makes an angle of  $45^\circ$  with the sidewall.

Length of Wingwall =  $(J - s - 1.0) 2.83$

Length =  $(9.0 - 1.33 - 1.0) 2.83 = 18.85$  ft Use 19'-0"

Required width of footing to prevent overturning

Fill Characteristics

Moist wt. =  $W = 110$  lbs/ft<sup>3</sup>

Angle of internal friction =  $\phi = 25^\circ$

Friction angle of soil =  $Z = \phi = 25^\circ$

Angle between horizontal and vertical wall =  $\theta = 90^\circ$

Angle between horizontal and fill slope =  $i = 19^\circ-27'$

Try 6.0 ft width of footing at upstream end

See equation 6.2-1 (EH, Sec. 6)

$$P = \frac{wH^2}{2} \frac{\sin^2 (\theta - \phi)}{\sin^2 \theta \sin (\theta + Z) \left[ 1 + \sqrt{\frac{\sin (Z + \phi) \sin (\phi - i)}{\sin (\theta + Z) \sin (\theta - i)}} \right]^2}$$

$$P = \frac{wH^2}{2} \frac{\sin^2 65^\circ}{\sin^2 90^\circ \cos 25^\circ \left[ 1 + \sqrt{\frac{\sin 50^\circ \sin 5^\circ - 33'}{\cos 25^\circ \sin 70^\circ - 33'}} \right]^2}$$

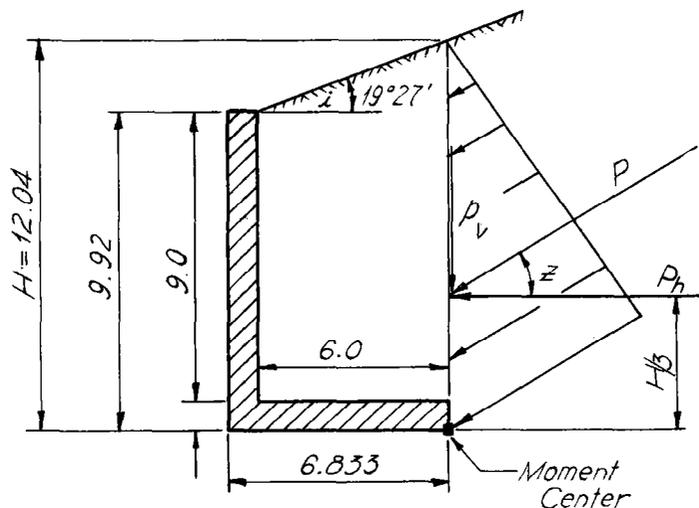
$$P = \frac{wH^2}{2} \frac{0.821}{(1.0)(0.906) \left[ 1 + \sqrt{\frac{0.766 \cdot 0.0967}{0.906 \cdot 0.943}} \right]^2} = \frac{wH^2}{2} (0.54)$$

$$P = 55 \cdot (12.04)^2 \cdot 0.54 = 4300 \text{ lbs}$$

$$P_h = P \cos Z = 4300 \cdot 0.906 = 3900 \text{ lbs}$$

$$P_v = P \sin Z = 4300 \cdot 0.423 = 1850 \text{ lbs}$$

$$H = 9.92 + 6 \tan i = 9.92 + 2.12 = 12.04, \quad H/3 = 4.01$$



Taking Moments

Part	Weight	Force	Arm	Moment
Footing $6.0 \cdot 0.92 \cdot 150 =$	828		3.0	2,480
Wall $9.92 \cdot 0.83 \cdot 150 =$	1,230		6.42	7,900
Earth $\square 6.0 \cdot 9.0 \cdot 110 =$	5,940		3.0	17,820
Earth $\triangle 3.0 \cdot 2.12 \cdot 110 =$	700		2.0	1,400
$P_v$	1,850		0.0	0
$P_h$		3,900	4.01	15,640
	10,548			45,240

$z = 45,240 \div 10,548 = 4.29 \text{ ft}$ . ∴ stable from overturning as resultant falls within the middle third of base ( $6.83 \cdot 2/3 = 4.55 \text{ ft}$ )

Try 1.0 ft width of footing at downstream end and check overturning at midsection.

Footing at midsection =  $(6.0 + 1.0) \div 2 = 3.5$  ft wide

Base width at midsection =  $3.5 + 0.83 = 4.33$  ft

$H = [(9.0 + 2.33) \div 2] + 0.92 + 3.5 \tan i$

$H = 5.67 + 0.917 + 1.23 = 7.82$  ft

Ht. of wall above footing = 5.67 ft

$P = (wH^2 \div 2) \cdot 0.54 = [(110 \cdot 7.82^2) \div 2] \cdot 0.54 = 1810$  lbs

$P_h = P \cos Z = 1810 \cdot 0.906 = 1640$  lbs

$P_v = P \sin Z = 1810 \cdot 0.423 = 765$  lbs

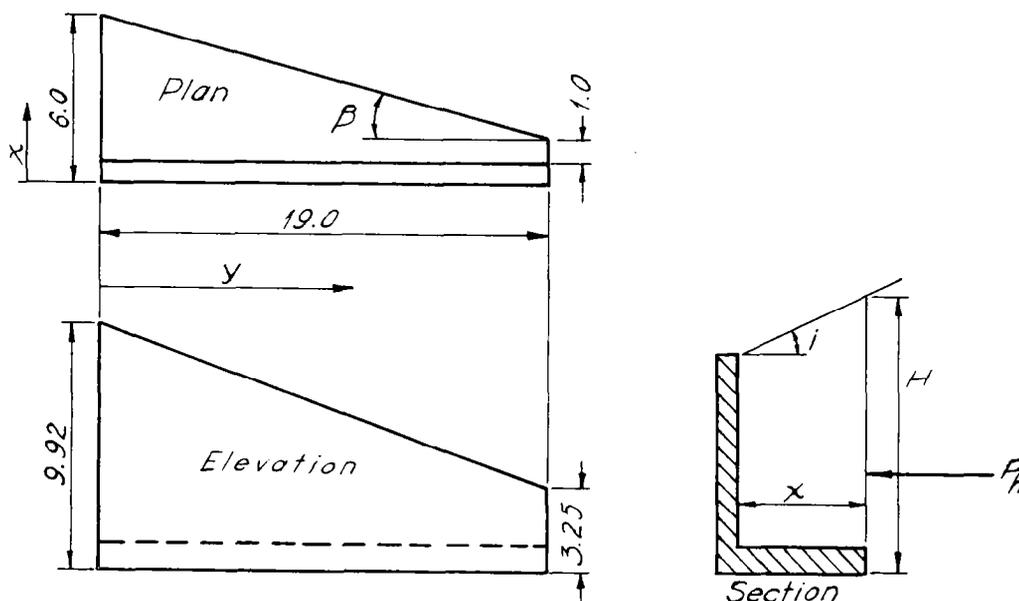
Taking Moments

Part	Weight	Force	Arm	Moment
Footing $3.5 \cdot 0.92 \cdot 150$	482		1.75	844
Wall $6.58 \cdot 0.833 \cdot 150$	822		3.92	3,220
Earth $\square 3.5 \cdot 5.67 \cdot 110$	2,180		1.75	3,820
Earth $\triangle 1.75 \cdot 1.23 \cdot 110$	238		0.412	98
$P_v$	765		0.0	0
$P_h$		1,640	2.61	4,280
	4,487		2.73	12,262

$4.33 \cdot \frac{2}{3} = 2.88 > 2.73$ ; hence, OK safe from overturning

Sliding

To prevent the wingwall from sliding, the wingwall footing will be tied into the apron and reinforced to carry the horizontal moment set up by the earth pressures on the wingwall, tending to rotate the wingwall about its junction with the sidewall.



Determine Horizontal Moment

From overturning calculations

$$P_h = P \cos Z = 0.906 P$$

$$P = (wH^2 \div 2) 0.54$$

$$\therefore P_h = 0.906 (110 H^2 \div 2) 0.54 = 26.92 H^2$$

$$H = 9.92 + x \cos i - \left( \frac{9.92 - 3.25}{19.0} \right) y = 9.92 + 0.353 x - 0.351 y$$

$$x = 6 - \left( \frac{6.0 - 1.0}{19} \right) y = 6 - 0.263 y$$

$$H = 9.92 + 2.12 - 0.093 y - 0.351 y = 12.04 - 0.444 y$$

$$\therefore P_h = 26.92 (12.04 - 0.444 y)^2$$

$$P_h = 3902 - 287.8 y + 5.31 y^2$$

$$\text{Total } P_h = P_{ht} = P_h dy = \int_0^{19} (3902 - 287.8 y + 5.31 y^2) dy$$

$$M = P_h y dy = \int_0^{19} (3902 - 287.8 y + 5.31 y^2) y dy$$

$$M = \left[ \frac{3902 y^2}{2} - \frac{287.8 y^3}{3} + \frac{5.31 y^4}{4} \right]_0^{19}$$

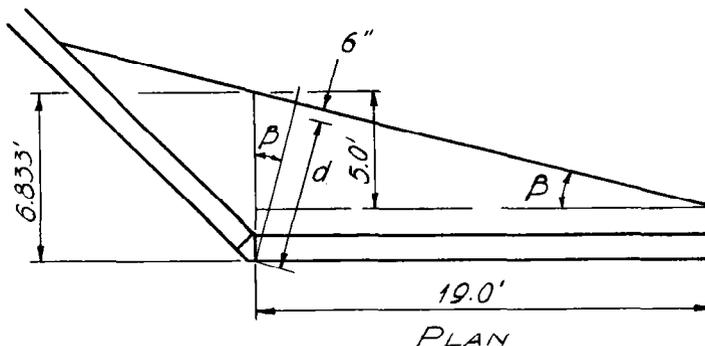
$$M = 704,300 - 662,600 + 173,000 = 214,700 \text{ ft lbs}$$

Determine effective depth

$$\tan \beta = 5 \div 19 = 0.263 \quad \therefore \beta = 14^\circ - 45'$$

$$\cos \beta = 0.967$$

$$d = (12 \cdot 6.83 \cos \beta) - 6 = 70.6 \text{ in} \quad (\text{Use } 70.0 \text{ in})$$



Req'd. steel ( $A_s = M \text{ (ft kip)} \div ad$ )

$$A_s = 214.7 \div (1.44 \cdot 70) = 2.13 \text{ in}^2$$

Use 2 No. 10 bars ( $A_s = 2.54$ ) place one bar 4 in. from edge and the other 10 in. from edge.

Design of Vertical Wall

Take  $Z$ , angle of friction between earth and concrete = 0  
(See equation 6.2-3, EH, Sec. 6)

$$P = \frac{wH^2}{2} \frac{\cos^2 \phi}{\left(1 + \sqrt{\frac{\sin \phi \sin (\phi - i)}{\sin (\theta - i)}}\right)^2} = \frac{wH^2}{2} \frac{\cos^2 25^\circ}{\left(1 + \sqrt{\frac{\sin 25^\circ \sin 5^\circ - 33'}{\sin 70^\circ - 33'}}\right)^2}$$

$$H = 9.0 \text{ ft}$$

$$P = \frac{110 (9)^2}{2} \frac{0.821}{\left(1 + \sqrt{\frac{0.423 \cdot 0.0967}{0.943}}\right)^2} = 4455 \cdot 0.562 = 2510 \text{ lbs}$$

$$M = P \frac{H}{3} = 2510 \cdot 3 = 7530 \text{ ft lbs, } V = P = 2510 \text{ lbs}$$

Assuming the same equivalent fluid pressure,  $w$ , at all points along the wall, build up  $M$  and  $A_s$  curves ( $d = 7 \frac{1}{2}$  in)

$$P = wH^2 \div 2 \quad \therefore w = 2P \div H^2$$

$$w = 2 \cdot 2510 \div 81 = 62.0 \text{ lbs/ft}^3$$

$$M = wH^3 \div 6 = 10.33 H^3$$

H	H <sup>3</sup>	M	A <sub>s</sub>
9.0	729	7530	0.67
8.0	512	5300	0.47
7.0	343	3550	0.31
6.0	216	2240	0.193
5.0	125	1290	0.108
4.0	64	662	<0.1
3.0	27	279	<0.1

$$d = 7.5 \text{ in}$$

Vertical Steel (Unexposed face)At Upstream End

$$M = 7530 \text{ ft lbs, } V = 2510 \text{ lbs}$$

Req'd.  $A_s = 0.67 \text{ in}^2$ , Req'd.  $\Sigma O = 1.29 \text{ in}$  (M will govern)

Use No. 6 at 8 ( $A_s = 0.66$ ,  $\Sigma O = 3.53 \text{ in}$ )

Find where No. 4 at 8 ( $A_s = 0.29$ ) can be used. From  $A_s$  curve

$$H = 6.9 \text{ ft, } \therefore \text{Length of No. 6 bar} = 9.0 - 6.9 + 3.42 - 0.25 + 1.0$$

$$= 6.27; \text{ Use } 6'-3'' \text{ and place } 3'' \text{ above bottom of toewall.}$$

Find point along wall where No. 6 at 12 ( $A_s = 0.44$ ) can be used

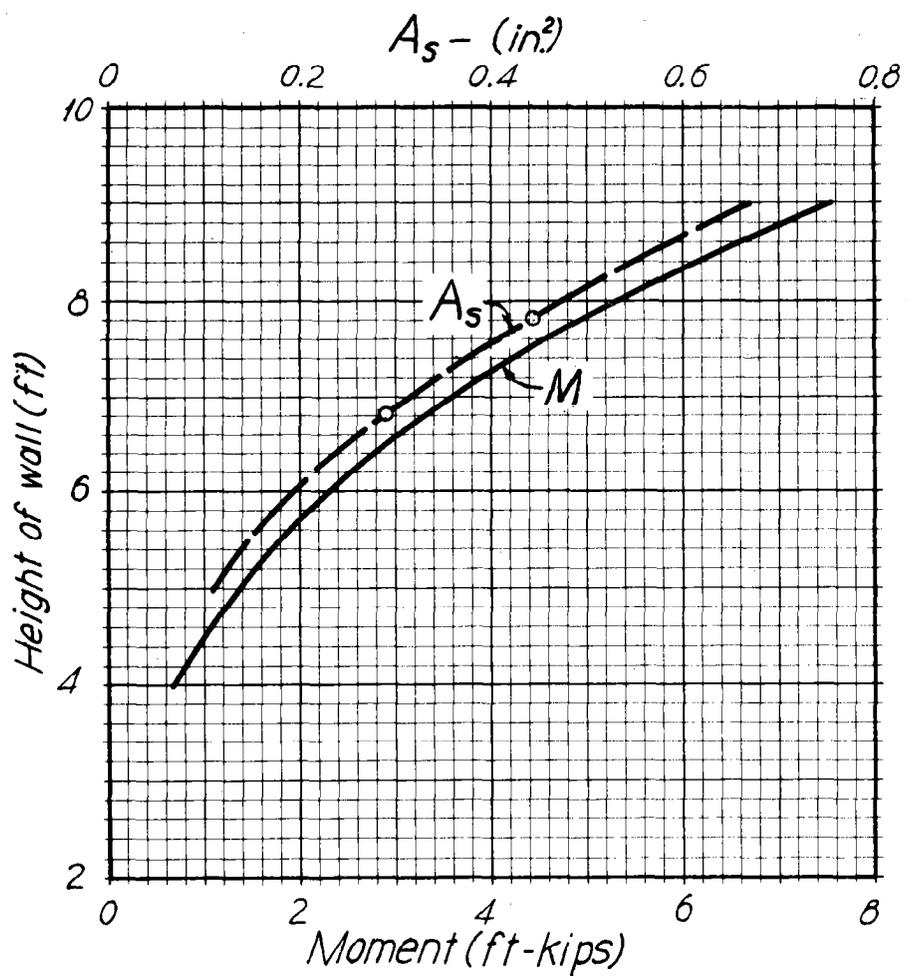
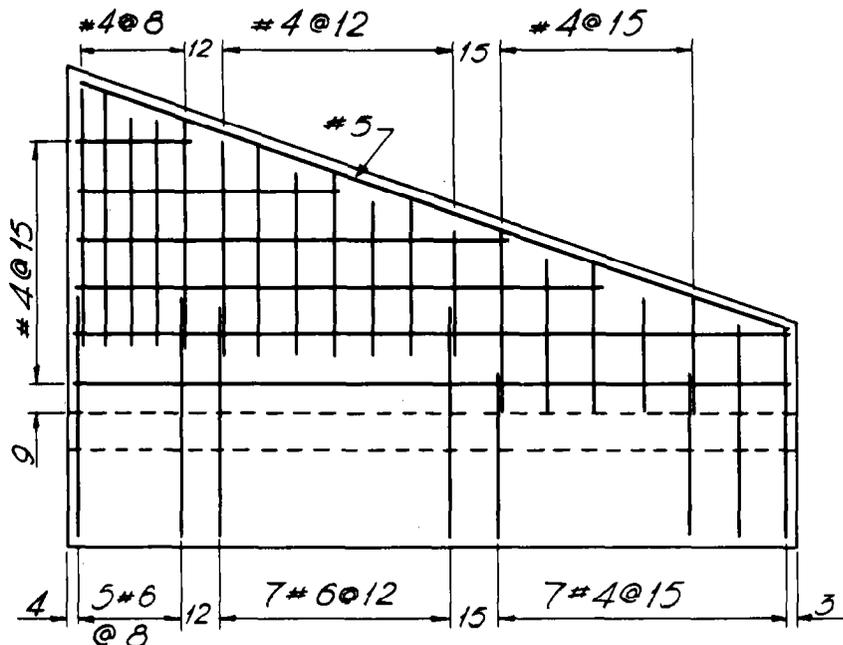
From  $A_s$  curve  $H = 7.85$

$$\text{Dist. from upstream end} = (9.0 - 7.85) \left[ \frac{19}{9.0 - 2.33} \right] = 3.28 \text{ ft}$$

Find where No. 4 at 12 ( $A_s = 0.20$ ) can be used.

# WINGWALL DESIGN

## Unexposed face of wingwall



$$H = 6.1 \text{ .'. Length of No. 6 bars} = 7.85 - 6.1 + 3.42 - 0.25 + 1.0 \\ = 5.92. \text{ Use } 6'-0''$$

Find point along wall where No. 4 at 15 ( $A_s = 0.16$ ) can be used

From  $A_s$  curve  $H = 5.70$

$$\text{Dist. from upstream end} = (9.0 - 5.70) \left[ \frac{19.0}{9.0 - 2.33} \right] = 9.40 \text{ ft}$$

Horizontal Steel (Unexposed face)

Temp. steel--Use No. 4 at 15 (Start 9 in. above footing)

Steel in Exposed face

Horizontal steel--Use No. 5 at 15 Temp. steel

Vertical steel--Use No. 4 at 15 Tie steel. Set on construction joint.

Design of Footing

Load on upstream section (See page 4.12)

$$p = \frac{V}{A} \left[ 1 \pm \frac{6e}{d} \right]$$

$V$  = vertical weight,  $A$  = area of base,  $e$  = eccentricity,  $d$  = length of section,  $e = z - (d \div 2) = 4.29 - (6.83 \div 2) = 0.873$

$$p_1 = \frac{10,548}{6.83} \left[ 1 + \frac{6(0.873)}{6.83} \right]$$

$$p_1 = 1546 (1 + 0.766) = 2730 \text{ lbs/ft}^2$$

$$p_2 = 1546 (1 - 0.766) = 362 \text{ lbs/ft}^2$$

$$p_3 = 362 + (6 \div 6.83)(2730 - 362)$$

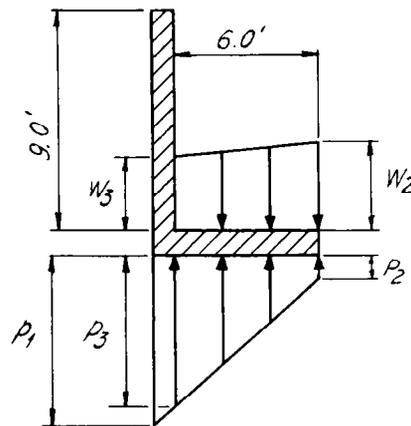
$$p_3 = 362 + 2080 = 2442 \text{ lbs/ft}^2$$

$$W_2 = (11.12 \cdot 110) + (0.92 \cdot 150)$$

$$W_2 = 1360 \text{ lbs/ft}^2$$

$$W_3 = (9.0 \cdot 110) + (0.92 \cdot 150)$$

$$W_3 = 1128 \text{ lbs/ft}^2$$



Moment and Shear

$$M = \frac{W_3 \cdot (6.0)^2}{2} + \frac{(W_2 - W_1)}{2} \frac{2}{3} (6.0)^2 - \frac{p_2(6.0)^2}{2} - \frac{(p_3 - p_2)}{2} \frac{1}{3} (6.0)^2$$

$$M = \frac{1128 \cdot 36}{2} + \frac{232 \cdot 36}{3} - \frac{362 \cdot 36}{2} - \frac{2080 \cdot 36}{6}$$

$$M = 20,300 + 2780 - 6510 - 12,480 = + 4090 \text{ ft lbs}$$

$$V = \frac{(W_2 + W_3)}{2} 6.0 - \frac{(P_2 + P_3)}{2} 6.0 = (W_2 + W_3 - P_2 - P_3) 3.0$$

$$V = 3.0 (1360 + 1128 - 362 - 2442) = 316 \cdot 3 = 948 \text{ lbs}$$

$$d = 7 \frac{1}{2} \text{ in}$$

Req'd.  $A_s = 0.36 \text{ in}^2$ . Use No. 6 at 12 ( $A_s = 0.44$ ) to midsection then use No. 4 at 12.; Bent into downstream face of toewall.

Use No. 4 at 15 parallel to wall as tie steel.

### Design of Headwall Extension

Due to the fact that the headwall extension is articulate, and the earth pressures on both sides of the wall will be about equal, the wall will be reinforced with the minimum steel requirements.

#### Vertical Steel (both faces)

Use No. 4 at 15

$$\begin{aligned} \text{Length of bars in first pour} &= (2'-6") + (0'-11") + (0'-4") + \\ &\quad (1'-3") - (0'-3") = 4'-9" \text{ (provides 1'-3" lap)} \end{aligned}$$

#### Horizontal Steel

Use No. 4 at 15, Length = 15'-3"

#### Steel in footing ( $\perp$ to wall)(both faces)

Use No. 4 at 15, Length = 3'-6"







## 7. MASONRY DROP SPILLWAYS

General. A comparative cost analysis will usually indicate that masonry construction should be confined to relatively low drop spillways. A reinforced concrete apron should be used in cases where masonry is selected as the construction material for the walls. A reinforced concrete apron is more durable than masonry and can be designed to resist uplift forces, whereas a masonry apron must resist uplift by its weight alone.

Design Procedures and Aids. The over-all dimensions of a masonry drop spillway and the loadings for the integral parts of the structure are determined in the same manner as for a reinforced concrete drop spillway. The required base widths for the gravity walls for various loads and loading conditions are shown on drawing ES-64, page 7.17.

The curves on this drawing are sufficiently accurate for the majority of conditions of loads and loadings. The top width of the wall has a negligible effect on the required base width. The weight of earth is not a factor in Case 2, drawing ES-64, page 7.17, and its effect is negligible in Case 1. In Case 3, for earth weights greater than 100 pounds per cubic foot, the curves are on the safe side. For weights of earth less than 100 pounds per cubic foot, the base width for Case 3 should be computed by the equation shown on drawing ES-64. The curves are not valid for a masonry weight other than 150 pounds per cubic foot.

To facilitate construction and to maintain smooth surfaces, the batter on the walls may be handled as follows:

The headwall extensions will have a vertical face on the downstream side and a minimum batter of 1 in 10 on the upstream side. The headwall extension will be designed for a differential pressure of 5 pounds per cubic foot acting in the downstream direction.

The headwall will have the same batter on the upstream side as the headwall extensions. The thickness of the headwall at crest elevation will be a function of the depth of the weir and the required batter on the headwall extensions.

The sidewalls and wingwalls will have vertical faces on the unexposed sides.

Reinforced Concrete Apron and Sill Design. The following design procedure should not be used where  $F + h$  exceeds 12 feet.

The apron may be designed as a series of beams in both directions. In the transverse direction, the beams should be designed for two conditions: (1) simply supported at the sidewalls and continuous over the longitudinal sills, and (2) fixed at the sidewalls and continuous over the longitudinal sills. In the longitudinal direction, the beams should be considered as (1) simply supported at both supports, and (2) fixed at both supports.

The upward pressure on the base area may be taken as the weight of the structure, plus the weights of the water and earth on the structure, with the assumption that it is uniformly distributed over the base area. The headwall extensions beyond the unexposed faces of the sidewalls and the wingwalls should not be included in the structure weight or base area. The maximum net load produced with or without flow over the structure should be taken as the design load.

Due to the relatively short span in the longitudinal direction, which makes it uneconomical to attempt to reduce reinforcing steel areas as the moments decrease, it is not necessary to construct moment and shear diagrams. The steel in the top face of a 12-inch slice should be designed for a moment of one-eighth  $wl^2$ , and should extend 1'-0" past the toe of the headwall batter and be bent into the upstream face of the toewall. The steel in the bottom face of a 12-inch slice should be designed for a moment of one-twelfth  $wl^2$  and a shear of one-half  $wl$ . The bottom steel should be bent into the downstream faces of the toewall and cutoff wall. For convenience, the bottom steel should consist of two sets of bars spliced at about the midpoint between the toe of the headwall batter and the transverse sill.

It is usually advantageous to construct the moment and shear curves in the transverse direction. A table of moment and shear coefficients are given to facilitate the construction of these diagrams when the longitudinal sills divide the apron into three equal spans. (See drawing ES-56, page 4.27, if spans are not equal.)

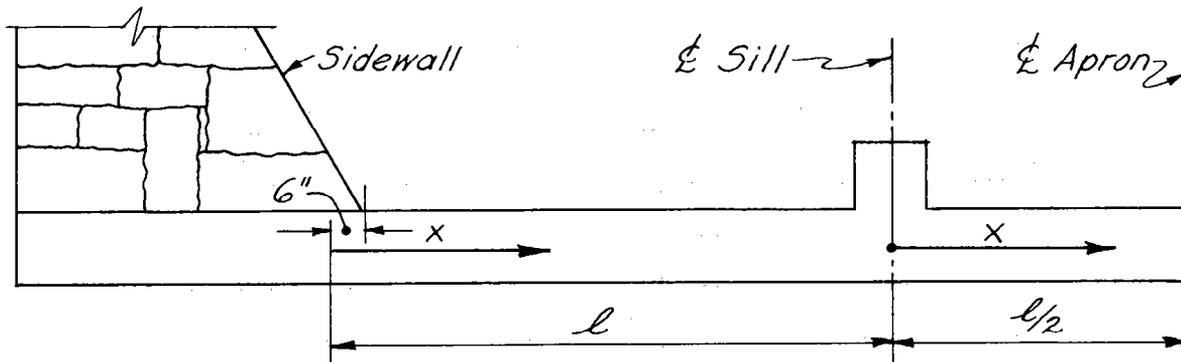


FIGURE 7.1

Moment and Shear Coefficients in Transverse Direction				
$M = C w l^2, \quad V = C_1 w l$				
in which				
$w = \text{load in lb/ft}^2 = \text{lb/ft on 12 in slice}$				
$l = \text{span length in ft}$				
$\frac{x}{l}$	Simply Supported at Sidewall		Fixed at Sidewall	
	C	$C_1$	C	$C_1$
0	0	-0.4	-0.0833	-0.5
0.1	+0.035	-0.3	-0.03833	-0.4
0.2	+0.060	-0.2	-0.00333	-0.3
0.3	+0.075	-0.1	+0.02167	-0.2
0.4	+0.080	0	+0.03667	-0.1
0.5	+0.075	+0.1	+0.04167	0
0.6	+0.060	+0.2	+0.03667	+0.1
0.7	+0.035	+0.3	+0.02167	+0.2
0.8	0	+0.4	-0.00333	+0.3
0.9	-0.045	+0.5	-0.03833	+0.4
Centerline Sill	-0.100	+0.6 -0.5	-0.0833	+0.5 -0.5
0.1	-0.055	-0.4	-0.03833	-0.4
0.2	-0.020	-0.3	-0.00333	-0.3
0.3	+0.005	-0.2	+0.02167	-0.2
0.4	+0.020	-0.1	+0.03667	-0.1
Centerline Apron	+0.025	0	+0.04167	0

- C denotes tension in the bottom face

-  $C_1$  denotes shear in the downward direction.

TABLE 7.1

The longitudinal sills may be designed as beams for two conditions; namely, simply supported at both ends and fixed at both ends. The design load should be taken as the maximum reaction at the sill found in the transverse apron design minus the weight of the sill.

The transverse sill together with the toewall may be designed as a beam for two conditions; namely, simply supported at both ends and fixed at both ends. Its loads are the concentrated loads from the longitudinal sills, and the uniform load equal to the base pressure minus the weight of the transverse sill and toewall.

Example 7.1

Design of Masonry Drop Spillway. Assume the design discharge is 225 cfs,  $F = 6'-0''$ , and that the site conditions limit the weir length to  $16'-0''$ .

Hydraulic Design

Weir Dimensions:  $Q = 225$  cfs,  $L = 16'-0''$

$$\text{Equation 3.6 (page 3.7)} \quad h = \left[ \frac{Q(1.10 + 0.01 F)}{CL} \right]^{2/3}$$

$$h = \left[ \frac{225(1.10 + 0.06)}{3.1 \cdot 16} \right]^{2/3} = (5.26)^{2/3}$$

$$h = 3.02 \text{ ft} \quad \text{Use } \underline{3'-0''}$$

Height of Transverse Sill:

$$s = \frac{1}{2} d_c$$

$$d_c = \sqrt[3]{\frac{Q^2}{L^2 g}} = \sqrt[3]{\frac{(225)^2}{(16)^2 \cdot 32.2}} = 1.83 \text{ ft}$$

$$s = \frac{1.83}{2} = 0.915 \text{ ft} \quad \text{Use } \underline{1'-0''}$$

$$\underline{\text{Height of Headwall}} = F + s = 6.0 + 1.0 = \underline{7.0 \text{ ft}}$$

$$\underline{\text{Height of Sidewall}} = 2.5 d_c + s = 2.5 \cdot 1.83 + 1.0 = 5.58 \text{ ft} \quad \text{Use } \underline{6'-0''}$$

Minimum Length of Apron:  $h \div F = 3 \div 6 = 0.50$ ; From ES-67 (page 5.3)

$$L_B \div F = 2.28 (h \div F) + 0.52 = 1.66$$

$$L_B = 1.66 \cdot 6 = 9.96$$

Use  $L_B = 9'-9''$  to maintain 2 to 1 fill slopes

Minimum Length of Headwall Extension =  $3h + 2.0'$

$$3h + 2.0 = 3 \cdot 3 + 2.0 = 11.0 \text{ ft}$$

Note: To maintain 2 to 1 fill slopes, the length was increased to 12'-9''.

Tailwater Depth and Depth of Water on Apron

The tailwater depth from the downstream channel design is assumed to be 2.4 ft; therefore, the depth of water on the apron is equal to  $2.4 + 1.0 = \underline{3.4 \text{ ft} = d_x}$ .

### Stability Design

The foundation and backfill material will be a sand-silt-clay mixture with the following characteristics:

Undisturbed dry wt = 100 lbs/ft<sup>3</sup> with 40 percent voids

Effective submerged wt =  $100 - (1.0 - 0.4) 62.4 = 62.5$  lbs/ft<sup>3</sup>

Compacted dry wt = 120 lbs/ft<sup>3</sup> with 28 percent voids =  $W_1$

Effective submerged wt =  $120 - (1.0 - 0.28) 62.4 = 75.1$  lbs/ft<sup>3</sup> =  $W_2$

Angle of internal friction =  $\phi = 30^\circ$ .

The channel above the structure will be a graded channel to crest elevation. It is anticipated that the water table will be above the apron elevation after construction; therefore, an ample drain will be provided through the headwall with its outlet at the elevation of the top of the transverse sill, or 1.0 ft above the top of the apron. It is estimated that the drain will maintain the water-table elevation at the headwall at its outlet elevation during periods of no flow, and at about tailwater elevation during periods of flow. These assumptions make the period of no flow the critical period for seepage under the structure.

#### Determine Depth of Cutoff Wall and Toewall

Wt creep ratio = 5.5

Head causing seepage = 1.75 ft to bottom of apron

Required wt creep distance =  $5.5 \cdot 1.75 = 9.63$  ft

Neglecting the horizontal creep distance

$4t = 9.63$  ft or  $t = 2.41$  ft; therefore, use minimum depth of 2'-6".

#### Determine Base Width of Headwall

With flow (produces max.)

$y_0 = 7.0$ ,  $y_1 = 3.6$ ,  $y_2 = 3.4$ ,  $W_1 = 120$ ,  $W_2 = 75.1$ ,  $d_x = 3.4$

The equation below gives the equivalent fluid pressure for a triangular load diagram.

$$w = \frac{6}{y_0^3} \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \left[ 31.2 y_0^2 (h - 0.5) + \frac{W_1 y_1}{2} (y_1 y_2 + \frac{y_1^2}{3} + y_2^2) + \frac{W_2 y_2^3}{6} \right] + 10.4 (y_2^3 - d_x^3) \right\}$$

(This computation can also be done as shown in Example 4.1, page 4.6).

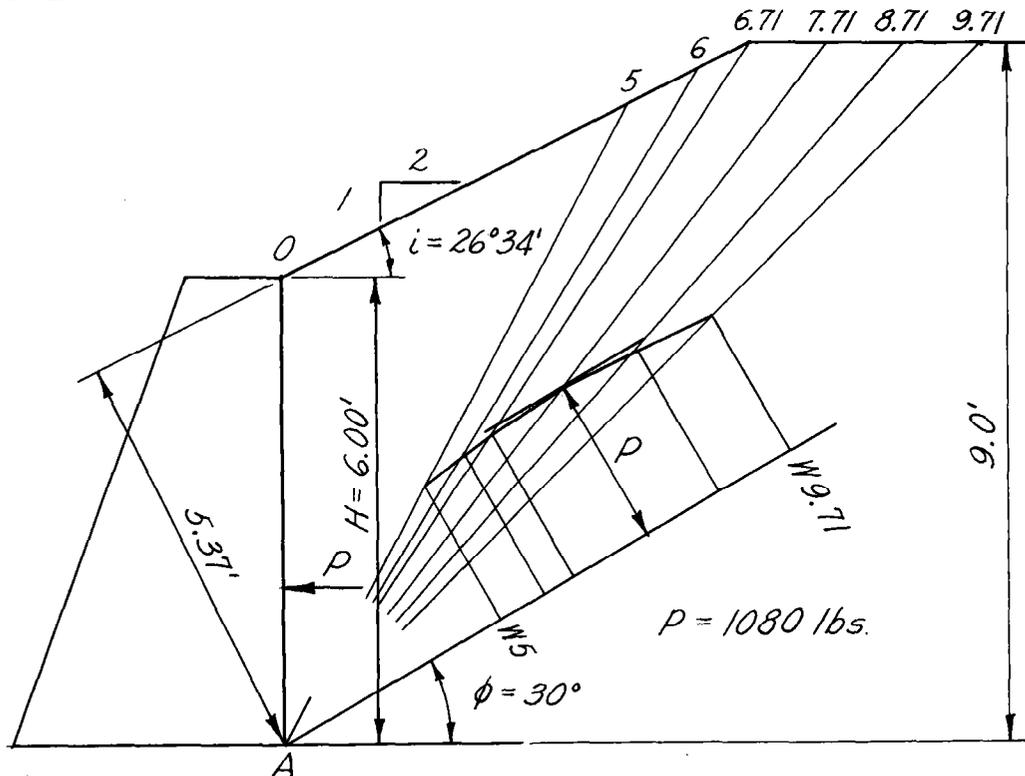
$$w = \frac{6}{343} \left\{ \frac{1}{3} \left[ 3820 + 216 (12.24 + 4.32 + 11.56) + 492 \right] \right\}$$

$$w = \frac{2}{343} (3820 + 6100 + 492) = \frac{2}{343} (10,412) = 60.8 \text{ lbs/ft}^3$$

From Case 1, drawing ES-64 (page 7.17),  $C = 4.2 \text{ ft}$  Use 4'-3"

### Determine Base Width of Sidewall

The construction below is a graphic solution for  $w = 60.0 \text{ lbs/ft}^3$  and  $H = 6.0$



(See subsection 2.2.2 of Structural Design, Section 6)

$$\text{Ave. ht. of fill} = \sqrt[3]{\frac{(6)^3 + (11)^3}{2}} = 9.2 \text{ ft} \quad \text{Use } 9'-0''$$

Slice	Weight of Slice	Accumulated Weight
0-A-5	$5.37 \cdot 2.5 \cdot 120 = 1612$	1612
5-A-6	$5.37 \cdot 0.5 \cdot 120 = 322$	1934
6-A-6.71	$5.37 \cdot 0.355 \cdot 120 = 229$	2163
6.71-A-7.71	$9.00 \cdot 0.50 \cdot 120 = 540$	2703
7.71-A-8.71	$9.00 \cdot 0.50 \cdot 120 = 540$	3243
8.71-A-9.71	$9.00 \cdot 0.50 \cdot 120 = 540$	3783

$$P = 1080 \text{ lbs} = \frac{wH^2}{2} = \frac{w(6)^2}{2} \quad \therefore w = \frac{2 \cdot 1080}{36} = 60.0 \text{ lbs/ft}^3$$

From Case 2, drawing ES-64 (page 7.17),  $C = 3.5$  Use 3'-6"

Determine Base Width of Headwall Extension

$$H = 10.0 \text{ ft}, \quad w = 5 \text{ lbs/ft}^3$$

$$\text{Minimum width} = b + (H \div 10) = 1.25 + 1.0 = 2.25 \text{ ft}$$

This is greater than shown on Case 3, drawing ES-64 (page 7.17), therefore, use  $C = 2'-3"$ .

Determine Base Widths for Wingwall

$H = 6.0 \text{ ft}$  and  $2.0 \text{ ft}$ . Use  $w = 60 \text{ lbs/ft}^3$ , the same as for the sidewall.

When  $H = 6.0$ ,  $C = 3'-6"$ , same as sidewall.

When  $H = 2.0$ ,  $C = 1.2 \text{ ft}$ , from Case 2, drawing ES-64.

To maintain the same batter along the wall, use  $2'-0"$ .

Determine Weight of Structure excluding the wingwalls and the headwall extension beyond the unexposed faces of the sidewalls.

$$\text{Weight of masonry} = 150 \text{ lbs/ft}^3$$

$$\text{Weight of concrete} = 150 \text{ lbs/ft}^3$$

$$\text{Weight of earth} = 120 \text{ lbs/ft}^3$$

	<u>Volume</u>
Hdwl. Ext. $\frac{2.25 + 1.25}{2} \cdot 10 \cdot 2.75 \cdot 2$	= 96.30
Hdwl. (approx.) $\frac{4.25 + 1.55}{2} \cdot 7 \cdot 16.0$	= 325.00
Hdwl. (approx.) $(0.3 \cdot 1.5) + (0.75 \cdot 1.133) \frac{3}{4} \cdot 7 \cdot 2$	= 12.60
Sidewall $\frac{1.25 + 3.5}{2} \cdot 6 \cdot 8.5 \cdot 2$	= 242.00
Sidewall $2 \cdot 1.25 \cdot 6 \cdot 2$	= 30.00
Sidewall $2 \cdot \frac{1.7 \cdot 2.25}{2} \cdot \frac{2}{3} \cdot 6$	= 15.30
Long. sill $\frac{7.5 + 7.9}{2} \cdot 0.75 \cdot 1.0 \cdot 2.0$	= 11.55
Trans. sill $\frac{1.75}{2} \cdot 1.0 \cdot 15.0$	= 13.13
Apron $21.5 \cdot 12.75 \cdot 0.75$	= 205.60
Cutoff wall $21.5 \cdot 2.5 \cdot 0.75$	= 40.30
Cutoff fillet $21.5 \cdot 1/2 \cdot 1/4$	= 2.69

	Volume
Toewall $16.5 \cdot 2.5 \cdot 0.75$	= 30.93
Toewall fillet $16.5 \cdot 1/2 \cdot 1/4$	= 2.06
(Concrete and Masonry)	1027.46 cu ft
Earth on batter = $0.7 \cdot 3.5 \cdot 21.5$	= 52.6 cu ft
Weight = $1027.46 \cdot 150 + 52.6 \cdot 120 = 160,500$ lbs	
Corresponding base area = $21.5 \cdot 12.75 = 274.0$ ft <sup>2</sup>	
Ave. pressure on base = $\frac{160,500}{274} = 585$ lbs/ft <sup>2</sup>	

UpliftWithout flow

Check neglecting cutoffs

$$\text{Uplift per ft width of structure} = \frac{1.75 \cdot 62.4}{2} \cdot 12.75 = 696 \text{ lbs/ft}$$

$$\text{Total uplift} = 696 \cdot 21.5 = 14,980 \text{ lbs}$$

$$\text{Ratio of } \frac{\text{Wt}}{\text{Uplift}} = \frac{160,500}{14,980} = 10.70 \quad \text{OK}$$

With flow

Check neglecting cutoffs

$$\text{Uplift per ft width of structure} = 4.15 \cdot 62.4 \cdot 12.75 = 3300 \text{ lbs/ft}$$

$$\text{Total uplift} = 3300 \cdot 21.5 = 71,000 \text{ lbs}$$

Wt. of water on structure

$$\text{Open apron area} = 7.5 \cdot 14.5 = 108.8 \text{ ft}^2$$

$$\text{Area at H}_2\text{O surface} = 17.0 \cdot 9.5 = 161.6 \text{ ft}^2$$

$$\text{Gross H}_2\text{O volume on apron (approx.)} = \frac{108.8 + 161.6}{2} \cdot 3.4 = 460 \text{ ft}^3$$

$$\text{Volume of sills} = 11.55 \text{ ft}^3$$

$$\text{Net H}_2\text{O on apron} = 460 - 11 = 449 \text{ ft}^3$$

$$\text{H}_2\text{O on headwall} = 1.83 \cdot 16 \cdot 1.55 = 45.4 \text{ ft}^3$$

$$\begin{aligned} \text{H}_2\text{O on headwall batter} &= (0.70 \cdot 1.83 \cdot 16) + \\ &\quad \left( \frac{0.70 + 0.88}{2} \cdot 1.83 \cdot 5.5 \right) = 28.5 \text{ ft}^3 \end{aligned}$$

$$\text{Total volume of H}_2\text{O on structure} = 523 \text{ ft}^3$$

$$\text{Wt. of H}_2\text{O} = 523 \cdot 62.4 = 32,700 \text{ lbs}$$

$$\text{Total weight} = 32,700 + 160,500 = 193,200 \text{ lbs}$$

$$\text{Ratio of } \frac{\text{Wt}}{\text{Uplift}} = \frac{193,200}{71,000} = 2.72 \quad \text{OK}$$

### Check Sliding

Neglect sliding resistance of cutoff wall and toewall.

Neglect the weight and sliding resistance of the wingwalls and the head-wall extensions beyond the unexposed faces of the sidewalls.

$$\phi = 30^\circ$$

$$\text{Wt. of dry earth} = 120 \text{ lbs/ft}^3$$

$$\text{Effective weight of submerged earth} = 75.1 \text{ lbs/ft}^3$$

$$\text{Depth of submerged earth} = 3.40 + 0.75 = 4.15 \text{ ft}$$

$$\text{Depth of dry earth} = 7.75 - 4.15 = 3.6 \text{ ft}$$

### Downstream force

$$p_a = wH \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right) = wH \left( \frac{1 - 0.5}{1 + 0.5} \right) = \frac{wH}{3}$$

$$p_1 = 120 \cdot 3.6 \cdot \frac{1}{3} = + 144 \text{ lbs (dry earth)}$$

$$p_2 = 62.4 \cdot 4.15 = + 259 \text{ lbs (H}_2\text{O)}$$

$$p_3 = 75.1 \cdot 4.15 \cdot \frac{1}{3} = + 104 \text{ lbs (submerged earth)}$$

$$p_4 = 62.4 \cdot 2.5 \cdot \frac{1}{3} = + 52 \text{ lbs (surcharge due to headwater)}$$

$$p_5 = 62.4 \cdot 4.15 = - 259 \text{ lbs (H}_2\text{O)}$$

$$p_6 = 62.4 \cdot 2.5 = + 156 \text{ lbs (headwater on headwall extension)}$$

$p_2 = p_5$ , therefore they cancel

$l$  = width of structure considered = 21.5 ft

$$\text{Downstream force} = \left[ \frac{3.6 p_1}{2} + 4.15 p_1 + \frac{4.15 p_3}{2} + 7.75 p_4 \right] + (l - 16) \left( \frac{2.5 p_6}{2} \right)$$

$$\text{D.F.} = 21.5 \left[ 5.95 (144) + 4.15 (52) + 7.75 (52) \right] + 5.5 (2.5)(78)$$

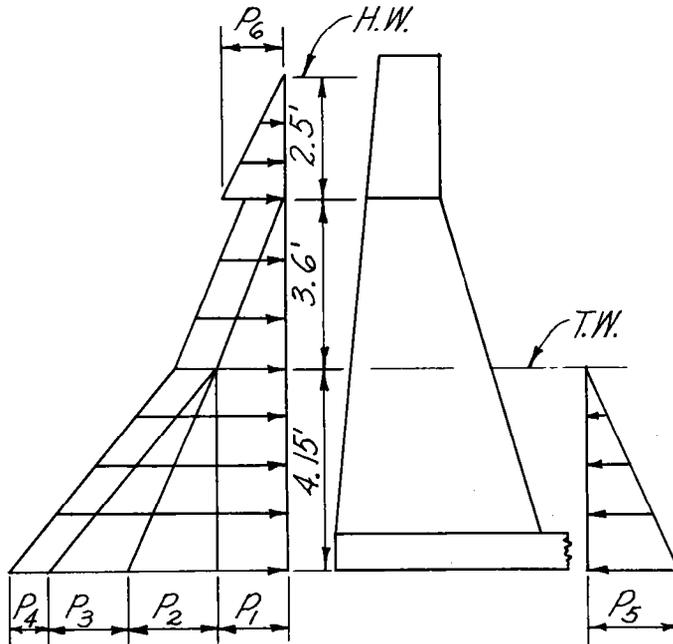
$$\text{D.F.} = 21.5 (1476) + 1073 = 31,725 + 1073 = 32,800 \text{ lbs}$$

$$\text{Net wt.} = (\text{total wt.} - \text{uplift}) = 193,200 - 71,000 = 122,200 \text{ lbs}$$

$$\text{Frictional resistance} = f (\text{net wt.}) = 0.35 \cdot 122,200 = 42,750 \text{ lbs}$$

$$\text{Ratio} = \frac{42,750}{32,800} = 1.30$$

OK



### Apron Design

#### Determine design load

##### Without flow

$$\text{Base area} = 21.5 \cdot 12.75 = 274.0 \text{ ft}^2$$

$$\text{Gross weight} = 160,500 \text{ lbs (see page 7.8)}$$

$$\text{Net load} = \frac{160,500}{274} - (0.75 \cdot 150) = 585 - 113 = 472 \text{ lbs/ft}^2$$

##### With flow

$$\text{Gross weight} = 193,200 \text{ lbs (see page 7.9)}$$

$$\text{Net load} = \frac{193,200}{274} - 113 - (3.4 \cdot 62.4) = 705 - 325 = 380 \text{ lbs/ft}^2$$

Use 472 lbs/ft<sup>2</sup>

#### Longitudinal steel design (Use Class B concrete)

$$l = 8.0 \text{ ft}$$

Top steel  $d = 8 - 2.5 = 5.5$  in

$$M = \frac{1}{8} w l^2 = \frac{1}{8} \cdot 472 \cdot (8.0)^2 = 3776 \text{ ft lbs}$$

$$A_s = 0.46 \text{ in}^2 \quad \underline{\text{Use No. 5 at 8 } (A_s = 0.46)}$$

Length = 10'-3" with vertical leg = 1'-3" bent into upstream face of toewall.

Bottom steel  $d = 9 - 3.5 = 5.5$  in

$$M = \frac{1}{12} w l^2 = \frac{1}{12} \cdot 472 \cdot (8.0)^2 = 2520 \text{ ft lbs}, A_s = 0.30 \text{ in}^2$$

$$V = \frac{1}{2} w l = \frac{1}{2} \cdot 472 \cdot 8.0 = 1890 \text{ lbs}, \quad \Sigma_o = 1.30 \text{ in}$$

$$\underline{\text{Use No. 5 at 12 } (A_s = 0.31, \Sigma_o = 1.96)}$$

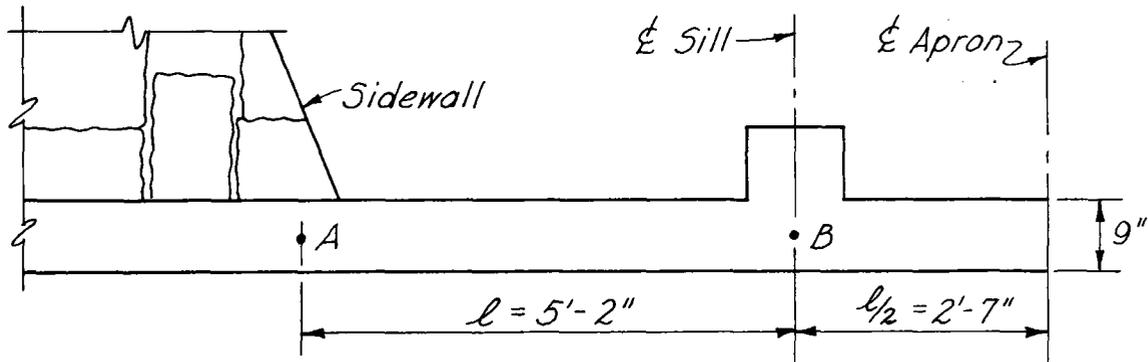
Splice two bars

Length = 12'-0" with vertical leg = 2'-6" bent into downstream face of cutoff wall.

Length = 5'-3" with vertical leg = 1'-3" bent into downstream face of toewall.

Transverse steel design

$$l = 5'-2" = 5.17 \text{ ft}$$

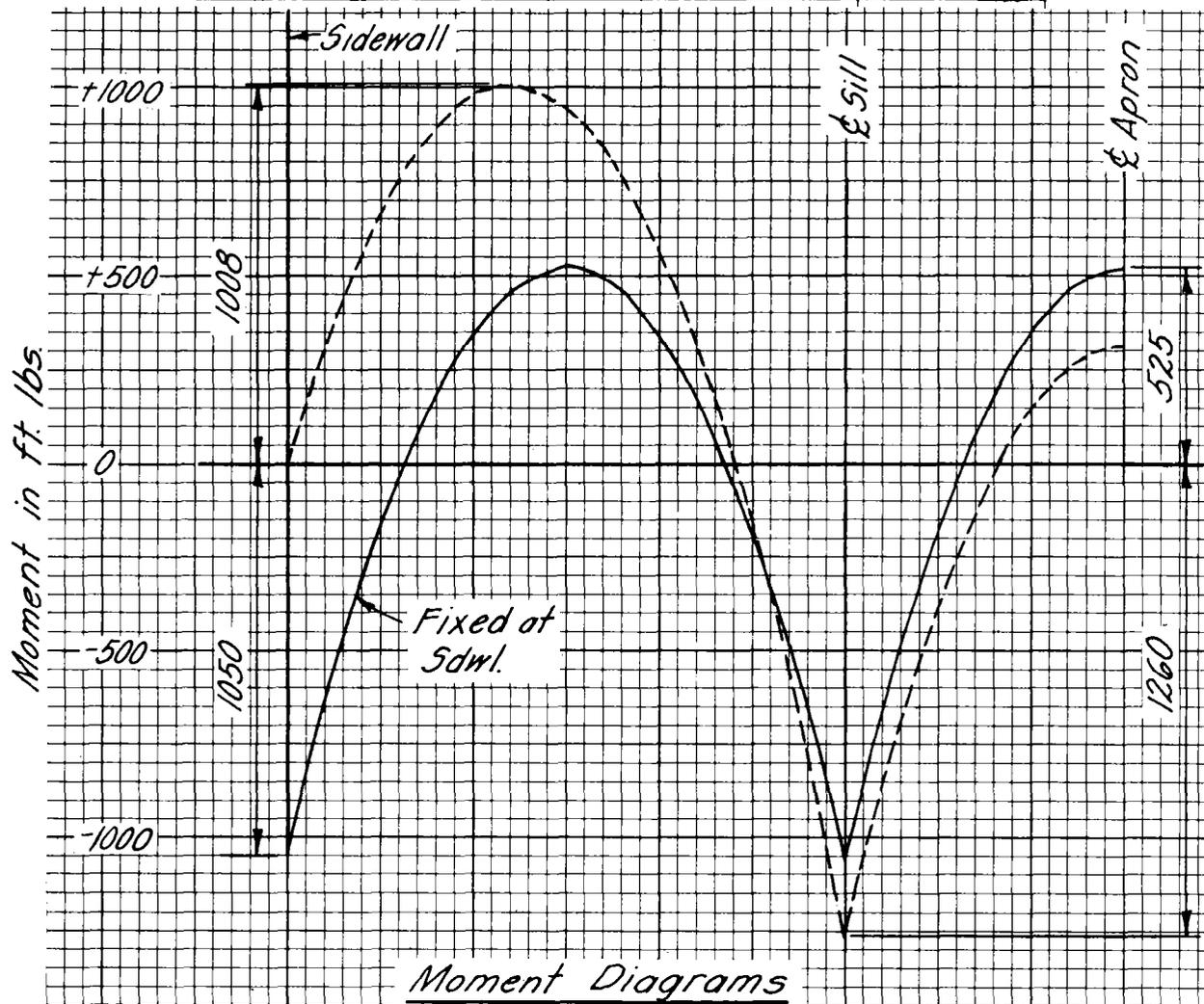


Moments

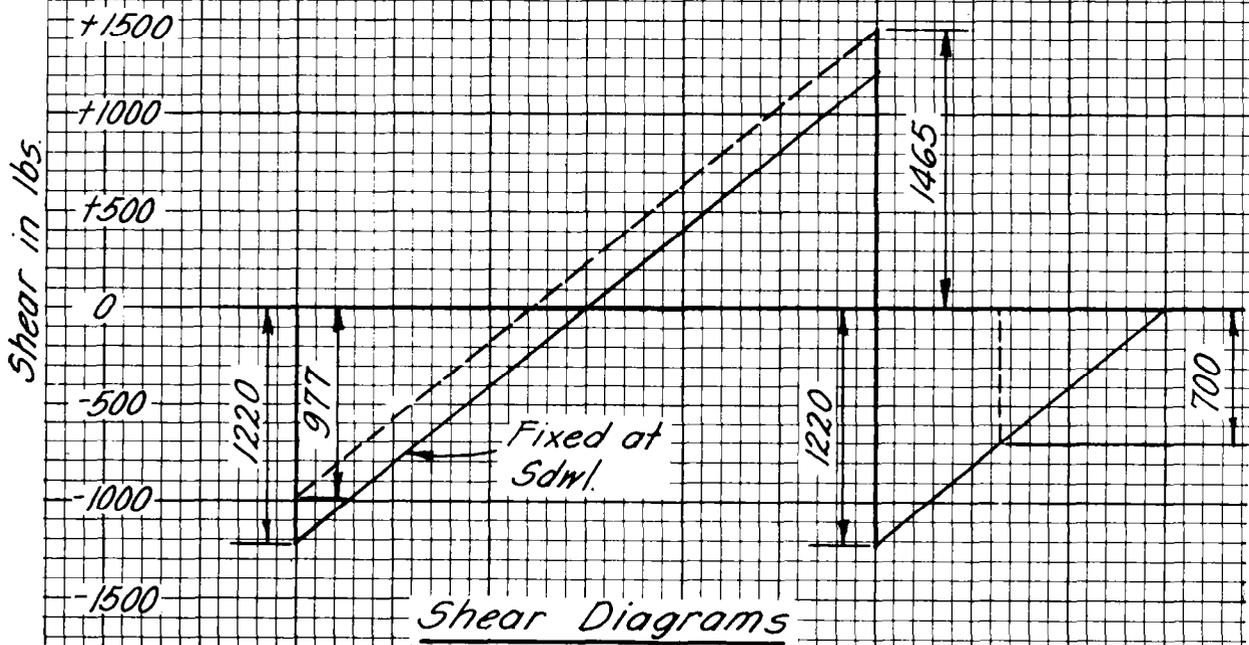
$$M = Cwl^2, \quad wl^2 = 472 (5.17)^2 = 12,600$$

$\frac{x}{l}$	Simply supported at sidewall	Fixed at sidewall
At sdwl.	0	- 1050
0.1	+ 441	- 483
0.2	+ 756	- 42
0.3	+ 945	+ 273
0.4	+ 1008	+ 462
0.5	+ 945	+ 525
0.6	+ 756	+ 462
0.7	+ 441	+ 273
0.8	0	- 42
0.9	- 567	- 483
At sill	- 1260	- 1050
0.1	- 693	- 483
0.2	- 252	- 42
0.3	+ 63	+ 273
0.4	+ 252	+ 462
Centerline apron	+ 315	+ 525
	$V_{AB} = - 0.4 wl$ $V_{AB} = - 977 \text{ lbs}$ $V_{BA} = + 0.6 wl$ $V_{BA} = + 1465 \text{ lbs}$ $R_B = 1.1 wl$ $R_B = 2685 \text{ lbs}$	$V_{AB} = V_{BA}$ $= 0.5 wl$ $= 1220 \text{ lbs}$

# APRON DESIGN (TRANSVERSE DIRECTION)



Moment Diagrams



Shear Diagrams

Bottom steel  $d = 9 - 3.5 - 0.25 - 0.25 = 5.0$  (assuming No. 4 bars)

$$- M = 1050 \text{ ft lbs at sdwl}; \quad A_s = 0.135 \text{ in}^2$$

$$- M = 1260 \text{ ft lbs at sill}; \quad A_s = 0.163 \text{ in}^2$$

$$V = 1220 \text{ lbs at sdwl}; \quad \Sigma_o < 1.00 \text{ in}$$

$$V = 1465 \text{ lbs at sill}; \quad \Sigma_o = 1.12 \text{ in}$$

Use No. 4 at 15 ( $A_s = 0.16$ ,  $\Sigma_o = 1.26$ ) continuous across apron.

Top steel  $d = 8 - 2 \frac{1}{2} - \frac{3}{8} - \frac{1}{4} = 4 \frac{7}{8}$  in (assuming No. 4 bars)

$$+ M = 1008 \text{ ft lbs in end spans}; \quad A_s = 0.133 \text{ in}^2$$

$$+ M = 525 \text{ ft lbs in center span}; \quad A_s < 0.10 \text{ in}^2$$

$$V = 977 \text{ lbs in end spans}; \quad \Sigma_o < 1.00 \text{ in}$$

$$V = 700 \text{ lbs in center span}; \quad \Sigma_o < 1.00 \text{ in}$$

Use No. 4 at 18 ( $A_s = 0.13$ ,  $\Sigma_o = 1.05$ ) continuous across apron.

#### Longitudinal Sill Design

$$w = \text{Max } R_B = 2685 \text{ lbs/ft}, \quad \ell = 8.0 \text{ ft}$$

$$d = 18 - 3.5 = 14.5 \text{ in}$$

#### Bottom steel

$$- M = \frac{1}{12} w \ell^2 = \frac{1}{12} \cdot 2685 (8)^2 = 14,320 \text{ ft lbs}; \quad A_s = 0.645 \text{ in}^2$$

$$V = \frac{1}{2} w \ell = 10,740 \text{ lbs}; \quad \Sigma_o = 2.84 \text{ in}$$

Use two No. 6 bars ( $A_s = 0.88$ ,  $\Sigma_o = 4.71$ )

Length = 11'-3" with vertical leg = 1'-3" bent into downstream face of toewall.

#### Top steel

$$M = \frac{1}{8} w \ell^2 = \frac{1}{8} \cdot 2685 (8)^2 = 21,500 \text{ ft lbs}; \quad A_s = 0.98 \text{ in}^2$$

Use two No. 7 bars ( $A_s = 1.20$ )

Length = 10'-9" with vertical leg 1'-3" bent into upstream face of toewall.

#### Transverse Sill Design

##### Uniform load

$$\text{Load up} = 585 \cdot 0.75 = 439 \text{ lbs/ft}$$

$$\text{Load down} = 150 \cdot 0.75 \cdot 4.25 = 478 \text{ lbs/ft}$$

$$\text{Net load} = 478 - 439 = 39 \text{ lbs/ft}$$

(Due to the fact that this load is small and acts in the opposite direction of the concentrated loads, it will be neglected.)

### Concentrated loads

Loads = 10,740 lbs (V from longitudinal sill design) acting at 1/3 points.

$$\text{Bottom steel } d = (4'-3") - 4" = 3'-11" = 47 \text{ in}$$

End fixed

$$-M = \left[ Z(1-Z)^2 + Z^2(1-Z) \right] P\ell = Z(1-Z)P\ell \text{ (See ES-17, Engineering Handbook, Section 6 on Structural Design.)}$$

$$Z = 1/3, \quad P = 10,740, \quad \ell = 15.5$$

$$-M = 1/3(2/3)(10,740)(15.5) = 37,000 \text{ ft lbs}$$

$$A_s = \frac{M}{ad} = \frac{37.0}{1.44 \cdot 47} = 0.55 \text{ in}^2$$

$$V = P = 10,740 \text{ lbs}$$

$$\Sigma_o = \frac{8V}{7ud} = \frac{8 \cdot 10,740}{7 \cdot 300 \cdot 47} = 0.87$$

Use two No. 5 bars ( $A_s = 0.61$ ,  $\Sigma_o = 3.93$ )

$$\text{Top steel } d = 47 \text{ in}$$

Ends simply supported

$$M = \frac{\ell}{3} P = \frac{15.5}{3} \cdot 10,740 = 55,500 \text{ ft lbs}$$

$$A_s = \frac{M}{ad} = \frac{55.5}{1.44 \cdot 47} = 0.82 \text{ in}^2$$

Use two No. 6 bars ( $A_s = 0.88$ )

### Steel in Toewall

#### Vertical steel

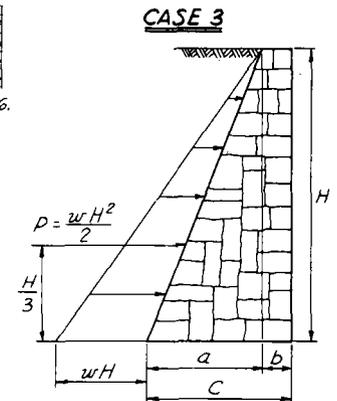
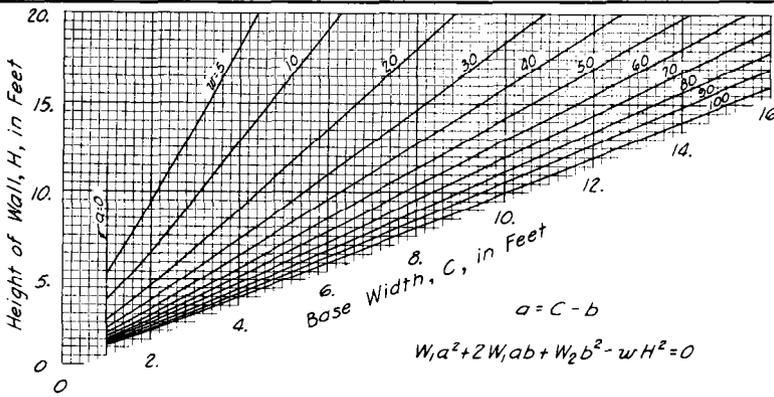
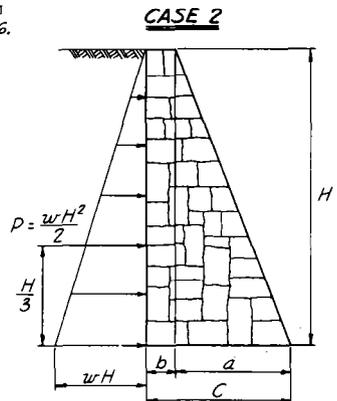
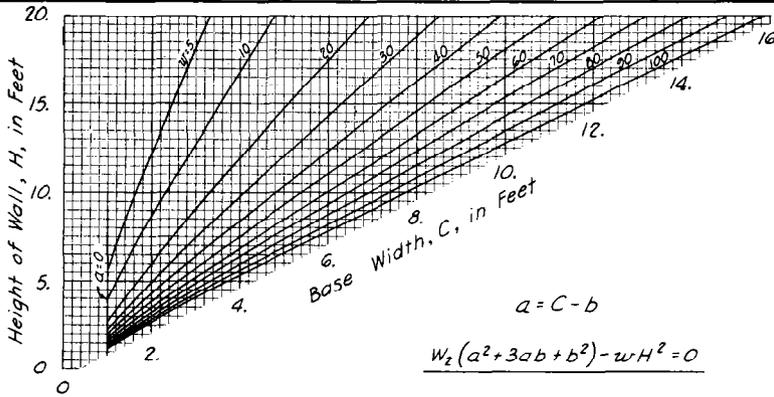
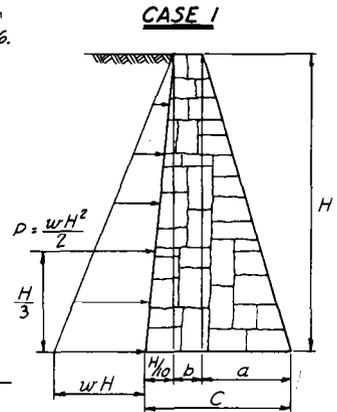
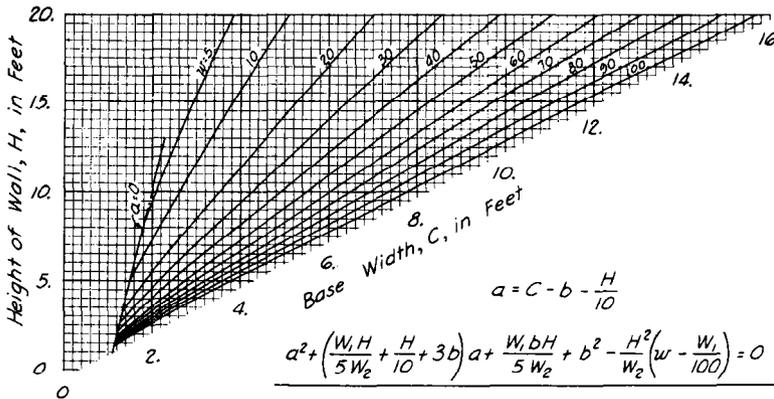
Upstream face - Use No. 5 at 15; length = 3'-9"

Downstream face - Use No. 4 at 12; length = 3'-9"





# DROP SPILLWAYS: REQUIRED BASE WIDTH FOR GRAVITY WALLS WITH VARIOUS LOADS AND LOADINGS



## NOTES

The curves are plotted for  $W_1 = 100 \text{ lbs/ft}^3$ ,  $W_2 = 150 \text{ lbs/ft}^3$ ,  $b = 1.0 \text{ ft}$  and are sufficiently accurate for reasonable variations of these values.

If more accurate results are desired for such variations, use the actual values of  $b$ ,  $W_1$ , and  $W_2$  and the appropriate equations shown above.

## SYMBOLS

$a$  = required batter in ft.  
 $b$  = top width of wall in ft.  
 $C$  = required base width of wall in ft.  
 $H$  = height of wall in ft.  
 $W_1$  = weight of earth in  $\text{lbs/ft}^3$   
 $W_2$  = weight of masonry in  $\text{lbs/ft}^3$   
 $w$  = equivalent fluid pressure in  $\text{lbs/ft}^2$   
 $P$  = horizontal earth pressure per foot length of wall.

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE  
 SOIL CONSERVATION SERVICE  
 Robert M. Salter, Chief  
 ENGINEERING STANDARDS UNIT

STANDARD DWG. NO.

ES-64

SHEET 1 OF 1

DATE 2-2-52