ENGINEERING HANDBOOK

hydraulics

section 5

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

PREFACE

This handbook is intended primarily for the use of Soil Conservation Service engineers. Much of the information will also be useful to engineers in other agencies and in related fields of work.

The aim of the handbook is to present in brief and usable form information on the application of engineering principles to the problems of soil and water conservation. While this information will generally be sufficient for the solution of most of the problems ordinarily encountered, full use should be made of other sources of reference material.

The scope of the handbook is necessarily limited to phases of engineering which pertain directly to the program of the Soil Conservation Service. Therefore, emphasis is given to problems involving the use, conservation, and disposal of water, and the design and use of structures most commonly used for water control. Typical problems encountered in soil and water conservation work are described, basic considerations are set forth, and all of the step-by-step procedures are outlined to enable the engineer to obtain a complete understanding of a recommended solution. These solutions will be helpful in training engineers and will tend to promote nation-wide uniformity in procedures. Since some phases of the field of conservation engineering are relatively new, it is expected that further experience may result in improved methods which will require revision of the handbook from time to time.

The handbook material has been prepared by M. M. Culp, Head of the Engineering Standards Unit of the Engineering Division, and his associates, Woody L. Cowan and Carroll A. Reese, under the general direction of the Engineering Council. The Council is made up of the regional engineers and the Chief of the Engineering Division at Washington. Under its direction the needs of engineers in all parts of the country have been considered and are reflected in the subject matter selected, the method of presentation, and the organization of the different sections.

Many sources of information have been utilized in developing the material. Original contributions and verbatim use of previously published materials are acknowledged in the text.

Engineering Council:

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Hydraulics - General

Introduction: The development of this section is based on the assumption that users will have available, as a working tool, a copy of the "Handbook of Hydraulics", Third Edition, by Horace W. King, McGraw-Hill Book Company. Those engineers whose work includes an appreciable amount of hydraulic computations will find time-saving, tabulated material in "Hydraulic Tables" by the War Department, Corps of Engineers, U. S. Government Printing Office. For brevity these two books are subsequently referred to as "King's Handbook" and "Hydraulic Tables."

A partial list of other widely used publications dealing with the practical phases of hydraulics and hydraulic structures is given below. The need for and the usefulness of these or other handbooks not listed will depend on the amount and type of work encountered in different work unit areas. Inclusion in this list is not a recommendation for the books listed nor a recommendation against any book not listed.

Handbook of Water Control - published by the R. Hardesty Mfg. Co.
Handbook of Culvert and Drainage Practice - published by Armco Culvert Mfg. Assn.
Handbook of Welded Steel Pipe - published by Armco Drainage and Metal Products, Inc., successors to R. Hardesty Mfg. Co.
Concrete Pipe Lines - published by American Concrete Pipe Assn.
Low Dams - by a Subcommittee of the National Resources Committee, U. S. Government Printing Office.
Hydraulic and Excavation Tables - Bureau of Reclamation, Department of Interior, U. S. Government Printing Office.

1. Symbols and Units

1.1 Symbols. The symbols used and their definitions are:

a = cross-sectional area.

b = bottom width of channel.

C = coefficient of discharge for weirs and orifices; constant in Hazen-Williams formula.

D = diameter of circular section.

d = depth of flow normal to channel bottom; diameter of pipe in feet.

d_a = average depth of flow in a channel reach.

d_c = critical depth of flow perpendicular to channel bottom.

d_i = diameter of pipe in inches.

d_m = mean depth of flow at a section.

d_n = depth of normal flow; that is, depth of uniform flow.

F = force.

g = acceleration of gravity.

H = total head.

H_e = specific energy head.

h_f = friction head.

h_p = pressure head.

h_v = velocity head.

I = volume of inflow to a reservoir.

i = rate of inflow to a reservoir.

K and K' = factors used in certain arrangements of Manning's formula and which vary with the ratios of specified linear dimensions of cross sections.

K = head loss coefficient. In most cases this symbol is used with a subscript to make it specific and where so used it is clearly defined.

L = length of channel or closed conduit; length of rectangular weir crest.

ℓ = length of a portion of a channel or closed conduit.

M = mass

n = coefficient of roughness in Manning's formula; an exponent.

O = volume of outflow from a reservoir.

o = rate of outflow from a reservoir.

P = total pressure force; a symbol used in a certain arrangement of Manning's formula, the value of which is $\dfrac{n^2}{2.2082\ ar^{4/3}}$

P_H = horizontal component of pressure force.

P_R = resultant pressure force.

P_V = vertical component of pressure force.

p = intensity of pressure per unit of area; wetted perimeter.

Q = total discharge; that is, volume of flow per unit of time.

Q_c = critical discharge.

Q_n = normal discharge.

q = discharge per unit of width.

q_c = critical discharge per unit of width.

q_n = normal discharge per unit of width.

R = Reynold's number

$R = \dfrac{s_o}{2}\dfrac{d\ell}{dd}$

r = hydraulic radius.

r_m = mean hydraulic radius in channel reach.

S = volume of temporary reservoir storage.

s = slope; that is, the tangent of the angle a line makes with the horizontal; the slope of the energy gradient in Manning's formula.

s_c = critical slope.

s_f = friction slope.

s_o = slope of channel bottom.

T = width of flow at the water surface; a conversion-time interval.

t = time.

V = volume.

v = mean velocity of flow.

v_a = velocity of approach.

v_c = critical velocity.

v_n = normal velocity; that is, velocity of uniform flow.

W = weight.

w = unit weight.

x = a horizontal distance or abscissa; an exponent; a variable; a time-conversion factor.

y = a vertical distance or ordinate; a variable.

$\bar{x}$, $\bar{y}$ = coordinates of the center of gravity of an area.

z = the elevation of a specified point above datum; the slope of the sides of trapezoidal sections expressed as a ratio of horizontal to vertical.

α (Greek alpha) = a kinetic energy correction factor.

β (Greek beta) = an angle defined specifically where used.

θ (Greek theta) = an angle defined specifically where used.

ν (Greek nu) = kinematic viscosity.

1.2 Units of the foot-pound-second system are used unless others are specified. Factors to be used in making conversions between various units and dimensions are available in Tables 1 to 11, "King's Handbook."

In many cases the conversion of units and dimensions is looked upon as a simple, unimportant process. The fact is that conversions are a frequent source of error in engineering computations. Valid equations must be expressed in corresponding units; that is, in a true equation there must be equality between both units and numbers. Engineers can materially reduce the chance of conversion errors by forming the habit of thinking in terms of equality of units as well as equality of numbers.

Problems often arise in which the correct relationship between units and dimensions is not readily visualized and becomes clear only by analysis. In these situations the quick selection of one or a series of conversion factors not expressed in equation form and tested for validity, may result in costly, systematic errors. As examples of the use of sound principles in the conversion process, consider the following:

Example 1:

1 cubic meter = Y gallons

Basically this intends to express an equality between two volumes; that is, two different linear dimensions raised to the third power. Since cubic meters can no more be equated to gallons than freight cars can be equated to bicycles, it is evident that some factor having dimensional as well as numerical value must be introduced if the expression is to be made a valid equation. Analysis shows that:

$$1\ \text{m}^3 \times \frac{35.3145}{1}\ \frac{\text{ft.}^3}{\text{m}^3} \times \frac{1728}{1}\ \frac{\text{in.}^3}{\text{ft.}^3} \times \frac{1}{231}\ \frac{\text{gal.}}{\text{in.}^3} = 264.17\ \text{gal.}$$

Note that all dimensions on the left cancel, leaving the unit, gallon; that is, corresponding units, on each side of the equation. The analysis results in a general equation for conversions between cubic meters and gallons:

$$\text{X m}^3 \times 264.17\ \frac{\text{gal.}}{\text{m}^3} = \text{Y gal.}$$

Example 2:

1 acre-foot per hour = Y gallons per minute

Step by step analysis results in a valid conversion equation consistent in both units and dimensions:

$$1\ \frac{\text{ac.-ft.}}{\text{hr.}} \times \frac{43560}{1}\ \frac{\text{ft.}^2}{\text{ac.}} \times \frac{1}{60}\ \frac{\text{hr.}}{\text{min.}} \times \frac{7.4805}{1}\ \frac{\text{gal.}}{\text{ft.}^3} = 5431\ \frac{\text{gal.}}{\text{min.}}$$

or

$$\text{X}\ \frac{\text{ac.-ft.}}{\text{hr.}} \times 5431\ \frac{\frac{\text{gal.}}{\text{min.}}}{\frac{\text{ac.-ft.}}{\text{hr.}}} = \text{Y}\ \frac{\text{gal.}}{\text{min.}}$$

Example 3:

1 cubic foot per second-day = Y acre feet

Analysis results in:

$$1\ \frac{\text{ft.}^3\text{-day}}{\text{sec.}} \times \frac{1}{43560}\ \frac{\text{ac.}}{\text{ft.}^2} \times \frac{24 \times 3600}{1}\ \frac{\text{sec.}}{\text{day}} = 1.9835\ \text{ac.ft.}$$

or

$$\text{X cfs.-d} \times 1.9835\ \frac{\text{ac.ft.}}{\text{cfs.-d}} = \text{Y ac.ft.}$$

Engineers who will approach conversion problems by the use of the principles illustrated above should secure the following benefits: (1) freedom from conversion errors; (2) savings in time required for both original and "check" computations; and (3) accuracy of conversion factor selection from standard tables or other sources.

2. Hydrostatics

2.1 Unit hydrostatic pressure varies directly with the depth and the unit weight of water and is expressed by the equation:

$$p = wh \qquad (5.2\text{-}1)$$

p = intensity of pressure per unit of area.
w = unit weight of water.
h = depth of submergence, or head.

Useful working equations are:

p, in p.s.i. = 0.433 h, in ft.
p, in $lb./ft.^2$ = 62.4 h, in ft.

In a body of water with free surface, the total unit pressure is the sum of the liquid pressure and the atmospheric pressure. The majority of hydraulic structures are built and operate under conditions such that atmospheric pressures are balancing forces which may be neglected. However, when significant, atmospheric pressure should be fully considered and its effect upon hydraulic operation and structural stability determined. Examples of structures whose operation or stability may be affected by atmospheric pressure are pipe lines having a portion of their length above the hydraulic grade line; weirs with nonadhering nappe which do not have the under side of the nappe free to the atmosphere.

2.2 Pressure loadings. The analysis of structures under pressure loads will, in most cases, be facilitated by the use of pressure diagrams. Since unit pressure varies directly with head, diagrams showing the variation of unit pressure in any plane take the form of triangles, trapezoids, or rectangles. Typical pressure diagrams and aides to working with such diagrams are shown on drawing ES-31.

2.3 Buoyancy. A submerged body is acted on by a vertical, buoyant force equal to the weight of the displaced water.

$$F_B = Vw \qquad (5.2\text{-}2)$$

F_B = buoyant force.

V = volume of the body.

w = unit weight of water.

If the unit weight of the body is greater than that of water, there is an unbalanced, downward force equal to the difference between the weight of the body and of an equal volume of water, and the body will sink. If the body has a unit weight less than that of water, the body will float with part of its volume below and part above the water surface in a position such that:

$$W = Vw \tag{5.2-3}$$

W = weight of the body.
V = volume of the body below the water surface, i.e. the volume of the displaced water.
w = unit weight of water.

Close examination should be made of the stability of hydraulic structures as it will be affected by: (1) whether the structure will be submerged; (2) whether wide variations in buoyant forces and net or effective weights are possible.

Porous materials, when submerged, are subject to different net weights and are acted on by different buoyant forces depending upon whether the voids are filled with air or water. Note the wide variation in the possible net weight of one cubic foot of treated structural timber weighing 55 lbs. under average atmospheric moisture conditions and having 30 percent voids:

1 ft.^{3} of structural timber, 30 percent voids	Before Saturation	After Saturation
W = weight in air, lbs.	55.	55 + (0.30 x 62.4) = 73.72
F_B = buoyant force when submerged, lbs.	62.4	62.4
$W-F_B$ = weight when submerged in water (net weight), lbs.	55 − 62.4 = − 7.4	73.72 − 62.4 = 11.32

The degree to which the factors discussed above are capable of affecting the net or stabilizing weight of a structure is illustrated by the following example:

Assume a timber crib diversion dam subject to complete submergence under normal flood flows. Materials, weights, and volumes are:

Material	Percent of Volume of the Dam	Unit Weights lbs/ft^3
Timber	12	55 in air
Timber		73 saturated
Loose stone, 30 percent voids	88	150 solid stone

Determine the net weight of one cubic yard of the dam when (1) not submerged; (2) submerged but timber not saturated; (3) submerged with timber saturated:

1. Compute cubic feet of timber, solid stone, and voids per cubic yard of dam:

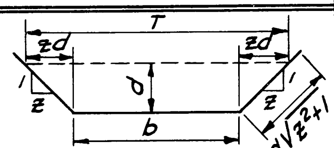
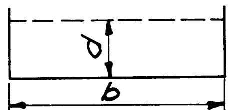
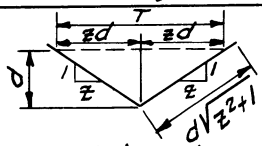
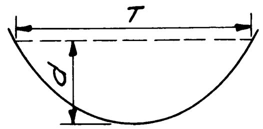
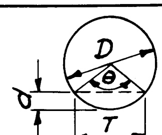
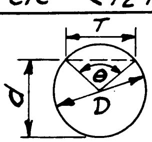
a. Timber: $0.12 \times 27 = 3.24$ ft.^{3}
b. Solid stone: $0.7 \times 0.88 \times 27 = 16.63$ ft.^{3}
c. Voids: $0.3 \times 0.88 \times 27 = \underline{7.13}$ ft.^{3}

27.00 ft.^{3}

2. Compute the net weights of one cubic yard of dam:

Material	Net Weights of Materials in lbs./cu.yd. of Dam		
	Not Submerged	Submerged	
		Timber not Saturated	Timber Saturated
Timber	3.24 x 55 = 178	3.24(55 − 62.4)= − 24	3.24(73 − 62.4)= 34
Stone	16.63 x 150 = 2494	16.63(150 − 62.4)= 1457	1457
Effective or stabilizing weight of dam per cu. yd.	= 2672	1433	1491

HYDRAULICS: ELEMENTS OF CHANNEL SECTIONS

Section	Area a	Wetted Perimeter p	Hydraulic Radius r	Top Width T
 Trapezoid	$bd+zd^2$	$b+2d\sqrt{z^2+1}$	$\dfrac{bd+zd^2}{b+2d\sqrt{z^2+1}}$	$b+2zd$
 Rectangle	bd	$b+2d$	$\dfrac{bd}{b+2d}$	b
 Triangle	zd^2	$2d\sqrt{z^2+1}$	$\dfrac{zd}{2\sqrt{z^2+1}}$	$2zd$
 Parabola	$\frac{2}{3}dT$	$T+\dfrac{8d^2}{3T}$ [1]	$\dfrac{2dT^2}{3T^2+8d^2}$ [1]	$\dfrac{3a}{2d}$
 Circle - < 1/2 full [2]	$\dfrac{D^2}{8}\left(\dfrac{\pi\theta}{180}-\sin\theta\right)$	$\dfrac{\pi D\theta}{360}$	$\dfrac{45D}{\pi\theta}\left(\dfrac{\pi\theta}{180}-\sin\theta\right)$	$D\sin\dfrac{\theta}{2}$ or $2\sqrt{d(D-d)}$
 Circle - > 1/2 full [3]	$\dfrac{D^2}{8}\left(2\pi-\dfrac{\pi\theta}{180}+\sin\theta\right)$	$\dfrac{\pi D(360-\theta)}{360}$	$\dfrac{45D}{\pi(360-\theta)}\left(2\pi-\dfrac{\pi\theta}{180}+\sin\theta\right)$	$D\sin\dfrac{\theta}{2}$ or $2\sqrt{d(D-d)}$

[1] Satisfactory approximation for the interval $0 < \frac{d}{T} \leqq 0.25$
When $d/T > 0.25$, use $p = \frac{1}{2}\sqrt{16d^2+T^2} + \frac{T^2}{8d}\sinh^{-1}\frac{4d}{T}$

[2] $\theta = 4\sin^{-1}\sqrt{d/D}$
[3] $\theta = 4\cos^{-1}\sqrt{d/D}$
} Insert θ in degrees in above equations

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
H. H. Bennett, Chief

ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-33
SHEET 1 OF 1
DATE 6-6-50

3. Fundamentals of Water Flow

3.1 Laminar and Turbulent Flow. Water flows with two distinctly different types of motion; laminar and turbulent.

When laminar flow occurs, the individual particles of water move along straight or orderly path lines. In straight conduits the path lines are straight and parallel; in irregular conduits or in passing obstacles they are orderly lines which do not intersect. With laminar motion, the mean velocity of flow varies directly with the slope of the hydraulic gradient.

In the case of turbulent flow, the water particles follow winding, irregular paths that are generally spiral in form. In addition to the main velocity in the direction of flow, there are transverse components of velocity. The mean velocity of flow varies with the square root of the slope of the hydraulic gradient.

The change from laminar to turbulent flow occurs at a velocity which is determined by the dimensions of the conduit and the viscosity of the water. In engineering practice the decision as to whether laminar or turbulent flow will occur in a given case is based on the Reynold's number value.

$$R = \frac{Lv}{\nu} \qquad (5.3\text{-}1)$$

R = Reynold's number.

L = a linear dimension of the conduit such as diameter of pipe or depth of flow.

v = mean velocity of flow.

ν (Greek nu) = kinematic viscosity.

Reynold's number is dimensionless; that is, it has the same value regardless of the system of consistent units used. The reports of various investigators indicate that in pipe flow Reynold's number values of 2000 or less characterize laminar motion, and 3000 or more turbulent motion with a transition range between these values. There are fewer reports of experiments with open flow, but it appears that Reynold's number values comparable to the above for open flow are about 500 to 1500 respectively. Reynold's number is the ratio of inertia force to viscous force and has broader significance than serving only as a criterion to distinguish between laminar and turbulent flow.

The type of motion with which water flows under different conditions has practical significance. Laminar flow is important to the hydraulic engineer because it is the type of motion with which percolation occurs. Problems dealing with the passage of water through soils, sands, gravels, or porous solids are solved by the application of the mechanics of laminar flow. Turbulent motion characterizes the flow in field hydraulic structures.

3.2 Continuity of Flow. When the discharge at a given cross section of a channel or pipe is constant with respect to time, the flow is steady. If steady flow occurs at all sections in a reach, the flow is continuous and

$$Q = a_1\ v_1 = a_2\ v_2 = a_3\ v_3 \qquad (5.3\text{-}2)$$

Q = discharge.

a = cross-sectional area.

v = mean velocity of flow.

1,2,3 = subscripts denoting different cross sections.

Equation (5.3-2) is known as the equation of continuity. The majority of our hydraulic problems deal with cases of continuous flow.

3.3 Energy and Head. Three forms of energy are normally considered in the analysis of problems in water flow: kinetic energy, potential energy, and pressure energy.

Kinetic energy exists by virtue of the velocity of motion and amounts to $Mv^2/2$, where M is any mass and v is velocity. Since $M = W/g$, the kinetic energy is $Wv^2/2g$, and when W = 1 lb. it has the value $v^2/2g$. Note that $v^2/2g$ being composed of the following units expresses velocity head only:

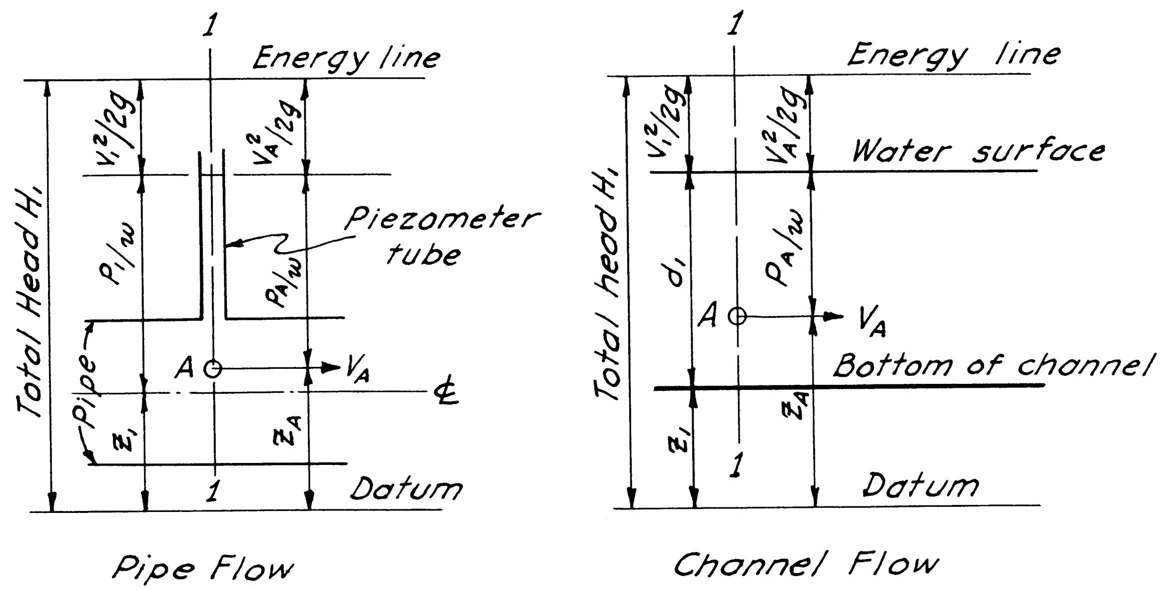
$$\frac{ft^2/sec^2}{ft/sec^2} = \frac{ft^2}{sec^2} \times \frac{sec^2}{ft} = ft.$$

However, it is directly proportional to the kinetic energy of the flowing water and is derived by assuming a weight of 1 lb; therefore, it is an expression of the kinetic energy in foot pounds per pound. If time is considered, the velocity head is also an expression of foot pounds per pound per second.

Potential energy is the ability to do work because of the elevation of a mass of water with respect to some datum. A mass of weight, W, at an elevation z feet, has potential energy amounting to Wz foot pounds with respect to the datum. The elevation head, z, expresses not only a linear quantity in feet, but also energy in foot pounds per pound.

A mass of water as such does not have pressure energy. Pressure energy is acquired by contact with other masses and is, therefore, transmitted to or through the mass under consideration. The pressure head, p/w, like the velocity and elevation heads, also expresses energy in foot pounds per pound.

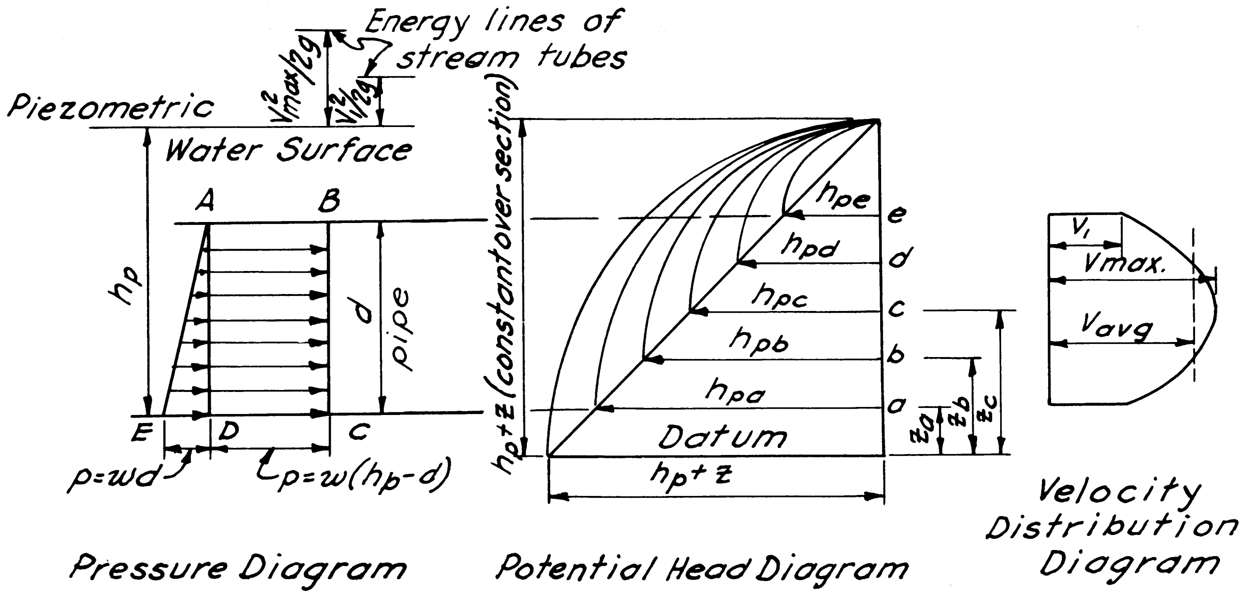
The relationship between the three forms of energy in pipe flow and in channel flow is shown by fig. 5.3-1. On the right in each case is shown the velocity head, pressure head, and elevation head for a stream tube at point A in section 1. On the left is shown the total head and the three separate energy heads for the section containing A. The distance from any stream tube to its energy line is the sum of pressure and velocity heads. If all stream tubes composing flow have equal energy at a given section, variations in the velocity heads of stream tubes must be balanced by equal and opposite changes in the pressure heads. Therefore, if all stream tubes are to have a common energy line at a section, two conditions must be satisfied: (1) pressure intensity must vary as a straight line in accordance with the hydrostatic law; (2) the flow must be parallel and the velocities of all stream tubes must be equal.



Energy and Head Relationships

FIG. 5.3-1

In pipe flow a change in pressure head causes a uniform change in pressure intensity throughout a given cross section. Therefore, changing hydrostatic head on a pipe system does not alter the pattern of motion, and the variation in the energy of the individual stream tubes composing flow at any cross section under a given hydrostatic head results only from the unequal velocities of the stream tubes. This is illustrated by fig. 5.3-2. The pressure diagram on the vertical diameter of a pipe is shown by ABCE. The complete pressure diagram is a truncated cylinder for which each vertical section is similar to ABCE. Variation of the pressure head, h_p, would change only the ABCD portion of the pressure diagram for which the unit pressure is uniform. Furthermore, potential energy, the sum of pressure and elevation heads, with respect to any datum is constant over the cross section, since variation in pressure head is balanced by an equal and opposite variation in elevation head. Variation in the velocities of the different stream tubes accounts for the variation in the energy of flow of the stream tubes at a given section.

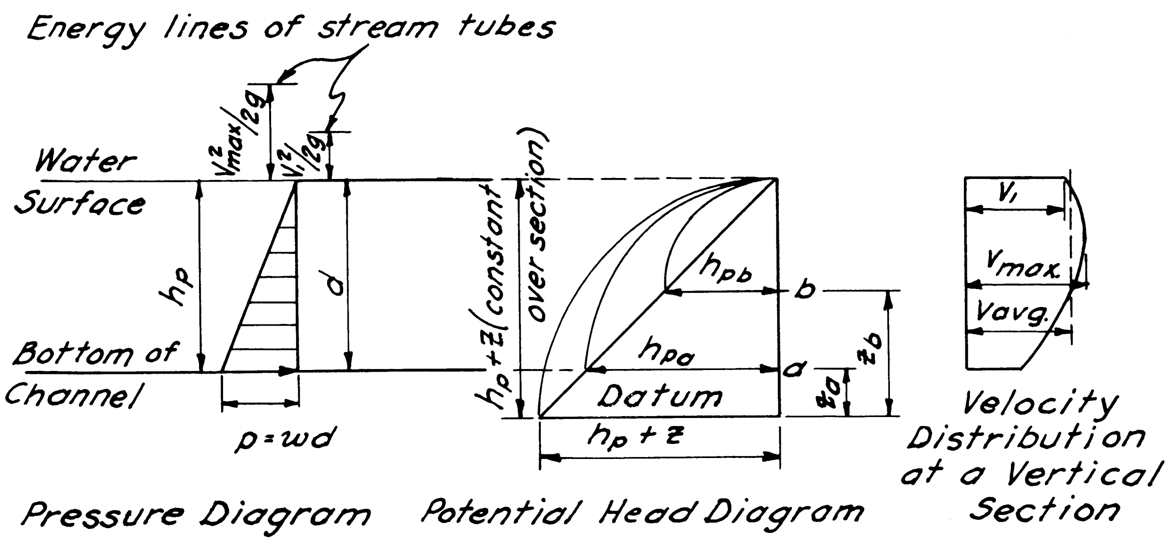


Energy in Pipe Flow at a Cross Section

FIG. 5.3-2

In open flow, pressure at the surface is atmospheric, and internal pressure cannot be changed without altering the pattern of flow. Curvilinear flow changes the internal pressure distribution through dynamic effect and, therefore, changes the flow pattern.

If open flow is parallel, the potential energy head is constant over any cross section and only the velocity head varies from one stream tube to another. This is illustrated by fig. 5.3-3.



Energy in Parallel Open Channel Flow at a Cross Section

FIG. 5.3-3

The above shows that in order to obtain a total head accurately representing the mean energy of flow, it is necessary to compute a weighted mean velocity head for addition to the constant potential head at a cross section. The equation expressing the weighted mean velocity head is:

$$h_v = \alpha \frac{v^2}{2g} \qquad (5.3\text{-}3)$$

h_v = weighted mean velocity head of flow at a cross section.
v = mean velocity of flow.
g = acceleration of gravity.
α (Greek alpha) = a kinetic energy correction factor, the value of which depends upon the distribution of velocity in the cross section of flow.

A method of determining α is given on page 260 of "King's Handbook." The value of α for relatively uniform velocity distribution is 1.05 to 1.10. Wide variations in velocity such as are found in obstructed flow or irregular alignment may produce values of α of 2.0 or greater. Problems may be encountered, therefore, in which a kinetic energy correction must be applied to velocity head if computations within reasonable limits of accuracy are to be made. In the majority of cases $v^2/2g$ is accepted as a sufficiently accurate expression of velocity head.

In pipe flow problems it is common practice to measure elevation head from the datum to the center line of the pipe, the pressure head from the center line to the piezometric surface, and the velocity head from the elevation established by the pressure head. In open channel flow the elevation head is measured from the datum to the bottom of the channel, pressure head is the depth of flow, and velocity head is measured from the water surface.

3.4 Bernoulli Theorem. This theorem is the application of the law of conservation of energy to fluid flow. It may be stated as follows: In frictionless flow the sum of the kinetic energy, pressure energy, and elevation energy is equal at all sections along a stream. In practice, friction and all other energy losses must be considered and the energy equation becomes:

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2 + h_f + h_\ell \qquad (5.3\text{-}4)$$

v = mean velocity of flow.
p = unit pressure.
w = unit weight of water.
g = acceleration of gravity.
z = elevation head.
h_ℓ = all losses in head other than by friction between sections 1 and 2.
h_f = head lost by friction between sections 1 and 2.
1 and 2 denote upstream and downstream sections respectively.

The energy equation and the equation of continuity are the two basic, simultaneous equations used in solving problems in water flow.

3.5 Hydraulic Gradient and Energy Gradient. The hydraulic grade line, or the hydraulic gradient, in open flow is the water surface, and in pipe flow it connects the elevations to which the water would rise in piezometer tubes along the pipe. The energy gradient is at a distance equal to the velocity head above the hydraulic gradient. In both open and pipe flow the fall of the energy gradient for a given length of channel or pipe represents the loss of energy by friction. When considered together, the hydraulic gradient and the energy gradient reflect not only the loss of energy by friction, but also the conversions between potential and kinetic energy.

In the majority of cases the end objective of hydraulic computations relating to flow in open channels is to determine the curve of the water surface. These problems involve three general relationships between the hydraulic gradient and the energy gradient. For uniform flow the hydraulic gradient and the energy gradient are parallel and the hydraulic gradient becomes an adequate basis for the determination of friction loss, since no conversion between kinetic and potential energy is involved. In accelerated flow the hydraulic gradient is steeper than the energy gradient, and in retarded flow the energy gradient is steeper than the hydraulic gradient. An adequate analysis of flow under these conditions cannot be made without consideration of both the energy gradient and the hydraulic gradient.

4. Open Channel Flow

4.1 Steady, Unsteady, Uniform, and Nonuniform Flow. Steady flow exists when the discharge passing a given cross section is constant with respect to time. The maintenance of steady flow in any reach requires that the rates of inflow and outflow be constant and equal. When the discharge varies with time, the flow is unsteady.

Service work will involve problems in unsteady flow in the analysis of discharge from conduits and spillways and in natural and improved channels where discharge varies during periods of runoff.

Steady flow includes two conditions of flow; uniform and nonuniform. Flow is steady and uniform when the mean velocity and the cross-sectional area are equal at all sections in a reach. Flow is steady and nonuniform when either the mean velocity or the cross-sectional area or both vary from section to section.

4.2 Elements of Cross Sections. The elements of cross sections required for hydraulic computations are:

- a, the cross-sectional area of flow;
- p, the wetted perimeter; that is, the length of the perimeter of the cross section in contact with the stream;
- $r = a/p$, the hydraulic radius, which is the cross-sectional area of the stream divided by the wetted perimeter.

General formulas for determining area, wetted perimeter, hydraulic radius, and top width in trapezoidal, rectangular, triangular, circular, and parabolic sections are given by drawing ES-33.

4.3 Manning's Formula. The most widely used open channel formulas express mean velocity of flow as a function of the roughness of the channel, the hydraulic radius, and the slope of the energy gradient. They are empirical equations in which the values of constants and exponents have been derived from experimental data. Manning's formula is one of the most widely accepted and commonly used of the open channel formulas:

$$v = \frac{1.486}{n} r^{2/3} s^{1/2} \qquad (5.4\text{-}1)$$

v = mean velocity of flow in ft. per sec.
r = hydraulic radius in ft.
s = slope of the energy gradient.
n = coefficient of roughness.

Manning's formula gives values of velocity consistent with experimental data and closely comparable to those computed by the Kutter-Chezy formula. For very flat slopes the Kutter-Chezy formula is considered to be preferable by some authorities. The Manning formula has the advantage of simplicity. The alignment chart, drawing ES-34, may be used to solve for v, r, s, and n when any three are known.

4.4 Coefficient of Roughness, n. The Manning formula is expressed so as to use the same n as is used in the Kutter formula. Table 93, p. 287, "King's Handbook", compares values of n which will make the Kutter-Chezy and Manning formulas equivalent. This table and many other comparisons between the two formulas show that Kutter's n need not be modified for use in Manning's formula when the slope is equal to or greater than 0.0001 and the hydraulic radius is between 1.0 and 20 or 30 feet.

The computed discharge for any given channel or pipe will be no more reliable than the value of n used in making the computation. The engineer, when he is selecting the value of n, is in fact estimating the resistance to flow of a given channel or pipe. This estimate affects the design discharge capacity and the cost and, therefore, requires careful consideration.

In the case of pipes and lined channels this estimate is easier to make, but it should be made with care. A given situation will afford specific information on such factors as size and shape of cross section, alignment of the pipe or channel, type and condition of the material forming the wetted perimeter. Knowledge of these factors, associated with the published results of experimental investigations and experience, make possible selections of n values within reasonably well-defined limits of probable error.

Natural channels and excavated channels, subject to various types and degrees of change, present a more difficult problem. The selection of appropriate values for design of drainage, irrigation, and other excavated channels is covered by handbook data relating to those subjects.

The value of n is influenced by several factors; those exerting the greatest influence are:

(1) The physical roughness of the bottom and sides of the channel. The types of natural material forming the bottom and sides and the degree of surface irregularity are the guides to evaluation. Soils made up of fine particles on smooth, uniform surfaces result in relatively low values on n. Coarse materials such as gravel or boulders and pronounced surface irregularity cause the higher values of n.

(2) Vegetation. The value of n should be an expression of the retardance to flow, as it will be affected by height, density, and type of vegetation. Consideration should be given to density and distribution of the vegetation along the reach and the wetted perimeter; the degree to which the vegetation occupies or blocks the cross-sectional area of flow at different depths; the degree to which the vegetation may be bent or "shingled" by flows of different depths.

(3) Variations in size and shape of cross section. Gradual and uniform increase or decrease in cross section size will not significantly affect n, but abrupt changes in size or the alternating of small and large sections call for the use of a somewhat larger n. Uniformity of cross-sectional shape will cause relatively little resistance to flow; whereas variation, particularly if it causes meandering of the major part of the flow from side to side of the channel, will increase n.

(4) Channel alignment. Curvature on relatively large radii and without frequent changes in direction of curvature will offer comparatively low resistance to flow. Severe meandering with the curves having relatively small radii will significantly increase n.

(5) Silting or scouring. Whether either or both of these processes are active and whether they are likely to continue or develop in the future is important. Active silting or scouring, since they result in channel variation of one form or another, will tend to increase n.

(6) Obstructions. Log jams and deposits of any type of debris will increase the value of n; the degree of effect is dependent on the number, type, and size of obstructions.

The value of n, in a natural or constructed channel in earth, varies with the season and from year to year; it is not a fixed value. Each year n increases in the spring and summer, as vegetation grows and foliage develops, and diminishes in the fall as the dormant season develops. The annual growth of vegetation, uneven accumulation of sediment in the channel, lodgment of debris, erosion and sloughing of banks, and other factors all tend to increase the value of n from year to year until the hydraulic efficiency of the channel is improved by clearing or clean-out.

All of these factors should be studied and evaluated with respect to kind of channel, degree of maintenance, seasonal requirements, and other considerations as a basis for making a determination of n. As a general guide to judgment, it can be accepted that conditions tending to induce turbulence will increase retardance; and those tending to reduce turbulence will reduce retardance. Table 5.4-1 lists values of n taken from various sources which will be useful as a guide to the value to be used in a given case.

TABLE 5.4-1. VALUES OF ROUGHNESS COEFFICIENT, n

Type of Conduit and Description	Values of n			References
	Min.	Design	Max.	
Pipe				
Cast-iron, coated	0.010	0.012 - 0.014	0.014	1
Cast-iron, uncoated	0.011	0.013 - 0.015	0.015	1
Wrought iron, galvanized	0.013	0.015 - 0.017	0.017	1
Wrought iron, black	0.012		0.015	1
Steel, riveted and spiral	0.013	0.015 - 0.017	0.017	1
Corrugated	0.021	0.025	0.0255	2
Wood stave	0.010	0.012 - 0.013	0.014	1
Neat cement surface	0.010		0.013	1
Concrete	0.010	0.012 - 0.017	0.017	1,6
Vitrified sewer pipe	0.010	0.013 - 0.015	0.017	1
Clay, common drainage tile	0.011	0.012 - 0.014	0.017	1
Lined Channels				
Metal, smooth semicircular	0.011		0.015	1,5
Metal, corrugated	0.0228	0.024	0.0244	2
Wood, planed	0.010	0.012	0.015	1,5
Wood, unplaned	0.011	0.013	0.015	1,5
Neat cement-lined	0.010		0.013	1,5
Concrete	0.012	0.014 - 0.016	0.018	1,5
Cement rubble	0.017		0.030	1,5
Vegetated, small channels, shallow depths				
Bermuda grass; long - 13", green	0.042			3
Long - 13", dormant	0.035		0.28	3
Short - 3", green	0.034			3
Short - 3", dormant	0.034			3
Sericea Lespedeza; long -16", green	0.076		0.22	3
Long - 16", dormant	0.050			3
Short - 2", green	0.033			3
Short - 2", dormant	0.034			3
Unlined Channels				
Earth; straight and uniform	0.017	0.0225	0.025	1
Dredged	0.025	0.0275	0.033	1
Winding and sluggish	0.0225	0.025	0.030	1
Stony bed, weeds on bank	0.025	0.035	0.040	1
Earth bottom, rubble sides	0.028	0.030 - 0.033	0.035	1

(Continued on next page)

TABLE 5.4-1 (Continued). VALUES OF ROUGHNESS COEFFICIENT, n

Type of Conduit and Description	Values of n			References
	Min.	Design	Max.	
Unlined Channels—Continued				
Rock cuts; smooth and uniform	0.025	0.033	0.035	1
Jagged and irregular	0.035		0.045	1
Natural Streams				
(1) Clean, straight banks, full stage, no rifts or deep pools	0.025		0.033	1,4
(2) Same as (1) but more weeds and stones	0.030		0.040	1,4
(3) Winding, some pools and shoals, clean	0.033		0.045	1,4
(4) Same as (3), lower stages, more ineffective slopes and sections	0.040		0.055	1,4
(5) Same as (3), some weeds and stones	0.035		0.050	1,4
(6) Same as (4), stony sections	0.045		0.060	1,4
(7) Sluggish reaches, rather weedy, very deep pools	0.050		0.080	1,4
(8) Very weedy reaches	0.075		0.150	1,4

REFERENCES:

1. "King's Handbook", pp. 182 and 268.
2. "Hydraulics of Corrugated Metal Pipes" by H. M. Morris, St. Anthony Falls Hydraulic Laboratory, University of Minnesota.
3. "Flow of Water in Channels Protected by Vegetative Linings" by W. O. Ree and V. J. Palmer; and USDA Technical Bulletin No. 967, February 1949.
4. "Low Dams" by National Resources Committee, U. S. Government Printing Office, Washington, D. C., pp. 227-233.
5. "The Flow of Water in Flumes" by Fred C. Scobey; USDA Technical Bulletin No. 393, Dec. 1933.
6. "Hydraulic Studies of Twenty-four Inch Culverts", studies by St. Anthony Falls Hydraulic Laboratory, University of Minnesota; The American Concrete Pipe Association; and the Portland Cement Association.
7. "The Flow of Water in Irrigation Channels" by Fred C. Scobey, USDA Bulletin 194, 1914.
8. "Flow of Water in Drainage Channels" by C. E. Ramser, USDA Technical Bulletin No. 129, 1929.
9. "Some Better Kutter's Formula Coefficients" by R. E. Horton, Engineering News, February 24, May 4, 1916.

4.5 Critical Flow. Critical flow is the term used to describe open channel flow when certain relationships exist between specific energy and discharge and between specific energy and depth of flow. Specific energy is the total energy head at a cross section measured from the bottom of the channel. The conditions described as critical flow are those which exist when the discharge is maximum for a given specific energy head, or stated conversely, those which exist when the specific energy head is minimum for a given discharge.

Consider the specific energy and discharge at a section in any channel, using the notation

Q = total discharge.
q = Q/T = discharge per unit width of channel.
a = cross-sectional area of flow.
d = depth of flow to the bottom of the section.
d_m = a/T = mean depth of flow.
T = top width of the stream.
v = mean velocity of flow.
g = acceleration of gravity.
H_e = specific energy head, i.e., the energy head referred to the bottom of channel.

The specific energy head (see Fig. 5.3-1) is:

$$H_e = d + \frac{v^2}{2g}$$

From equation (5.3-2) $v = Q/a$; therefore,

$$H_e = d + \frac{Q^2}{2ga^2} \qquad (5.4\text{-}2)$$

By solving this equation for the H_e at which Q is a maximum or the depth at which H_e is a minimum, the following general equation for critical flow in any channel may be obtained. (See "King's Handbook", pp. 372-373):

$$\frac{Q^2}{g} = \frac{a^3}{T} \qquad (5.4\text{-}3)$$

From equation (5.4-3) $Q^2/a^2 = ag/T$; and since $Q^2/a^2 = v^2$ and $a = d_mT$, the specific energy when flow is critical is:

$$H_e = d + \frac{a}{2T} = d + \frac{d_m}{2} \qquad (5.4\text{-}4)$$

Study of the specific energy diagram on drawing ES-35 will give a more thorough understanding of the relationships between discharge, energy, and depth when flow is critical. While studying this diagram, consider the following critical flow terms and their definitions:

Critical discharge - The maximum discharge for a given specific energy, or a discharge which occurs with minimum specific energy.

Critical depth - The depth of flow at which the discharge is maximum for a given specific energy, or the depth at which a given discharge occurs with minimum specific energy.

Critical velocity - The mean velocity when the discharge is critical.

Critical slope - That slope which will sustain a given discharge at uniform, critical depth in a given channel.

Subcritical flow - Those conditions of flow for which the depth is greater than critical and the velocity is less than critical.

Supercritical flow - Those conditions of flow for which the depth is less than critical and the velocity is greater than critical.

The curves show the variation of specific energy with depth of flow for several discharges in a channel of unit width. These curves are plotted from the equation, $H_e = d + (q^2 \div 2gd^2)$, by taking constant values of q, assuming d, and computing H_e. Similar curves for any discharge at a section of any form may be obtained from the general equation (5.4-2). Certain points, as illustrated by these curves, should be noted:

(a) There is a different critical depth for every discharge. In this graph all critical depths fall on the line defined by the equation $H_e = 3d_c/2$; in the general case, critical depths will fall on a curve defined by equation (5.4-4).

(b) In a specific energy diagram the pressure head and velocity head are shown graphically. The pressure head, depth in open channel flow, is represented by the horizontal scale as the distance from the vertical axis to the line along which $H_e = d$. The velocity head at any depth is represented by the horizontal distance from the line along which $H_e = d$, to the curve of constant q.

(c) For any discharge there is a minimum specific energy, and the depth of flow corresponding to this minimum specific energy is the critical depth. For any specific energy greater than this minimum there are two depths, sometimes called alternate stages, of equal energy at which the discharge may occur. One of these depths is in the subcritical range and the other is in the supercritical range.

(d) At depths of flow near the critical for any discharge, a minor change in specific energy will cause a much greater change in depth.

(e) Through the major portion of the subcritical range the velocity head for any discharge is relatively small when compared to specific energy, and changes in depth are approximately equal to changes in specific energy.

(f) Through the supercritical range the velocity head for any discharge increases rapidly as depth decreases; and changes in depth are associated with much greater changes in specific energy.

In addition to its importance in the discharge-energy relationship, critical velocity has significance as the velocity with which gravity waves travel in relatively shallow water. If, in the general equation (5.4-3) Q = av and the appropriate values for a channel of unit width are substituted, the critical velocity is found to be $\sqrt{gd_c}$. The velocity of propagation of gravity waves in shallow water is also $\sqrt{gd}$, d being the depth of water. Therefore, a wave may be propagated upstream in subcritical flow but not in supercritical flow.

4.5.1 General Formulas for Critical Flow. General formulas for critical flow in any section are:

$$\frac{Q^2}{g} = \frac{a^3}{T} \qquad (5.4\text{-}3)$$

$$H_e = d_c + \frac{d_m}{2} \qquad (5.4\text{-}4)$$

$$d_m = \frac{v_c^2}{g} \qquad (5.4\text{-}5)$$

$$d_m = \frac{Q_c^2}{a^2 g} \qquad (5.4\text{-}6)$$

$$v_c = \sqrt{gd_m} \qquad (5.4\text{-}7)$$

$$Q_c = a\sqrt{gd_m} \qquad (5.4\text{-}8)$$

Symbols used in these formulas are:

H_e = specific head.

Q_c = critical discharge.

q_c = Q_c/T = critical discharge per unit width of channel.

a = cross-sectional area.

T = width of water surface.

d_c = critical depth.

d_m = a/T = mean depth of critical flow.

v_c = critical velocity.

g = acceleration of gravity.

See drawing ES-33 for the symbols for channel dimensions.

4.5.2 Critical Flow Formulas for Rectangular Channels. See paragraph 4.5.1 for the symbols used in the following formulas:

$$H_e = 3/2\ d_c \tag{5.4-9}$$

$$d_c = 2/3\ H_e \tag{5.4-10}$$

$$d_c = \frac{v_c^2}{g} \tag{5.4-11}$$

$$d_c = \sqrt[3]{\frac{q_c^2}{g}} \tag{5.4-12}$$

$$d_c = \sqrt[3]{\frac{Q_c^2}{b^2 g}} \tag{5.4-13}$$

$$v_c = \sqrt{g d_c} \tag{5.4-14}$$

$$v_c = \sqrt[3]{g q_c} \tag{5.4-15}$$

$$v_c = \sqrt[3]{\frac{g Q_c}{b}} \tag{5.4-16}$$

$$q_c = d_c^{3/2}\ \sqrt{g} \tag{5.4-17}$$

$$Q_c = 5.67\ b d_c^{3/2} \tag{5.4-18}$$

$$Q_c = 3.087\ b H_e^{3/2} \tag{5.4-19}$$

Graphical solutions for Q_c or d_c in equation (5.4-18) may be made by the use of the alignment chart on drawing ES-24.

4.5.3 Critical Flow Formulas for Trapezoidal Channels. See paragraph 4.5.1 for the symbols used in the following formulas:

$$H_e = \frac{(3b + 5zd_c)\ d_c}{(2b + 4zd_c)} \tag{5.4-20}$$

HYDRAULICS: RELATIONSHIP BETWEEN DEPTH OF FLOW AND SPECIFIC ENERGY FOR RECTANGULAR SECTION

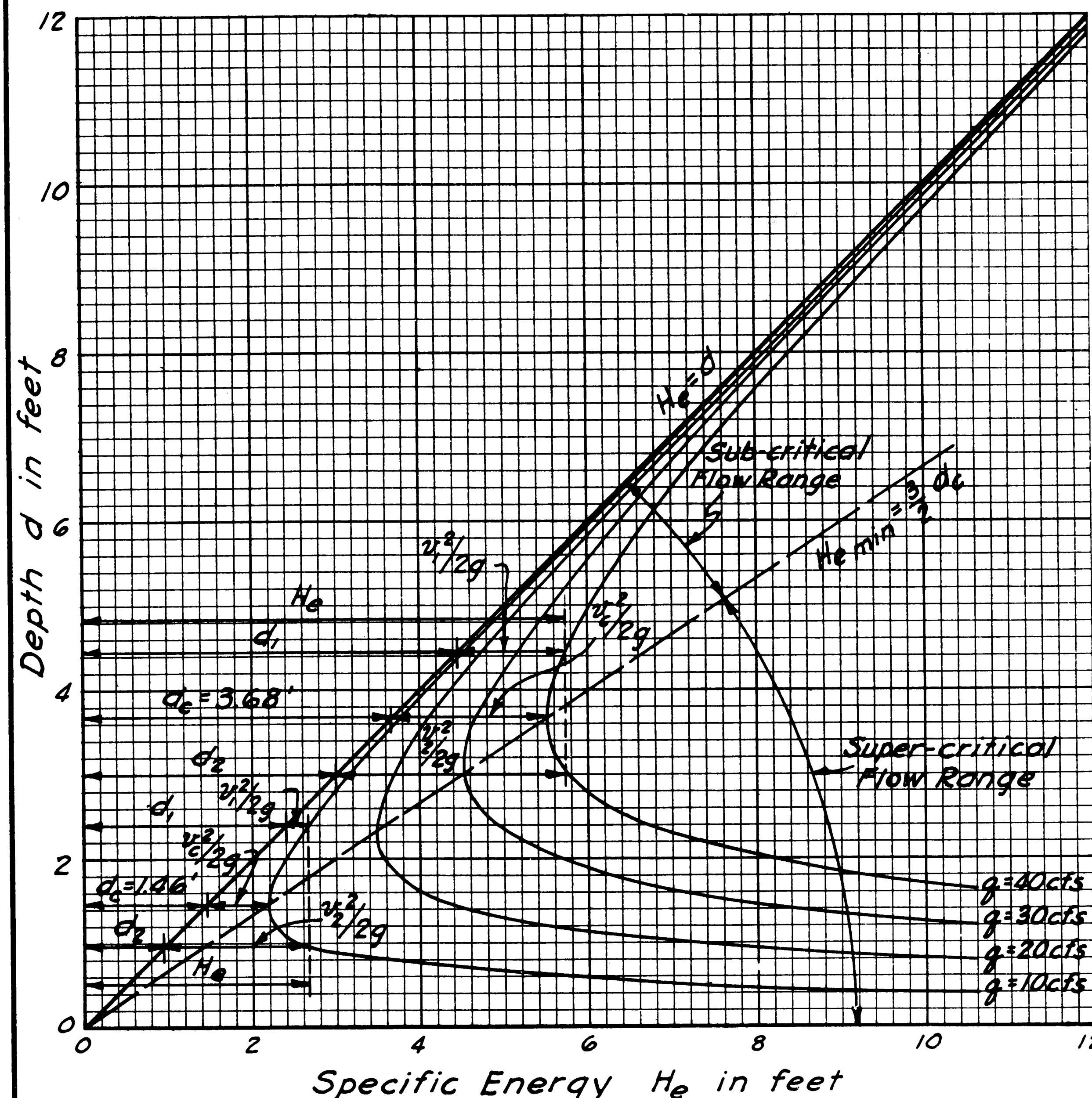
Equations:

$$H_e = d + \frac{v^2}{2g} = d + \frac{q^2}{2gd^2}$$ where q = discharge per unit width.

$$d_c = \left(\frac{q_c}{\sqrt{g}}\right)^{\frac{2}{3}} = \frac{2}{3} H_{e\,min.}$$ where d_c = critical depth

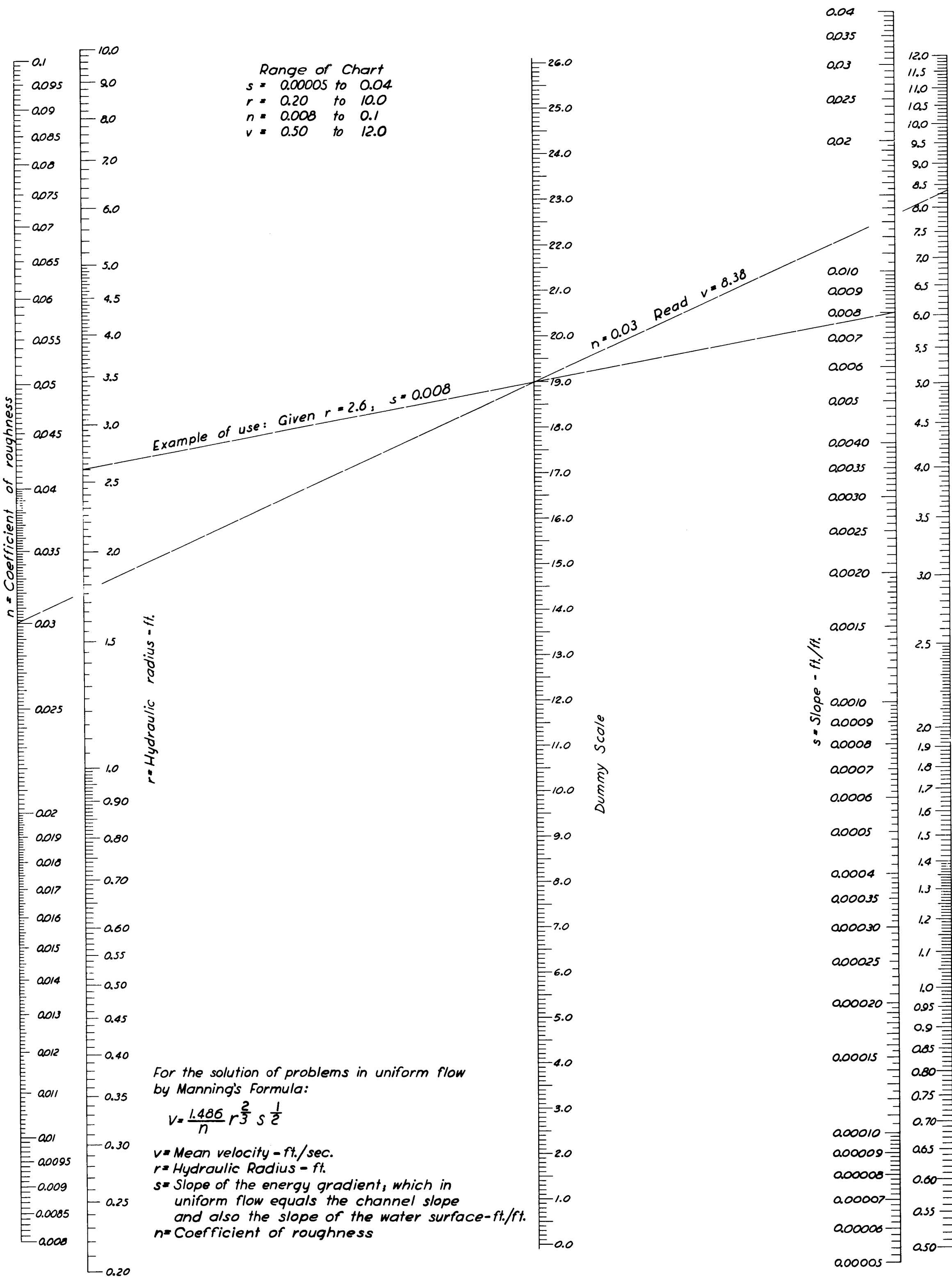
q_c = critical discharge per unit width

$H_{e\,min.}$ = minimum energy content



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 35 SHEET 1 OF 1 DATE 6-28-50

HYDRAULICS: MANNINGS FORMULA



For the solution of problems in uniform flow by Manning's Formula:

$$V = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

v = Mean velocity - ft./sec.
r = Hydraulic Radius - ft.
s = Slope of the energy gradient; which in uniform flow equals the channel slope and also the slope of the water surface - ft./ft.
n = Coefficient of roughness

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REVISED 3-31-51

HYDRAULICS: MANNINGS FORMULA

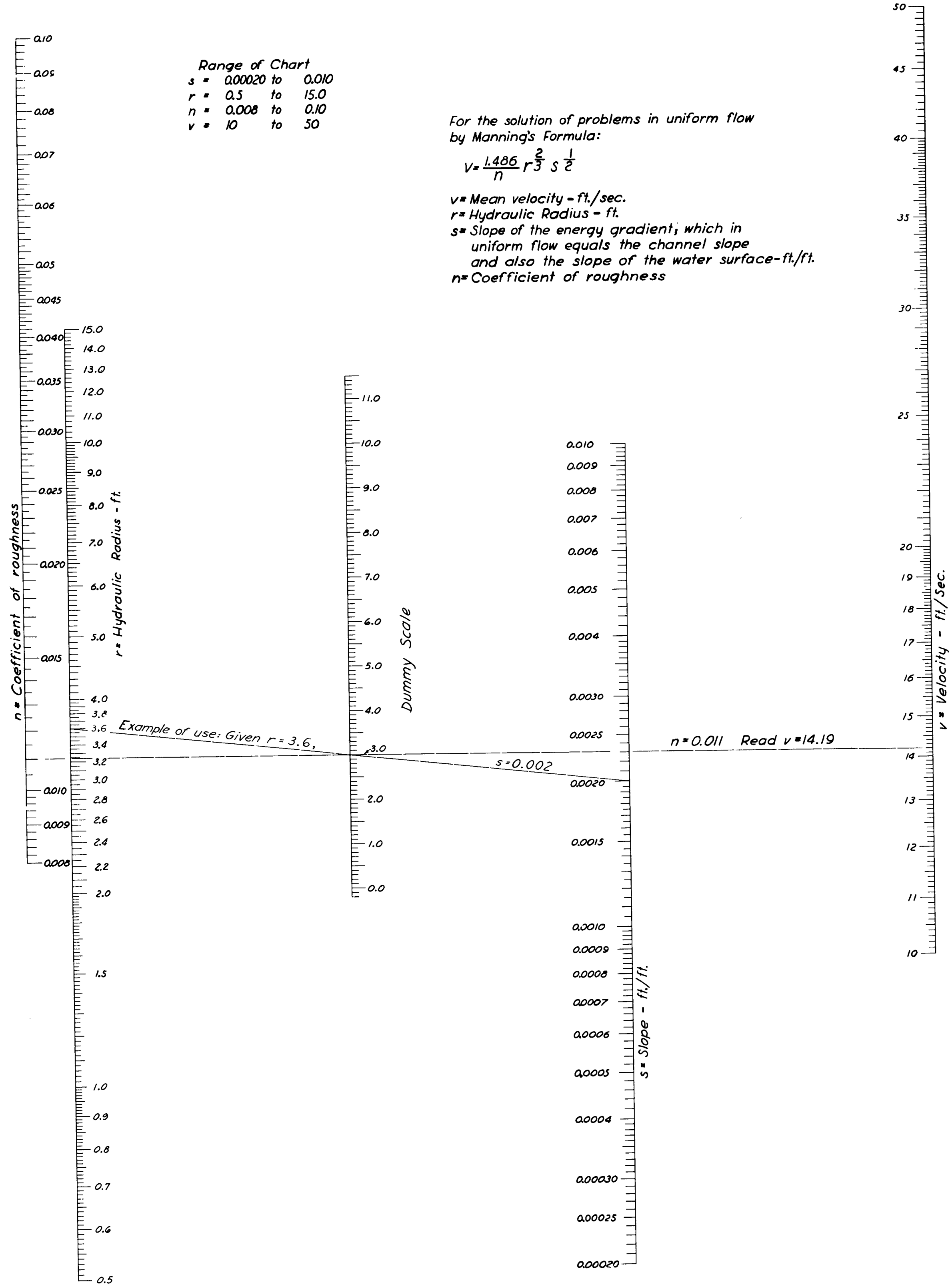
Range of Chart

s =	0.00020	to	0.010
r =	0.5	to	15.0
n =	0.008	to	0.10
v =	10	to	50

For the solution of problems in uniform flow by Manning's Formula:

$$v = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

v = Mean velocity - ft./sec.
r = Hydraulic Radius - ft.
s = Slope of the energy gradient; which in uniform flow equals the channel slope and also the slope of the water surface-ft./ft.
n = Coefficient of roughness

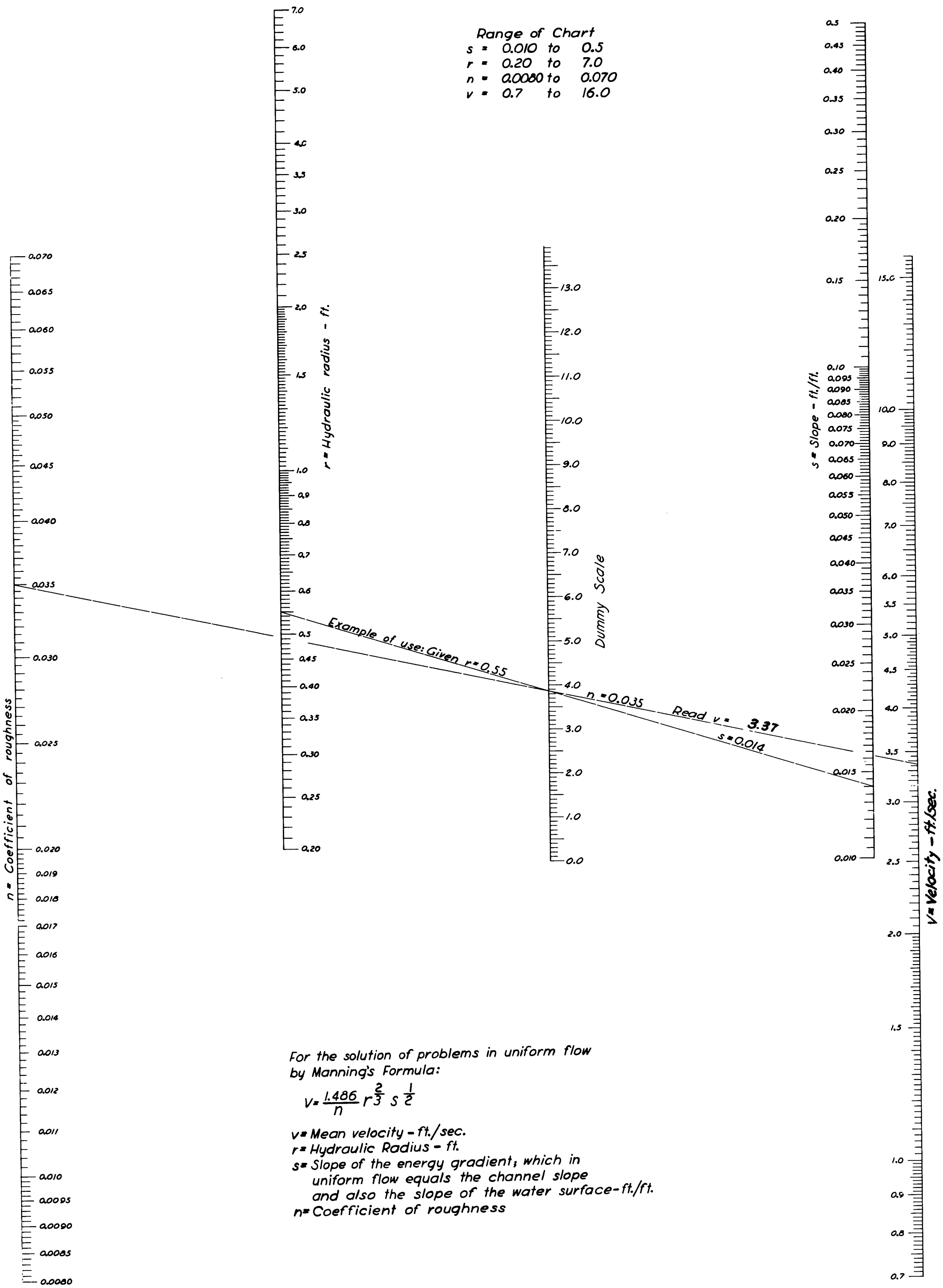


REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 34 SHEET 2 OF 4 DATE 7 - 11 - 50

REVISED 3-31-51

HYDRAULICS: MANNINGS FORMULA

Range of Chart
s = 0.010 to 0.5
r = 0.20 to 7.0
n = 0.0080 to 0.070
v = 0.7 to 16.0



For the solution of problems in uniform flow by Manning's Formula:

$$V = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

v = Mean velocity - ft./sec.
r = Hydraulic Radius - ft.
s = Slope of the energy gradient, which in uniform flow equals the channel slope and also the slope of the water surface - ft./ft.
n = Coefficient of roughness

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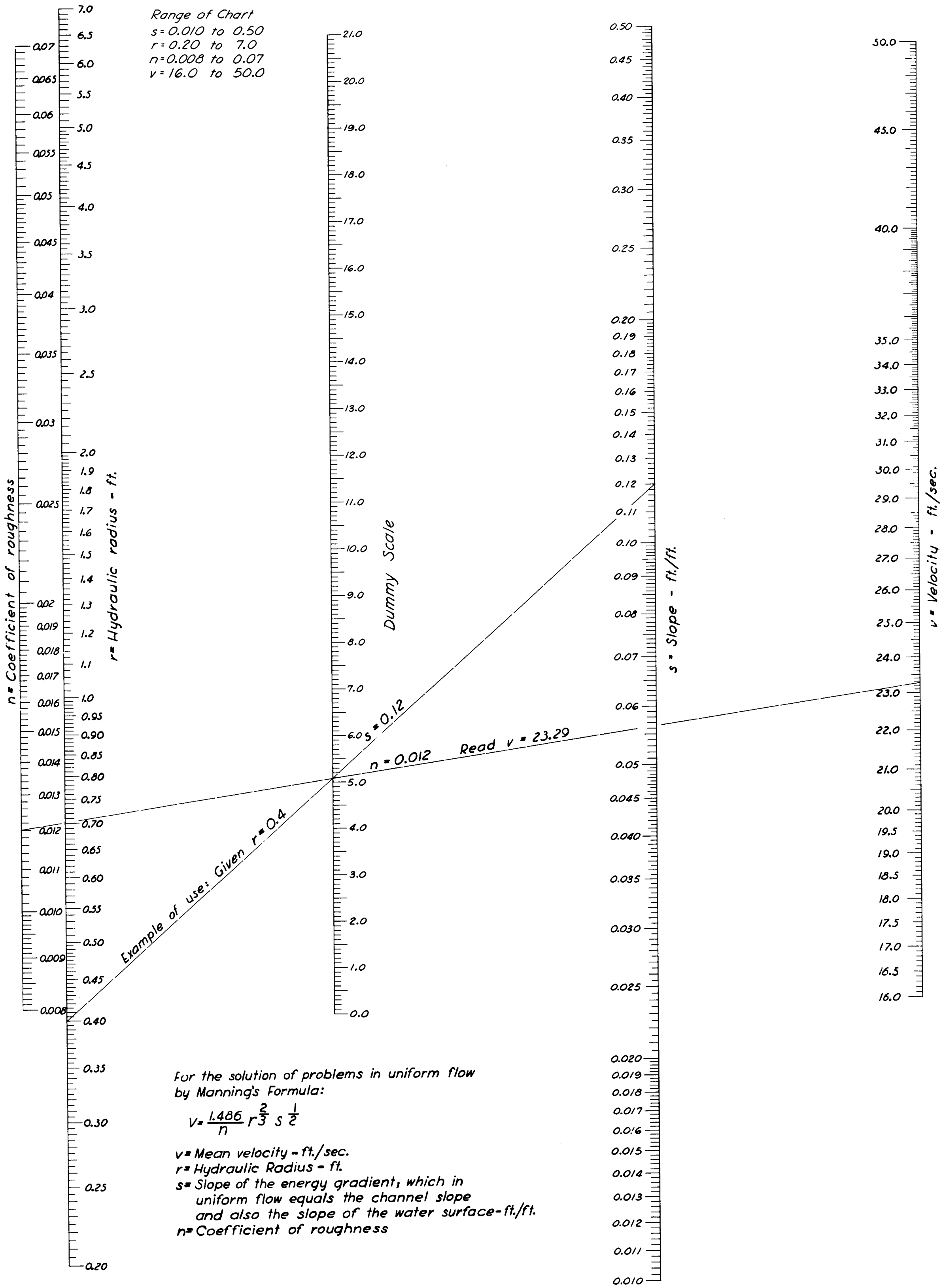
HYDRAULICS: MANNINGS FORMULA

Range of Chart
s = 0.010 to 0.50
r = 0.20 to 7.0
n = 0.008 to 0.07
v = 16.0 to 50.0



For the solution of problems in uniform flow by Manning's Formula:

$$V = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

v = Mean velocity - ft./sec.
r = Hydraulic Radius - ft.
s = Slope of the energy gradient; which in uniform flow equals the channel slope and also the slope of the water surface - ft./ft.
n = Coefficient of roughness

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HYDRAULICS: PRESSURE DIAGRAMS AND METHODS OF COMPUTING HYDROSTATIC LOADS

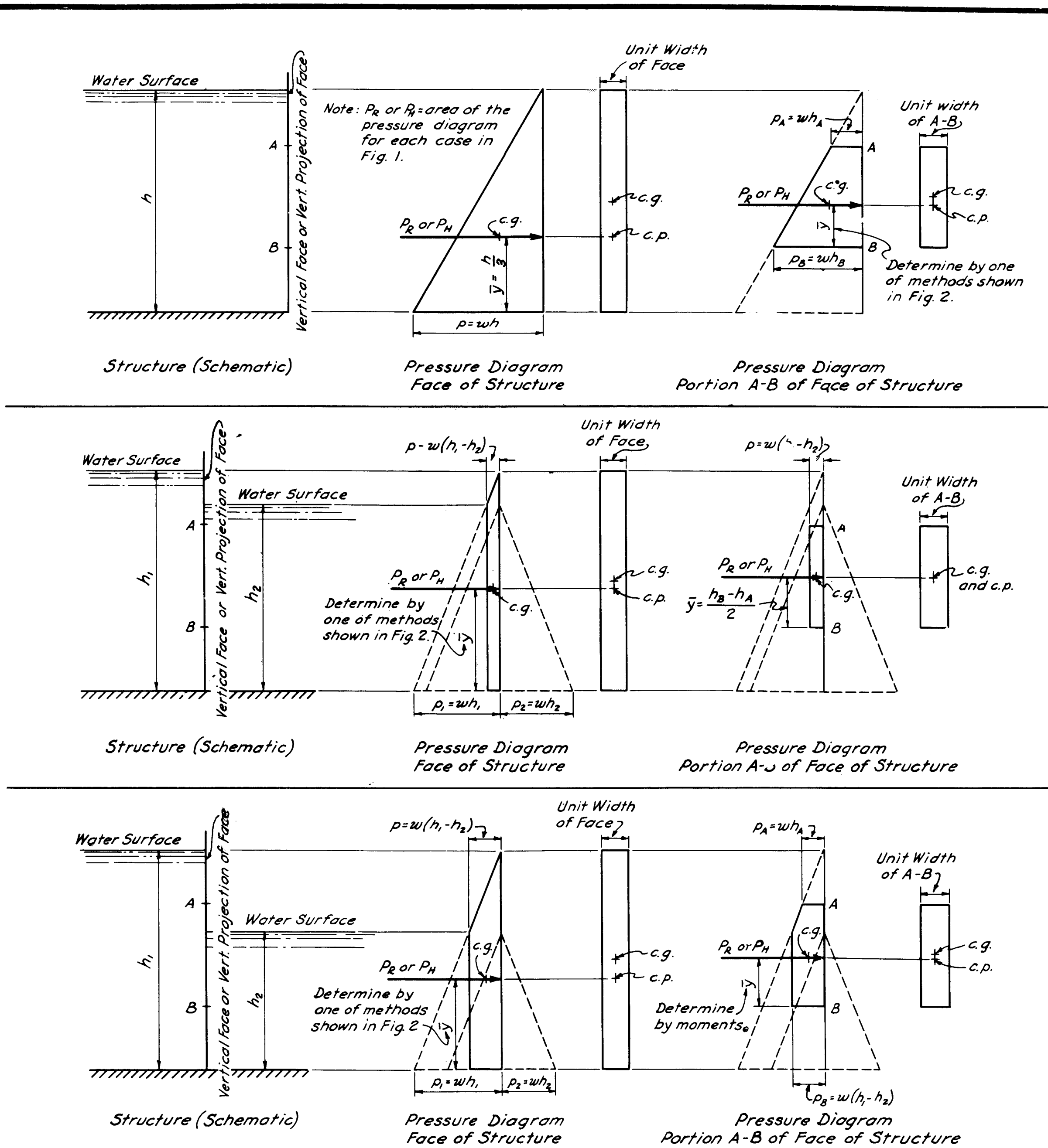

Figure 1

Determination of Center of Gravity of Trapezoid

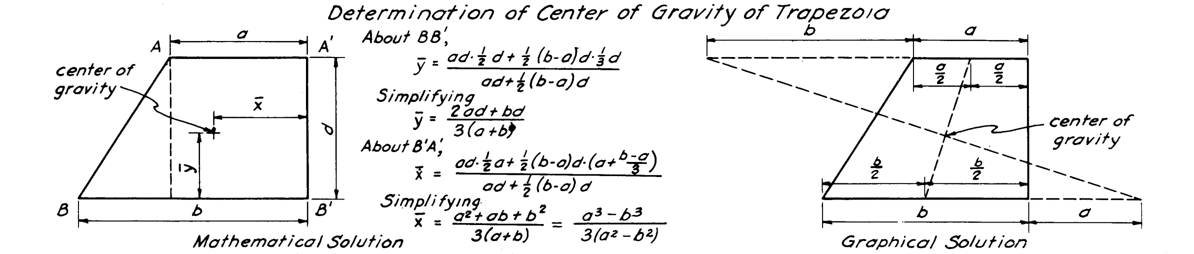

Figure 2

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 31 SHEET 1 OF 2 DATE 6-8-50

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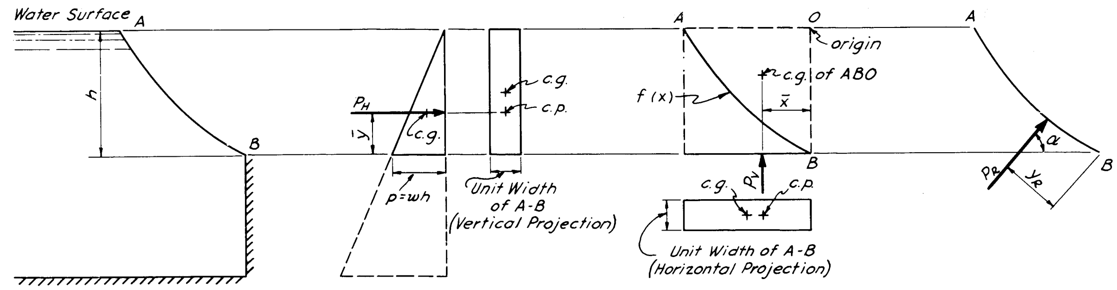
HYDRAULICS: PRESSURE DIAGRAMS AND METHODS OF COMPUTING HYDROSTATIC LOADS



Note: The following solution is general and is valid for, (1), any shape of portion A-B and, (2), water on either side of A-B. However, when water is on the right of A-B, the pressure forces P_H, P_V, and P_R act in directions opposite to those shown.

Structure (Schematic)

$$P_H = \frac{1}{2} wh^2$$

$$\bar{y} = \frac{h}{3}$$

Pressure Diagram of Horizontal Pressure Portion A-B

$$P_V = w A_{ABO}$$

$$A_{ABO} = \text{Area bounded by ABO} = \int_0^A f(x)\,dx$$

$$\bar{x} = \frac{\int_0^A x f(x)\,dx}{A_{ABO}}$$

Vertical Pressure Force Portion A-B

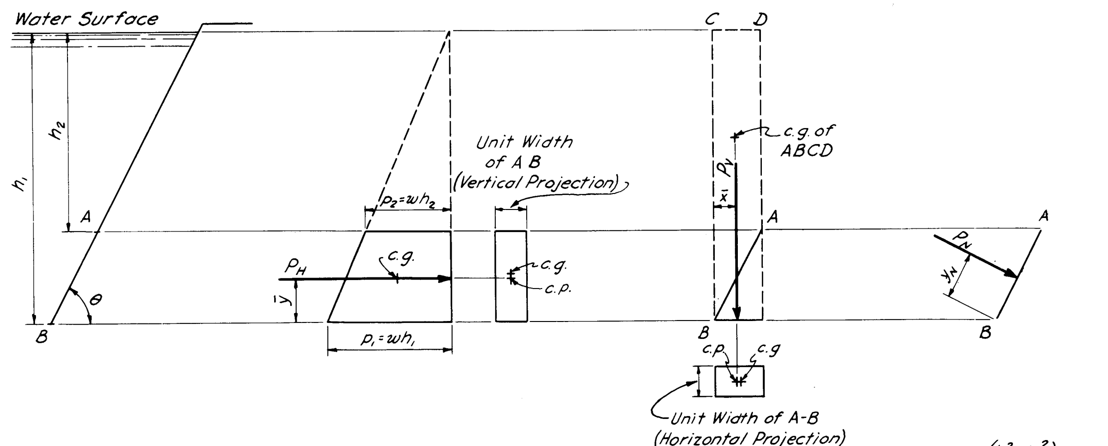
$$P_R = \sqrt{P_H^2 + P_V^2}$$

$$\sin\alpha = \frac{P_V}{P_R}$$

$$y_R = \frac{P_H \cdot \frac{h}{3} + P_V \cdot \bar{x}}{P_R}$$

Resultant Pressure Force Portion A-B

Figure 3



Structure (Schematic)

$$P_H = \frac{w}{2}(h_1^2 - h_2^2)$$

$$\bar{y} = \frac{(h_1 - h_2)(h_1 + 2h_2)}{3(h_1 + h_2)}$$

Pressure Diagram of Horizontal Pressure Portion A-B

$$P_V = \frac{w(h_1^2 - h_2^2)}{2\tan\theta}$$

$$\bar{x} = \frac{(h_1 - h_2)(h_1 + 2h_2)}{3(h_1 + h_2)\tan\theta}$$

Vertical Pressure Force Portion A-B

$$P_N = \frac{w(h_1^2 - h_2^2)}{2\sin\theta}$$

$$y_N = \frac{(h_1 - h_2)(h_1 + 2h_2)}{3(h_1 + h_2)\sin\theta}$$

Normal Pressure Force Portion A-B

Figure 4

Symbols and Definitions

- c.g. — center of gravity of area, as indicated.
- c.p. — center of pressure; i.e., point of action of a pressure force, or a component of a pressure force, on the face, or a projection of the face, of a structure.
- h — height of water above a point, as indicated in "Structure (Schematic)", or as indicated by subscript.
- p. — intensity of pressure at a point indicated by subscript, or at bottom of structure or portion of structure if no subscript is used.
- P_H — horizontal component of pressure force per foot width.
- P_V — vertical component of pressure force per foot width.
- P_N — normal pressure force per foot width.
- P_R — resultant pressure force per foot width.
- w — weight of water per cubic foot.
- $\bar{x}, \bar{y}$ — coordinates of c.g. of pressure diagram.
- y_N — distance from given point perpendicular to line of action of P_N.
- y_R — distance from given point perpendicular to line of action of P_R.

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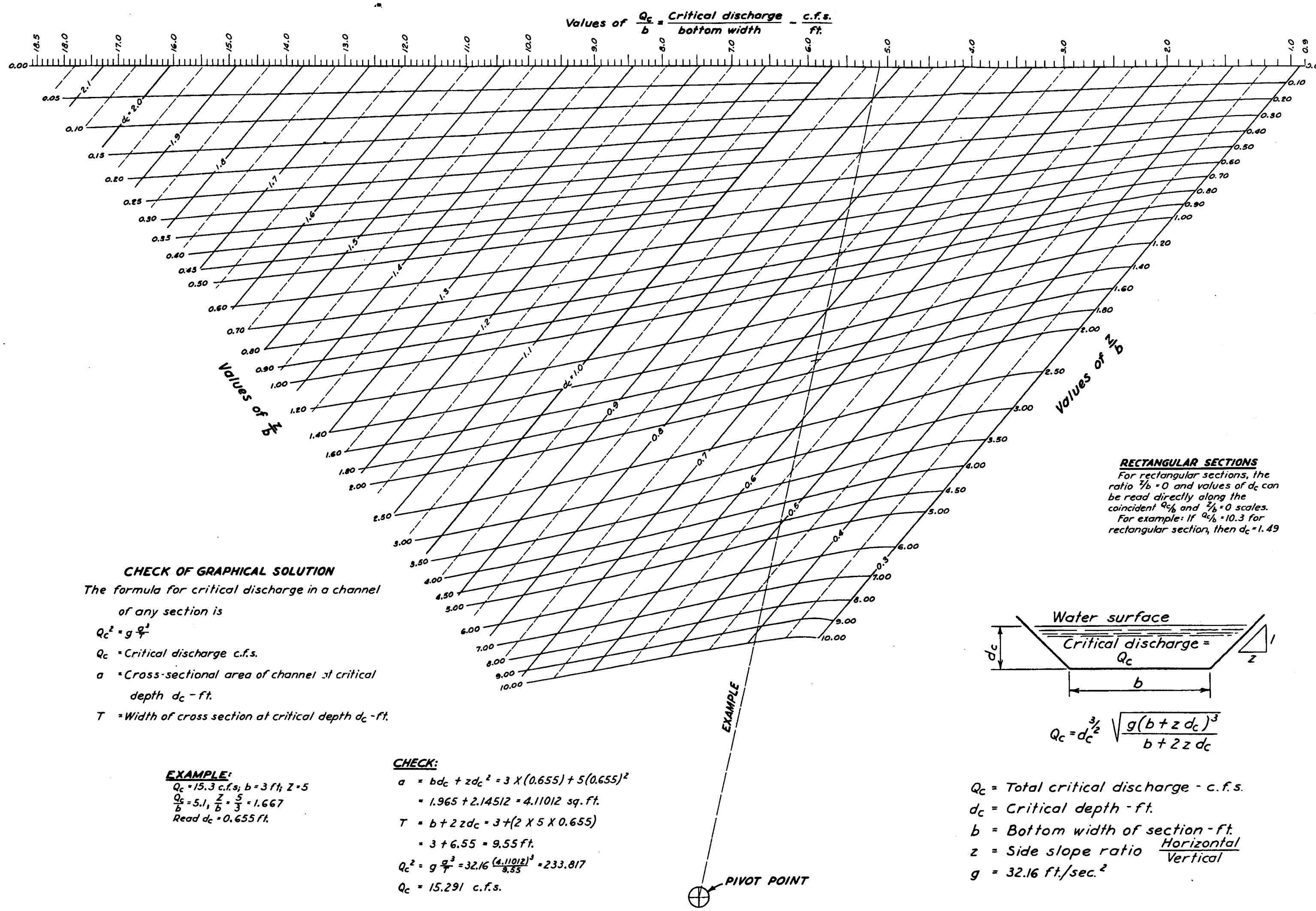
Revised 4-11-51

HYDRAULICS: CRITICAL DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR SECTIONS



RECTANGULAR SECTIONS

For rectangular sections, the ratio $z/b = 0$ and values of d_c can be read directly along the coincident Q_c/b and $z/b = 0$ scales.
For example: If $Q_c/b = 10.3$ for rectangular section, then $d_c = 1.49$

CHECK OF GRAPHICAL SOLUTION

The formula for critical discharge in a channel of any section is

$Q_c^2 = g \frac{a^3}{T}$

Q_c = Critical discharge c.f.s.

a = Cross-sectional area of channel at critical depth d_c - ft.

T = Width of cross section at critical depth d_c - ft.

EXAMPLE:

$Q_c = 15.3$ c.f.s.; $b = 3$ ft; $z = 5$

$\frac{Q_c}{b} = 5.1$, $\frac{z}{b} = \frac{5}{3} = 1.667$

Read $d_c = 0.655$ ft.

CHECK:

$a = bd_c + zd_c^2 = 3 \times (0.655) + 5(0.655)^2$

$= 1.965 + 2.14512 = 4.11012$ sq. ft.

$T = b + 2zd_c = 3 + (2 \times 5 \times 0.655)$

$= 3 + 6.55 = 9.55$ ft.

$Q_c^2 = g\frac{a^3}{T} = 32.16 \frac{(4.11012)^3}{9.55} = 233.817$

$Q_c = 15.291$ c.f.s.

$$Q_c = d_c^{3/2}\sqrt{\frac{g(b + z\,d_c)^3}{b + 2z\,d_c}}$$

Q_c = Total critical discharge - c.f.s.

d_c = Critical depth - ft.

b = Bottom width of section - ft.

z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$

g = 32.16 ft./sec.^{2}

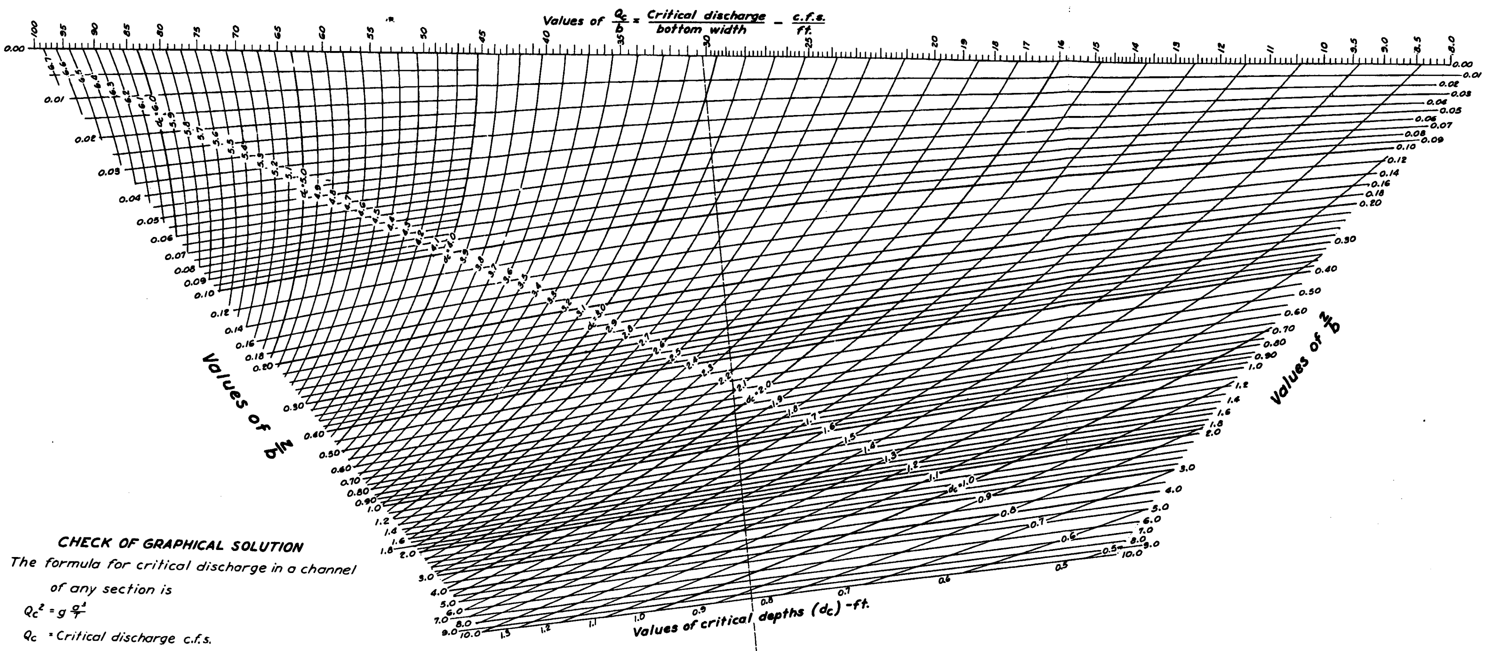
REFERENCE *This nomogram was developed by Paul D. Doubt of the Engineering Standards Unit.*

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
H. H. Bennett, Chief
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES - 24
SHEET 1 OF 3
DATE 5-2-50

REVISED 3-30-51

HYDRAULICS: CRITICAL DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR SECTIONS



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EXAMPLE:

$b = 10$ ft; $z = 4$, $Q_c = 302$ cfs

$\frac{Q_c}{b} = 30.2$; $\frac{z}{b} = 0.4$

Read $d_c = 2.26$ ft.

CHECK:

$a = bd_c + zd_c^2 = 10 \times 2.26 + 4(2.26)^2$

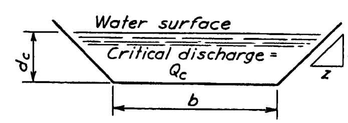
$= 22.6 + 20.43 = 43.03$ sq. ft.

$T = b + 2zd_c = 10 + (2 \times 4 \times 2.26)$

$= 10 + 18.08 = 28.08$ ft.

$Q_c^2 = g\frac{a^3}{T} = 32.16\frac{(43.03)^3}{28.08} = 91,252.5$

$Q_c = \sqrt{91,252.5} = 302.08$



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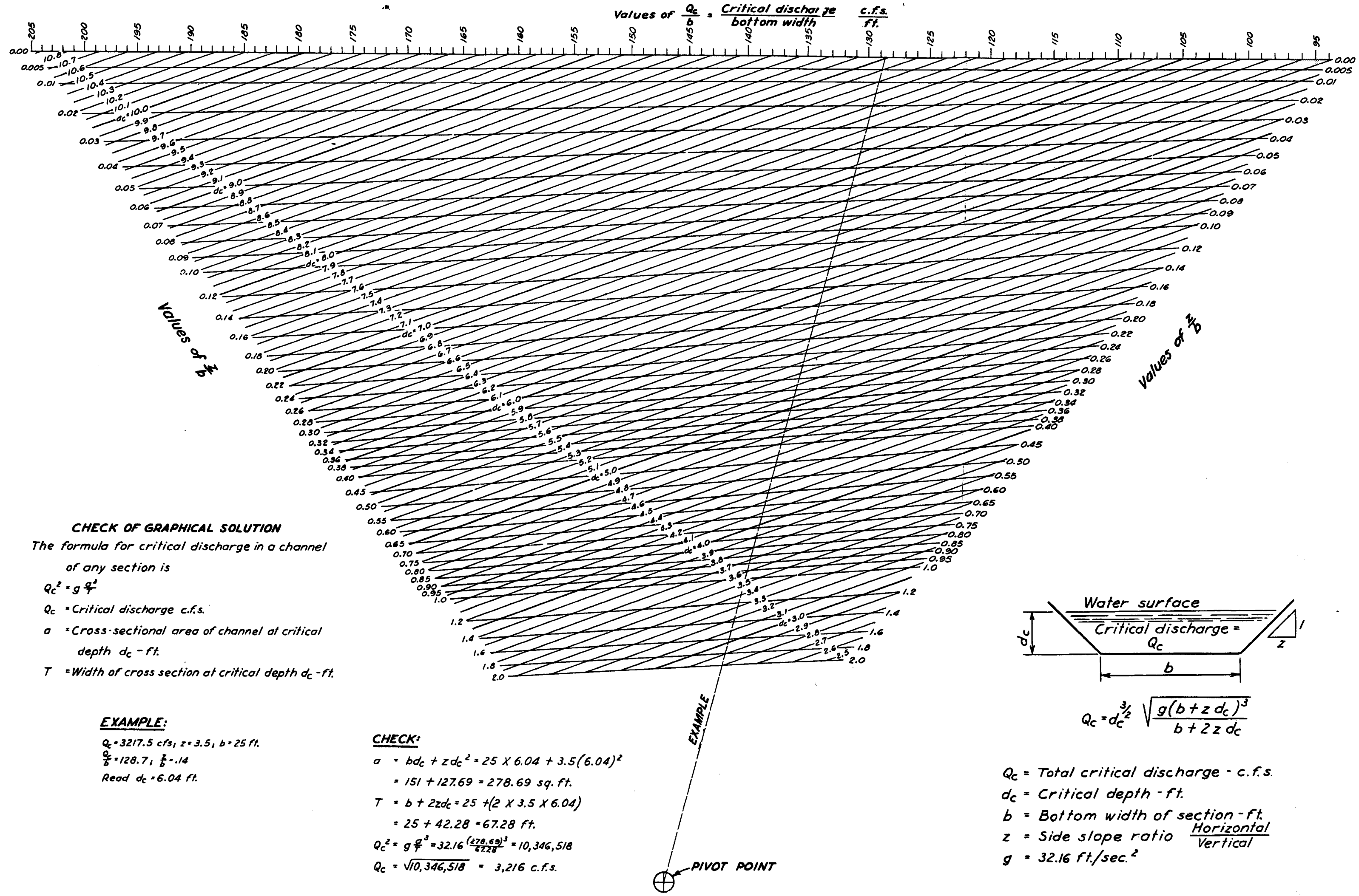

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EXAMPLE:

$Q_c = 3217.5$ cfs; $z = 3.5$; $b = 25$ ft.

$\frac{Q_c}{b} = 128.7$; $\frac{z}{b} = .14$

Read $d_c = 6.04$ ft.

CHECK:

$a = bd_c + zd_c^2 = 25 \times 6.04 + 3.5(6.04)^2$

$= 151 + 127.69 = 278.69$ sq. ft.

$T = b + 2zd_c = 25 + (2 \times 3.5 \times 6.04)$

$= 25 + 42.28 = 67.28$ ft.

$Q_c^2 = g\frac{a^3}{T} = 32.16\frac{(278.69)^3}{67.28} = 10,346,518$

$Q_c = \sqrt{10,346,518} = 3,216$ c.f.s.

$$Q_c = d_c^{3/2}\sqrt{\frac{g(b+zd_c)^3}{b+2zd_c}}$$

Q_c = Total critical discharge - c.f.s.

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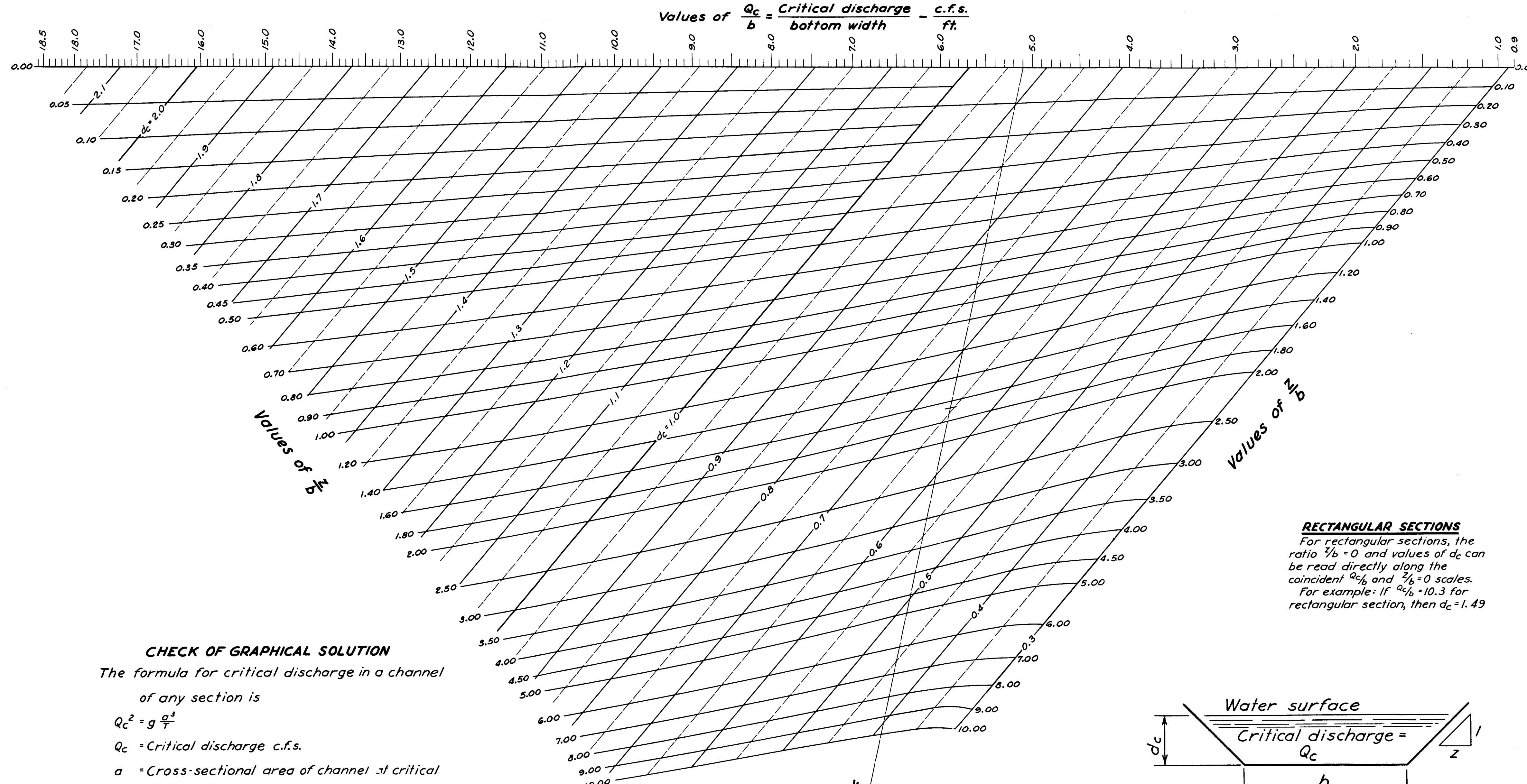

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PIVOT POINT

REFERENCE This nomogram was developed by Paul D. Doubt of the Engineering Standards Unit.

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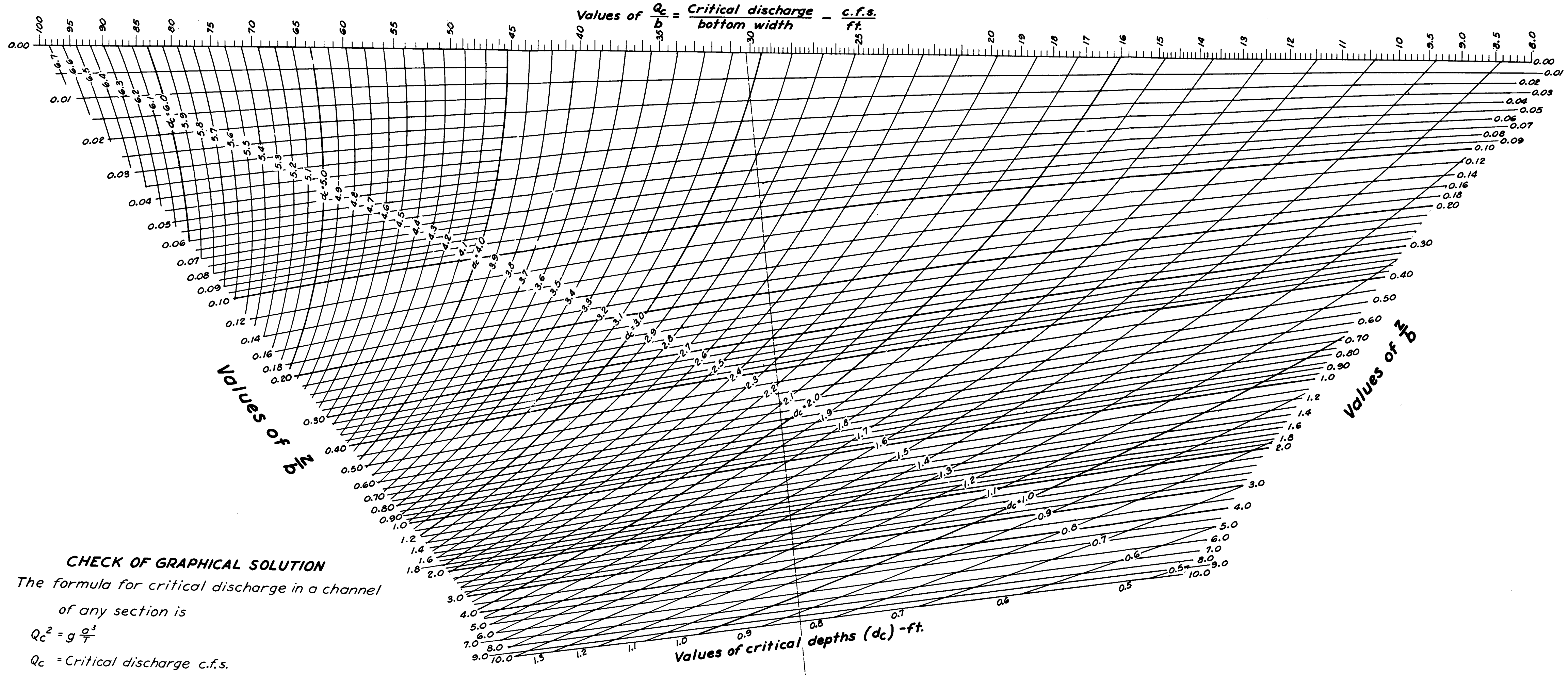

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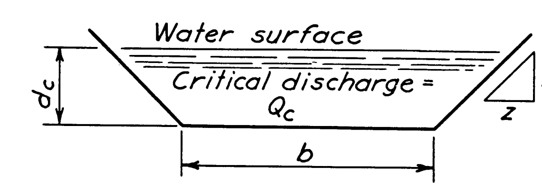

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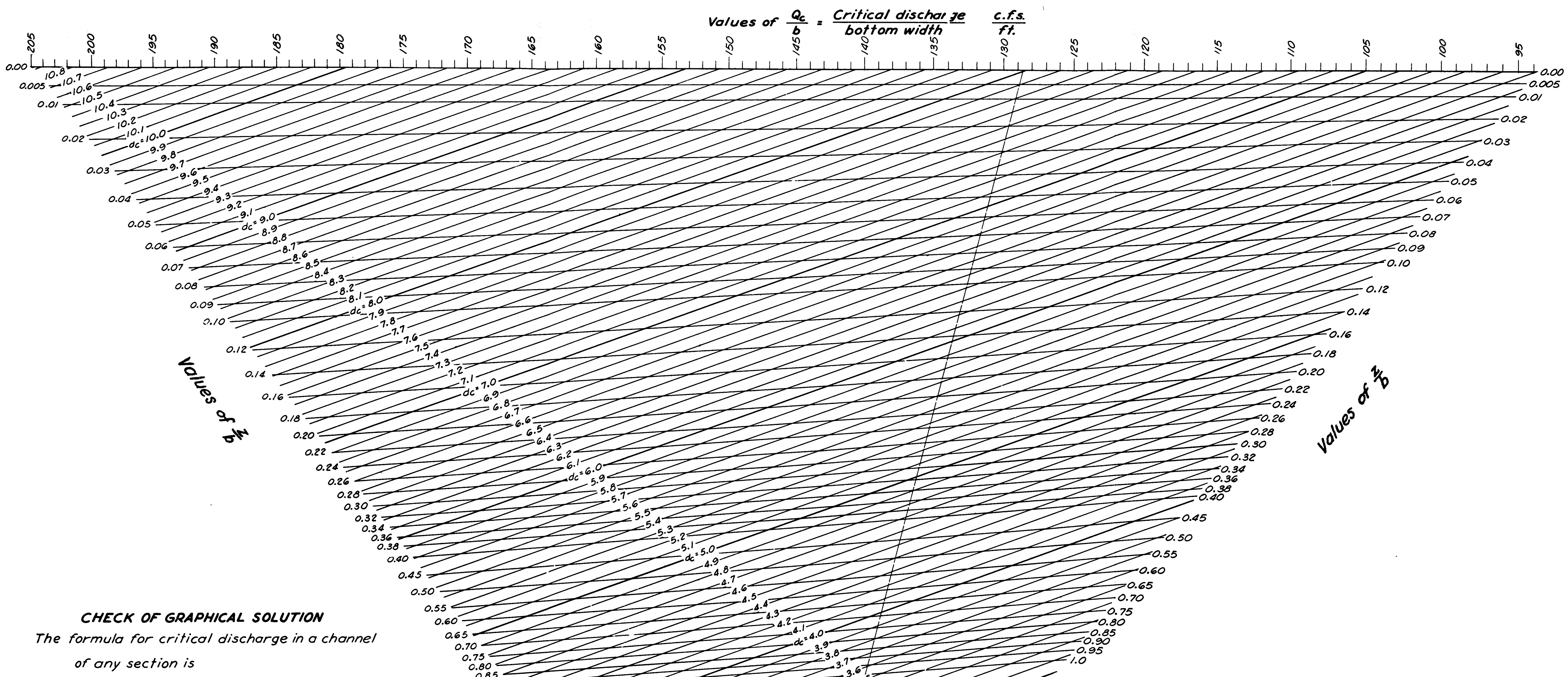

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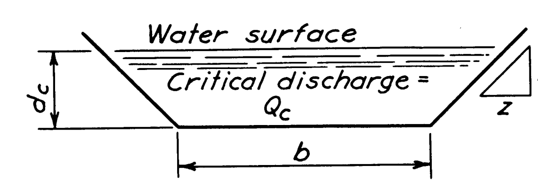

$$Q_c = d_c^{3/2}\sqrt{\frac{g(b + z\,d_c)^3}{b + 2z\,d_c}}$$

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$$d_c = \frac{v_c^2}{g} - \frac{b}{2z} + \sqrt{\frac{b^2}{4z^2} + \frac{v_c^4}{g^2}} \qquad (5.4\text{-}21)$$

$$v_c = \sqrt{\left(\frac{b + zd_c}{b + 2zd_c}\right) d_c g} \qquad (5.4\text{-}22)$$

$$Q_c = d_c^{3/2} \sqrt{\frac{g(b + zd_c)^3}{b + 2zd_c}} \qquad (5.4\text{-}23)$$

The use of these formulas will be materially simplified by tables referred to in "King's Handbook", pp. 382-384. Drawing ES-24 is an alignment chart to be used in solving for Q_c or d_c in equation (5.4-23).

4.5.4 Critical Slope. Critical slope is that slope which will sustain a given discharge in a given channel at uniform, critical depth. The relationships that must exist between discharge, energy, and depth in critical flow are expressed by equations (5.4-3) and (5.4-4). The slope, roughness coefficient, and shape of channel cross section determine whether flow will occur in accordance with these specific relationships. A channel of given cross section, slope, and roughness coefficient will carry only one discharge at uniform, critical depth; the uniform depths at which other discharges will occur are either greater or less than the critical depths for the respective discharges. The same fact stated in another manner is: A channel having a given cross section, roughness coefficient, and discharge will carry that discharge at uniform, critical depth if the channel slope is equal to the critical slope; if the channel slope is greater than the critical slope, the depth of flow will be less than critical; if the channel slope is less than critical, the depth of flow will be greater than critical.

From the critical flow formulas and Manning's formula,

$$v_c = \sqrt{gd_m} \quad \text{and}$$

$$v = \frac{1.486}{n} r^{2/3} s^{1/2}, \text{ then}$$

$$\frac{1.486}{n} r^{2/3} s_c^{1/2} = \sqrt{gd_m}$$

From which the critical slope, s_c, is:

$$s_c = 14.56 \frac{n^2 d_m}{r^{4/3}} \qquad (5.4\text{-}24)$$

In a given channel the following criteria apply:

If the channel slope = $14.56\ n^2 d_m / r^{4/3}$, depth of flow = d_c.

If the channel slope < $14.56\ n^2 d_m / r^{4/3}$, depth of flow > d_c.

If the channel slope > $14.56\ n^2 d_m / r^{4/3}$, depth of flow < d_c.

Formula (5.4-24) and the above criteria are useful for the following purposes: locating control sections, guiding the selection of channel section and grade in preliminary design so that the unstable conditions of uniform flow at critical depth may be avoided, determining the type of water surface curve that will occur in a given reach of channel.

4.5.5 Significance of Critical Flow in Design. Critical, subcritical and supercritical flow affect design in the following manner:

(a) Critical flow. Uniform flow at or near critical depth is unstable. This results from the fact that the unique relationship between energy head and depth of flow which must exist in critical flow is readily disturbed by minor changes in energy. Examine the curve for q = 40 c.f.s. on drawing ES-35. The critical depth is 3.68 feet and the corresponding energy head is 5.52. If the energy head is increased to 5.60, the depth may be 3.2 or 4.2. Those who have seen uniform flow at or near critical depth have observed the unstable, wavy surface that is caused by appreciable changes in depth resulting from minor changes in energy. In channel design these conditions must be recognized. Variations in channel roughness, cross section, slope, or minor deposits of sediment or debris may cause fluctuations in depth of flow that are significant to channel operation. In many cases it is desirable to base design computations on two or more values of n in order to establish the probable range of operating conditions. Because of the unstable flow, channels carrying uniform flow at or near critical depth should not be used unless the situation allows no alternative.

The critical flow principle is the basis for the design of control sections at which a definite stage-discharge relation is desired or required.

(b) Subcritical flow. Two general characteristics of subcritical flow are important. First, at all stages in the subcritical range, except those in the immediate vicinity of the critical, the velocity head is small in comparison with the depth of flow. Study of the curves of constant discharge, drawing ES-35, will make this point clear. Second, the velocities are less than wave velocity for the depths involved and a backwater curve will result from retardation of velocity. Thus, in the subcritical range we are concerned with cases in which the depth of flow is of greater importance than kinetic energy as represented by velocity head. In practice, this means that changes in channel cross section, slope, roughness, and alignment may be made without the danger of developing seriously disturbed flow conditions so long as the design assures that flow in the supercritical range will not be created for some discharges in the operational range.

However, in many cases the latitude in design which may be possible as a result of dealing with subcritical flow will be offset by limited head requiring that friction losses be held to a minimum.

(c) Supercritical flow. The design of structures to carry supercritical flow requires consideration of some of the most complex problems in hydraulics. In supercritical flow the velocity head may range from a value approximately equal to depth of flow to many times the depth of flow. Note from the curves on drawing ES-35 that the velocity head increases very rapidly with decreases in depth throughout the supercritical range. Supercritical velocities exceed the velocities at which gravity waves may be propagated upstream. Any obstruction of flow will result in a standing wave, and there will be no effect upon flow upstream from the obstruction. The fact that kinetic energy predominates in supercritical flow and cannot be dissipated through the development of a water surface curve extending upstream is of primary importance in design.

Channels involving changes of direction, contraction or expansion of cross section, or the joining of two flows at a confluence at which either or both of the flows may be supercritical require careful consideration. Changes in direction or channel contractions develop disturbances at the walls of the channel which take the form of standing waves reflected diagonally from wall to wall downstream from the disturbance points. The height of these standing waves may be several times the depth of flow immediately upstream from the origin of the disturbance. Confluences at which either flow or both flows may be supercritical also develop disturbances resulting in standing waves. In expanding channels the discharge may be incapable of following the channel walls because of the high velocities involved. This results in nonuniform depth and the development of a hydraulic jump which is unstable as to both location and height.

A number of the factors that must be determined as a basis for design of these high velocity structures cannot be evaluated through theoretical analyses only. General experimental results as well as experimentation with individual structures are required. Basic requirements for projects and structures should first be determined and tentative designs to meet these requirements selected. The tentative designs should then be perfected through model tests.

Water surface profiles applying to cases of supercritical flow in straight channels of uniform width can normally be determined with sufficient accuracy for design by standard methods of analysis. Most structures must have outlet velocities in the subcritical range to prevent erosion damage. The creation of the hydraulic jump by the use of stilling basins is an efficient means of dissipating the excessive energy in supercritical flow. Design criteria, based on thorough model investigation, are available for some types of stilling basins. An example is the SAF stilling basin. Under unusual conditions or when exacting requirements must be met, stilling basin designs should also be perfected by model tests.

4.6 The Hydraulic Jump. When water flowing at greater than critical velocity enters water with less than critical velocity and sufficient depth, a hydraulic jump develops. In the jump the depth increases from an original depth to a depth which is less than the higher of the two alternate depths of equal energy. The depth before the jump is always less than critical, and the depth after the jump is greater than critical.

Between cross sections located just upstream and downstream from the jump there occurs a loss of energy, a decrease in velocity, and an increase in hydrostatic pressure. "King's Handbook", pp. 373-378 and 406-412, gives a discussion of the energy and momentum conditions involved in the hydraulic jump and shows the derivation of general formulas.

In paragraph 4.5 the specific energy in flow is discussed and illustrated by the curves on drawing ES-35. The force of a flowing stream is the momentum force due to velocity plus the hydrostatic pressure force. The force equation is:

$$F_m = \frac{Q^2}{ga} + a\bar{y} \qquad (5.4\text{-}25)$$

F_m = the force of the stream.
Q = the discharge.
a = the cross-sectional area.
$\bar{y}$ = the depth to the center of gravity of the cross section.
g = the acceleration of gravity.

For a rectangular channel of unit width, equation (5.4-25) becomes:

$$F_m = \frac{q^2}{gd} + \frac{d^2}{2}$$

Drawing ES-36 shows the specific energy curve, the momentum force curve, and a sketch of a hydraulic jump for a discharge of 30 c.f.s. in a rectangular channel of unit width. The momentum force curve for any discharge in any type of channel would be similar to the one shown. Note that there is a depth at which the force of the flowing stream is minimum and this depth is the critical depth. When the force is greater than the minimum, there are two possible depths, called conjugate or sequent depths, of flow. One of these depths is less than critical, that is, in the supercritical range where the pressure force, because of shallow depth, is relatively low and the momentum force, because of high velocity, is relatively great. The other depth, the sequent depth, is in the subcritical range where the pressure force becomes more significant than the momentum force. The lesser of the two depths is the depth before a jump and the greater is the depth after a jump. The energy head lost in the jump is the difference between the energy heads for these two depths. As the two depths of equal force approach the critical depth, the energy loss in the jump decreases.

<u>4.6.1 Depth After the Jump</u>. Formulas from which depth before and after the jump in any type channel may be computed are:

HYDRAULICS: LOSS IN ENERGY HEAD DUE TO HYDRAULIC JUMP IN RECTANGULAR CHANNEL

Equations and Symbols:

$H_e = d + \frac{q^2}{2gd^2}$ = specific energy

$F_m = \frac{q^2}{gd} + \frac{d^2}{2}$ = momentum force

d = depth of flow.

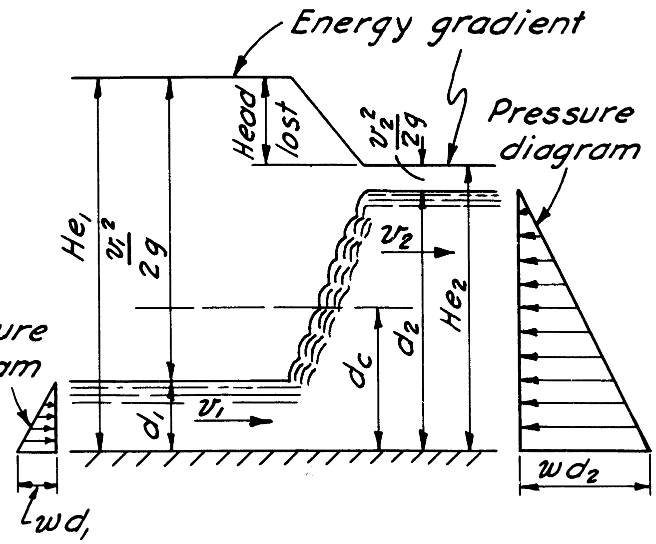
d_c = critical depth.

v = velocity of flow.

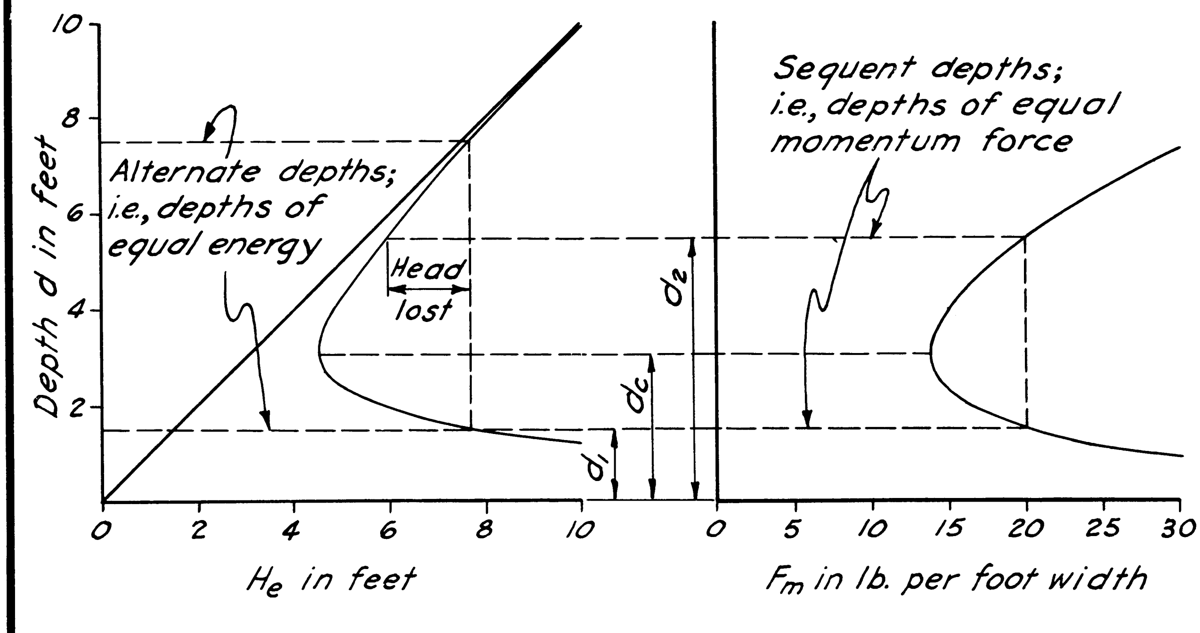
q = discharge per foot width.

w = weight of water per cubic foot.

1 & 2 = subscripts denoting section before and after jump respectively.



Hydraulic Jump



Specific Energy Diagram for q=30 c.f.s. per foot width

Momentum Force Diagram for q=30 c.f.s. per foot width

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$$v_1^2 = g \left[\frac{a_2\bar{y}_2 - a_1\bar{y}_1}{a_1\left(1 - \frac{a_1}{a_2}\right)} \right] \qquad (5.4\text{-}26)$$

$$Q^2 = g \left(\frac{a_2\bar{y}_2 - a_1\bar{y}_1}{\frac{1}{a_1} - \frac{1}{a_2}} \right) \qquad (5.4\text{-}27)$$

Depths before and after the jump in rectangular sections are given by:

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{2v_1^2 d_1}{g} + \frac{d_1^2}{4}} \qquad (5.4\text{-}28)$$

$$d_1 = -\frac{d_2}{2} + \sqrt{\frac{2v_2^2 d_2}{g} + \frac{d_2^2}{4}} \qquad (5.4\text{-}29)$$

Q = discharge.
v = mean velocity.
a = cross-sectional area of flow.
d = depth of flow.
$\bar{y}$ = depth to the center of gravity of the cross section of flow.
g = acceleration of gravity.
Subscripts 1 and 2 denote cross sections and depths before and after the jump respectively.

"King's Handbook", table 133, gives a limited number of values of depth after the jump in rectangular channels, and "Hydraulic Tables", table 3, gives a more complete series of these values. Values for $\bar{y}$ for trapezoidal and circular channels for use in equations (5.4-26) and (5.4-27) can be computed more readily by the use of "King's Handbook", tables 99 and 104.

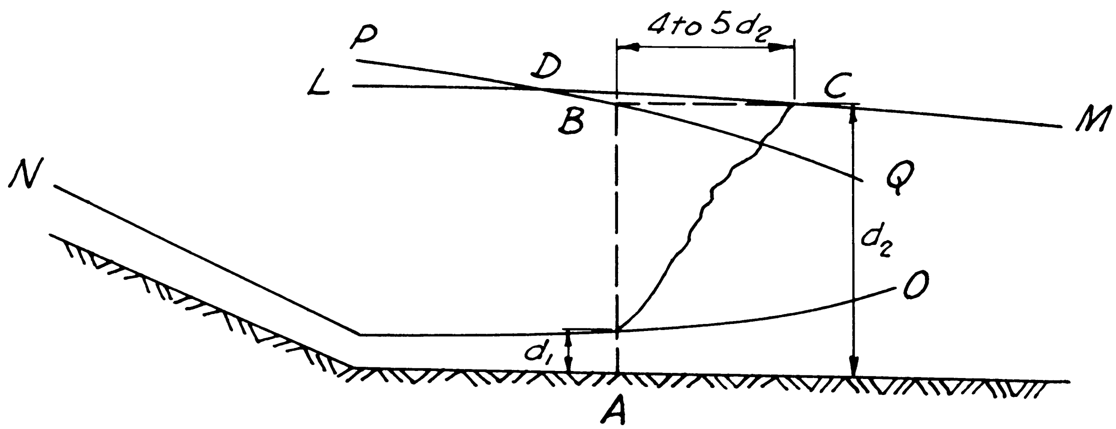
4.6.2 Location of the Jump. When structures involve the hydraulic jump, design will normally be made by criteria that will place the jump in a certain position and no specific estimate of jump location is necessary. However, there will be cases in which a determination of where the jump will occur will be valuable or required. The reliability of the determination depends on the accuracy with which friction loss can be estimated. The location of the jump is estimated by the following steps (see Fig. 5.4-1):

a. From a control section downstream from the jump compute and plot the water surface profile, LM, to a point upstream from the probable position of the jump. And from a control section upstream from the jump compute and plot the water surface profile, NO, to a point downstream from the jump. Methods of computing these profiles are given in paragraphs 4.7.4 and 4.7.5.

b. Through depths sequent to the depths of profile, NO, at 4 to 6 points along a reach certain to include the jump, draw the curve PQ. See paragraph 4.6.1 for methods of determining sequent depths.

c. The approximate location of the jump is D, the intersection of the sequent depth curve, PQ, and the tailwater curve, LM.

d. In most cases the approximate location of the jump, as determined by carrying out steps a, b, and c, will satisfy practical requirements. Some authorities suggest that a closer approximation of the jump location may be obtained as follows: Construct AB vertical and equal in length to the depth d_2 with the jump at A, and BC horizontal and equal in length to the length of jump which may be taken as 4 to 5 d_2. This construction must be in accordance with the horizontal and vertical scales to which the water surface profiles and the sequent depth curve are plotted. Note that the position of point C must meet three simultaneous requirements: First, it is on the tailwater profile; second, the depth d_2 at C is that which is sequent to d_1 at A; third, it is the length of the jump downstream from A.



Location of the Hydraulic Jump

FIG. 5.4-1

4.7 Water Surface Profiles. The main objective in the majority of open channel problems is to determine the profile of the water surface. Methods of computing water surface profiles are described for the general cases of uniform flow, accelerated flow, and retarded flow.

4.7.1 Uniform Flow. In uniform flow the force of gravity is balanced by the friction force. The slopes of the hydraulic gradient, the energy gradient, and the bottom of the channel are equal; also mean velocity, depth of flow, and area of flow are constant from section to section. Depth of uniform flow is called the normal depth. Flow may be uniform only when the channel is uniform in cross section. The relationships between the energy gradient, hydraulic gradient, and bottom of channel are shown by fig. 5.4-2.

The energy equation for sections 1 and 2 is:

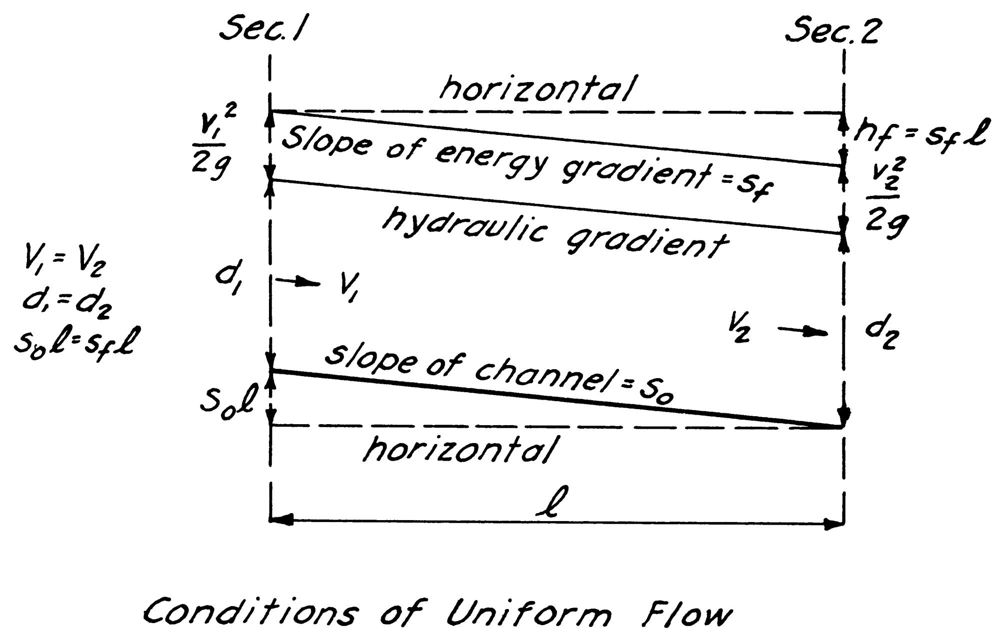
$$\frac{v_1^2}{2g} + d_1 + s_o \ell = \frac{v_2^2}{2g} + d_2 + h_f$$

Since the velocities and depths at sections 1 and 2, and channel slope and friction slope are equal, the friction head, h_f, expressed as a function of velocity, cross section elements, slope, and roughness coefficient, is the basis for the solution of uniform flow problems. Solutions may be made through the use of Manning's formula:

$$v = \frac{1.486}{n} r^{2/3} s^{1/2} \qquad (5.4\text{-}1)$$

$$Q = \frac{1.486}{n} ar^{2/3} s^{1/2} \qquad (5.4\text{-}30)$$

v = mean velocity in ft. per sec.
Q = discharge in cu. ft. per sec.
a = cross-sectional area of flow in sq. ft.
r = hydraulic radius in ft.
s = slope of the energy gradient or friction head in ft. per ft.
n = roughness coefficient.



Conditions of Uniform Flow

FIG. 5.4-2

"King's Handbook", pp. 279-283, gives a number of useful working forms of Manning's formula. The most frequently used of these forms are:

$$Q = \frac{K}{n} d^{8/3} s^{1/2} \qquad (5.4\text{-}31)$$

$$Q = \frac{K'}{n} (b \text{ or } T)^{8/3} s^{1/2} \qquad (5.4\text{-}32)$$

$$s = \frac{n^2 v^2}{2.2082 r^{4/3}} \qquad (5.4\text{-}33)$$

$$h_f = \frac{\ell n^2 v^2}{2.2082 r^{4/3}} \qquad (5.4\text{-}34)$$

K and K' = factors varying with the ratios of certain linear dimensions of cross sections.
b = bottom width of rectangular or trapezoidal sections.
d = depth of flow in any section.
h_f = total friction head lost in a reach.
T = width of water surface in parabolic sections.
ℓ = horizontal length of reach.

"King's Handbook" contains tables of values of K, K' and $1 \div (2.2082 r^{4/3})$. The utility of formulas (5.4-31) and (5.4-32) depends upon the availability of tables giving values of K and K', and the user should refer to King's Handbook for further discussion regarding application of these formulas to channel sections of different forms. Formulas (5.4-1), (5.4-30), (5.4-33), and (5.4-34) are useful in many problems and they are adapted to use with slide rule or longhand computations. It should be noted that King treats the triangular section as a special form of the trapezoidal section; therefore, K and other factors related to triangular sections are found in the tables for trapezoidal sections where the ratio of d/b = infinity. The alignment chart, drawing ES-34, may be used for graphical solutions for any one unknown in formula (5.4-1).

4.7.2 Accelerated and Retarded Flow. This subsection considers steady, nonuniform flow and attention is given to methods of determining water surface profiles under various conditions. Those who are interested in a more thorough treatment of nonuniform flow are referred to "Steady Flow in Open Channels" by Sherman M. Woodward and Chesley J. Posey, John Wiley and Sons, Inc.; and "Hydraulics of Open Channels" by Boris A. Bakhmeteff, McGraw-Hill Book Company.

In accelerated or retarded flow, as in uniform flow, the fall of the energy gradient represents the loss of head by friction. The fall in the water surface reflects both friction loss and the conversions between potential and kinetic energy. In analyzing nonuniform flow problems it is, therefore, necessary to consider both the hydraulic gradient and the energy gradient.

Refer to fig. 5.4-2. The equation of energy for sections 1 and 2 is:

$$\frac{v_1^2}{2g} + d_1 + s_o \ell = \frac{v_2^2}{2g} + d_2 + s_f \ell \qquad (5.4\text{-}35)$$

Solving for ℓ gives:

$$\ell = \frac{\left(\frac{v_2^2}{2g} + d_2\right) - \left(\frac{v_1^2}{2g} + d_1\right)}{s_o - s_f} \qquad (5.4\text{-}36)$$

v = mean velocity.
d = depth of flow.
s_o = slope of channel.
$s_f = h_f/\ell$ = friction slope, i.e., the slope of the energy gradient.
ℓ = length of reach.
Subscripts 1 and 2 denote upstream and downstream sections respectively.

Water surface profiles may be computed by the use of formula (5.4-35) or (5.4-36). The methods of use of both formulas are among the several step methods for backwater computations. In those cases where the water surface elevation, i.e., the depth, at a specific section is to be computed, the solution is by trial and error and formula (5.4-35) should be used. This method of solution can be applied in any type of channel regardless of whether it is uniform or nonuniform. Where the distances between a series of depths along a uniform channel are to be computed, the solution is direct with formula (5.4-36), but it should be noted that this approach may be used only in uniform channels.

The general procedure for computing depths at given locations is: (1) Determine the location of a control section and the depth of flow at that section. (2) Take a relatively short reach of selected length and assume the depth at the upper or lower end depending on whether the computations are to proceed upstream or downstream. (3) Evaluate s_f and the velocity heads and substitute the values in formula (5.4-35). If the equation balances, the assumed depth is the correct depth; if it does not balance, a new trial must be made by assuming a new depth in (2) and repeating (3). (4) Continue these trial and error determinations by reaches until the depths at the given locations have been computed.

Procedure for computing successive distances along the channel to selected depths is: (1) Determine the location of a control section and the depth of flow at that section. (2) Select a depth at the upper or lower end of a reach of length, ℓ, depending on whether the computations are to be carried upstream or downstream. (3) Evaluate s_f and the velocity heads, substitute these values in formula (5.4-36), and compute ℓ. (4) Continue these computations by repeating (1), (2), and (3).

In computing water surface profiles for the design of improved channels, particularly lined channels, by either of these procedures the change in velocity in a reach should be held to a maximum of 15 to 20 percent, that is, neither v_1 nor v_2 should be allowed to vary from the other by more than 15 or 20 percent. This can be done in the trial and error computation of depth by keeping the selected reaches sufficiently short. When distances between depths are being computed by formula (5.4-36) the value of d_1 or d_2, whichever is being selected, can be taken so that neither v_1 nor v_2 is greater or less than the other by more than 15 to 20 percent.

It is recommended that in all cases the computations for surface profiles be carried upstream when the depth of flow is greater than critical and downstream when the depth of flow is less than critical. The first step in the analysis of flow in a channel should be to locate all control sections for the discharges to be investigated. This step sets out the portions of the channel in which depths of flow will be greater or less than critical and spots the stations or sections from which computations should be carried upstream and downstream.

Note that velocity head plus depth, $(v^2 \div 2g) + d$, at sections 1 and 2 is the specific energy at those sections. Inspection of the specific energy diagrams on drawing ES-35 will show that when depth of flow is less than critical, specific energy increases as depth decreases; and when depth of flow is greater than critical, specific energy increases as depth increases. Formula (5.4-36) may be written:

$$\ell = \frac{H_{e2} - H_{e1}}{s_o - s_f}$$

When step computations in a uniform channel of considerable length are to be made, it will often be worthwhile to plot the specific energy diagram for the discharge or discharges to be considered. This diagram may be used as a guide to the selections of depths for successive steps when either formula (5.4-35) or (5.4-36) is being used.

Evaluation of s_f may be made by one of the following formulas:

$$s_f = \frac{n^2 \, v_m^2}{2.2082 \, r_m^{4/3}} \qquad (5.4\text{-}37)$$

$$s_f = \frac{Q^2 \, n^2}{2.2082 \, a_m^2 \, r_m^{4/3}} \qquad (5.4\text{-}38)$$

$$s_f = \left(\frac{Qn}{Kd_a^{8/3}}\right)^2 \qquad (5.4\text{-}39)$$

$$s_f = \left(\frac{Qn}{K' \, b^{8/3}}\right)^2 \qquad (5.4\text{-}40)$$

n = roughness coefficient.

$v_m = \frac{v_1 + v_2}{2}$ = mean velocity in a reach.

$r_m = \frac{r_1 + r_2}{2}$ = mean hydraulic radius in a reach.

$a_m = \frac{a_1 + a_2}{2}$ = mean area of flow in a reach.

$d_a = \frac{d_1 + d_2}{2}$ = average depth in a reach.

b = bottom width of rectangular or trapezoidal channel.

Q = discharge.

K and K' = factors varying with the ratios of certain linear dimensions of cross sections.

In computations for uniform channels, formulas (5.4-39) and (5.4-40) are time savers. This is particularly true of formula (5.4-40) since for a given Q the value of $(Qn \div b^{8/3})^2$ is constant and s_f is obtained by multiplying $(1/K')^2$ for the various values of d_a/b by this constant. King's Handbook contains tables of values of K, K', and $(1/K')^2$ for various types of channels. In nonuniform and natural channels s_f may be computed by formula (5.4-37) or (5.4-38). When a high degree of accuracy is not required, s_f may be obtained from the alignment chart, drawing ES-34, by entering the chart with the appropriate values of v_m, r_m, and n.

A general guide to the analysis of flow conditions in the cases most commonly dealt with in channel design is given by drawing ES-38.

4.7.3 Examples - Uniform Flow. In the majority of cases we will know, or will have selected, the type of cross section and the roughness coefficient and will want to determine either discharge, velocity, channel dimensions, or slope. Methods of solving a number of practical problems are illustrated by examples. The following table summarizes the factors known and to be determined in the various examples; Q and v are discharge and velocity; b, T, d, and z are channel dimensions (see drawing ES-33).

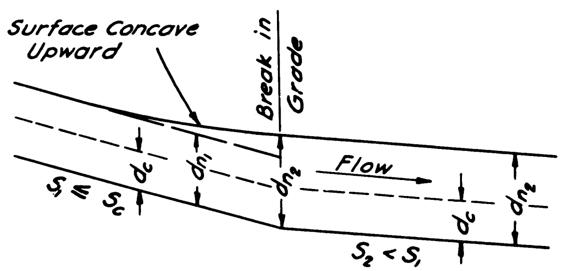
Q	v	b or T	d	z	s	n	Type of Channel	Example No.
0	0	X	X	X	X	X	Trapezoidal	1
0	0		X	X	X	X	Triangular	2
X	0	X	0		X	X	Rectangular	3
X	0	X	0	X	X	X	Trapezoidal	4
X	0		0	X	X	X	Triangular	5
X	0	X	0	X	X	X	Parabolic	6
0	0	X	X	X	X	X	Trapezoidal	7
X	0	0	X	X	X	X	Trapezoidal	8
X	0	0	X		X	X	Parabolic	9
X	X	0	0	X	X	X	Trapezoidal	10
X	X	0	0		X	X	Rectangular	11
X	X		0	0	X	X	Triangular	12
X	X	0	0		X	X	Parabolic	13

X - Known

0 - To be determined

HYDRAULICS: SURFACE PROFILES IN UNIFORM CHANNELS

A

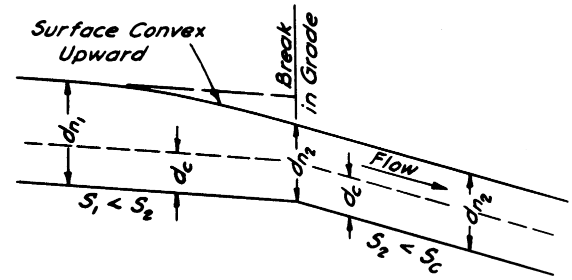


Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is concave upward and is asymptotic to uniform flow surface.

Flow is retarded and sub-critical

Determine the concave upward surface profile by computing upstream from the break in grade starting with the normal depth corresponding to s_2.

B

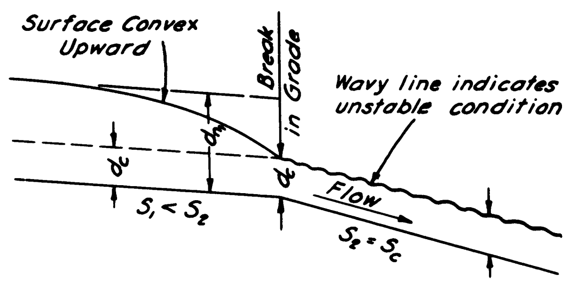


Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is convex upward and is asymptotic to uniform flow surface.

Flow is accelerated and sub-critical

Determine the convex upward surface profile by computing upstream from the break in grade starting with the normal depth corresponding to s_2.

C

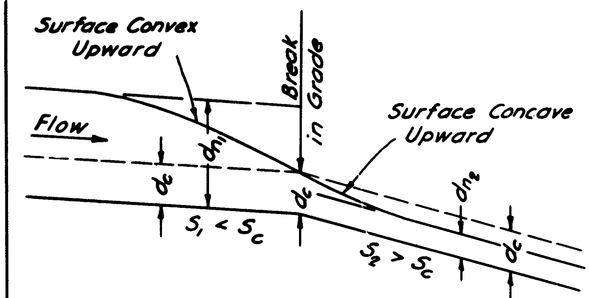


Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is convex upward and is asymptotic to uniform flow surface.

Flow is accelerated and changes from sub-critical to critical flow. This case can occur for only one discharge for a given channel cross-section, slope and roughness coefficient.

Determine the convex upward surface profile by computing upstream from the break in grade starting with the normal depth corresponding to s_2 which is also d_c.

D



Surface profile immediately downstream from the break in grade is concave upward and asymptotic to the uniform flow surface. Surface profile immediately upstream from the break in grade is convex upward and asymptotic to the uniform flow surface.

Flow is accelerated and progresses from sub-critical through critical to super-critical.

Determine the concave upward surface profile by computing downstream from the break in grade, starting with the critical depth. Determine the convex upward surface profile by computing upstream from the break in grade, starting with the critical depth.

NOTE: SURFACE PROFILES ILLUSTRATED ARE BASED ON THE ASSUMPTION THAT THE REACHES OF s AND s_2 ARE SUFFICIENTLY LONG TO PRODUCE UNIFORM FLOW, THUS THE HORIZONTAL SCALE MUST BE VISUALIZED AS BEING GREATLY CONDENSED.

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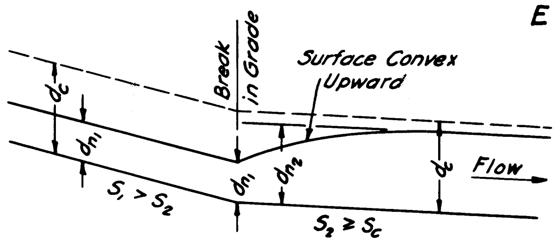
REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-38 SHEET 1 OF 5 DATE 6-18-50

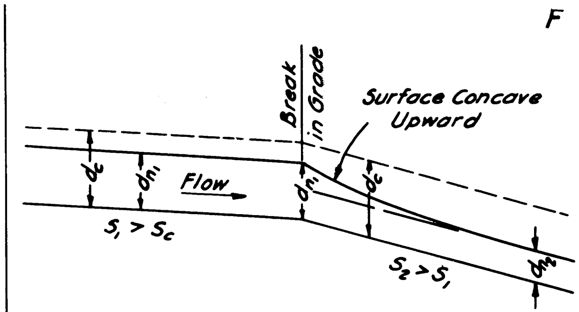
HYDRAULICS: SURFACE PROFILES IN UNIFORM CHANNELS



Surface profile immediately downstream from the break in grade is convex upward and asymptotic to the uniform flow surface. Surface profile is straight, and uniform flow exists throughout the reach upstream from the break in grade.

Flow is retarded and super-critical.

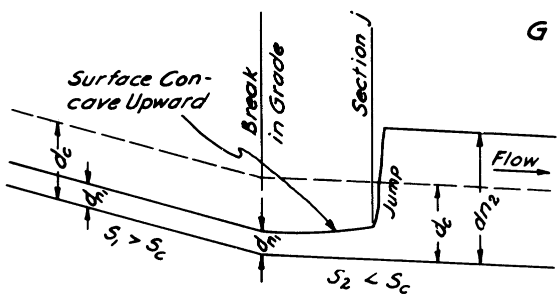
Determine the convex upward surface profile by computing downstream from the break in grade, starting with the normal depth of s_1.



Surface profile immediately downstream from the break in grade is concave upward and asymptotic to the uniform flow surface. Surface profile is straight, and uniform flow exists throughout the reach upstream from the break in grade.

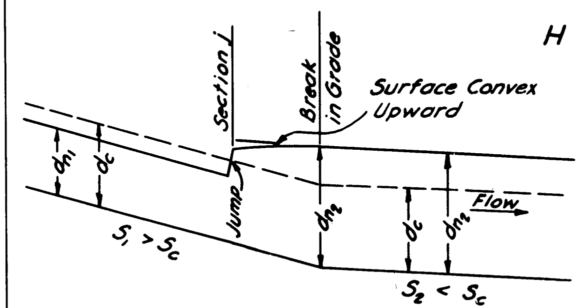
Flow is accelerated and super-critical.

Determine the concave upward surface profile by computing downstream from the break in grade, starting with the normal depth of s_1.



Surface profile is straight, and uniform flow exists downstream from the jump. Surface profile immediately downstream from the break in grade is concave upward. Surface profile is straight, and uniform flow exists upstream from the break in grade.

Flow is retarded and changes abruptly from super-critical to sub-critical.



Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is convex upward. Surface profile is straight, and uniform flow exists immediately upstream from the jump.

Flow is retarded and changes abruptly from super-critical to sub-critical.

The criterion to determine whether the jump occurs downstream or upstream from the break in grade is:

$$a_1 \bar{y}_1 + \frac{Q^2}{ga_1} > a_2 \bar{y}_2 + \frac{Q^2}{ga_2} \qquad\qquad a_1 \bar{y}_1 + \frac{Q^2}{ga_1} < a_2 \bar{y}_2 + \frac{Q^2}{ga_2}$$

See 4.6.1 for nomenclature

Determine the concave upward surface profile by computing downstream from the break in grade, starting with the normal depth corresponding to s_1.

The location of the jump is at the section j within the reach containing the concave upward surface profile satisfying the relation

$$a_j \bar{y}_j + \frac{Q^2}{ga_j} = a_2 \bar{y}_2 + \frac{Q^2}{ga_2}$$

Determine the convex upward surface profile by computing upstream from the break in grade, starting with the normal depth corresponding to s_2.

The location of the jump is at the section j within the reach containing the convex upward surface profile satisfying the relation

$$a_1 \bar{y}_1 + \frac{Q^2}{ga_1} = a_j \bar{y}_j + \frac{Q^2}{ga_j}$$

NOTE: SURFACE PROFILES ILLUSTRATED ARE BASED ON THE ASSUMPTION THAT THE REACHES OF s_1 AND s_2 ARE SUFFICIENTLY LONG TO PRODUCE UNIFORM FLOW, THUS THE HORIZONTAL SCALE MUST BE VISUALIZED AS BEING GREATLY CONDENSED.

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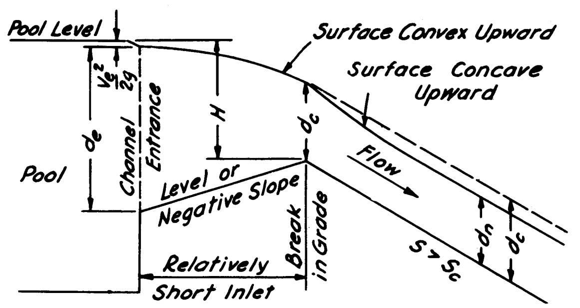
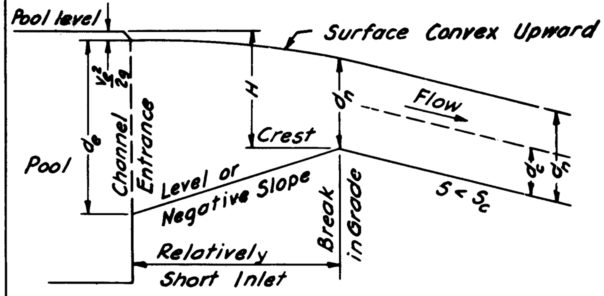
J



Surface profile immediately downstream from the break in grade is concave upward and asymptotic to the uniform flow surface. Surface profile immediately upstream from the break in grade is convex upward.

Flow is accelerated and progresses from sub-critical through critical to super-critical.

K



Surface profile is straight, and uniform flow exists downstream from the break in grade. Surface profile immediately upstream from the break in grade is convex upward.

Flow is accelerated and sub-critical.

General solution for the discharge at a given pool elevation: Each discharge over the crest has its own pool elevation which may be determined. 1st – Determine whether flow conditions are shown by J or K. 2nd – Find two trial discharges; one which has a pool elevation slightly higher and the other slightly lower than the actual pool elevation, H. 3rd – Interpolate for the correct discharge between the two trial discharges.

1. Establish, closely, whether the downstream slope $s <$ or $> s_c$ by:
   A. Solve for a test discharge Q_t, assuming no loss of head from the channel entrance to the crest when the flow over the crest is critical.
   (Q_t is chosen to balance the equation $d_c + \frac{Q_t^2}{2ga_c^2} = H$; values of d_c and a_c are the critical quantities corresponding to Q_t at the crest.)
   B. For Q_t, compute s_c. If $s > s_c$, proceed on the left under J; if $s < s_c$, proceed on the right under K.

Under J:

2. A. Take $Q_1 = Q_t$ for first trial discharge.
   a. Compute, for the discharge Q_1, the convex upward surface profile to the channel entrance starting with the critical depth d_c at the crest; value of d_c is critical corresponding to Q_1.
   b. Obtain the pool level by adding the velocity head and depth at the channel entrance. The pool elevation, H_1 for the trial discharge, Q_1, will be found to be higher than the actual pool elevation, H.
   B. Select the second trial discharge $Q_2 < Q_1$ such that its pool elevation H_2 is slightly less than H. (Determine H_2 by the same method as stated for H_1 in 2.A.a. and b.)
3. Interpolate for Q, the correct discharge lying between the trial discharges Q_1 and Q_2, given H, H_1, H_2, and observe whether $s > s_c$ for Q; if $s < s_c$, solve by method under K.
4. If $s > s_c$, determine the convex upward surface profile to the channel entrance by computing upstream from the break in grade starting with the critical depth, d_c, corresponding to Q.
5. Determine the concave upward surface profile by computing downstream from the break in grade starting with d_c corresponding to Q.

Under K:

2. A. Solve for the first trial discharge Q_1 by assuming no loss of head from the channel entrance to the crest when the flow over the crest is normal depth for Q_1 downstream from the break in grade. (Q_1 is chosen to balance the equation $d_n + \frac{Q_1^2}{2ga_n^2} = H$; values of d_n and a_n are the normal quantities corresponding to Q_1 on the slope, s.)
   a. Compute, for the discharge Q_1, the convex upward surface profile to the channel entrance starting with d_n at the crest, d_n being the normal depth corresponding to Q_1 on the slope, s.
   b. Obtain the pool level by adding the velocity head and depth at the channel entrance. The pool elevation, H_1 for the trial discharge, Q_1, will be found to be higher than the actual pool elevation, H.
   B. Select the second trial discharge $Q_2 < Q_1$ such that its pool elevation H_2 is slightly less than H. (Determine H_2 by the same method as stated for H_1 in 2.A.a. and b.)
3. Interpolate for Q, the correct discharge, lying between the trial discharges Q_1 and Q_2, given H, H_1, H_2, and observe whether $s < s_c$ for Q; if $s > s_c$, solve by method under J.
4. Determine the convex upward surface profile through the inlet by computing upstream from the break in grade, starting with the normal depth, d_n, corresponding to Q in the channel downstream from the break in grade.

NOTE: SURFACE PROFILES ILLUSTRATED ARE BASED ON THE ASSUMPTION THAT THE REACH OF s IS SUFFICIENTLY LONG TO PRODUCE UNIFORM FLOW, THUS THE HORIZONTAL SCALE MUST BE VISUALIZED AS BEING GREATLY CONDENSED. THE INLET MAY BE A NON-UNIFORM CHANNEL.

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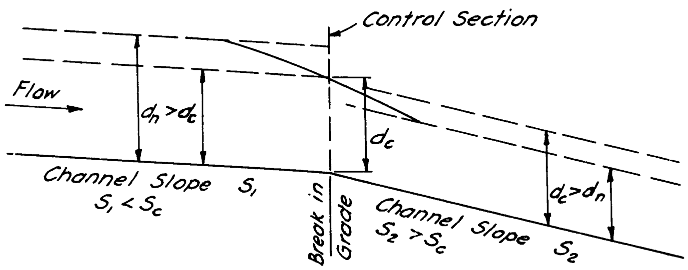
CONTROL SECTIONS

Definition of control section: A flow section at which, for a discharge or a range of discharge, there is a fixed relation between stage or depth of flow and discharge. These conditions are most commonly met at those sections where critical depth occurs. Control section, as used here, does not mean control of discharge. Examples are: (1) a weir; (2) a cross-section of a channel at which the depth of flow is critical, thus establishing that,

$$Q_c = 5.67\ bd_c^{3/2} \quad \text{and} \quad Q_c = d_c^{3/2}\sqrt{\frac{g(b + zd_c)^3}{b + 2zd_c}}$$

for rectangular and trapezoidal channels respectively. See the subsection 4.5 on Critical Flow.

Control sections are the starting points, both as to station location and elevation of water surface, from which water surface profiles are computed. Consider the following sketch of a break in grade in a uniform channel.



When there is steady flow in a uniform channel with constant roughness coefficient, acceleration or retardation can be caused only by a change in slope. The slopes upstream and downstream from the break in grade determine whether critical depth will occur and, therefore, whether the break in grade is a control section. Refer to paragraph 4.5.4 on Critical Slope and to formula (5.4-24).

$$s_c = \frac{14.56\ n^2\ d_m}{r^{4/3}}$$

s_c = critical slope; d_m = mean depth when flow is critical; r = hydraulic radius corresponding to critical depth; n = roughness coefficient.

Method of Determining Control Sections

Assume a break in grade in a uniform channel as sketched above, to determine whether the break is a control section when the discharge is Q.

1st. Compute d_c corresponding to Q. If the channel is rectangular or trapezoidal, make this computation with the alignment chart, Drawing No. ES-24. When the channel is another form see subsection 4.5 for the formula to be used in computing d_c.

2nd. Compute d_m corresponding to d_c by: d_m = cross-sectional area ÷ width of flow surface. Refer to Drawing No. ES-33, Elements of Channel Sections; also see "King's Handbook", Table 98, "Hydraulic Tables", Tables 4-14 inclusive.

3rd. Compute r corresponding to d_c. "King's Handbook", Tables 97, 101, and 105; "Hydraulic Tables", Tables 4 to 14 incluvive.

4th. Compute the critical slope, s_c,
   (a) by formula (5.4-24) given above
   (b) or by computing v_c, (Q ÷ area corresponding to d_c, or by appropriate formula in subsection 4.5) entering the alignment chart, Drawing No. ES-34, with v_c, r, and n, and reading s_c.

If $s_1 < s_c < s_2$, d_c occurs at the break in grade and it is a control section. But if $s_1 < s_c > s_2$, or if $s_1 > s_c$, d_c cannot occur and the break in grade is not a control section.

In a given channel s_1 and s_2 are fixed. It is important to remember that d_c and s_c vary with discharge; therefore, it may be found that a break in grade is a control section for some discharges but not all discharges in the operational range.

Revised 4-19-51

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

ENGINEERING STANDARDS UNIT

Example

Given: Concrete lined, rectangular channel, depth 7.5 ft.; width, 20 ft.; $s_1 = 0.0025$; $s_2 = 0.0055$; $n = 0.018$

To determine: If the break in grade between s_1 and s_2 is a control section when the discharge is: 2000, 1000, 500, and 200 c.f.s.

Compute the following tabulated values under instructions given above. In other than rectangular channel, d_m and $d_m/r^{4/3}$ would be required.

Q	Q/b	d_c	r	$r^{4/3}$	$d_c/r^{4/3}$	s_c	Conclusions
2000	100	6.77	4.04	6.43	1.057	0.00498	$s_1 < s_c < s_2$, break is control section.
1000	50	4.27	2.99	4.31	0.993	0.00468	$s_1 < s_c < s_2$, break is control section.
500	25	2.69	2.12	2.72	0.988	0.00467	$s_1 < s_c < s_2$, break is control section.
200	10	1.46	1.27	1.38	1.057	0.00499	$s_1 < s_c < s_2$, break is control section.

$$s_c = 14.56\, n^2 \frac{d_c}{r^{4/3}} = 14.56\,(0.018)^2 \frac{d_c}{r^{4/3}} = 0.00472\, d_c/r^{4/3}$$

STEPS IN ANALYSIS

An analysis of flow in a channel having a number of breaks in grade should be made in the following steps:

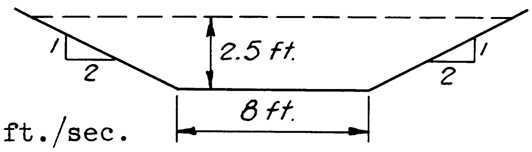
1st. Determine the control sections and the depths of flow at those sections for each discharge to be investigated. This sets out the reaches in which the depth of flow will be greater or less than critical and defines the starting points for surface profile computations.

2nd. Compute the surface profiles for each discharge. Carry computations upstream in the reaches where the depth of flow is greater than critical and downstream where the depth of flow is less than critical.

Revised 4-19-51

EXAMPLE 1

Given: Trapezoidal section



$n = 0.02$, $s = 0.006$

To determine: Q in c.f.s. and v in ft./sec.

Solution by formula (5.4-31):

1. $d/b = \frac{2.5}{8} = 0.313$

2. From King's Handbook, table 112, p. 331, K = (by interpolation) 5.96

3. From King's Handbook, table 111, p. 324, $d^{8/3} = 2.5^{8/3} = 11.51$

4. From King's Handbook, table 108, p. 311, or by slide rule, $s^{1/2} = 0.006^{1/2} = 0.0775$

5. $Q = \frac{K}{n} d^{8/3} s^{1/2} = \frac{5.96}{0.02} \times 11.51 \times 0.0775 = 266$ c.f.s.

6. $v = \frac{Q}{a} = 266 \div [(8 \times 2.5)+(2 \times 2.5^2)] = 8.19$ ft./sec.

Solution by formula (5.4-30) without tables or other work aides:

1. $a = (2.5 \times 8)+(2 \times 2.5^2) = 32.5$

2. $p = 8 + 2\sqrt{5^2 + 2.5^2} = 19.18$

3. $r = 32.5 \div 19.18 = 1.695 \qquad r^{2/3} = 1.422$

4. $s^{1/2} = 0.006^{1/2} = 0.0775$

5. $Q = \frac{1.486}{n} ar^{2/3} s^{1/2} = \frac{1.486}{0.02} \times 32.5 \times 1.422 \times 0.0775 = 266$ c.f.s.

6. $v = \frac{Q}{a} = 266 \div 32.5 = 8.19$ ft./sec.

Solution by alignment chart, drawing ES-34:

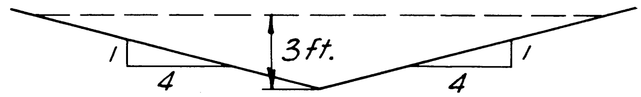
1. $r = \frac{a}{p} = 32.5 \div 19.18 = 1.695$

2. Enter chart with $r = 1.695$, $s = 0.006$, $n = 0.02$, and read $v = 8.19$ ft./sec.

3. $Q = av = 32.5 \times 8.19 = 266$ c.f.s.

EXAMPLE 2

Given: Triangular section



$n = 0.025$, $s = 0.006$

To determine: Q in c.f.s. and v in ft./sec.

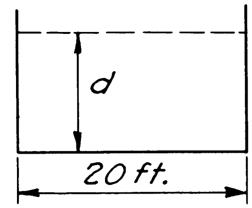
Solution by formula (5.4-31) using King's Handbook tables:

1. $d/b = \frac{3.0}{0.0}$ = infinity
2. K = 3.67, table 112, p. 335
3. $d^{8/3} = 3^{8/3} = 18.70$, table 111, p. 324
4. $s^{1/2} = 0.006^{1/2} = 0.0775$, table 108, p. 311
5. $Q = \frac{K}{n} d^{8/3} s^{1/2} = \frac{3.67}{0.025} \times 18.70 \times 0.0775 = 213$ c.f.s.
6. $v = \frac{Q}{a} = 213 \div 36.0 = 5.91$ ft./sec.

Solution by formula (5.4-1) using King's Handbook tables:

1. d/b = infinity
2. $r = cd = 0.485 \times 3.0 = 1.455$ c, from table 97, p. 296
3. $r^{2/3} = 1.455^{2/3} = 1.284$, from table 109, p. 312
4. $s^{1/2} = 0.006^{1/2} = 0.0775$
5. $v = \frac{1.486}{n} r^{2/3} s^{1/2} = \frac{1.486}{0.025} \times 1.284 \times 0.0775 = 5.91$ ft./sec.
6. $Q = av = 36 \times 5.91 = 213$ c.f.s.

EXAMPLE 3



Given: Rectangular section

$Q = 600$ c.f.s.
$n = 0.025$
$s = 0.0004$

To determine: d in ft. and v in ft./sec.

Solution using formula (5.4-32) in the form $Q = \frac{K'}{n} b^{8/3} s^{1/2}$ with King's Handbook tables:

1. $b^{8/3} = 20^{8/3} = 2950$, table 111, p. 327; $0.0004^{1/2} = 0.02$, table 108, p. 311.

2. $$K' = \frac{Qn}{b^{8/3}\ s^{1/2}} = \frac{600 \times 0.025}{2950 \times 0.02} = 0.254$$

3. In table 113, p. 336, column for vertical sides, find $K' = 0.254$, and find, by interpolation, $d/b = 0.448$. Then $d/20 = 0.448$ and $d = 20 \times 0.448 = 8.96$ ft.

4. $$v = \frac{Q}{bd} = \frac{600}{8.96 \times 20} = 3.35 \text{ ft./sec.}$$

Solution using formula (5.4-30) and slide rule only:

1. $$ar^{2/3} = \frac{Qn}{1.486\ s^{1/2}} = \frac{600 \times 0.025}{1.486 \times 0.02} = 505$$

2. By assuming values of d, compute $ar^{2/3}$ until one value lower and one higher than 505 is found, as follows:

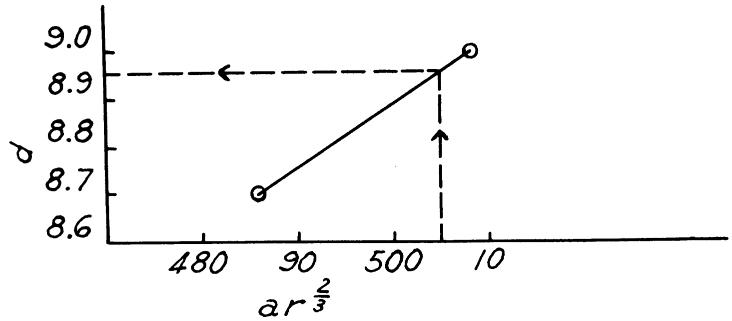
$$a = 20d; \quad r = \frac{20d}{20 + 2d}$$

Trial	d	a	20 + 2d	r	$r^{2/3}$	$ar^{2/3}$
1	8.0	160.0	36.0	4.45	2.71	434
2	8.7	174.0	37.4	4.65	2.79	486
3	9.0	180.0	38.0	4.74	2.82	508

Trials 2 and 3 bracket the value sought.

Example 3 - Continued

3. Plot d versus $ar^{2/3}$ for trials 2 and 3 as follows:



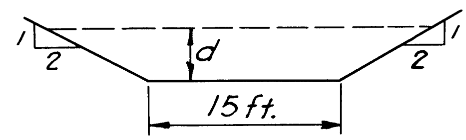
Enter this plot with $ar^{2/3}$ = 505 and read d = 8.95 ft.

4. $v = \frac{Q}{bd} = \frac{600}{8.95 \times 20} = 3.35$ ft./sec.

Note: "Hydraulic Tables" makes rapid computations possible in step 2.

EXAMPLE 4

Given: Trapezoidal section



$Q = 300$ c.f.s.
$n = 0.02$
$s = 0.0009$

To determine: d in ft. and v in ft./sec.

Solution by formula (5.4-32) in the form $Q = \frac{K'}{n} b^{8/3} s^{1/2}$ with King's Handbook tables:

1. $b^{8/3} = 15^{8/3} = 1370$, table 111, p. 326
2. $s^{1/2} = 0.0009^{1/2} = 0.03$
3. $K' = \dfrac{Qn}{b^{8/3}\ s^{1/2}} = \dfrac{300 \times 0.02}{1370 \times 0.03} = 0.146$
4. In table 113, p. 336, column for side slopes 2:1, find $K' = 0.150$ for $d/b = 0.23$ and $K' = 0.139$ for $d/b = 0.22$; by interpolation, $d/b = 0.226$ when $K' = 0.146$.
5. $d = 15 \times 0.226 = 3.39$ ft.
6. $v = Q/a = 300 \div [(3.39 \times 15) + (2 \times 3.39^2)] = 4.06$ ft./sec.

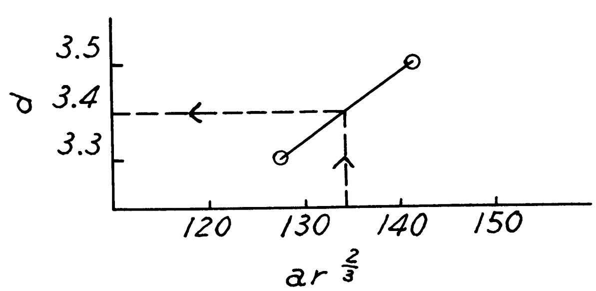
Solution using slide rule and formula (5.4-30):

1. $ar^{2/3} = \dfrac{Qn}{1.486\ s^{1/2}} = \dfrac{300 \times 0.02}{1.486 \times 0.03} = 134.5$
2. By assuming values of d, compute $ar^{2/3}$ for a value higher and lower than 134.5

   $a = 15d + 2d^2$; $p = 15 + 4.47d$; $r = (15d + 2d^2) \div (15 + 4.47d)$

Trial	d	a	15 + 4.47d	r	$r^{2/3}$	$ar^{2/3}$
1	3.0	63.0	28.42	2.21	1.698	107.0
2	3.5	77.0	30.65	2.51	1.847	142.0
3	3.3	71.3	29.76	2.39	1.788	127.5

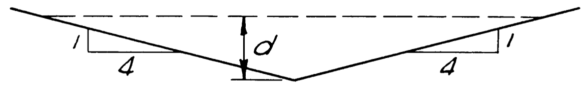
3. Plot d versus $ar^{2/3}$ for trials 2 and 3; where $ar^{2/3} = 134.5$, $d = 3.40$ ft.



4. $v = Q/a = 300 \div [(15 \times 3.4) + (2 \times 3.4^2)] = 300 \div 74.1 = 4.05$ ft./sec.

EXAMPLE 5

Given: Triangular section



$Q = 400$ c.f.s.
$n = 0.025$
$s = 0.005$

To determine: d in ft. and v in ft./sec.

Solution using formula (5.4-31).

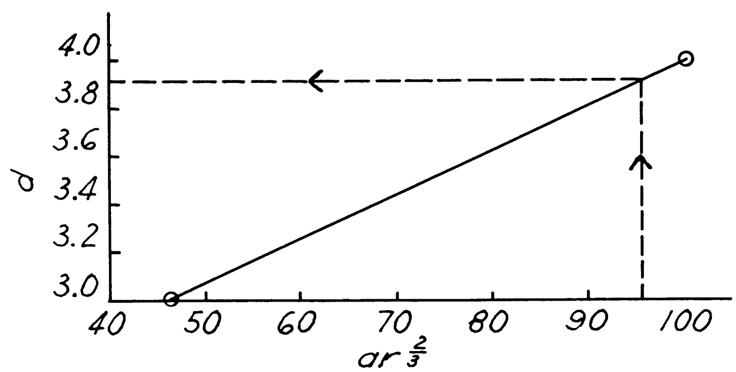
1. d/b = infinity
2. From King's Handbook, table 112, p. 335, col. $z = 4$, $K = 3.67$
3. $d^{8/3} = \frac{Qn}{K\ s^{1/2}} = \frac{400 \times 0.025}{3.67 \times 0.0707} = 38.55; \quad d = 38.55^{3/8} = 3.93$ ft.
4. $v = Q/a = 400 \div (4 \times 3.93^2) = 6.48$ ft./sec.

Solution using formula (5.4-30):

1. $ar^{2/3} = \frac{Qn}{1.486\ s^{1/2}} = \frac{400 \times 0.025}{1.486 \times 0.0707} = 95.3$
2. Compute the following table assuming d:
   $a = zd^2 = 4d^2; \ p = 2d\sqrt{z^2+1} = 2d\sqrt{17} = 8.24d; \ r = \frac{4d^2}{8.24d} = 0.485d$

d	a	r	$r^{2/3}$	$ar^{2/3}$
2	16	0.97	0.98	15.7
3	36	1.455	1.28	46.1
4	64	1.94	1.556	99.6

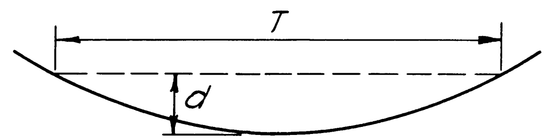
3. Plot $ar^{2/3}$ for $d = 3$ and $d = 4$; enter with $ar^{2/3} = 95.3$ and read $d = 3.92$ ft.



4. $v = Q/a = 400 \div (4 \times 3.92^2) = 6.50$ ft./sec.

EXAMPLE 6

Given: Parabolic section



$Q = 400$ c.f.s.
$n = 0.025$
$s = 0.005$

To determine: d and v when width of water surface T is 10 ft., 20 ft., and 30 ft.

Solution by formula (5.4-32) and King's Handbook tables:

1. $Q = \frac{K'}{n} T^{8/3} s^{1/2}$ or $K' = \frac{Qn}{T^{8/3} s^{1/2}}$

When T = 10, $K' = \frac{400 \times 0.025}{464 \times 0.0707} = 0.305$

When T = 20, $K' = \frac{400 \times 0.025}{2950 \times 0.0707} = 0.048$

When T = 30, $K' = \frac{400 \times 0.025}{8690 \times 0.0707} = 0.0163$

2. From table 118, p. 358, by interpolation

When K' = 0.305, d/T = 0.746 and d = 10 x 0.746 = $\underline{7.46}$ ft.
When K' = 0.048, d/T = 0.198 and d = 20 x 0.198 = $\underline{3.96}$ ft.
When K' = 0.0163, d/T = 0.1002 and d = 30 x 0.1002 = $\underline{3.01}$ ft.

3. $a = (2/3)Td$ $\quad v = Q \div a$

$v = 400 \div (2/3 \times 10 \times 7.46) = \underline{8.03}$ ft./sec.
$v = 400 \div (2/3 \times 20 \times 3.96) = \underline{7.60}$ ft./sec.
$v = 400 \div (2/3 \times 30 \times 3.01) = \underline{6.65}$ ft./sec.

Solution by formula (5.4-30):

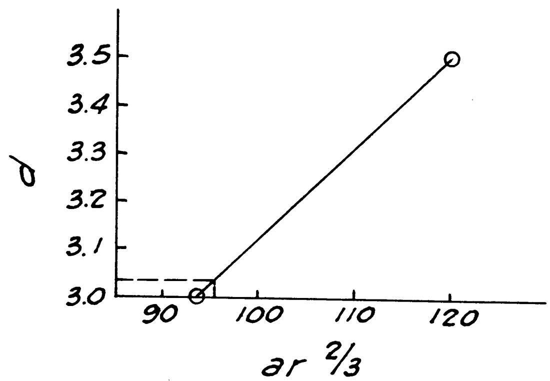
1. $ar^{2/3} = \frac{Qn}{1.486 \times s^{1/2}} = \frac{400 \times 0.025}{1.486 \times 0.0707} = 95.3$

2. Compute the following table for T = 30

$a = (2/3)Td = 20d$ $\qquad r = \frac{T^2 d}{1.5T^2 + 4d^2} = \frac{900 d}{1350 + 4d^2}$

d	a	r	$r^{2/3}$	$ar^{2/3}$
2	40	1.318	1.202	48.1
3	60	1.945	1.558	93.5
3.5	70	2.255	1.72	120.4

3. Plot d versus $ar^{2/3}$, enter with $ar^{2/3}$ = 95.3 and read d = 3.02 ft.

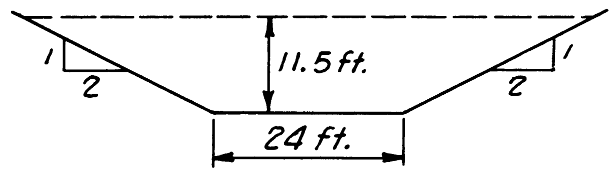


4. $v = \frac{Q}{(2/3)dT} = 400 \div (20 \times 3.02) = 6.63$ ft./sec.

EXAMPLE 7

Given: A concrete lined floodway

Trapezoidal section



$n = 0.016$
$s = 0.005$

To determine:

(a) Maximum Q when the freeboard is 1.0 ft., i.e., d = 10.5 ft.

(b) The depth, d, for any Q less than maximum.

Solution by formula (5.4-30):

1. Compute a table as follows:

Columns	1	2	3	4	5	6
	d	r	$r^{2/3}$	a	$ar^{2/3}$	Q
	10.5	6.66	3.54	472.5	1673.	11,000.
	8.5	5.62	3.16	348.5	1101.	7,230.
	6.4	4.48	2.72	235.52	641.	4,210.
	4.4	3.30	2.22	144.32	320.	2,100.
	2.4	1.99	1.58	69.12	109.2	717.
	1.4	1.24	1.15	37.52	43.1	283.
	0	0	0	0	0	0

Col. 1: Assume values of d.

Col. 2: From "Hydraulic Tables", table 10, p. 164.

Col. 3: From "Hydraulic Tables", table 19, p. 294, or "King's Handbook", table 109, p. 312.

Col. 4: From "Hydraulic Tables", table 10, p. 164.

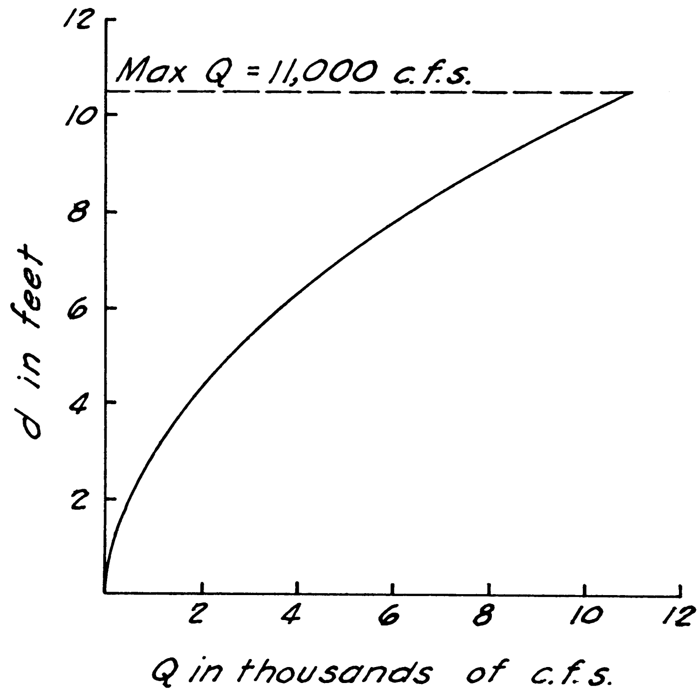
Col. 5: Products of the values in cols. 3 and 4 by slide rule or calculating machine if preferred.

Col. 6: $Q = ar^{2/3} \frac{1.486}{n} s^{1/2}$, $\frac{1.486}{n} s^{1/2} = \frac{1.486}{0.016} \times 0.0707 = 6.57$;

therefore, the values in col. 6 = the values in col. 5 x 6.57.

2. Maximum Q = 11,000 c.f.s.

3. Plot d versus Q, from which d for any given Q may be read.



Solution using formula (5.4-1) and "Hydraulic Tables", taking n as 0.0175 instead of 0.016:

1. Compute a table as follows:

Columns	1	2	3	4	5
	d	r	a	v	Q
	10.5	6.66	472.5	21.26	10,050.
	8.5	5.62	348.5	18.97	6,620.
	6.4	4.48	235.52	16.32	3,840.
	4.4	3.30	144.32	13.31	1,920.
	2.4	1.99	69.12	9.50	656.
	1.4	1.24	37.52	6.96	261.
	0	0	0	0	0

Col. 1: Assume values of d.

Col. 2: Tabulate values of r from table 10, p. 164.

Col. 3: Tabulate values of a from table 10, p. 164.

Col. 4: Tabulate values of v for r, s, and n from table 28, p. 378. Interpolation is required.

Col. 5: Product of the values in cols. 3 and 4 (Q = av).

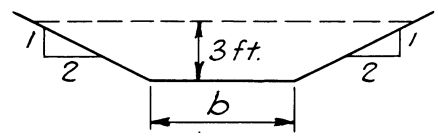
2. Maximum Q = 10,050 c.f.s.
3. From the table developed in step 1, plot a graph of d versus Q, from which d for any given Q may be read.

Note: Cols. 1, 2, and 3 could be tabulated as above, and v in col. 4 computed by the alignment chart, drawing ES-34.

EXAMPLE 8

Given: Trapezoidal section

$Q = 500$ c.f.s.
$s = 0.009$
$n = 0.025$



To determine: b in ft. and v in ft./sec.

Solution using formula (5.4-31) with "King's Handbook" tables:

1. $K = \dfrac{Qn}{d^{8/3}\ s^{1/2}} = \dfrac{500 \times 0.025}{3^{8/3} \times 0.009^{1/2}} = \dfrac{500 \times 0.025}{18.7 \times 0.0949} = 7.04$

2. In table 112, p. 331, column for z = 2-1, find K = 7.08 for d/b = 0.25, and K = 6.87 for d/b = 0.26. Interpolating d/b = 0.252 for K = 7.04

   $b = 3.0 \div 0.252 = 11.90$ ft.

3. $v = \dfrac{Q}{bd + zd^2} = 500 \div [(11.90 \times 3.0)+(2 \times 9)] = 9.31$ ft./sec.

Solution using formula (5.4-30):

1. $ar^{2/3} = \dfrac{Qn}{1.486\ s^{1/2}} = (500 \times 0.025) \div (1.486 \times 0.0949) = 88.8$

2. Compute the following table:

Columns	1	2	3	4	5
	b	a	r	$r^{2/3}$	$ar^{2/3}$
	10	48.00	2.05	1.613	77.4
	12	54.00	2.12	1.650	89.1

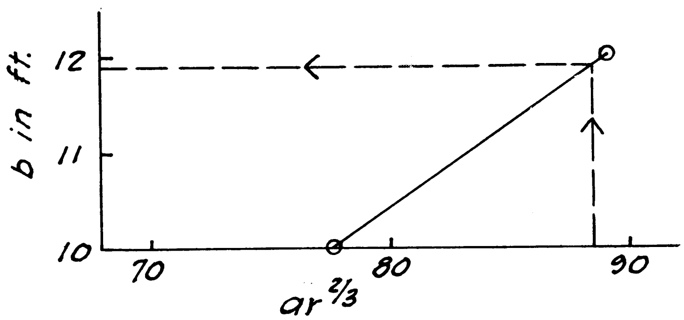
Col. 1: Assume values of b

Cols. 2 and 3: Values of a and r from "Hydraulic Tables", table 10, p. 163. These values may be computed if tables are not available.

Col. 4: From "Hydraulic Tables", table 19, or from "King's Handbook", table 109.

Col. 5: Product of cols. 2 and 4.

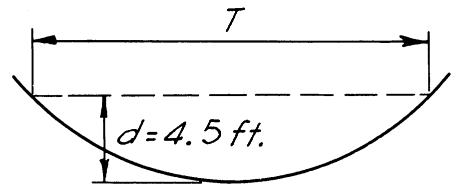
3. Plot b versus $ar^{2/3}$, enter with $ar^{2/3} = 88.8$, and read b = 11.90 ft.



4. $v = \frac{Q}{a} = 500 \div [(11.90 \times 3)+(2 \times 9)] = 9.31$ ft./sec.

Note: The solutions for b and v in rectangular sections are similar to those given above.

EXAMPLE 9



Given: Parabolic section

$Q = 600$ c.f.s.
$n = 0.03$
$s = 0.009$

To determine: T in ft. and v in ft./sec.

Solution by formula (5.4-31) using King's Handbook:

1. $K = \dfrac{Qn}{d^{8/3} \times s^{1/2}} = \dfrac{600 \times 0.03}{55.2 \times 0.09487} = 3.44$

2. In table 117, p. 358, by interpolation, $d/T = 0.2058$

   $T = 4.5 \div 0.2058 = 21.8$ ft.

3. $v = \dfrac{Q}{a} = 600 \div (2/3 \times 21.8 \times 4.5) = 9.2$ ft./sec.

This may also be solved by formula (5.4-30) by:

1. Compute $ar^{2/3} = \dfrac{Qn}{1.486\ s^{1/2}} = 127.7$

2. Holding d constant and assuming T, compute values of $ar^{2/3}$ above and below 127.7.

3. Plotting T versus $ar^{2/3}$ and entering the plot with 127.7 to find T.

4. $v = \dfrac{Q}{2/3\ dT} = \dfrac{600}{3T}$

Cases Where Both d and b or d and T are Required

In conservation work, cases are frequently encountered in which Q, n, s, and shape of cross section of a waterway are known but where the allowable velocity is limited by the soils at the waterway site. These cases require the determination of channel dimensions and the solution is tedious if approached on a trial and error basis. The general approach to these problems is given in the following 3 steps:

Step 1. Compute the required r by Manning's formula from the known n, v, and s.

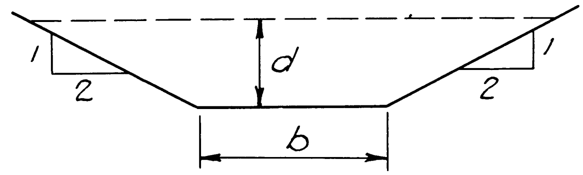
Step 2. Compute the required area by $a = Q/v$.

Step 3. Express a and r in terms of the channel dimensions z, d, b, or T, and solve for the dimensions by these two simultaneous equations. Only two unknowns are involved in any case. In trapezoidal channels z is selected or known, leaving b and d as the required values. In rectangular and triangular channels the two unknowns are b and d, and z and d, respectively. Parabolic channels require that T and d be determined.

EXAMPLE 10

Given: Trapezoidal section

$Q = 500$ c.f.s.
$v = 4.0$ ft./sec.
$n = 0.03$
$s = 0.005$



To determine: b and d in ft.

1. $$r = \left(\frac{vn}{1.486\ s^{1/2}}\right)^{3/2} = \left(\frac{4.0 \times 0.03}{1.486 \times 0.07071}\right)^{3/2} = 1.22$$

2. $a = Q/v = 500 \div 4.0 = 125$ ft.^{2}

3. Completing step 3 (see preceding page) gives a general equation for depth in trapezoidal channels:

$$d = \frac{\frac{x}{r} \pm \sqrt{\left(\frac{x}{r}\right)^2 - 4x}}{2} \qquad (5.4\text{-}41)$$

x varies with z and a and is to be evaluated by:

$$x = \frac{a}{2\sqrt{z^2+1} - z} \qquad (5.4\text{-}42)$$

(a) Evaluating x:

$$x = \frac{125}{2\sqrt{5} - 2.0} = \frac{125}{2.47} = 50.61$$

(b) Computing d:

$$d = \frac{\frac{50.61}{1.22} \pm \sqrt{\left(\frac{50.61}{1.22}\right)^2 - (4 \times 50.61)}}{2} = \frac{41.50 \pm 39.00}{2} = \underline{1.25} \text{ or } 40.25$$

(d = 40.25 would obviously not give a practical channel section.)

(c) Compute b:

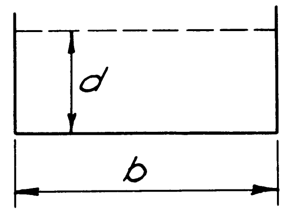
$$b = \frac{a}{d} - zd = \frac{125}{1.25} - (2 \times 1.25) = 97.50 \text{ ft.}$$

(d) Check:

$$r = \frac{bd + zd^2}{b + 2d\sqrt{z^2+1}} = \frac{(97.5 \times 1.25) + 2(1.25)^2}{97.5 + 2.5\sqrt{5}} = \frac{125.13}{103.1} = \underline{1.21} \quad \text{O.K.}$$

EXAMPLE 11

Given: Rectangular section



$Q = 300$ c.f.s.
$v = 5.0$ ft./sec.
$s = 0.005$
$n = 0.03$

To determine: b and d in ft.

1. $$r = \left(\frac{vn}{1.486\ s^{1/2}}\right)^{3/2} = \left(\frac{5.0 \times 0.03}{1.486 \times 0.07071}\right)^{3/2} = (1.428)^{3/2} = 1.705$$

2. $a = Q/v = 300 \div 5.0 = 60$ ft.^{2}

3. Compute d and b. See step 3, example 10:

$$d = \frac{\frac{x}{r} \pm \sqrt{\left(\frac{x}{r}\right)^2 - 4x}}{2}$$

In rectangular sections $x = \frac{a}{2} = \frac{60}{2} = 30$

$$d = \frac{\frac{30}{1.705} \pm \sqrt{310 - 120}}{2} = \frac{17.6 \pm 13.8}{2} = \underline{1.90} \quad \text{or} \quad 15.70$$

(d = 15.7 is not a practical section.)

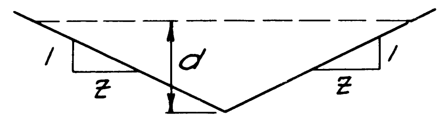
$b = a/d = 60 \div 1.90 = \underline{31.6}$

Check:

$$r = \frac{bd}{b + 2d} = \frac{31.6 \times 1.90}{31.6 + (2 \times 1.90)} = \frac{60}{35.4} = \underline{1.695} \quad \text{O.K.}$$

EXAMPLE 12

Given: Triangular section



$Q = 300$ c.f.s.
$v = 6.0$ ft./sec.
$n = 0.03$
$s = 0.0075$

To determine: d and z

1. $r = \left(\frac{vn}{1.486\ s^{1/2}}\right)^{3/2} = \left(\frac{6.0 \times 0.03}{1.486 \times 0.0866}\right)^{3/2} = (1.40)^{3/2} = 1.656$

2. $a = Q/v = 300 \div 6.0 = 50$ ft.^{2}

3. This step in the solution for triangular sections is to be accomplished by:

   (a) Entering the graph on drawing ES-39 with the value of a/r^2 and reading z.

   (b) Computing d from $d = \sqrt{\frac{a}{z}}$

Carrying out the solution:

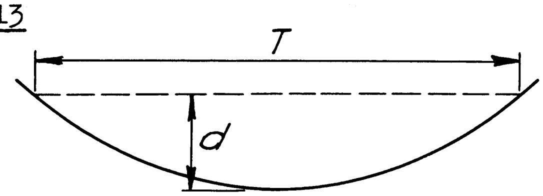
(a) $a/r^2 = 50 \div (1.656)^2 = 18.25$. From drawing ES-39, $z = \underline{4.30}$

(b) $d = \sqrt{\frac{a}{z}} = \sqrt{\frac{50}{4.3}} = \sqrt{11.63} = \underline{3.41}$

Check:

$$r = \frac{zd}{2\sqrt{z^2 + 1}} = \frac{4.3 \times 3.41}{2 \times 4.41} = \underline{1.66} \quad \text{O.K.}$$

EXAMPLE 13



Given: Parabolic section

$Q = 300$ c.f.s.
$v = 5.0$ ft./sec.
$n = 0.035$
$s = 0.008$

To determine: d and T

1. $$r = \left(\frac{vn}{1.486\ s^{1/2}}\right)^{3/2} = \left(\frac{5.0 \times 0.035}{1.486 \times 0.08944}\right)^{3/2} = (1.317)^{3/2} = 1.51$$

2. $a = Q/v = 300 \div 5.0 = 60$ ft.^{2}

3. This step in the solution of parabolic channels is to be accomplished by:

   (a) Entering the graph on drawing ES-41 with the value of a/r^2 and reading the value of $x = d/T$.

   (b) Computing T from $T = \sqrt{\frac{3a}{2x}}$

   (c) Computing d from $d = xT$

Carrying out the solution:

(a) $a/r^2 = 60 \div 1.51^2 = 26.3$. From drawing ES-41, $x = 0.058$.

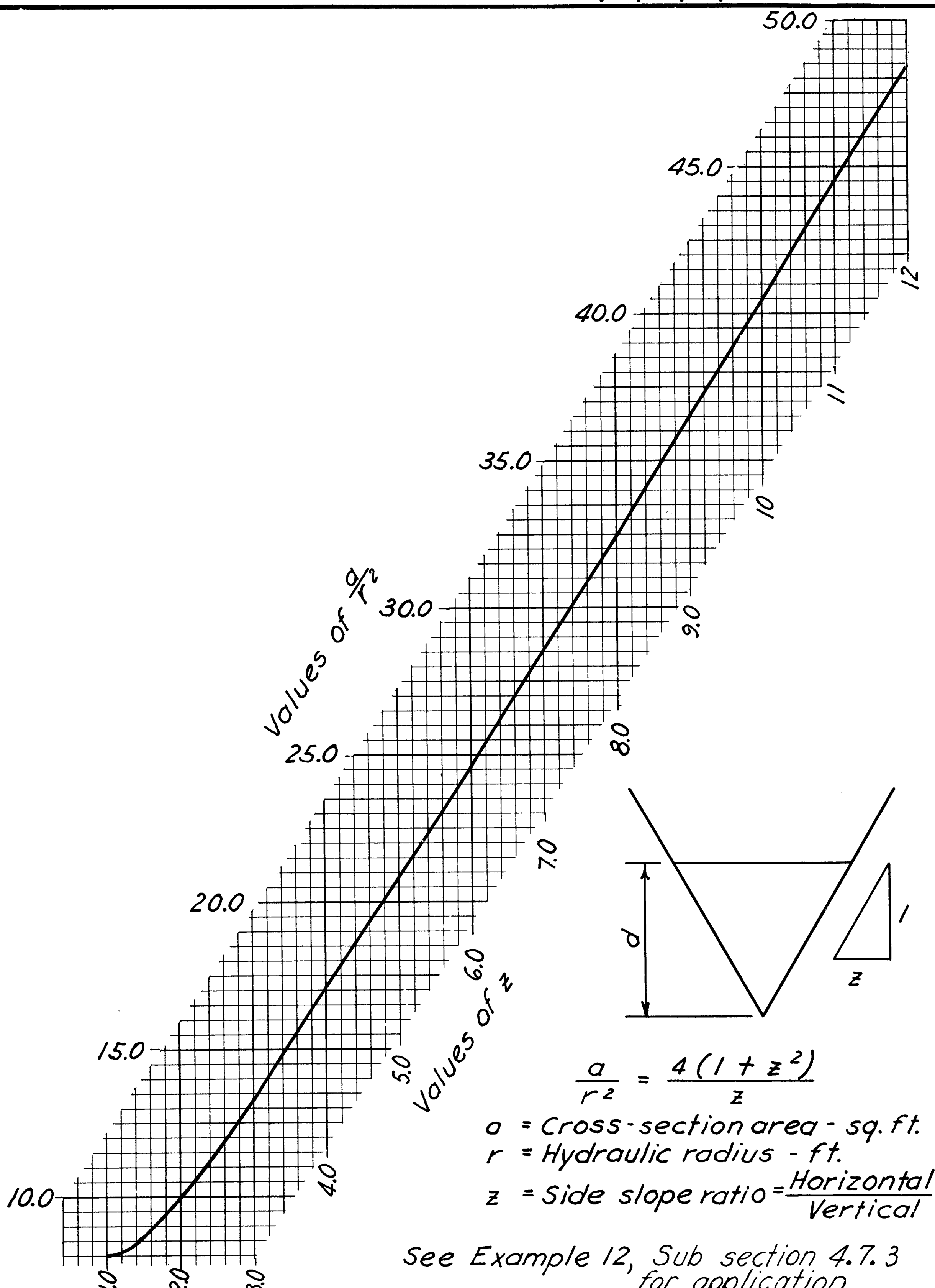
(b) $$T = \sqrt{\frac{3a}{2x}} = \sqrt{\frac{180}{0.116}} = \sqrt{1550} = 39.4 \text{ ft.}$$

(c) $d = x\,T = 0.058 \times 39.4 = 2.29$ ft.

Check:

$$r = \frac{2dT^2}{3T^2 + 8d^2} = \frac{4.58 \times 1550}{(3 \times 1550)+(8 \times 5.24)} = \frac{7110}{4691.9} = \underline{1.51} \quad \text{O.K.}$$

HYDRAULICS: GRAPH FOR DETERMINING SIDE SLOPE z OF A TRIANGULAR CHANNEL WITH Q, v, n, s, GIVEN



$$\frac{a}{r^2} = \frac{4(1 + z^2)}{z}$$

a = Cross-section area - sq. ft.

r = Hydraulic radius - ft.

z = Side slope ratio = $\frac{\text{Horizontal}}{\text{Vertical}}$

See Example 12, Sub section 4.7.3 for application

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 39 SHEET 1 OF 1 DATE 7-17-50

HYDRAULICS: GRAPH FOR DETERMINING DIMENSIONS OF A PARABOLIC CHANNEL WITH Q, v, n, s GIVEN

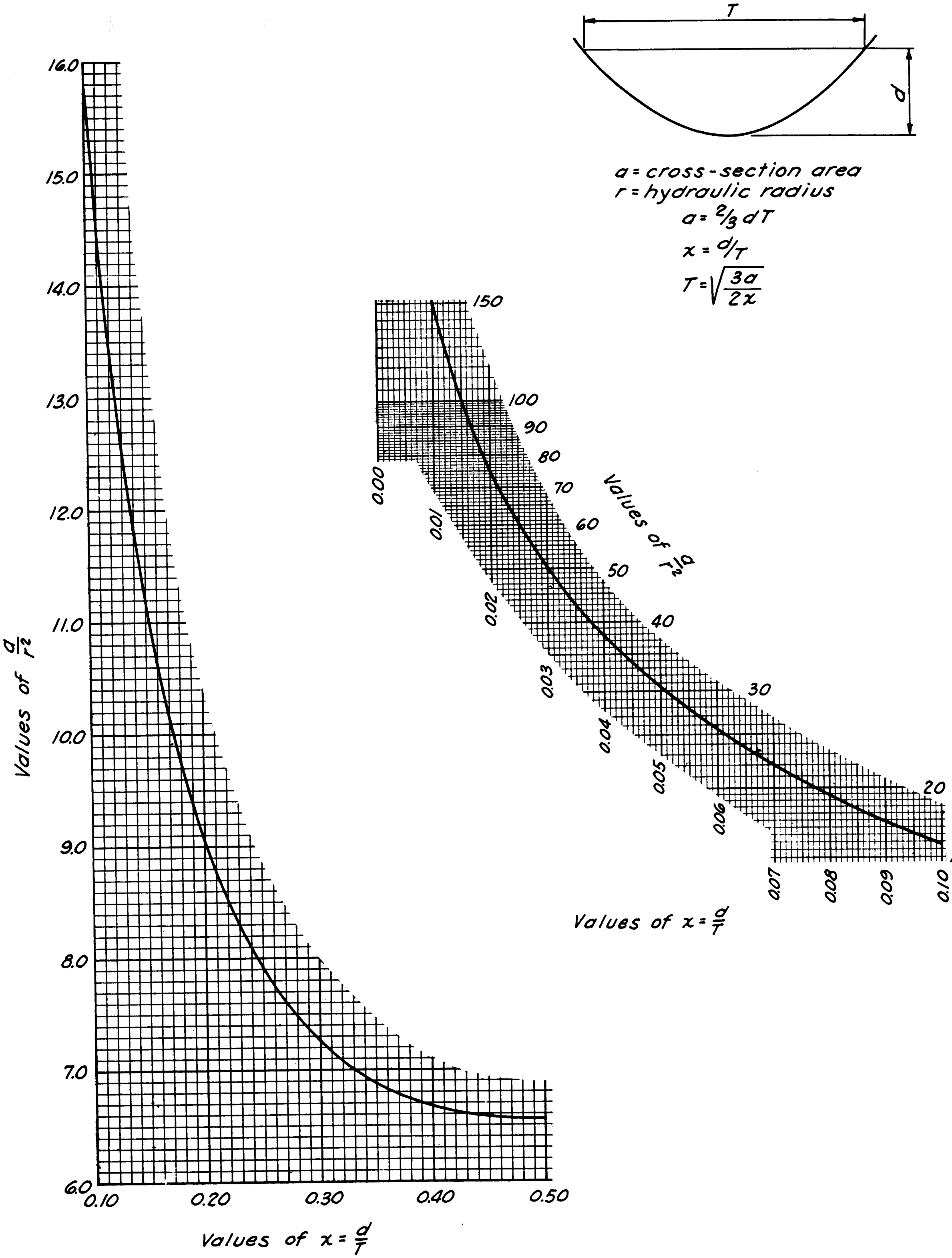
See Example 13, Subsection 4.7.3 for application

a = cross-section area
r = hydraulic radius

$a = \frac{2}{3} dT$

$x = d/T$

$T = \sqrt{\frac{3a}{2x}}$



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-41 SHEET 1 OF 1 DATE 7-21-50

Revised 12-27-50

4.7.4 Examples - Nonuniform Flow in Uniform Channels.

EXAMPLE 1

Given: Concrete, trapezoidal channel, $Q = 1000$ c.f.s., $z = 1.0$, $b = 10$ ft., $n = 0.014$. Station numbers increase in the direction of flow. A break in grade is located at Sta. 33 + 50; the slope upstream from 33 + 50, designated as s_1, is 0.001; the slope downstream from 33 + 50, designated as s_2, is 0.004. A profile of the channel is shown at the end of this example.

To determine: The water surface profile upstream and downstream from the break in grade at Sta. 33 + 50 by computing distance to selected depths using formula (5.4-36).

Solution: (See subsection 4.7.2 and drawing ES-38.)

1. Determine whether Sta. 33 + 50 is a control section.

   (a) Compute critical depth:

   $$Q/b = 1000 \div 10 = 100, \text{ and } z/b = 1 \div 10 = 0.1$$

   From drawing ES-24, $d_c = 5.58$ ft.

   Or using King's Handbook, formula (57), p. 383,

   $$K'_c = \frac{Q}{b^{5/2}} = \frac{1000}{10^{5/2}} = \frac{1000}{316.2} = 3.162$$

   From table 125, p. 436, by interpolation, when $K'_c = 3.162$, $d_c/b = 0.558$ and $d_c = 10 \times 0.558 = 5.58$.

   (b) Compute critical slope: $s_c = 14.56 \dfrac{n^2\ d_m}{r^{4/3}}$

   $$d_m = \frac{a}{T} = \frac{(b + d_c)d_c}{b + 2d_c} = \frac{(10 + 5.58)5.58}{10 + (2 \times 5.58)} = 4.11$$

   $$r = \frac{(b + d_c)d_c}{b + 2d\sqrt{2}} = \frac{(10 + 5.58)5.58}{10 + (2.83 \times 5.58)} = 3.37$$

   $$s_c = \frac{14.56 \times 0.014^2 \times 4.11}{3.37^{4/3}} = \frac{0.0117}{5.05} = 0.00232$$

   $0.001 < 0.00232 < 0.004$; i.e., $s_1 < s_c < s_2$; therefore, Sta. 33 + 50 is a control section.

   Or enter alignment chart, drawing ES-34, v_c, r, n, and read s_c. Use values r and v_c corresponding to d_c.

2. Compute the normal depths (uniform flow) on the slopes upstream and downstream from 33 + 50. This is not necessary, but knowing these depths, facilitates the selections of reaches and trial depths in the computation of the surface profile.

   Use formula (5.4-32) in the form $K' = \dfrac{Qn}{b^{8/3}\ s^{1/2}}$ and King's Handbook, table 113, p. 336.

   (a) For s = 0.001, $K' = \dfrac{1000 \times 0.014}{10^{8/3}\ s^{1/2}} = \dfrac{0.03018}{0.001^{1/2}} = 0.955$

   1:1 side slopes, d/b = 0.698, d = 6.98

   (b) For s = 0.004, $K' = 0.03018 \div 0.004^{1/2} = 0.477$

   d/b = 0.482, d = 4.82

   Upstream from 33 + 50 the depth increases from 5.58(d_c) to 6.98; downstream the depth decreases to 4.82. See Example 4, subsection 4.7.3 for other methods of determining normal depth.

3. Compute the water surface profile downstream and upstream from Sta. 33 + 50 by computing distances to selected depths using formula (5.4-36).

$$\ell = \frac{\left(\frac{v_2^2}{2g} + d_2\right) - \left(\frac{v_1^2}{2g} + d_1\right)}{s_o - s_f}$$

The table on the following page lists the computations which are carried downstream and upstream because depths are respectively less than and greater than critical.

(a) Cols. 5, 6, and 7 record velocity, velocity head, and specific energy corresponding to the selected depths in col. 4.

(b) The differences in specific energy at successive sections are tabulated in parentheses. This is the numerator of formula (5.4-36).

(c) Computations for the friction slope, s_f, between two successive sections are tabulated in cols. 8, 9, and 10. Friction slope is evaluated by the equation,

$$s_f = \left[(Qn) \div b^{8/3}\right]^2 \quad (1 \div K')^2$$

where $(1/K')^2$ is determined for the average depth (d_a) between successive sections. Tabulated values for $(1/K')^2$ corresponding to d_a/b are taken from "King's Handbook", table 114, pp. 341-356.

(d) Col. 11 is the denominator of formula (5.4-36) and is the difference between friction slope and slope of channel bottom.

(e) Col. 12 lists ℓ, determined by dividing the values in parentheses in col. 7 by the values in col. 11.

Remarks: If reasonably accurate results are to be obtained in computing ℓ by equation (5.4-36), it will frequently be necessary to evaluate $H_e = d + (v^2 \div 2g)$ to the nearest 4th or 5th decimal place. Note that this may be necessary to obtain significant figures in both the numerator and denominator.

COMPUTATIONS FOR WATER SURFACE PROFILE

Example 1 - Subsection 4.7.4

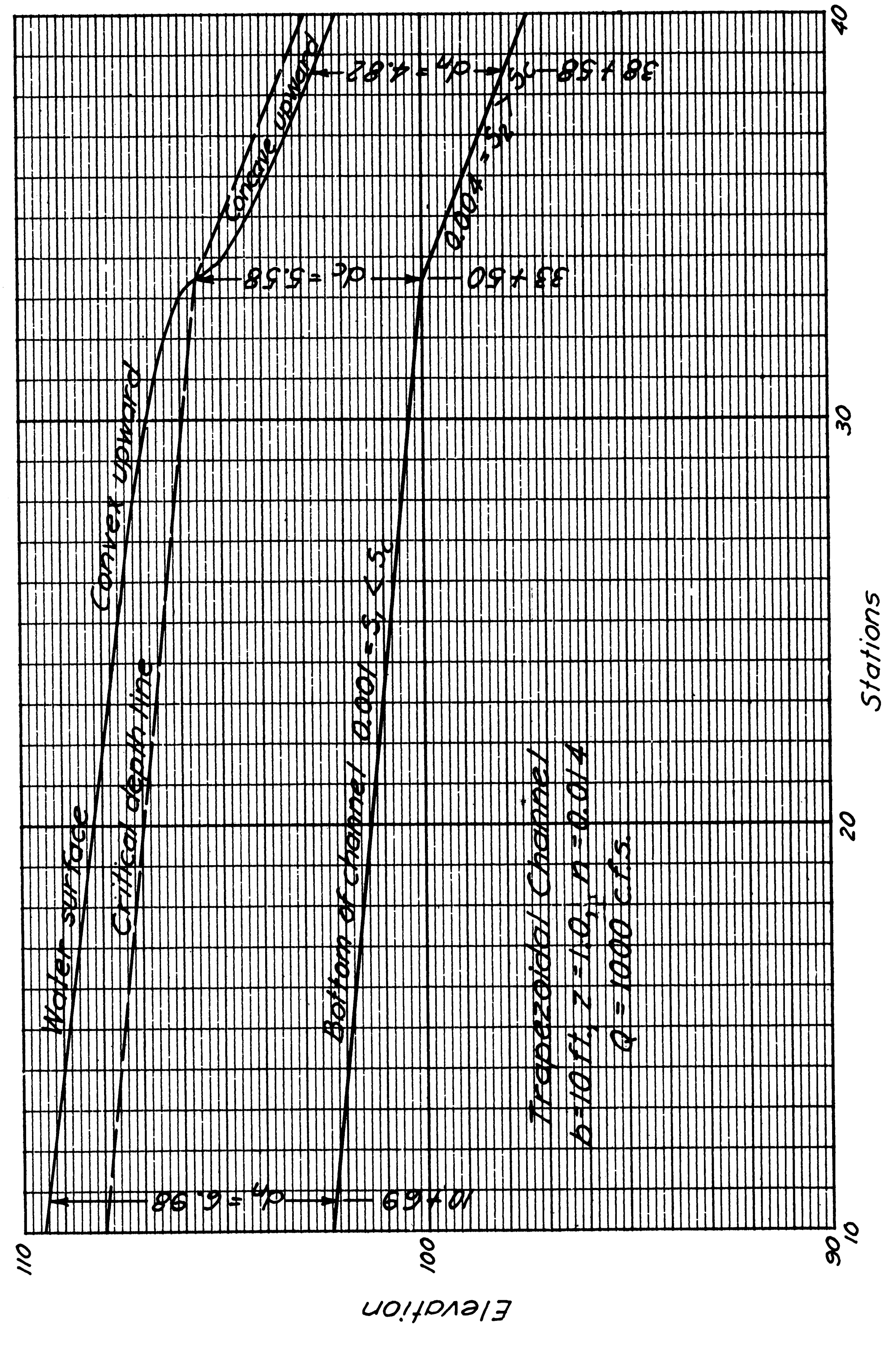
1	2	3	4	5	6	7	8	9	10	11	12
Sta.	Elevation: Bottom Channel	Elevation: Water Surface	d	v	$\frac{v^2}{2g}$	$H_e = d + \frac{v^2}{2g}$	$\frac{d_a}{b}$	$\left(\frac{1}{K'}\right)^2$	s_f	$s_o - s_f$	ℓ
Downstream from Sta. 33 + 50											
33+50	100.00	105.58	5.58	11.503	2.057	7.637					
						(0.011)	0.549	2.71	0.00247	0.00153	7
33+57	99.97	105.37	5.40	12.025	2.248	7.648					
						(0.041)	0.530	3.09	0.00281	0.00119	34
33+91	99.84	105.04	5.20	12.652	2.489	7.689					
						(0.075)	0.510	3.56	0.00324	0.00076	99
34+90	99.44	104.44	5.00	13.333	2.764	7.764					
						(0.103)	0.491	4.09	0.00372	0.00028	368
38+58	97.97	102.79	4.82	13.999	3.047	7.867					
Upstream from Sta. 33 + 50											
33+50	100.00	105.58	5.58	11.503	2.057	7.637					
						−(0.014)	0.569	2.38	0.00217	−0.00117	12
33+38	100.01	105.81	5.80	10.912	1.851	7.651					
						−(0.036)	0.590	2.07	0.00188	−0.00088	41
32+97	100.05	106.05	6.00	10.417	1.687	7.687					
						−(0.0873)	0.615	1.78	0.00162	−0.00062	141
31+56	100.19	106.49	6.30	9.738	1.4743	7.7743					
						−(0.1208)	0.645	1.48	0.00135	−0.00035	345
28+11	100.54	107.14	6.60	9.127	1.2951	7.8951					
						−(0.1916)	0.679	1.22	0.00111	−0.00011	1742
10+69	102.28	109.26	6.98	8.437	1.1067	8.0867					

Q = 1000 c.f.s. $\quad$ z = 1.0

b = 10 ft. $\quad$ s_1 = 0.001

n = 0.014 $\quad$ s_2 = 0.004

$$\left(\frac{Qn}{b^{8/3}}\right)^2 = 0.00091 \qquad s_f = \left(\frac{Qn}{b^{8/3}}\right)^2 \left(\frac{1}{K'}\right)^2$$



PROFILE

Example 1, Subsection 4.7.4

EXAMPLE 2

Given: A concrete, rectangular channel, Q = 1240 cfs., b = 20 ft., n = 0.014. At sta. 27 + 30 the channel slope changes from 0.018 to 0.0015. The slope of 0.0015 is of sufficient length to assure that uniform flow will occur in the reach downstream from 27 + 30. Bottom of channel elevation at 27 + 30 is 100.00. A profile is shown at the end of this example.

To determine: The location of the hydraulic jump when depth of flow at sta. 27 + 30 is 3.1 ft.

Solution: (See subsection 4.6.2 and drawing ES-38.)

1. Compute the tailwater curve for the flow in the reach downstream from 27 + 30. One of the given conditions is that this reach is long enough to assure the occurrence of uniform flow; therefore, the tailwater curve is a straight line parallel to the channel bottom and at the depth, d_n, for 1240 cfs.

   (a) Compute d_n using formula (5.4-32):

   $$K' = \frac{Qn}{b^{8/3}\ s^{1/2}} = \frac{1240 \times 0.014}{20^{8/3} \times (0.0015)^{1/2}} = \frac{17.36}{114.2} = 0.152$$

   From King's Handbook, table 113, p. 336, by interpolating, find

   $d/b = 0.309$; therefore, $d_n = 20 \times 0.309 = \underline{6.18}$ ft.

2. Compute and plot the water surface profile downstream from 27 + 30 starting with d = 3.10. The jump must occur before the depth has increased to d_c.

   $$d_c = (q^2 \div g)^{1/3} = (Q \div b)^{2/3} \div g^{1/3} = (1240 \div 20)^{2/3} \div 32.2^{1/3} = 4.93$$

   d_c may also be computed from drawing ES-24.

   Use formula (5.4-36):

   $$\ell = \frac{\left(\frac{v_2^2}{2g} + d_2\right) - \left(\frac{v_1^2}{2g} + d_1\right)}{s_o - s_f}$$

   The table on the following page lists the computations:

   Col. 1 starts with sta. 27 + 30 given; other stations are obtained by accumulating the computed distances in col. 12.

   Cols. 2, 3, 5, and 6 are self-explanatory.

COMPUTATION OF WATER SURFACE PROFILE

Example 2 - Subsection 4.7.4

Rectangular Channel:

Q = 1240 c.f.s. n = 0.014
b = 20 ft. s = 0.0015

1	2	3	4	5	6	7	8	9	10	11	12
Sta.	Bottom Channel Elev.	Water Surface Elev.	d	v	$\frac{v^2}{2g}$	$d + \frac{v^2}{2g}$	$\frac{d_a}{b}$	$\left(\frac{1}{K'}\right)^2$	s_f	$s_o - s_f$	ℓ
27+30	100.00	103.10	3.10	20.000	6.219	9.319					
						(0.532)	0.160	295	0.01026	0.00876	61
27+91	99.91	103.21	3.30	18.788	5.487	8.787					
						(0.409)	0.170	246	0.00856	0.00706	58
28+49	99.82	103.32	3.50	17.714	4.878	8.378					
						(0.439)	0.1825	199	0.00694	0.00544	81
29+30	99.70	103.50	3.80	16.316	4.139	7.939					
						(0.351)	0.200	152	0.00529	0.00379	93
30+23	99.55	103.75	4.20	14.762	3.388	7.588					
						(0.194)	0.225	107	0.00372	0.00222	87
31+10	99.43	104.23	4.80	12.917	2.594	7.394					

$$s_f = \left(\frac{Qn}{b^{8/3}}\right)^2 \left(\frac{1}{K'}\right)^2 \qquad \left(\frac{Qn}{b^{8/3}}\right)^2 = \left(\frac{1240 \times 0.014}{2950}\right)^2 = 0.0000348$$

Col. 4 starts with d = 3.10 given; other values of d are selected so as to hold the change in velocity, v, about 10 to 15 percent.

s_f is evaluated by formula (5.4-40) which may be written,

$$s_f = \left(\frac{1}{K'}\right)^2 \left(\frac{Qn}{b^{8/3}}\right)^2$$

Col. 7 lists specific energy, with the differences in specific energy at successive sections shown in parentheses.

Col. 8 lists the average of appropriate pairs of depths in col. 4 divided by bottom width.

Col. 9 lists $(1/K')^2$ for the d_a/b values found in King's Handbook, table 114, p. 341.

Col. 10 lists the values in col. 9 multiplied by the constant,

$$\left(\frac{Qn}{b^{8/3}}\right)^2 = 0.0000348$$

Col. 11 is self-explanatory.

Col. 12 is computed by dividing the values in parentheses in col. 7 by the values in col. 11.

3. Compute and plot a curve of sequent depths, i.e., depths after jump corresponding to those shown by the profile computed in step 2. These depths after jump may be computed by formula (5.4-28) or they may be determined by interpolation using "Hydraulic Tables", table 3, p. 16, or "King's Handbook", table 133, p. 444.

Sta.	d_1	v_1	d_2	Bottom Channel Elev.	Sequent Depth Elev.
27 + 30	3.10	20.00	7.37	100.00	107.37
27 + 91	3.30	18.788	7.02	99.91	106.93
28 + 49	3.50	17.714	6.70	99.82	106.52
29 + 30	3.80	16.316	6.26	99.70	105.96
30 + 23	4.20	14.762	5.73	99.55	105.28
31 + 10	4.80	12.917	5.06	99.43	104.49

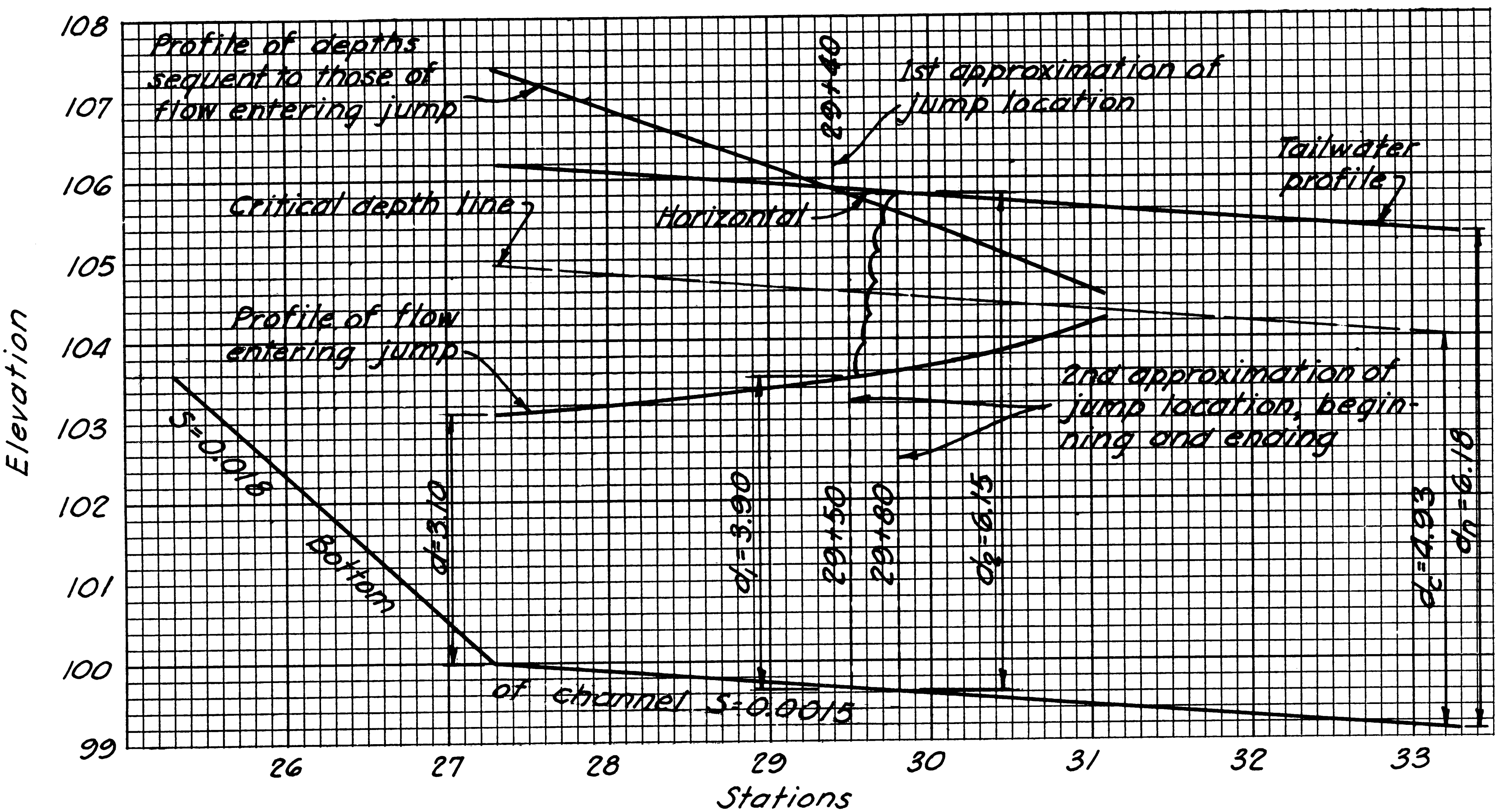
4. The profiles of tailwater, flow entering jump, and sequent depths are plotted at the end of this example. Note that the vertical scale is exaggerated in relation to the horizontal scale.

(a) An approximate location of the jump is sta. 29 + 40 where the profiles of tailwater and sequent depths intersect. On the profile shown on the following page, this is noted as the first approximate jump location.

(b) A second approximation of the jump location, determined by the procedure in step d, subsection 4.6.2, is also shown on the profile. At 29 + 50 the depth of flow entering the jump, d_1, is 3.90; and at 29 + 80 the tailwater depth, d_2, is about 6.15, which is the depth sequent to 3.90, making the length of the jump 30 to 35 ft. or approximately 5 times 6.15.

Note that the accuracy with which water surface profiles may be determined depends on: (1) whether the roughness coefficient, n, accurately represents the condition of the channel; (2) whether the friction losses, s_f, are carefully evaluated. Since the position of the jump depends on the profile of flow entering the jump and the tailwater profile, it is apparent that a minor error in n may result in an appreciable error in the determination of jump location. In short, the estimated position of a jump in a channel not designed to stabilize the jump should be accepted only as an approximation.

Rectangular concrete channel
b= 20 ft.; n=0.014; Q=1240 c.f.s.



PROFILE

Example 2, Subsection 4.7.4

4.7.5 Examples - Nonuniform Channels and Natural Channels.

EXAMPLE 1

Given: A rectangular, concrete spillway. Maximum discharge 2000 cfs; roughness coefficient, 0.014. Width of the channel varies uniformly from 30 ft. at entrance to 20 ft. at crest. A profile is shown on the computation sheet.

To determine: The water surface profile from the crest upstream to the spillway entrance and the pool elevation for the discharge 2000 cfs.

Solution: (See subsection 4.7.2) The slope downstream from the crest, sta. 1 + 00, is given as greater than s_c for 2000 cfs; therefore, critical depth occurs at 1 + 00. The water surface profile rises in the upstream direction from the crest through the nonuniform width inlet. The pool elevation for 2000 cfs. is equal to the elevation of the water surface profile at 0 + 00 plus the velocity head at 0 + 00.

1. Compute d_c at the crest, sta. 1 + 00, where the section is 20 ft. wide.

$$Q/b = 2000 \div 20 = 100 \quad \text{and} \quad z/b = 0$$

From drawing ES-24, d_c = 6.77 ft.

2. Compute the water surface profile from 1 + 00 to 0 + 00 using the energy equation (5.4-35).

$$\frac{v_1^2}{2g} + d_1 + s_o \ell = \frac{v_2^2}{2g} + d_2 + s_f \ell$$

(a) Since subscripts 1 and 2 denote upstream and downstream sections respectively, d_2 and v_2 are known for each successive reach as the computations progress upstream.

(b) The length, ℓ, is selected for each reach.

(c) In each reach assume d_1 and compute $v_1 = Q/bd_1$. Note that b changes from section to section.

(d) Evaluate $v_1^2/2g$ and $v_2^2/2g$.

(e) Evaluate $s_f = \dfrac{v_m^2 \; n^2}{2.2082 \; r_m^{4/3}}$

$$v_m = \frac{v_1 + v_2}{2} \qquad \text{and} \qquad r_m = \frac{r_1 + r_2}{2}$$

(f) Determine whether the assumed d_1 balances equation (5.4-35); if not, take additional trial depths until a balance is obtained.

The table on the following page lists computations made in accordance with the above instructions.

Cols. 1, 2, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 18, and 19 are self-explanatory.

Col. 3 lists both d_1 and d_2 for a given reach. For example, in the reach from 1 + 00 to 0 + 90, d_2 = 6.77; and 9.80, 9.66, and 9.67 are trial values of d_1; 9.67 gives a balance for the energy equation, and it becomes d_2 for the reach 0 + 90 to 0 + 80. Values of d_1 giving a balance in the energy equation are marked with subscript 1.

Col. 11 lists values taken from King's Handbook, table 107, p. 309.

Col. 16 lists values of the left-hand member of the energy equation.

Col. 17 lists values of the right-hand member of the energy equation. Note that the values of col. 16 and col. 17 are essentially equal for the trial set of computations for d_1 marked with the subscript 1, thus indicating that the assumed depth is correct.

Note that s_o is negative in this example since the channel slope is adverse to the direction of flow.

COMPUTATIONS FOR WATER SURFACE PROFILE, 1 + 00 to 0 + 00
Example 1 - Subsection 4.7.5

Sta.	b	Depth d	Area a	Velocity $v = \frac{Q}{a}$	$\frac{v^2}{2g}$	$H_e = \frac{v^2}{2g} + d$	$r = \frac{a}{p}$	Elevations: Channel Bottom	Elevations: Surface Profile
1+00	20	6.77	135.40	14.7710	3.3921	10.1621	4.04	1109.20	1115.97
0+90	21	9.80	205.80	9.7182	1.4683	11.2683	5.06		
0+90		9.66	202.86	9.8590	1.5112	11.1712	5.03		
0+90		9.67_1	203.07	9.8488	1.5081	11.1781	5.03	1108.20	1117.87
0+80	22	11.20	246.40	8.1169	1.0243	12.2243	5.55		
0+80		11.15_1	245.30	8.1533	1.0335	12.1835	5.54	1107.20	1118.35
0+60	24	14.00	336.00	5.9524	0.5509	14.5509			
0+60		13.60	326.40	6.1275	0.5837	14.1837	6.37		
0+60		13.61_1	326.64	6.1229	0.5828	14.1928	6.38	1105.20	1118.81
0+00	30	20.00	600.00	3.3333	0.1727	20.1727	8.57		
0+00		20.03_1	600.90	3.3283	0.1722	20.2022	8.57	1099.20	1119.23

v_m	r_m	$\frac{1}{2.2082 r_m^{4/3}}$	$(nv_m)^2$	s_f	$s_f \ell$	$s_o \ell$	$\frac{v_2^2}{2g} + d_2 - s_o \ell = H_{e2} - s_o \ell$	$\frac{v_1^2}{2g} + d_1 - s_f \ell = H_{e1} - s_f \ell$
12.24	4.55	0.0601	0.0295	0.00177	0.0177	-1.00	11.1621	11.2506
12.31	4.53	0.0604	0.0296	0.00179	0.0179			11.1533
12.31	4.53	0.0604	0.0297	0.00179	0.0179_1			11.1602
8.98	5.29	0.0491	0.0158	0.00078	0.0078	-1.00	12.1781	12.2165
9.00	5.29	0.0491	0.0159	0.00079	0.0079_1			12.1756
						-2.00	14.1835	
7.14	5.95	0.0420	0.0100	0.00042	0.0084			14.1753
7.14	5.96	0.0419	0.0100	0.00042	0.0084_1			14.1844
4.73	7.47	0.0310	0.00438	0.000135	0.0081	-6.00	20.1928	20.1647
4.73	7.47	0.0310	0.00438	0.000135	0.0081_1			20.1941

$$\frac{v_2^2}{2g} + d_2 - s_o \ell = \frac{v_1^2}{2g} + d_1 - s_f \ell$$

$\frac{v_e^2}{2g}$ at sta. 0 + 00 = 0.17

Pool elev. = 1119.40

Q = 2000 cfs
n = 0.014
s_o = -0.10

$$v_m = \frac{v_1 + v_2}{2} \qquad r_m = \frac{r_1 + r_2}{2} \qquad s_f = \frac{(n\, v_m)^2}{2.2082\, r_m^{4/3}}$$

Check: Consider the energy equation between stations 0 + 00 and 1 + 00.

$$\frac{v_1^2}{2g} + d_1 + s_o \ell = \frac{v_2^2}{2g} + d_2 + \Sigma\, s_f \ell$$

At sta. 0 + 00 (s_o is negative)

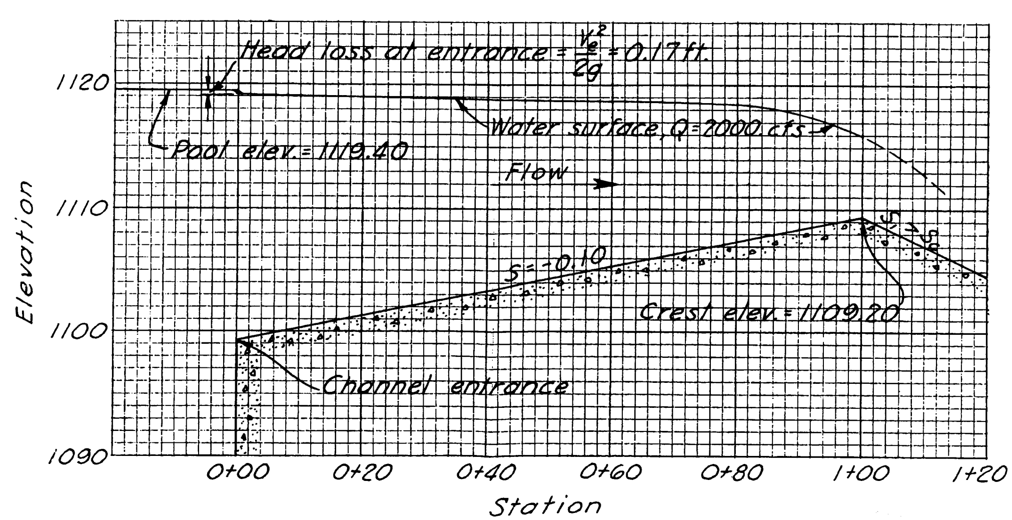
$$\frac{v_1^2}{2g} + d_1 + s_o \ell = 0.1722 + 20.03 - (0.1 \times 100) = 10.2022$$

At sta. 1 + 00

$$\frac{v_2^2}{2g} + d_2 + \Sigma\, s_f \ell$$

$= 3.3921 + 6.77 + (0.0179 + 0.0079 + 0.0084 + 0.0081)$

$= \underline{10.2024}$



COMPUTATIONS FOR WATER SURFACE PROFILE, 1 + 00 to 0 + 00

Example 1 - Subsection 4.7.5

Sta.	b	Depth d	Area a	Velocity $v = \frac{Q}{a}$	$\frac{v^2}{2g}$	$H_e = \frac{v^2}{2g} + d$	$r = \frac{a}{p}$
1+00	20	6.77	135.40	14.7710	3.3921	10.1621	4.04
0+90	21	9.80	205.80	9.7182	1.4683	11.2683	5.06
0+90		9.66	202.86	9.8590	1.5112	11.1712	5.03
0+90		9.67_1	203.07	9.8488	1.5081	11.1781	5.03
0+80	22	11.20	246.40	8.1169	1.0243	12.2243	5.55
0+80		11.15_1	245.30	8.1533	1.0335	12.1835	5.54
0+60	24	14.00	336.00	5.9524	0.5509	14.5509	
0+60		13.60	326.40	6.1275	0.5837	14.1837	6.37
0+60		13.61_1	326.64	6.1229	0.5828	14.1928	6.38
0+00	30	20.00	600.00	3.3333	0.1727	20.1727	8.57
0+00		20.03_1	600.90	3.3283	0.1722	20.2022	8.57

v_m	r_m	$\frac{1}{2.2082 r_m^{4/3}}$	$(nv_m)^2$	s_f	$s_f \ell$	$s_o \ell$	$\frac{v_2^2}{2g} + d_2 - s_o \ell = H_{e2} - s_o \ell$	$\frac{v_1^2}{2g} + d_1 - s_f \ell = H_{e1} - s_f \ell$
12.24	4.55	0.0601	0.0295	0.00177	0.0177	-1.00	11.1621	11.2506
12.31	4.53	0.0604	0.0296	0.00179	0.0179			11.1533
12.31	4.53	0.0604	0.0297	0.00179	0.0179_1			11.1602
8.98	5.29	0.0491	0.0158	0.00078	0.0078	-1.00	12.1781	12.2165
9.00	5.29	0.0491	0.0159	0.00079	0.0079_1			12.1756
						-2.00	14.1835	
7.14	5.95	0.0420	0.0100	0.00042	0.0084			14.1753
7.14	5.96	0.0419	0.0100	0.00042	0.0084_1			14.1844
4.73	7.47	0.0310	0.00438	0.000135	0.0081	-6.00	20.1928	20.1647
4.73	7.47	0.0310	0.00438	0.000135	0.0081_1			20.1941

Elevations: Channel Bottom	Elevations: Surface Profile
1109.20	1115.97
1108.20	1117.87
1107.20	1118.35
1105.20	1118.81
1099.20	1119.23

$$\frac{v_2^2}{2g} + d_2 - s_o \ell = \frac{v_1^2}{2g} + d_1 - s_f \ell$$

$\frac{v_e^2}{2g}$ at sta. 0 + 00 = 0.17

Pool elev. = 1119.40

Q = 2000 cfs
n = 0.014
s_o = -0.10

$$v_m = \frac{v_1 + v_2}{2} \qquad r_m = \frac{r_1 + r_2}{2} \qquad s_f = \frac{(n\,v_m)^2}{2.2082\, r_m^{4/3}}$$

Check: Consider the energy equation between stations 0 + 00 and 1 + 00.

$$\frac{v_1^2}{2g} + d_1 + s_o \ell = \frac{v_2^2}{2g} + d_2 + \Sigma\, s_f \ell$$

At sta. 0 + 00 (s_o is negative)

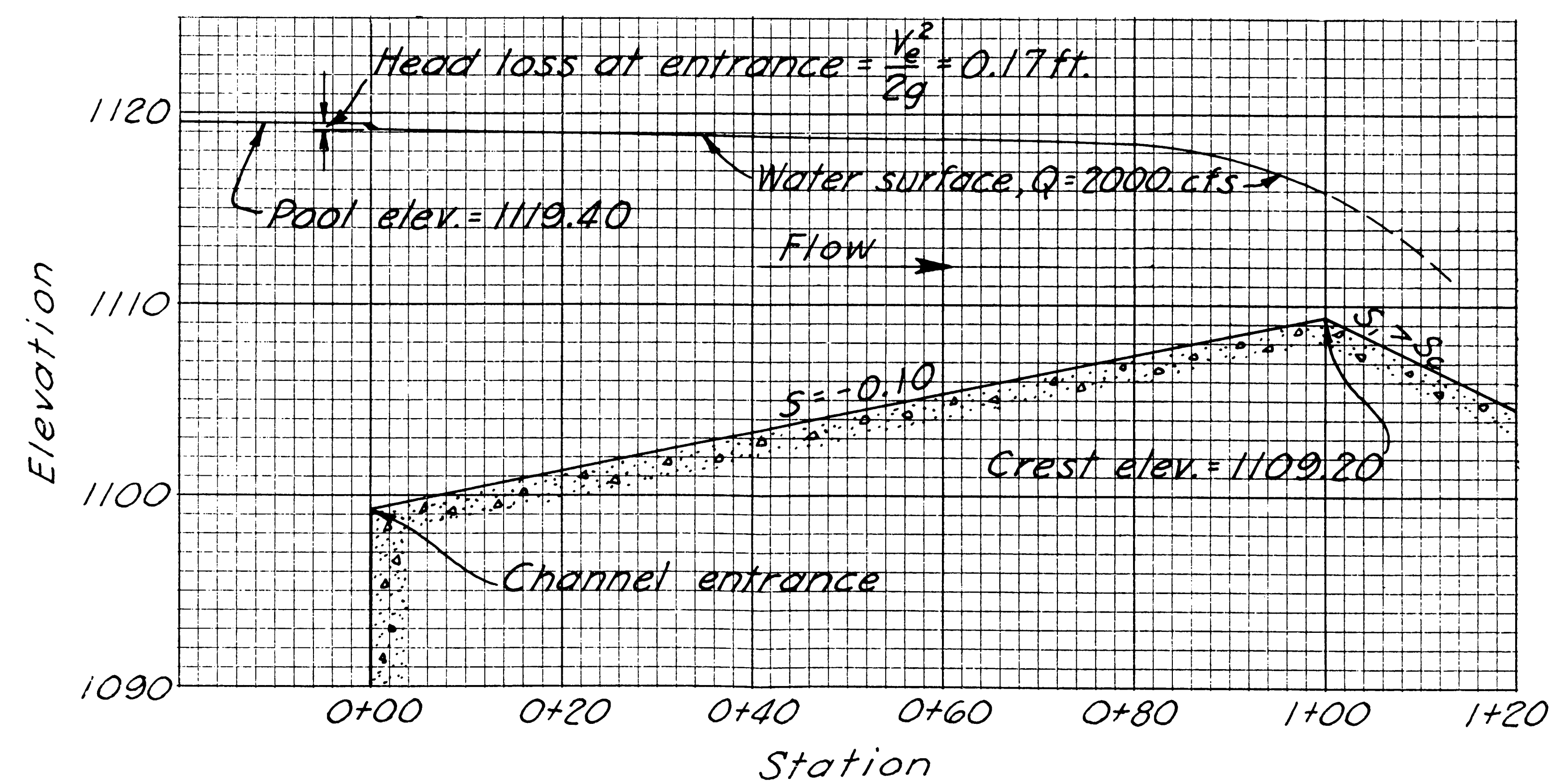
$$\frac{v_1^2}{2g} + d_1 + s_o \ell = 0.1722 + 20.03 - (0.1 \times 100) = 10.2022$$

At sta. 1 + 00

$$\frac{v_2^2}{2g} + d_2 + \Sigma . s_f \ell$$

$$= 3.3921 + 6.77 + (0.0179 + 0.0079 + 0.0084 + 0.0081)$$

$$= \underline{10.2024}$$



EXAMPLE 2

In dealing with natural channels, it is common practice to use an average slope and an average cross section for a reach in the open channel formulas. A further simplification is normally made by the assumption that the variations in velocity head may be neglected. The decision as to whether this latter assumption may be applied in a given case should be made by the engineer from a consideration of the conditions involved. If the changes in velocity head cannot be neglected, the water surface profile may be computed by the method illustrated in Example 1 of this subsection.

When velocity head may be neglected, the method illustrated by the following example results in a direct solution. This method requires that stream profile, cross sections, and roughness coefficients be obtained by field surveys and that water surface elevations at the lower end of the stretch of stream for given discharges be known or determinable. The method is taken from: "Graphical Calculation of Backwater Eliminates Solution by Trial" by Francis F. Escoffier, Engineering News Record, June 27, 1946.

Given: A portion of a natural stream is shown in plan and profile on following pages. Cross sections are available at stations 23 + 00, 26 + 00, 29 + 00, 33 + 50, 38 + 00, 42 + 00, and 46 + 00. The cross section at station 38 + 00, which is typical, is shown on a following page.

To determine: The water surface profile for Branches E and E-2 for the three sets of conditions stated in the following table:

Sets of Conditions	Discharges - cfs			Elev. Water Surface 46 + 00
	Branch E-1	Branch E-2	Branch E	
A	1800	600	2400	1181.50
B	1200	400	1600	1180.75
C	600	200	800	1180.20

Theory:

The energy equation (5.4-35) between sections at the upper and lower ends of a reach is:

$$\frac{v_1^2}{2g} + d_1 + s_o \ell = \frac{v_2^2}{2g} + d_2 + s_f \ell$$

Assuming the change in velocity heads to be negligible results in:

$$\frac{v_1^2}{2g} = \frac{v_2^2}{2g} \; ; \quad \text{then} \quad d_1 - d_2 + s_o \ell = s_f \ell$$

From Manning's formula:

$$Q = \frac{1.486}{n}\, ar^{2/3}\, s_f^{1/2}, \text{ and since } s_f = \frac{h_f}{\ell}$$

$$\frac{h_f}{\ell} = Q^2 \left(\frac{n}{1.486\ ar^{2/3}}\right)^2$$

Placing $\left(\frac{n}{1.486\ ar^{2/3}}\right)^2 = P$ results in $h_f = Q^2 P \ell$

$$\therefore\ d_1 - d_2 + s_o\ell = h_f = Q^2 P \ell$$

h_f = loss of head due to friction.

Since the value of P varies with n and water surface elevation at each cross section, assume an average for a reach as valid.

$P = 1/2\ (P_2 + P_1)$, in which

P_2 = the value of P at the lower end of a reach.

P_1 = the value of P at the upper end of a reach.

$$\therefore\ h_f = 1/2\ (P_2 + P_1)\, Q^2 \ell \quad \text{or} \quad \frac{h_f}{P_2 + P_1} = \frac{Q^2 \ell}{2}$$

Values of P_2 and P_1 versus water surface elevation are plotted as in fig. 5.4-3.

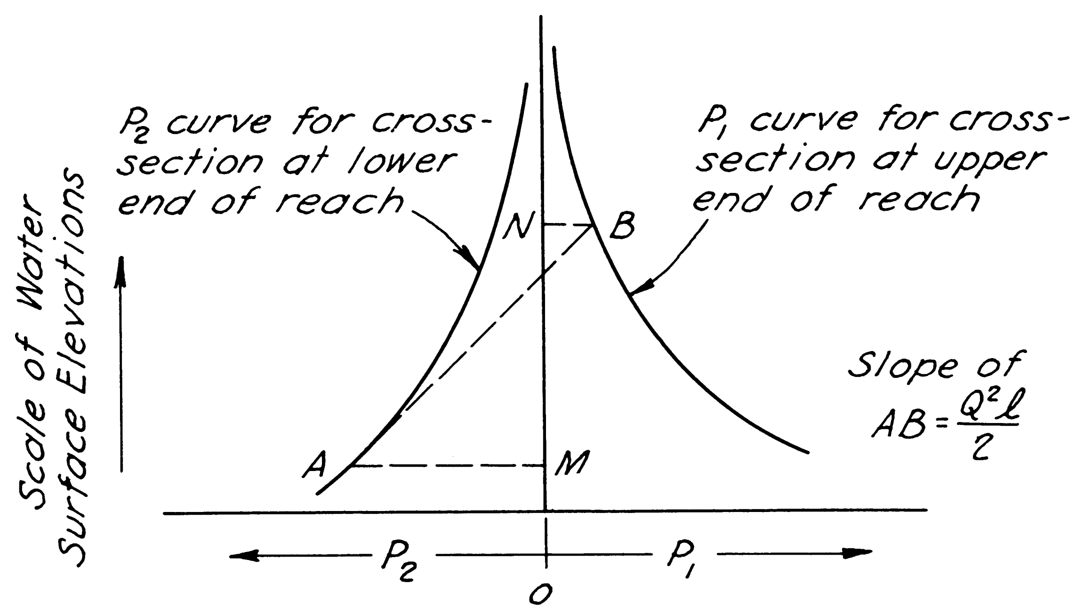


FIG. 5.4-3

If the known elevation of the water surface for a discharge, Q, at the lower end of a reach, point M, is projected to A, the elevation of the water surface at the upper end of the reach, point N, may be determined by constructing AB on the correct slope. By inspection,

$$MA = P_2 \;,\; NB = P_1 \;,\; \text{and the slope of AB is } \frac{MN}{MA + NB} = \frac{h_f}{P_2 + P_1}$$

Since $h_f \div (P_2 + P_1) = (Q^2 \ell) \div 2$, the slope at which AB is to be drawn is $(Q^2 \ell) \div 2$.

Solution:

1. Compute the P values for each cross section. Table I lists the computations for P at one foot intervals of elevation in the cross section at 38 + 00; other cross sections would be treated in a similar manner. The cross section is given on a following page. Column headings in the table are self-explanatory.

   (a) Values of $(1.486/n)ar^{2/3}$ are first computed in each part of the cross section and totaled at each elevation for the entire cross section.

   In a cross section not subdivided,

$$P = \left(\frac{n}{1.486\ ar^{2/3}}\right)^2 \quad \text{and} \quad \frac{1}{\sqrt{P}} = \frac{1.486}{n}\ ar^{2/3}$$

   In a subdivided cross section,

$$P_T = \frac{1}{\left(\frac{1}{\sqrt{P_a}} + \frac{1}{\sqrt{P_b}} + \frac{1}{\sqrt{P_c}} + \cdots\right)^2} = \frac{1}{\left(\sum_1^n \frac{1}{\sqrt{P_i}}\right)^2}$$

   P_T = the P value for an entire cross section at any elevation.

   P_a, P_b, P_c, etc. = the P values for parts of the cross section at any elevation.

   $n \geq 2$, the total number of parts of a cross section at any elevation.

   (b) This method makes it possible to apply different n values in selected subdivisions of a channel cross section.

   (c) In a subdivided part of a channel only the perimeter of the channel in contact with water should be used in computing r. That portion of the perimeter where water is in contact with water should be excluded in computing r.

2. Plot the values of P for each cross section as P_2 and P_1 as shown on the work sheet. P values for the cross section at 38 + 00 are plotted twice as P_2 values for reach 3 on the left side of the sheet, and as P_1 values for reach 2 on the right. P values at 42 + 00, 33 + 50, 29 + 00, and 26 + 00 are also plotted as both P_2 and P_1 values for appropriate reaches. The P values for the cross sections at 46 + 00 and 23 + 00 are plotted only once; those at 46 + 00 as P_2 values for reach 1, and those at 23 + 00 as P_1 values for reach 6.

3. Compute values of $Q^2\ell \div 2$ for the reaches and sets of discharge conditions:

Reach				A Condition		B Condition		C Condition	
No.	From	To	ℓ	Q	$\frac{Q^2\ell}{2} \times 10^{-8}$	Q	$\frac{Q^2\ell}{2} \times 10^{-8}$	Q	$\frac{Q^2\ell}{2} \times 10^{-8}$
1	46+00	42+00	400	2400	11.52	1600	5.12	800	1.28
2	42+00	38+00	400	2400	11.52	1600	5.12	800	1.28
3	38+00	33+50	450	2400	12.96	1600	5.76	800	1.44
4	33+50	29+00	450	2400	12.96	1600	5.76	800	1.44
5	29+00	26+00	300	600	0.54	400	0.24	200	0.06
6	26+00	23+00	300	600	0.54	400	0.24	200	0.06

4. Determine the three water surface profiles using: (a) the given water surface elevations at 46 + 00; (b) the $Q^2\ell \div 2$ values tabulated above; (c) the work sheet on which the P curves are plotted.

Examine the set-up of the work sheet. The scales of P_2, P_1, and water surface elevation and the plotting of the P curves will be readily understood. Note the slope scale, i.e., the scale of $Q^2\ell \div 2$ values, on the right and the reference point near the bottom center. The value of the slope scale unit in relation to the reference point is determined as follows:

$$\text{slope} = \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{h_f}{P_2 + P_1} = \frac{Q^2\ell}{2}$$

The vertical distance = 1 scale unit = 1 foot.

Each horizontal unit has the value 1×10^{-9}. The horizontal distance from the reference point to the slope scale = 10 scale units = $10 \times 1 \times 10^{-9}$ = 10^{-8}; therefore, the value of one unit on the slope scale is:

$$\text{slope} = \frac{1}{10^{-8}} = 1 \times 10^{8}$$

To solve the water surface profile for the "C" condition proceed as indicated below.

On the P_2 curve for reach 1, station 46 + 00, locate the point for water surface elevation 1180.20, Q = 800 cfs. From the table of $Q^2\ell \div 2$ values, take 1.28×10^8 for "C" condition, reach 1, and find this value on the slope scale, thus defining a slope line. Draw a line parallel to this slope line from the point of elevation 1180.20 on the P_2 curve for station 46 + 00 to intersect the P_1 curve for reach 1, station 42 + 00. This intersection establishes the water surface elevation, 1181.23, at the upper end of reach 1 and the lower end of reach 2. Project this water surface elevation to the left to the P_2 curve, reach 2, station 42 + 00, and repeat the above procedure for reaches 2 through 6 to complete the water surface profile for "C" condition. Note that the discharge changes to 200 cfs at reach 5, and that this fact has been recognized in the computation of $Q^2\ell \div 2$ values.

Water surface profiles for "A" and "B" conditions are determined in the same manner and the three profiles are plotted on the last page of this example.

Remarks:

This and other methods for determining backwater curves require that the water surface elevation at the lower end of a stretch of stream be known or that it be determined through some type of stage-discharge relationship. In some cases a gaging station or control point makes it possible to meet these requirements, but in most cases they cannot be met directly by determinations at a single section. In such cases a series of several contiguous reaches should be established downstream from the stretch of stream under consideration, and several profiles for any required value of Q based on assumed water surface elevations at the downstream end of the lowest reach should then be computed upstream. These profiles will converge in the upstream direction making it possible to determine or closely approximate the water surface elevations for given discharges at the lower end of the stretch.

An important advantage of this method is that when the P values have been computed and the work sheet constructed, only a short time is required to make the graphical solution for the water surface profile of any discharge.

TABLE I - COMPUTATIONS OF "P" VALUES FOR SECTION AT STA. 38 + 00
Example 2 - Subsection 4.7.5

Elev.	Part a (n = 0.075)					Part b (n = 0.030)					Part c (n = 0.050)				
	a ft^2	p ft	r ft	$r^{2/3}$	$\frac{1.486}{n} ar^{2/3} = \frac{1}{\sqrt{P_a}}$	a ft^2	p ft	r ft	$r^{2/3}$	$\frac{1.486}{n} ar^{2/3} = \frac{1}{\sqrt{P_b}}$	a ft^2	p ft	r ft	$r^{2/3}$	$\frac{1.486}{n} ar^{2/3} = \frac{1}{\sqrt{P_c}}$
1178	0					1.8	13.1	.137	0.266	23.7	0				
1179	0					29.0	33.2	.874	0.914	1310.	0				
1180	0					59.0	35.4	1.665	1.405	4100.	0				
1181	0					93.4	37.6	2.48	1.83	8450.	0				
1182	0					130.2	39.8	3.27	2.20	14200.	0				
1183	3.4	8.6	0.396	0.540	36.4	168.	40.	4.20	2.60	21600.	29.6	58	0.511	0.64	560.
1184	49.4	55.6	0.888	0.924	905.	206.	40.	5.15	2.98	30400.	90.8	65	1.40	1.25	3370.
1185	105.8	60.0	1.762	1.46	3060.	244.	40.	6.10	3.34	40300.	159.2	72	2.21	1.70	8050.
1186	165.4	62.6	2.64	1.91	6250.	282.	40.	7.05	3.68	51400.	232.8	78	2.99	2.07	14300.

Elev.	$\frac{1}{\sqrt{P_T}}$	$\sqrt{P_T} \times 10^6$	$P_T \times 10^{12}$
1178	23.7	42300.	179×10^7
1179	1310.	764.	584,000.
1180	4100.	244.	59,500.
1181	8450.	118.2	14,000.
1182	14200.	70.5	4,970.
1183	22196.	45.15	2,040.
1184	34675.	28.85	832.
1185	51410.	19.48	379.
1186	71950.	13.9	193.

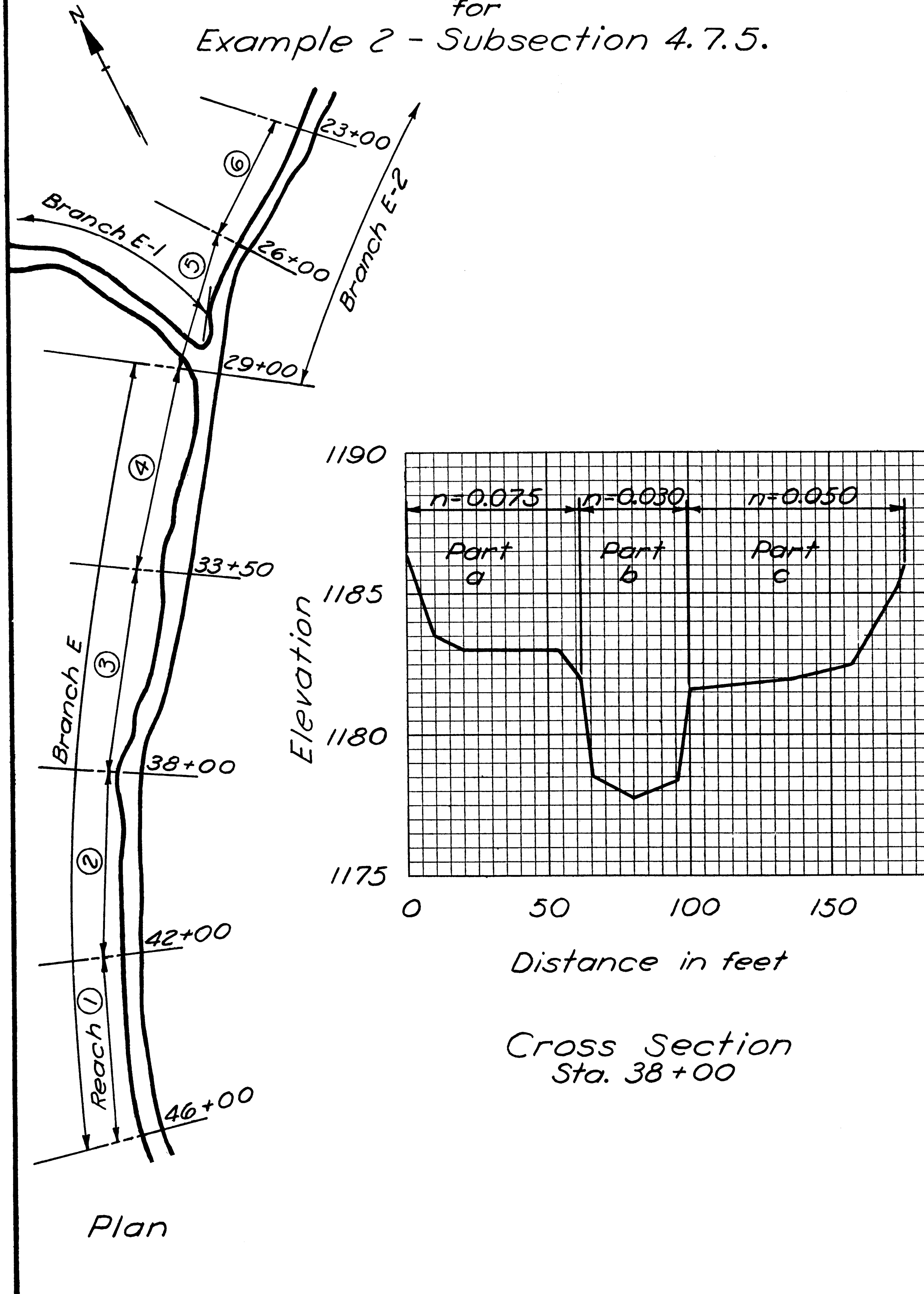
a = area of stream

p = wetted perimeter

r = hydraulic radius

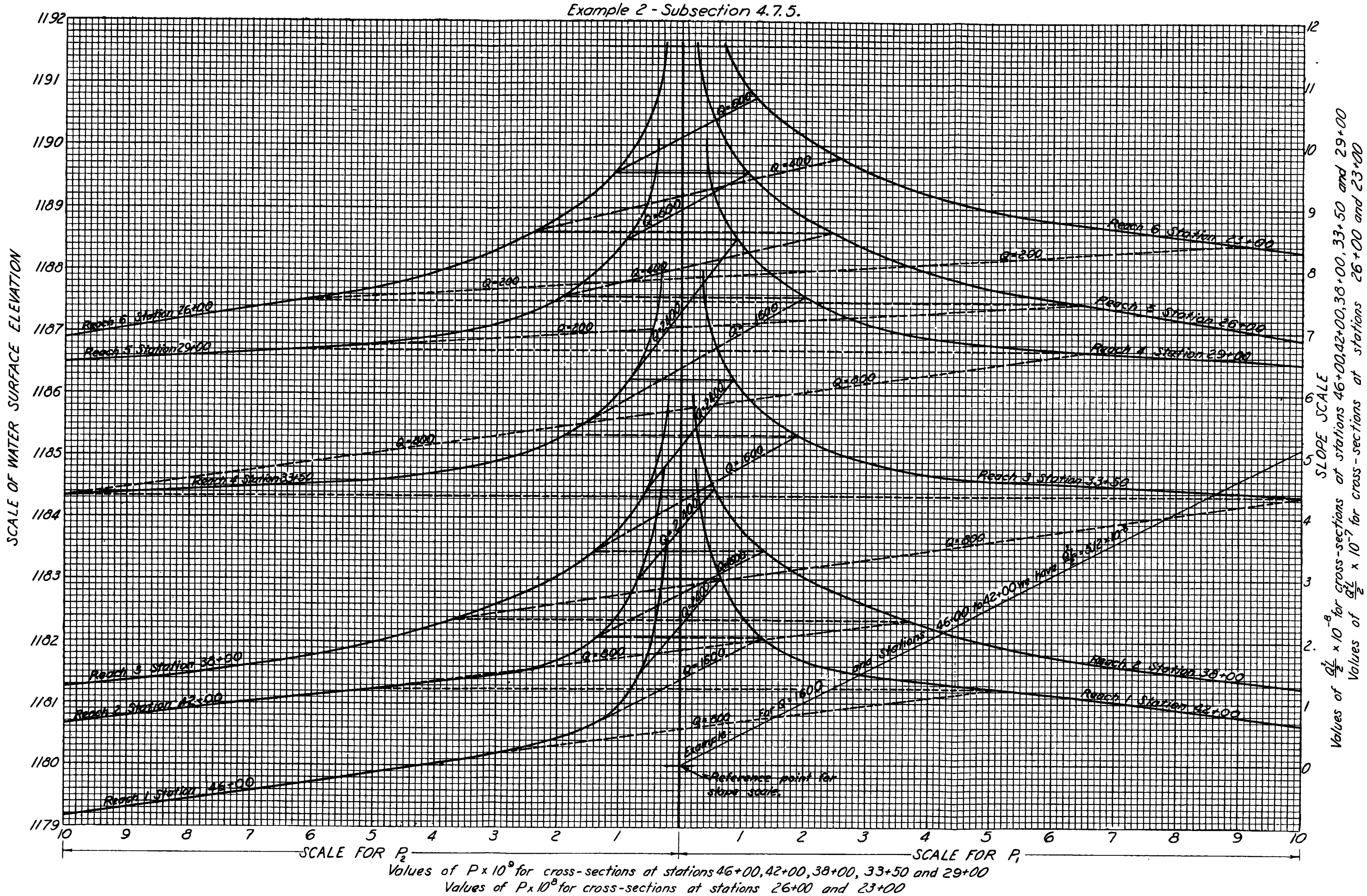
$$\frac{1}{\sqrt{P_T}} = \sum \frac{1.486}{n} ar^{2/3} \text{ at each elevation}$$

Sketch Plan and Typical Cross Section for Example 2 - Subsection 4.7.5.



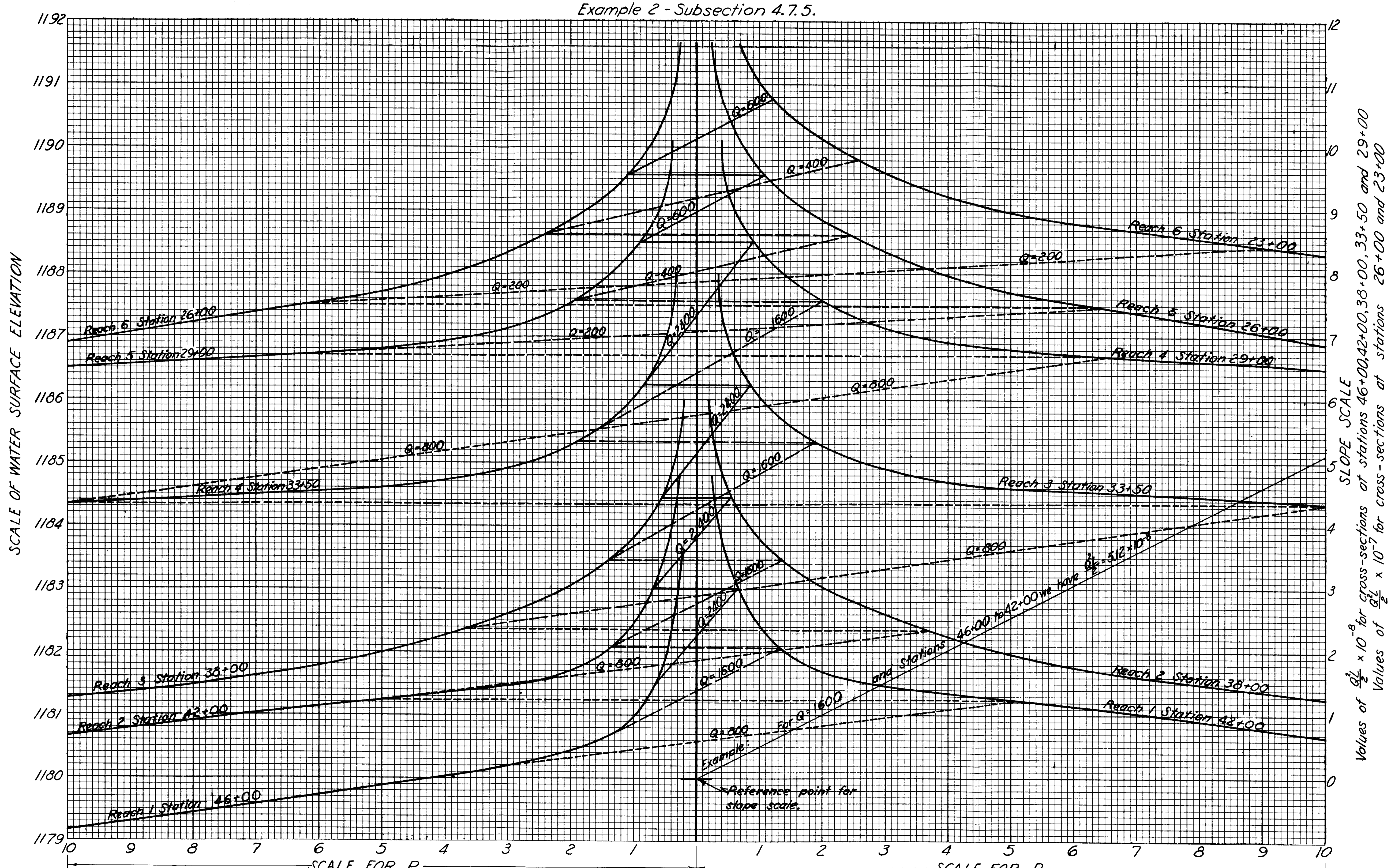
WORK SHEET

Example 2 - Subsection 4.7.5.



WORK SHEET

Example 2 - Subsection 4.7.5.

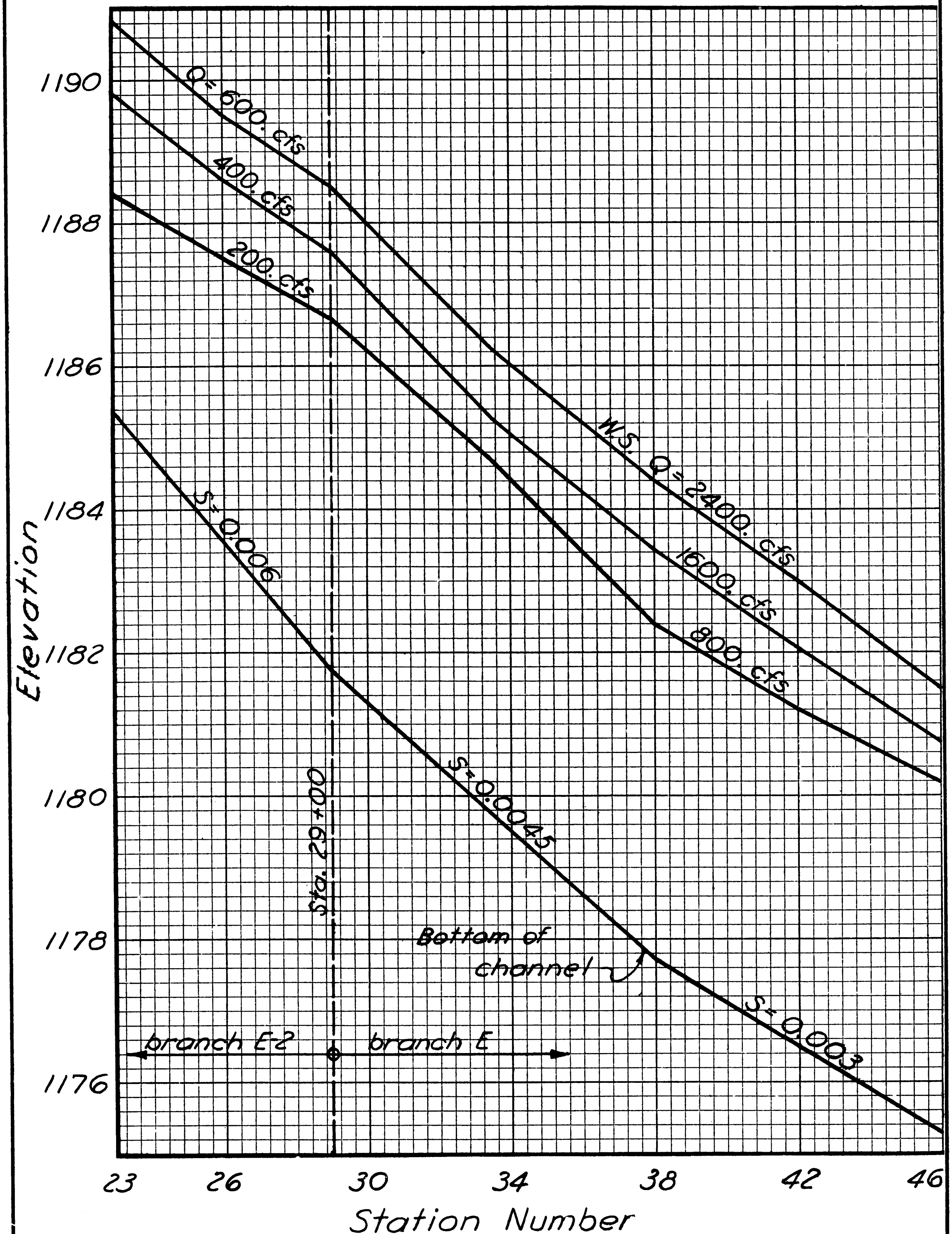


Values of $P\times10^9$ for cross-sections at stations 46+00, 42+00, 38+00, 33+50 and 29+00

Values of $P\times10^8$ for cross-sections at stations 26+00 and 23+00

5-4-60

Water Surface Profiles for Example 2 - Subsection 4.7.5.



5. Pipe Flow

5.1 General. Pipe flow exists when a closed conduit of any form is flowing full of water. In pipe flow the cross-sectional area of flow is fixed by the cross section of the conduit and the water surface is not exposed to the atmosphere. The internal pressure in a pipe may be equal to, greater than, or less than the local atmospheric pressure.

The principles of pipe flow apply to the hydraulics of such structures as culverts, drop inlets, regular and inverted siphons, and various types of pipe lines.

If pipe flow exists for a range of discharges under study, the cross-sectional area of flow and the hydraulic radius remain constant for any particular cross section and the velocity is directly proportional to the discharge.

5.2 Fundamentals. The following discussions under subsection 3, Fundamentals of Water Flow, should be reviewed: 3.1, Laminar and Turbulent Flow; 3.2, Continuity of Flow; 3.3, Energy and Head; 3.4, Bernoulli Theorem; and 3.5, Hydraulic Gradient and Energy Gradient.

As pointed out in paragraphs 3.1, the possibility of laminar flow in pipes when Reynold's number exceeds 3000 is very remote. For our purposes this value will be assumed to be the lower limit for turbulent flow.

Reynold's number for pipes is:

$$R = \frac{vd}{\nu} \qquad (5.5\text{-}1)$$

where

R = Reynold's number
v = mean velocity in pipe in ft. per sec.
d = diameter of pipe in feet
ν = (nu) kinematic viscosity in ft.^{2} per sec.
d_i = diameter of pipe in inches.

The kinematic viscosity and other physical properties of water are given in table 5.5-1 for atmospheric pressure. The kinematic viscosity is not affected appreciably by the variation in pressures that can normally be expected in our work.

For example: Based on the above assumption of a minimum value of R = 3000, compute the minimum permissible velocity in a 2-inch diameter pipe at which water with a temperature of 50° F. will flow turbulently.

$$v = \frac{R\nu}{d} = \frac{3000 \times 1.41 \times 10^{-5}}{\left(\frac{2}{12}\right)} = 0.254 \text{ fps}$$

TABLE 5.5-1

Temperature Degrees Fahrenheit	Specific Weight lbs./ft.^{3}	Kinematic Viscosity ft.^{2}/sec.	Vapor Pressure psi
32	62.4	1.93×10^{-5}	0.08
40	62.4	1.67×10^{-5}	0.11
50	62.4	1.41×10^{-5}	0.17
60	62.4	1.21×10^{-5}	0.26
70	62.3	1.05×10^{-5}	0.36
80	62.2	0.93×10^{-5}	0.51
90	62.1	0.82×10^{-5}	0.70
100	62.0	0.74×10^{-5}	0.96

To assure turbulent flow, the value of R should be equal to or greater than 3000. Assuming a value of kinematic viscosity, ν, of 0.0000105 for water at 70° F., substituting $(Q \div a)$ for v, $0.7854d^2$ for a, and reducing d to d_i gives a minimum discharge for turbulent flow in terms of the diameter of the pipe.

$$Q_{min.} = 2.0617 \times 10^{-3}\ d_i \qquad (5.5\text{-}2)$$

5.3 Friction Loss. The loss of energy or head resulting from turbulence created at the boundary between the sides of the conduit and the flowing water is called friction loss.

In a straight length of conduit, flowing full, with constant cross section and uniform roughness, the rate of loss of head by friction is constant and the energy gradient has a slope, in the direction of flow, equal to the friction head loss per foot of conduit.

Of the many equations that have been developed to express friction loss, the following two equations have been selected for inclusion herein; they are widely used and reliable.

5.3.1 Manning's Formula. The general form of this equation is:

$$v = \frac{1.486}{n} r^{2/3} s^{1/2} \qquad (5.5\text{-}3)$$

Nomenclature:

- a = cross-sectional area of flow in ft.^{2}
- d = diameter of pipe in feet.
- d_i = diameter of pipe in inches.
- g = acceleration of gravity = 32.2 ft. per sec.^{2}
- H_1 = loss of head in feet due to friction in length, L.
- K_c = head loss coefficient for any conduit.
- K_p = head loss coefficient for circular pipe.
- L = length of conduit in feet.
- n = Manning's roughness coefficient.

p = wetted perimeter in feet.
r = hydraulic radius in feet = (a ÷ p) = (d ÷ 4) for round pipe.
s = loss of head in feet per foot of conduit = slope of energy grade and hydraulic grade lines in straight conduits of uniform cross section = (H_1 ÷ L).
v = mean velocity of flow in ft. per sec.
Q = discharge or capacity in ft.^{3} per sec.

Starting with equation (5.5-3) solve for s, multiply numerator and denominator of right side of equation by 2g, and substitute (H_1 ÷ L) for s. The result is:

$$H_1 = K_c L \frac{v^2}{2g} \tag{5.5-4}$$

where

$$K_c = \frac{29.164\ n^2}{r^{4/3}} \tag{5.5-5}$$

Adaption of this equation (5.5-5) to circular pipes involves the substitution of (d ÷ 4) for r and the change from d to d_i.

$$K_p = \frac{5087\ n^2}{{d_i}^{4/3}} \tag{5.5-6}$$

Tables of values for K_p and K_c for the usual ranges of variables encountered are given in drawing ES-42. The 1/3, 2/3, 3/2, 3/4, and 4/3 powers of numbers frequently used in Manning's formula can be found from drawing ES-37.

In some cases it is useful to consider the conditions of flow in a straight conduit of uniform cross section and roughness coefficient when the conduit is on neutral slope. Neutral slope is defined as that slope of a conduit at which the friction loss per foot (H_1 ÷ L) is equal to the slope of the conduit, i.e., when the conduit is parallel to the hydraulic gradient and energy gradient. In a conduit of given cross section, roughness coefficient and slope, this condition occurs for only one discharge. In figure 5.5-1 the conduit is on neutral slope when

$$\sin \theta = \frac{H_1}{L} = K_c \frac{v^2}{2g} \tag{5.5-7}$$

and

$$s_n \text{ (neutral slope)} = \tan \theta = \frac{K_c \frac{v^2}{2g}}{\sqrt{1 - \left(K_c \frac{v^2}{2g}\right)^2}} \tag{5.5-8}$$

In any case of pipe flow in a conduit of uniform cross section, the loss of head per foot of conduit is:

$$s = K_c \frac{v^2}{2g} \tag{5.5-9}$$

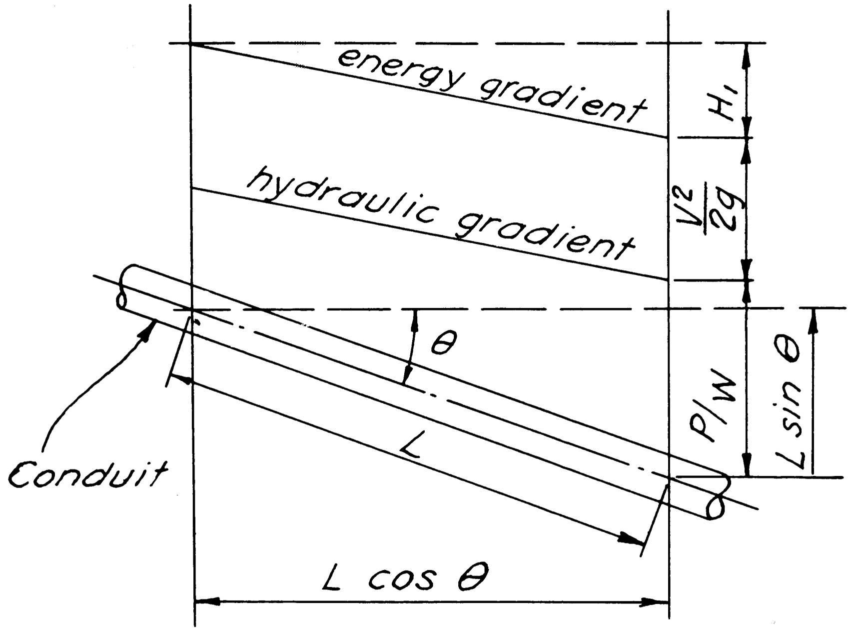


FIG. 5.5-1

The total head loss in a given length of conduit can be determined from equation (5.5-4) with the proper value of K_p or K_c selected from drawing ES-42.

King's Handbook, pp. 188 and 189, gives a number of convenient working forms of Manning's formula and references to tables that will facilitate their use. Four of these are:

$$H_1 = 2.87\ n^2\ \frac{Lv^2}{d^{4/3}} \tag{5.5-10}$$

$$H_1 = 4.66\ n^2\ \frac{LQ^2}{d^{16/3}} \tag{5.5-11}$$

$$d = \left(\frac{2.159\ Qn}{s^{1/2}}\right)^{3/8} \tag{5.5-12}$$

$$d_i = \left(\frac{1630\ Qn}{s^{1/2}}\right)^{3/8} \tag{5.5-13}$$

Drawing ES-54, which is based on equation (5.5-13), may be used to determine d_i, s, or Q, when two of these quantities and n are known.

5.3.2 Hazen-Williams Formula. As generally used, this formula is:

$$v = 1.318\ C\ r^{0.63}\ s^{0.54} \tag{5.5-14}$$

Notation is the same as given in subsection 5.3.1 with the addition of C, the coefficient of roughness in Hazen-Williams formula.

HYDRAULICS: HEAD LOSS COEFFICIENTS FOR CIRCULAR AND SQUARE CONDUITS FLOWING FULL

HEAD LOSS COEFFICIENT, K_p, FOR CIRCULAR PIPE FLOWING FULL $\quad K_p = \dfrac{5087\, n^2}{d_i^{4/3}}$

Pipe diam. inches	Flow area sq. ft.	MANNING'S COEFFICIENT OF ROUGHNESS "n"															
		0.010	0.011	0.012	0.013	0.014	0.015	0.016	0.017	0.018	0.019	0.020	0.021	0.022	0.023	0.024	0.025
6	0.196	0.0467	0.0565	0.0672	0.0789	0.0914	0.1050	0.1194	0.1348	0.151	0.168	0.187	0.206	0.226	0.247	0.269	0.292
8	0.349	.0318	.0385	.0458	.0537	.0623	.0715	.0814	.0919	.1030	.1148	.1272	.140	.154	.168	.183	.199
10	0.545	.0236	.0286	.0340	.0399	.0463	.0531	.0604	.0682	.0765	.0852	.0944	.1041	.1143	.1249	.136	.148
12	0.785	.0185	.0224	.0267	.0313	.0363	.0417	.0474	.0535	.0600	.0668	.0741	.0817	.0896	.0980	.1067	.1157
14	1.069	.0151	.0182	.0217	.0255	.0295	.0339	.0386	.0436	.0488	.0544	.0603	.0665	.0730	.0798	.0868	.0942
15	1.23	.0138	.0166	.0198	.0232	.0270	.0309	.0352	.0397	.0446	.0496	.0550	.0606	.0666	.0727	.0792	.0859
16	1.40	.0126	.0153	.0182	.0213	.0247	.0284	.0323	.0365	.0409	.0455	.0505	.0556	.0611	.0667	.0727	.0789
18	1.77	.01078	.0130	.0155	.0182	.0211	.0243	.0276	.0312	.0349	.0389	.0431	.0476	.0522	.0570	.0621	.0674
21	2.41	.00878	.01062	.0126	.0148	.0172	.0198	.0225	.0254	.0284	.0317	.0351	.0387	.0425	.0464	.0506	.0549
24	3.14	.00735	.00889	.01058	.0124	.0144	.0165	.0188	.0212	.0238	.0265	.0294	.0324	.0356	.0389	.0423	.0459
27	3.98	.00628	.00760	.00904	.01061	.0123	.0141	.0161	.0181	.0203	.0227	.0251	.0277	.0304	.0332	.0362	.0393
30	4.91	.00546	.00660	.00786	.00922	.01070	.01228	.0140	.0158	.0177	.0197	.0218	.0241	.0264	.0289	.0314	.0341
36	7.07	.00428	.00518	.00616	.00723	.00839	.00963	.01096	.0124	.0139	.0154	.0171	.0189	.0207	.0226	.0246	.0267
42	9.62	.00348	.00422	.00502	.00589	.00683	.00784	.00892	.01007	.01129	.0126	.0139	.0154	.0169	.0184	.0201	.0218
48	12.57	.00292	.00353	.00420	.00493	.00572	.00656	.00747	.00843	.00945	.01053	.01166	.0129	.0141	.0154	.0168	.0182
54	15.90	.00249	.00302	.00359	.00421	.00488	.00561	.00638	.00720	.00808	.00900	.00997	.01099	.0121	.0132	.0144	.0156
60	19.63	.00217	.00262	.00312	.00366	.00424	.00487	.00554	.00626	.00702	.00782	.00866	.00955	.01048	.0115	.0125	.0135

HEAD LOSS COEFFICIENT, K_c, FOR SQUARE CONDUIT FLOWING FULL $\quad K_c = \dfrac{29.16\, n^2}{r^{\frac{4}{3}}}$

Conduit Size feet	Flow area sq. ft.	MANNING'S COEFFICIENT OF ROUGHNESS "n"				
		0.012	0.013	0.014	0.015	0.016
2×2	4.00	0.01058	0.01242	0.01440	0.01653	0.01880
$2\frac{1}{2}\times2\frac{1}{2}$	6.25	0.00786	0.00922	0.01070	0.01228	0.01397
3×3	9.00	.00616	.00723	.00839	.00963	.01096
$3\frac{1}{2}\times3\frac{1}{2}$	12.25	.00502	.00589	.00683	.00784	.00892
4×4	16.00	.00420	.00493	.00572	.00656	.00746
$4\frac{1}{2}\times4\frac{1}{2}$	20.25	.00359	.00421	.00488	.00561	.00638
5×5	25.00	.00312	.00366	.00425	.00487	.00554
$5\frac{1}{2}\times5\frac{1}{2}$	30.25	.00275	.00322	.00374	.00429	.00488
6×6	36.00	.00245	.00287	.00333	.00382	.00435
$6\frac{1}{2}\times6\frac{1}{2}$	42.25	.00220	.00258	.00299	.00343	.00391
7×7	49.00	.00199	.00234	.00271	.00311	.00354
$7\frac{1}{2}\times7\frac{1}{2}$	56.25	.00182	.00213	.00247	.00284	.00323
8×8	64.00	.00167	.00196	.00227	.00260	.00296
$8\frac{1}{2}\times8\frac{1}{2}$	72.25	.00154	.00180	.00209	.00240	.00273
9×9	81.00	.00142	.00167	.00194	.00223	.00253
$9\frac{1}{2}\times9\frac{1}{2}$	90.25	.00133	.00156	.00180	.00207	.00236
10×10	100.00	.00124	.00145	.00168	.00193	.00220

$$H_1 = (K_p \text{ or } K_c)\, L\, \frac{v^2}{2g}$$

Nomenclature:

- a = Cross-sectional area of flow in sq. ft.
- d_i = Inside diameter of pipe in inches.
- g = Acceleration of gravity = 32.2 ft. per sec.
- H_1 = Loss of head in feet due to friction in length L.
- K_c = Head loss coefficient for square conduit flowing full.
- K_p = Head loss coefficient for circular pipe flowing full.
- L = Length of conduit in feet.
- n = Manning's coefficient of roughness.
- Q = Discharge or capacity in cu. ft. per sec.
- r = Hydraulic radius in feet.
- v = Mean velocity in ft. per sec.

Example 1: Compute the head loss in 300 ft. of 24 in. diam. concrete pipe flowing full and discharging 30 c.f.s. Assume $n = 0.015$

$$v = \frac{Q}{a} = \frac{30}{3.14} = 9.55 \text{ f.p.s.};\quad \frac{v^2}{2g} = \frac{(9.55)^2}{64.4} = 1.42 \text{ ft.}$$

$$H_1 = K_p L \frac{v^2}{2g} = 0.0165 \times 300 \times 1.42 = 7.03 \text{ ft.}$$

Example 2: Compute the discharge of a 250 ft., 3×3 square conduit flowing full if the loss of head is determined to be 2.25 ft. Assume $n = 0.014$.

$$H_1 = K_c L \frac{v^2}{2g};\quad \frac{v^2}{2g} = \frac{H_1}{K_c L} = \frac{2.25}{0.00839 \times 250} = 1.073 \text{ ft.}$$

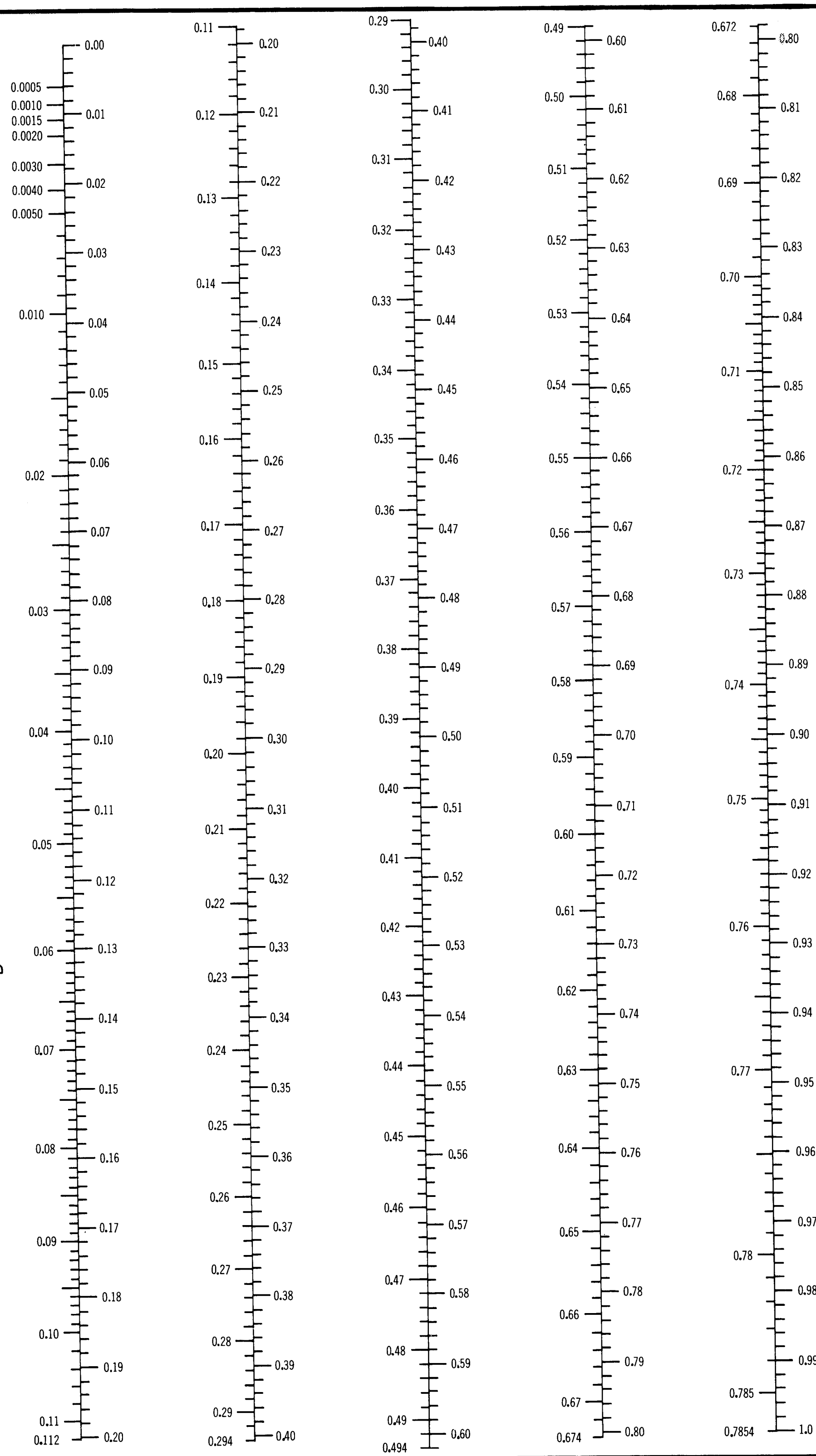
$$v = \sqrt{64.4 \times 1.073} = 8.31\ ;\quad Q = 9 \times 8.31 = 74.8 \text{ cfs.}$$

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 42 SHEET 1 OF 1 DATE 7-17-50

Revised 4-11-51

HYDRAULICS: FLOW AREAS ($a - ft^2$) IN CIRCULAR CONDUITS FOR VARIOUS DEPTHS OF FLOW (d - feet)

Values of $\frac{a}{D^2}$ are found on the left hand side of each scale line



Values of $\frac{d}{D}$ are found on the right hand side of each scale line

REFERENCE

This drawing was prepared by Richard M. Matthews of the Design Section.

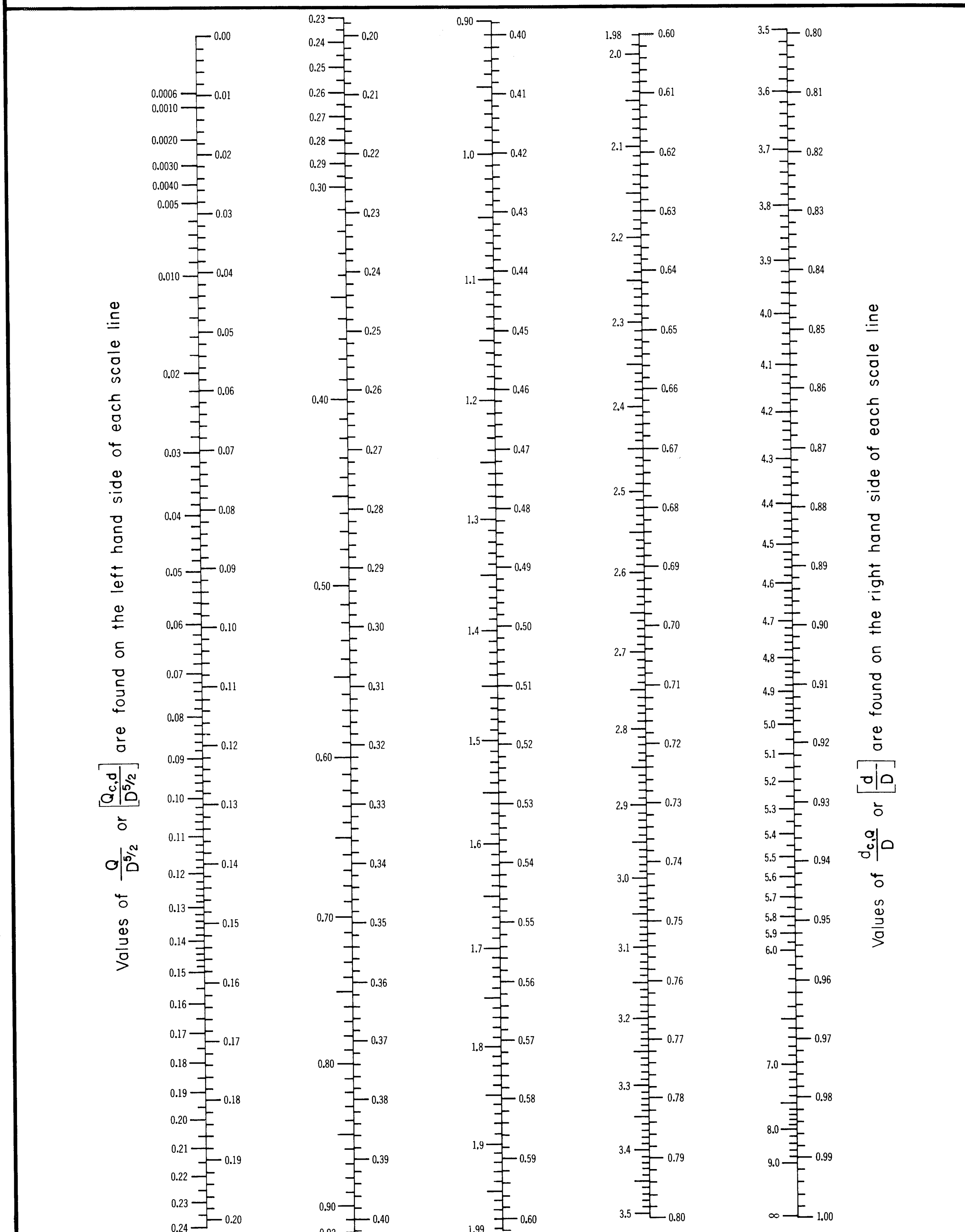
U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DRAWING NO. ES-97

SHEET 1 OF 7

DATE 1-6-55

HYDRAULICS: CIRCULAR CONDUITS; Critical discharges ($Q_{c,d}$ — cfs) corresponding to various depths (d — in ft) and critical depths ($d_{c,Q}$ — ft) corresponding to various discharges (Q — cfs)

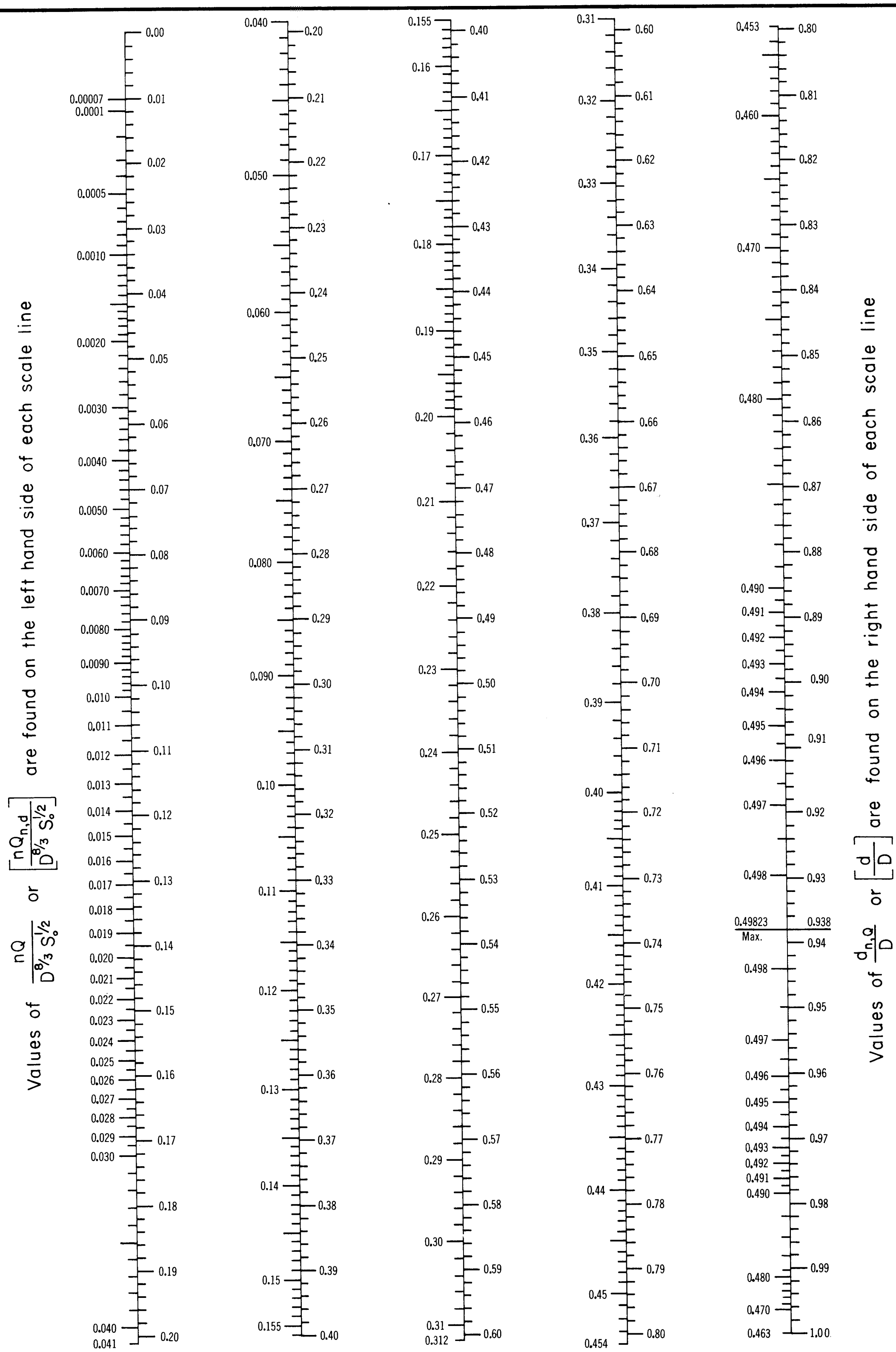


REFERENCE
This drawing was prepared by Richard M. Matthews of the Design Section.

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DRAWING NO.
ES-97
SHEET 2 OF 7
DATE 1-6-55

HYDRAULICS: CIRCULAR CONDUITS; Normal discharges ($Q_{n,d}$ – cfs) corresponding to various depths (d – in ft) and normal depths ($d_{n,Q}$ – ft) corresponding to various discharges (Q – cfs)



REFERENCE
This drawing was prepared by Richard M. Matthews of the Design Section.

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ENGINEERING DIVISION - DESIGN SECTION

STANDARD DRAWING NO.
ES-97
SHEET 3 OF 7
DATE 1-6-55

HYDRAULICS: FLOW IN CIRCULAR CONDUITS; Definition of symbols and formulas.

DEFINITION OF SYMBOLS

a = Cross-sectional area of flow in sq ft
$a_{c,Q}$ = Critical area corresponding to the discharge Q in ft^2 = area corresponding to $d_{c,Q}$
$a_{n,Q}$ = Normal area corresponding to the discharge Q in ft^2 = area corresponding to $d_{n,Q}$
d = Depth of flow in ft
$d_{c,Q}$ = Critical depth corresponding to the discharge Q in ft
$d_{n,Q}$ = Normal depth corresponding to the discharge Q in ft
D = Inside diameter of circular conduits in ft
D_r = Required inside diameter with freeboard of circular conduit in ft
g = Acceleration due to gravity = 32.16 ft/sec^2
n = Manning's coefficient of roughness
Q = Discharge in cfs
$Q_{c,d}$ = Critical discharge in cfs
$Q_{n,d}$ = Normal discharge in cfs
r = Hydraulic radius in ft
s_c = Critical slope in ft/ft
s_o = Bottom slope of conduit in ft/ft
T = Top width of flow in ft
v = Velocity of flow in ft/sec
$v_{c,Q}$ = Critical velocity corresponding to the discharge Q in ft/sec = $\dfrac{Q}{a_{c,Q}}$
$v_{n,Q}$ = Normal velocity corresponding to the discharge Q in ft/sec = $\dfrac{Q}{a_{n,Q}}$

FORMULAS

$$\frac{a}{D^2} = \frac{1}{4}\left[\cos^{-1}\frac{D-2d}{D} - \left(\frac{D-2d}{D}\right)\sin\left(\cos^{-1}\frac{D-2d}{D}\right)\right]$$

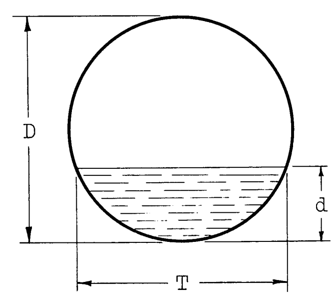
$$T = D\sin\left(\cos^{-1}\frac{D-2d}{D}\right)$$

$$\frac{Q_c^2}{g} = \frac{a_c^3}{T}$$

$$\frac{Q_c}{D^{5/2}} = \sqrt{\frac{g\left[\cos^{-1}\frac{D-2d_c}{D} - \left(\frac{D-2d_c}{D}\right)\sin\left(\cos^{-1}\frac{D-2d_c}{D}\right)\right]^3}{(4)^3\left[\sin\left(\cos^{-1}\frac{D-2d_c}{D}\right)\right]}}$$

$$Q_n = \frac{1.486}{n}\,a_n r^{2/3}\,s_o^{1/2}$$

$$\frac{n\,Q_n}{D^{8/3}\,s_o^{1/2}} = \frac{1.486\left[\cos^{-1}\frac{D-2d_n}{D} - \left(\frac{D-2d_n}{D}\right)\sin\left(\cos^{-1}\frac{D-2d_n}{D}\right)\right]^{5/3}}{(4)^{5/3}\left[\cos^{-1}\frac{D-2d_n}{D}\right]^{2/3}}$$



CIRCULAR SECTION

CONVERSIONS

$$\frac{\text{one cubic foot}}{\text{second}} = \frac{448.8\ \text{gallons}}{\text{minute}}$$

HYDRAULICS: FLOW IN CIRCULAR CONDUITS; Definition of symbols and formulas.

DEFINITION OF SYMBOLS

a = Cross-sectional area of flow in sq ft
$a_{c,Q}$ = Critical area corresponding to the discharge Q in ft^2 = area corresponding to $d_{c,Q}$
$a_{n,Q}$ = Normal area corresponding to the discharge Q in ft^2 = area corresponding to $d_{n,Q}$
d = Depth of flow in ft
$d_{c,Q}$ = Critical depth corresponding to the discharge Q in ft
$d_{n,Q}$ = Normal depth corresponding to the discharge Q in ft
D = Inside diameter of circular conduits in ft
D_r = Required inside diameter with freeboard of circular conduit in ft
g = Acceleration due to gravity = 32.16 ft/sec^2
n = Manning's coefficient of roughness
Q = Discharge in cfs
$Q_{c,d}$ = Critical discharge in cfs
$Q_{n,d}$ = Normal discharge in cfs
r = Hydraulic radius in ft
s_c = Critical slope in ft/ft
s_o = Bottom slope of conduit in ft/ft
T = Top width of flow in ft
v = Velocity of flow in ft/sec
$v_{c,Q}$ = Critical velocity corresponding to the discharge Q in ft/sec = $\dfrac{Q}{a_{c,Q}}$
$v_{n,Q}$ = Normal velocity corresponding to the discharge Q in ft/sec = $\dfrac{Q}{a_{n,Q}}$

FORMULAS

$$\frac{a}{D^2} = \frac{1}{4}\left[\cos^{-1}\frac{D-2d}{D} - \left(\frac{D-2d}{D}\right)\sin\left(\cos^{-1}\frac{D-2d}{D}\right)\right]$$

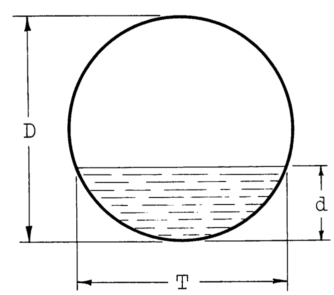
$$T = D\sin\left(\cos^{-1}\frac{D-2d}{D}\right)$$

$$\frac{Q_c^2}{g} = \frac{a_c^3}{T}$$

$$\frac{Q_c}{D^{5/2}} = \sqrt{\frac{g\left[\cos^{-1}\frac{D-2d_c}{D} - \left(\frac{D-2d_c}{D}\right)\sin\left(\cos^{-1}\frac{D-2d_c}{D}\right)\right]^3}{(4)^3\left[\sin\left(\cos^{-1}\frac{D-2d_c}{D}\right)\right]}}$$

$$Q_n = \frac{1.486}{n} a_n r^{2/3} s_o^{1/2}$$

$$\frac{n\,Q_n}{D^{8/3}\,s_o^{1/2}} = \frac{1.486\left[\cos^{-1}\frac{D-2d_n}{D} - \left(\frac{D-2d_n}{D}\right)\sin\left(\cos^{-1}\frac{D-2d_n}{D}\right)\right]^{5/3}}{(4)^{5/3}\left[\cos^{-1}\frac{D-2d_n}{D}\right]^{2/3}}$$



CIRCULAR SECTION

CONVERSIONS

$$\frac{\text{one cubic foot}}{\text{second}} = \frac{448.8\text{ gallons}}{\text{minute}}$$

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DRAWING NO. ES-97 SHEET 4 OF 7 DATE 1-13-55

EXAMPLE 1

Given: A circular conduit having a diameter D = 2.0 ft.

Determine: The cross-sectional flow area a corresponding to a depth of flow d = 0.86 ft.

Solution: Solving for the flow area a when $d \div D = 0.86 \div 2.0 = 0.43$, the corresponding value for $a \div D^2$ is 0.323. (From Sheet 1)

$$a = (0.323)(2.0)^2 = 1.292 \text{ ft}^2$$

EXAMPLE 2

Given: A circular conduit having a diameter D = 2.0 ft.

Determine: The depth of flow d corresponding to the cross-sectional area $a = 1.10 \text{ ft}^2$.

Solution: Solving for the depth of flow d when $a \div D^2 = 1.10 \div (2.0)^2 = 0.275$, the corresponding value for $d \div D$ is 0.381. (From Sheet 1)

$$d = (0.381)(2.0) = 0.762 \text{ ft}$$

EXAMPLE 3

Given: A circular conduit having a diameter D = 2.0 ft.

Determine: The critical discharge $Q_{c,d}$ corresponding to the depth of flow d = 0.86 ft.

Solution: Solving for the critical discharge $Q_{c,d}$ when $d \div D = 0.86 \div 2.0 = 0.43$, the corresponding value for $Q_{c,d} \div D^{5/2}$ is 1.046. (From Sheet 2)

$$Q_{c,d} = (1.046)(2.0)^{5/2} = 5.917 \text{ cfs}$$

EXAMPLE 4

Given: A circular conduit having a diameter D = 2.0 ft.

Determine: The critical depth of flow $d_{c,Q}$ corresponding to the discharge Q = 5.60 cfs.

Solution: Solving for the critical depth $d_{c,Q}$ when $Q \div D^{5/2} = 5.60 \div (2.0)^{5/2} = 0.9915$, the corresponding value for $d_{c,Q} \div D$ is 0.418. (From Sheet 2)

$$d_{c,Q} = (0.418)(2.0) = 0.836 \text{ ft}$$

EXAMPLE 5

Given: A concrete circular conduit having a diameter D = 2.0 ft, n = 0.015, and $s_o = 0.0025$.

Determine: The normal discharge $Q_{n,d}$ corresponding to a depth of flow d = 0.86 ft.

Solution: Solving for the normal discharge $Q_{n,d}$ when $d \div D = 0.86 \div 2.0 = 0.43$, the corresponding value for $nQ_{n,d} \div D^{8/3}s_o^{1/2}$ is 0.178. (From Sheet 3)

$$Q_{n,d} = \frac{(0.178)(2.0)^{8/3}(0.0025)^{1/2}}{0.015} = 3.768 \text{ cfs}$$

EXAMPLE 6

Given: A concrete circular conduit having a diameter D = 2.0 ft, n = 0.015, and $s_o = 0.0025$.

Determine: The normal depth of flow $d_{n,Q}$ corresponding to the discharge Q = 3.60 cfs.

Solution: Solving for the normal depth $d_{n,Q}$ when $nQ \div D^{8/3}s_o^{1/2} = (0.015)(3.60) \div (2.0)^{8/3}(0.0025)^{1/2} = 0.1701$, the corresponding value for $d_{n,Q} \div D$ is 0.419. (From Sheet 3)

$$d_{n,Q} = (0.419)(2.0) = 0.838 \text{ ft}$$

EXAMPLE 7

Given: A semi-circular metal flume having a diameter of D = 1.91 ft. Manning's roughness coefficient n = 0.012.

Determine: The critical slope of this flume for a discharge of Q = 1700 gpm = 3.788 cfs.

Solution: The critical slope s_c corresponding to a discharge Q is that bottom slope $s_o = s_c$ which will cause the critical depth $d_{c,Q}$ and the normal depth $d_{n,Q}$ to be equal. Solving for critical depth $d_{c,Q}$ corresponding to Q = 3.788 cfs.

When $\frac{Q}{D^{5/2}} = \frac{3.788}{(1.91)^{5/2}} = 0.7513$, the corresponding value for $\frac{d_{c,Q}}{D}$ is 0.3620. (From Sheet 2)

When $\frac{d_{n,Q}}{D} = 0.3620$, the corresponding value for $\frac{nQ}{D^{8/3}s_c^{1/2}}$ is 0.1298, (From Sheet 3) or

$$s_c = \left[\frac{nQ}{0.1298D^{8/3}}\right]^2 = \left[\frac{(0.012)(3.788)}{(0.1298)(1.91)^{8/3}}\right]^2 = 0.003883 \text{ ft/ft}$$

EXAMPLE 8

Given: The problem of designing a straight, semi-circular metal flume (n = 0.012). The flume is to convey a maximum discharge of Q = 1700 gpm = 3.788 cfs with a minimum freeboard of 6 percent of the diameter D of the flume and a bottom slope s_o which will permit a maximum velocity v equal to 80 percent of the critical velocity $v_{c,Q}$ corresponding to the discharge Q is to be provided. The actual velocity v is not to exceed 4 ft/sec. The flume is to be constructed so that the depth of flow at the outlet end of the flume will be equal to the calculated normal depth $d_{n,Q}$ corresponding to the discharge Q if $v_{n,Q} \leq 4$ ft/sec. This insures that normal flow conditions will exist in the flume (see ES-38, Case A).

Determine: a(1) The required diameter D_r of the flume which will satisfy, simultaneously, the stated freeboard and velocity criteria.

(2) The diameter D of a standard size flume to be used.

b(1) The critical velocity $v_{c,Q}$ corresponding to the discharge Q in the flume having the diameter D.

(2) The maximum bottom slope s_o of the flume having a diameter D determined in a(2).

(3) The normal depth of flow $d_{n,Q}$ corresponding to the discharge Q.

(4) The percent freeboard of diameter D.

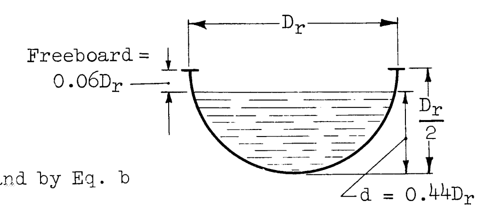
c(1) The minimum slope s_o of the flume having a diameter D determined in a(2).

(2) The actual velocity existing in the flume with the minimum slope.

(3) The ratio $v/v_{c,Q}$.

Solution: a(1) Solving for the required diameter D_r of the flume which satisfies the stated freeboard and velocity criteria. The depth of flow is 6 percent of D_r less than $\frac{D_r}{2}$ or $d = \frac{D_r}{2} - 0.06D_r$ or $\frac{d}{D_r} = 0.44$. The critical velocity $v_{c,Q}$ corresponding to the discharge Q is

$$v_{c,Q} = \frac{Q}{a_{c,Q}} \quad \text{(Eq. a)}$$



The actual velocity v is

$$v = \frac{Q}{a} \quad \text{(Eq. b)}$$

When $\frac{d}{D_r} = 0.44$, the corresponding value of $\frac{a}{D_r^2}$ is 0.333 and by Eq. b

$$v = \frac{Q}{(0.333)D_r^2} \quad \text{(Eq. c)}$$

On substituting for v and $v_{c,Q}$ (see Eqs. a and c) and the stated criterion $v = 0.80v_{c,Q}$, obtain (It will be assumed that v is less than 4 ft/sec)

$$\frac{Q}{(0.333)D_r^2} = 0.80\frac{Q}{a_{c,Q}} \quad \text{or} \quad \frac{a_{c,Q}}{D_r^2} = 0.80(0.333) = 0.2664$$

When $\frac{a_{c,Q}}{D_r^2} = 0.2664$, the corresponding value of $\frac{d_{c,Q}}{D_r}$ is 0.3727. When $\frac{d_{c,Q}}{D_r} = 0.3727$, the corresponding value of $\frac{Q}{D_r^{5/2}}$ is 0.795.

$$D_r^{5/2} = \frac{Q}{0.795} = \frac{3.788}{0.795} = 4.7648$$

$$D_r = 1.867 \text{ ft}$$

Concluded on Sheet 7

Continuation from Sheet 6

There exists only one value of D_r which will satisfy both the criteria (1) $d = 0.44D_r$ and (2) $v = 0.80v_{c,Q}$ simultaneously, for a given value of Q, n, and cross-sectional shape.

(2) The value of D_r is not a standard diameter size. The next standard diameter D size greater than D_r is to be chosen. Use 36-inch sheeting or D = 1.91 ft. By changing the diameter D to a greater number than D_r, both criteria cannot be satisfied simultaneously. Two extremes in the bottom slopes can be obtained by:

(1) fixing the maximum velocity in accordance to the stated criteria and allowing the freeboard to increase above the criterion.

(2) fixing the minimum freeboard to 6 percent of D and allowing the velocity to be decreased from that given by the criterion.

b(1) Solving for the critical velocity $v_{c,a}$ corresponding to the discharge Q. When $\frac{Q}{D^{5/2}} = \frac{3.788}{(1.91)^{5/2}} = 0.75125$, the corresponding value for $\frac{d_{c,Q}}{D}$ is 0.3620. (From Sheet 2) When $\frac{d_{c,Q}}{D} = 0.3620$, the corresponding value for $\frac{a_{c,Q}}{D^2}$ is 0.2565.

$$a_{c,Q} = 0.2565D^2 = 0.2565\ (1.91)^2 = 0.9357\ ft^2$$

$$v_{c,Q} = \frac{Q}{a_{c,Q}} = \frac{3.788}{0.9357} = 4.048\ ft/sec$$

(2) The maximum bottom slope is determined by fixing the maximum permissible velocity.

$$v_{n,Q} = 0.80v_{c,Q} = 0.80\ (4.048) = 3.238\ ft/sec$$

$$a_{n,Q} = \frac{Q}{v_{n,Q}} = \frac{3.788}{3.238} = 1.170\ ft^2$$

When $\frac{a_{n,Q}}{D^2} = \frac{1.170}{(1.91)^2} = 0.3207$, the corresponding value for $\frac{d_{n,Q}}{D}$ is 0.4277. (From Sheet 1) When $\frac{d_{n,Q}}{D} = 0.4277$, the corresponding value for $\frac{nQ}{D^{8/3}\ s_o^{1/2}}$ is 0.1763, (From Sheet 3) or

$$s_o = \left[\frac{nQ}{D^{8/3}(0.1763)}\right]^2 = \left[\frac{(0.012)(3.788)}{(1.91)^{8/3}(0.1763)}\right]^2 = 0.002105$$

(3) Solving for the normal depth of flow $d_{n,Q}$ corresponding to the discharge Q is (see above)

$$d_{n,Q} = 0.4277D = 0.4277\ (1.91) = 0.817\ ft$$

(4) The percent freeboard is

$$\frac{\left[\frac{D}{2} - d_{n,Q}\right]100}{D} = \frac{\left[\frac{1.91}{2} - 0.817\right]100}{1.91} = 7.23\%$$

c(1) When the minimum slope is desired the minimum velocity is required, i.e., the maximum flow area. The maximum flow area permissible by the stated criteria corresponds to a depth of flow equal to 0.44D. Solving for the minimum slope s_o of the flume having a depth of flow d = 0.44D. When $\frac{d}{D} = 0.44$, the corresponding value for $\frac{nQ_{n,d}}{D^{8/3}\ s_o^{1/2}}$ is 0.1853

$$s_o = \left[\frac{nQ}{D^{8/3}(0.1853)}\right]^2 = \left[\frac{(0.012)(3.788)}{(1.91)^{8/3}(0.1853)}\right]^2 = 0.001905$$

(2) Solving for the actual velocity at flow depth d = 0.44D. When $\frac{d}{D} = 0.44$, the corresponding value for $\frac{a}{D^2} = 0.333$.

$$a = 0.333D^2 = 0.333(1.91)^2 = 1.215\ ft^2$$

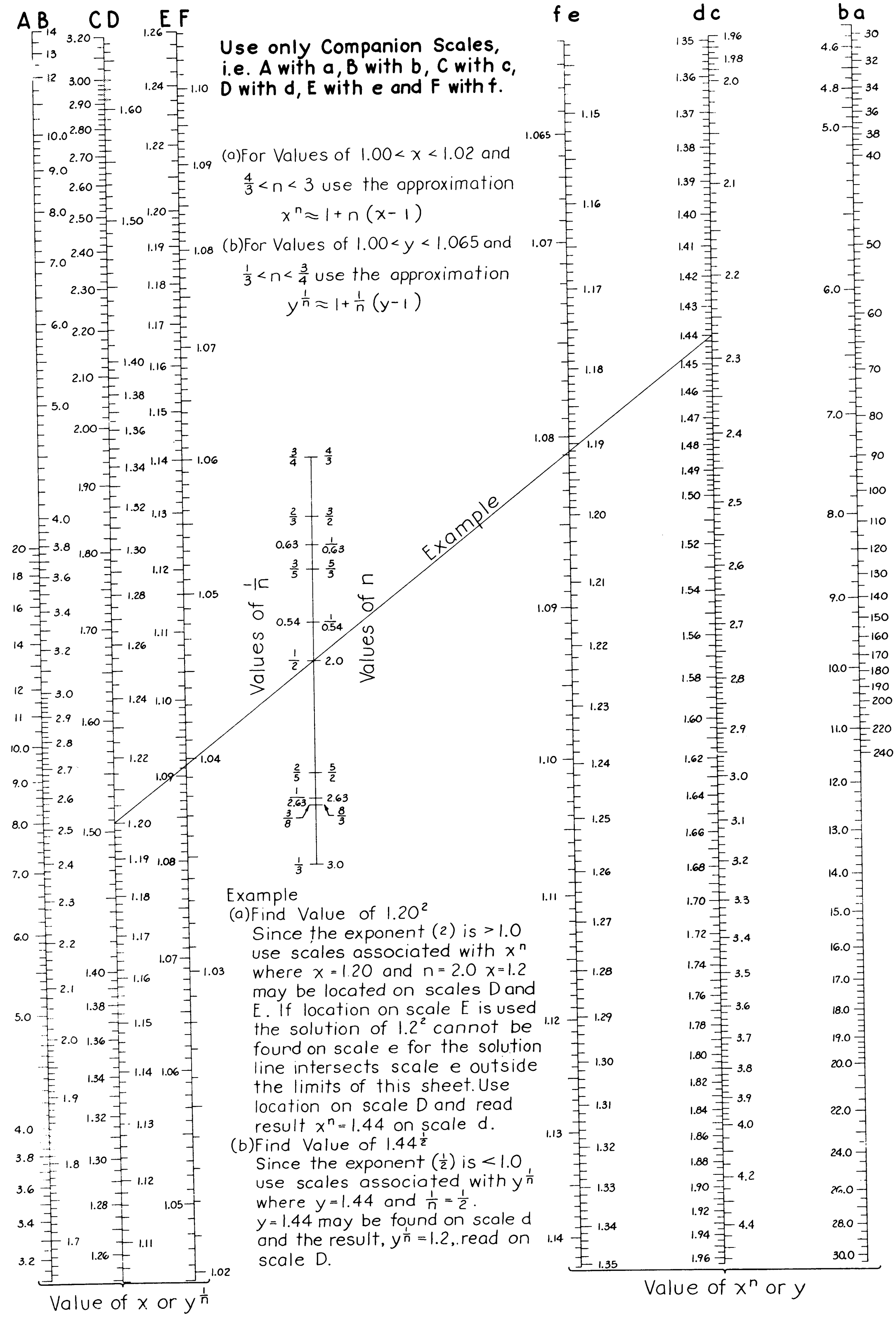
$$v = \frac{Q}{a} = \frac{3.788}{1.215} = 3.118\ ft/sec$$

(3) Solving for the ratio $\frac{v}{v_{c,Q}}$

$$\frac{v}{v_{c,Q}} = \frac{3.118}{4.048} = 77.03\%$$

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DRAWING NO. ES-97 SHEET 7 OF 7 DATE 1-14-55

HYDRAULICS: VALUES OF y^n AND $x^{1/n}$ FOR x AND y >1



Use only Companion Scales, i.e. A with a, B with b, C with c, D with d, E with e and F with f.

(a) For Values of $1.00 < x < 1.02$ and $\frac{4}{3} < n < 3$ use the approximation

$$x^n \approx 1 + n(x-1)$$

(b) For Values of $1.00 < y < 1.065$ and $\frac{1}{3} < n < \frac{3}{4}$ use the approximation

$$y^{\frac{1}{n}} \approx 1 + \frac{1}{n}(y-1)$$

Example

(a) Find Value of 1.20^2

Since the exponent (2) is >1.0 use scales associated with x^n where $x = 1.20$ and $n = 2.0$ $x = 1.2$ may be located on scales D and E. If location on scale E is used the solution of 1.2^2 cannot be found on scale e for the solution line intersects scale e outside the limits of this sheet. Use location on scale D and read result $x^n = 1.44$ on scale d.

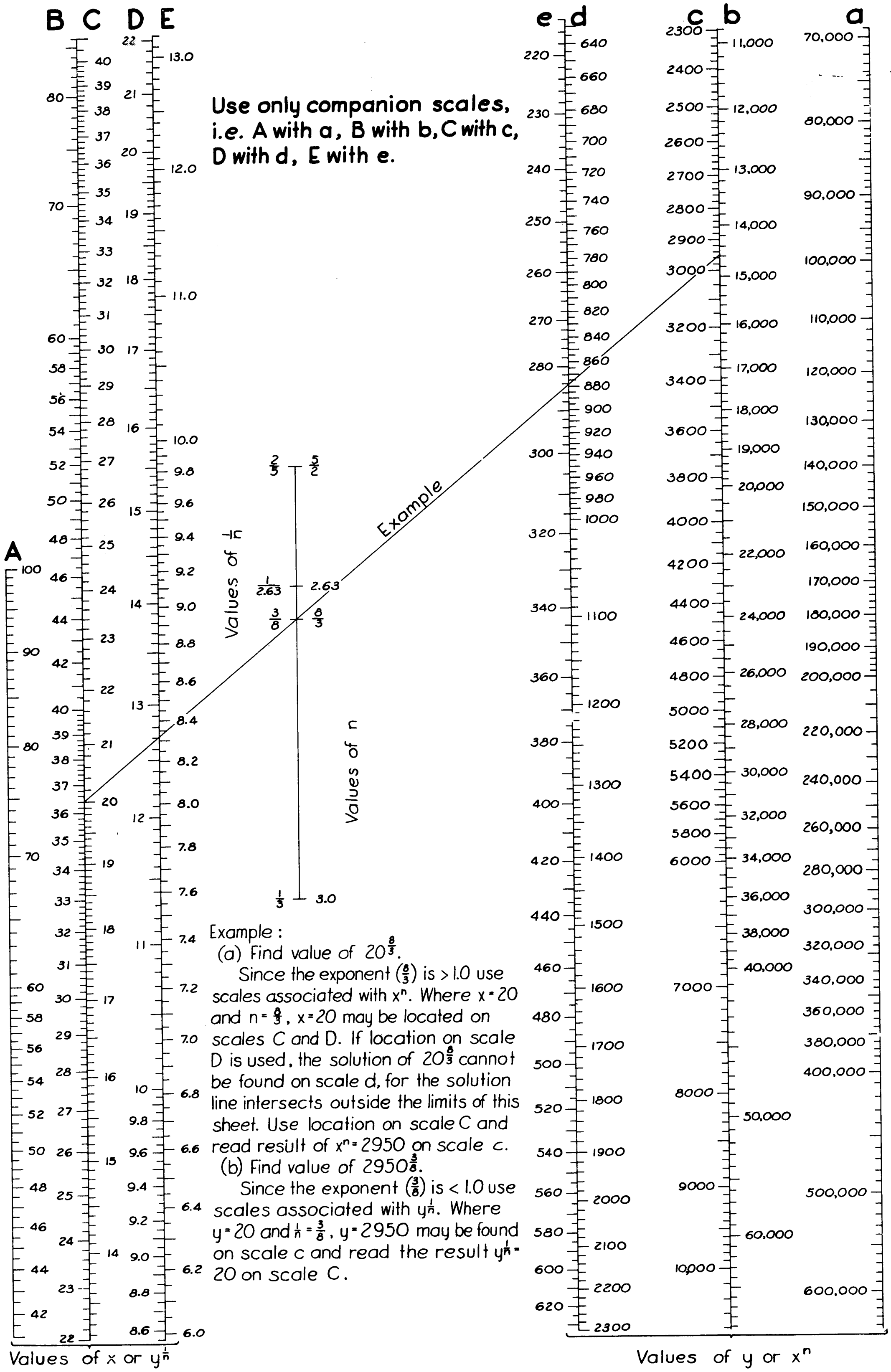
(b) Find Value of $1.44^{\frac{1}{2}}$

Since the exponent $(\frac{1}{2})$ is <1.0 use scales associated with $y^{\frac{1}{n}}$ where $y = 1.44$ and $\frac{1}{n} = \frac{1}{2}$. $y = 1.44$ may be found on scale d and the result, $y^{\frac{1}{n}} = 1.2$, read on scale D.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 37 SHEET 1 OF 3 DATE 6-3-50

Revised 6-4-51

HYDRAULICS: VALUES OF y^n AND $x^{1/n}$ FOR x AND y > 1



Use only companion scales, i.e. A with a, B with b, C with c, D with d, E with e.

Example :

(a) Find value of $20^{\frac{8}{3}}$.

Since the exponent ($\frac{8}{3}$) is > 1.0 use scales associated with x^n. Where x = 20 and n = $\frac{8}{3}$, x = 20 may be located on scales C and D. If location on scale D is used, the solution of $20^{\frac{8}{3}}$ cannot be found on scale d, for the solution line intersects outside the limits of this sheet. Use location on scale C and read result of x^n = 2950 on scale c.

(b) Find value of $2950^{\frac{3}{8}}$.

Since the exponent ($\frac{3}{8}$) is < 1.0 use scales associated with $y^{\frac{1}{n}}$. Where y = 20 and $\frac{1}{n} = \frac{3}{8}$, y = 2950 may be found on scale c and read the result $y^{\frac{1}{n}}$ = 20 on scale C.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 37 SHEET 2 OF 3 DATE 6 - 3 - 50

Revised 6-4-51

HYDRAULICS: VALUES OF y^n AND $x^{1/n}$ FOR x AND y<1

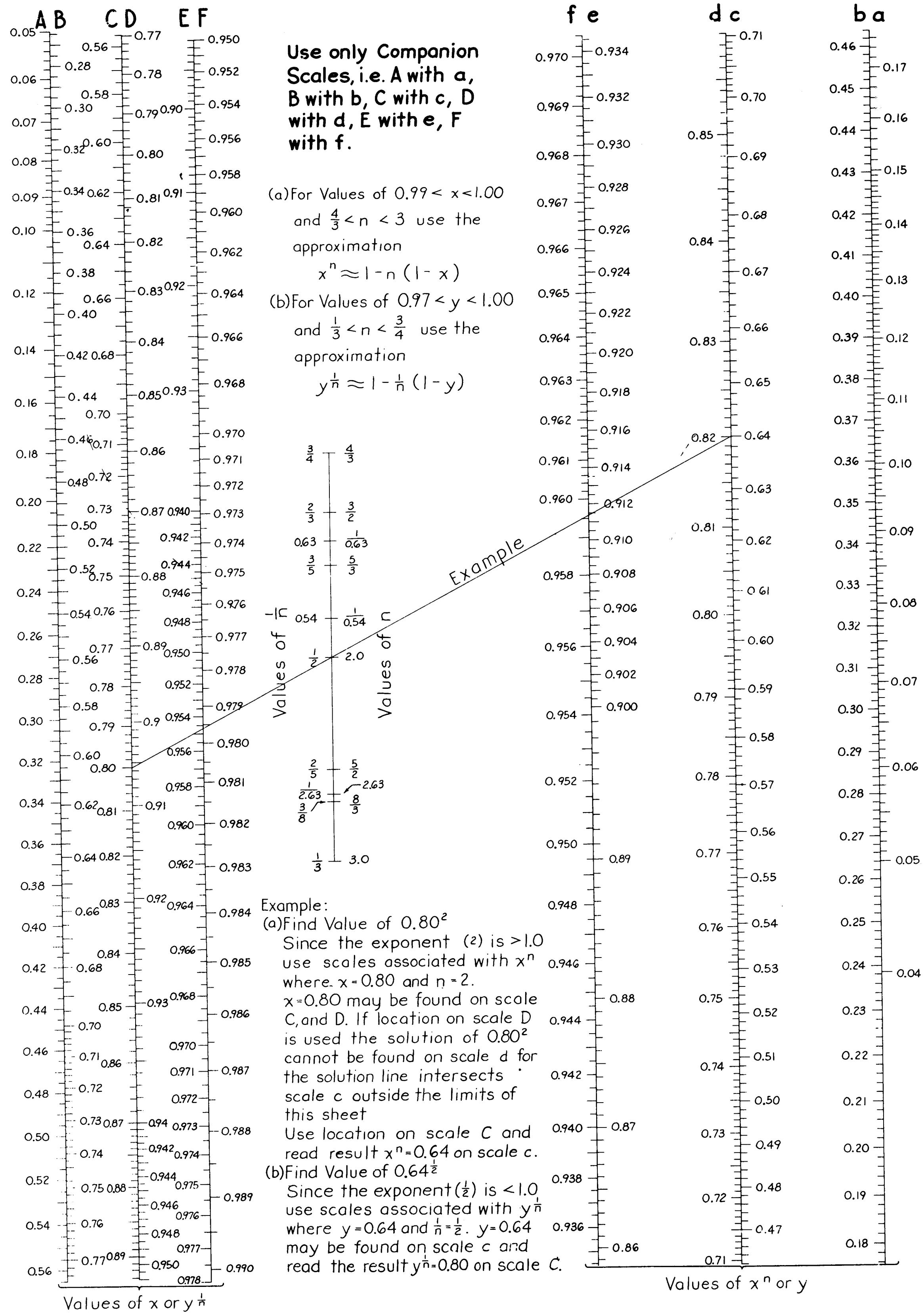
Use only Companion Scales, i.e. A with a, B with b, C with c, D with d, E with e, F with f.

(a) For Values of $0.99 < x < 1.00$ and $\frac{4}{3} < n < 3$ use the approximation

$$x^n \approx 1 - n(1 - x)$$

(b) For Values of $0.97 < y < 1.00$ and $\frac{1}{3} < n < \frac{3}{4}$ use the approximation

$$y^{\frac{1}{n}} \approx 1 - \frac{1}{n}(1 - y)$$



Example:

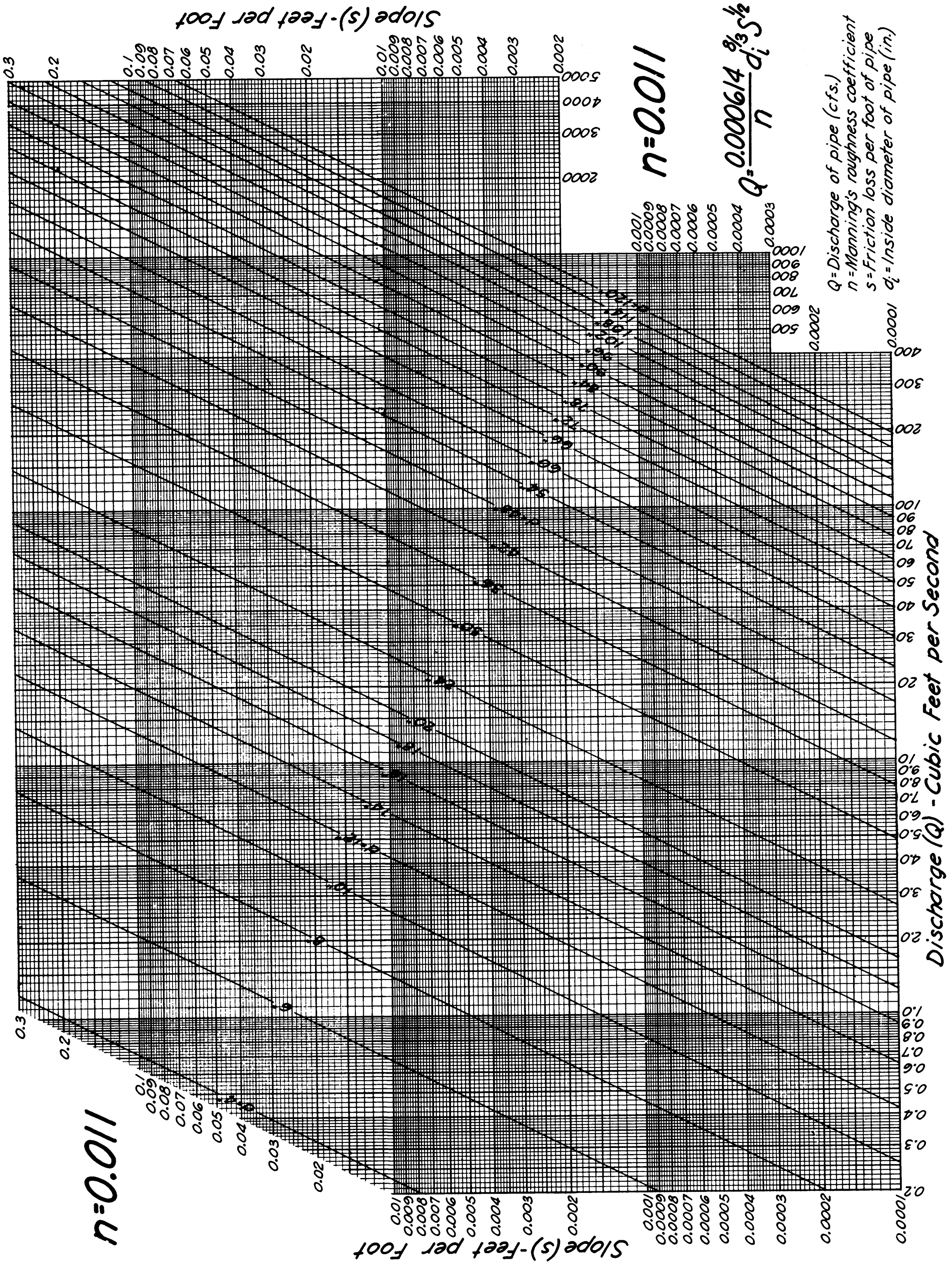
(a) Find Value of 0.80^2
Since the exponent (2) is >1.0 use scales associated with x^n where $x = 0.80$ and $n = 2$.
$x = 0.80$ may be found on scale C, and D. If location on scale D is used the solution of 0.80^2 cannot be found on scale d for the solution line intersects scale c outside the limits of this sheet.
Use location on scale C and read result $x^n = 0.64$ on scale c.

(b) Find Value of $0.64^{\frac{1}{2}}$
Since the exponent ($\frac{1}{2}$) is <1.0 use scales associated with $y^{\frac{1}{n}}$ where $y = 0.64$ and $\frac{1}{n} = \frac{1}{2}$. $y = 0.64$ may be found on scale c and read the result $y^{\frac{1}{n}} = 0.80$ on scale C.

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 37 SHEET 3 OF 3 DATE 6 - 3 - 50

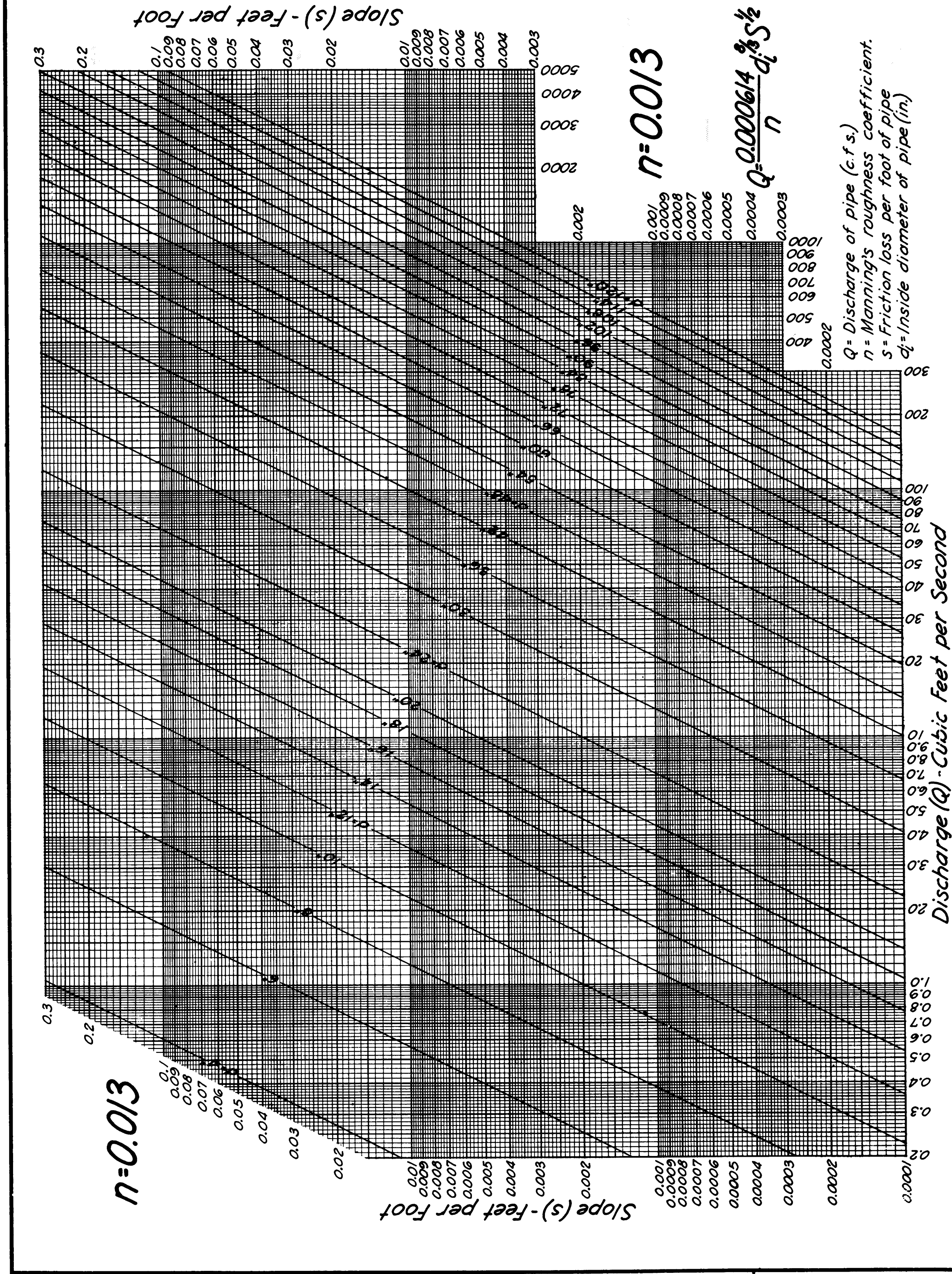
Revised 6-4-51

HYDRAULICS: DISCHARGE OF CIRCULAR PIPE FLOWING FULL



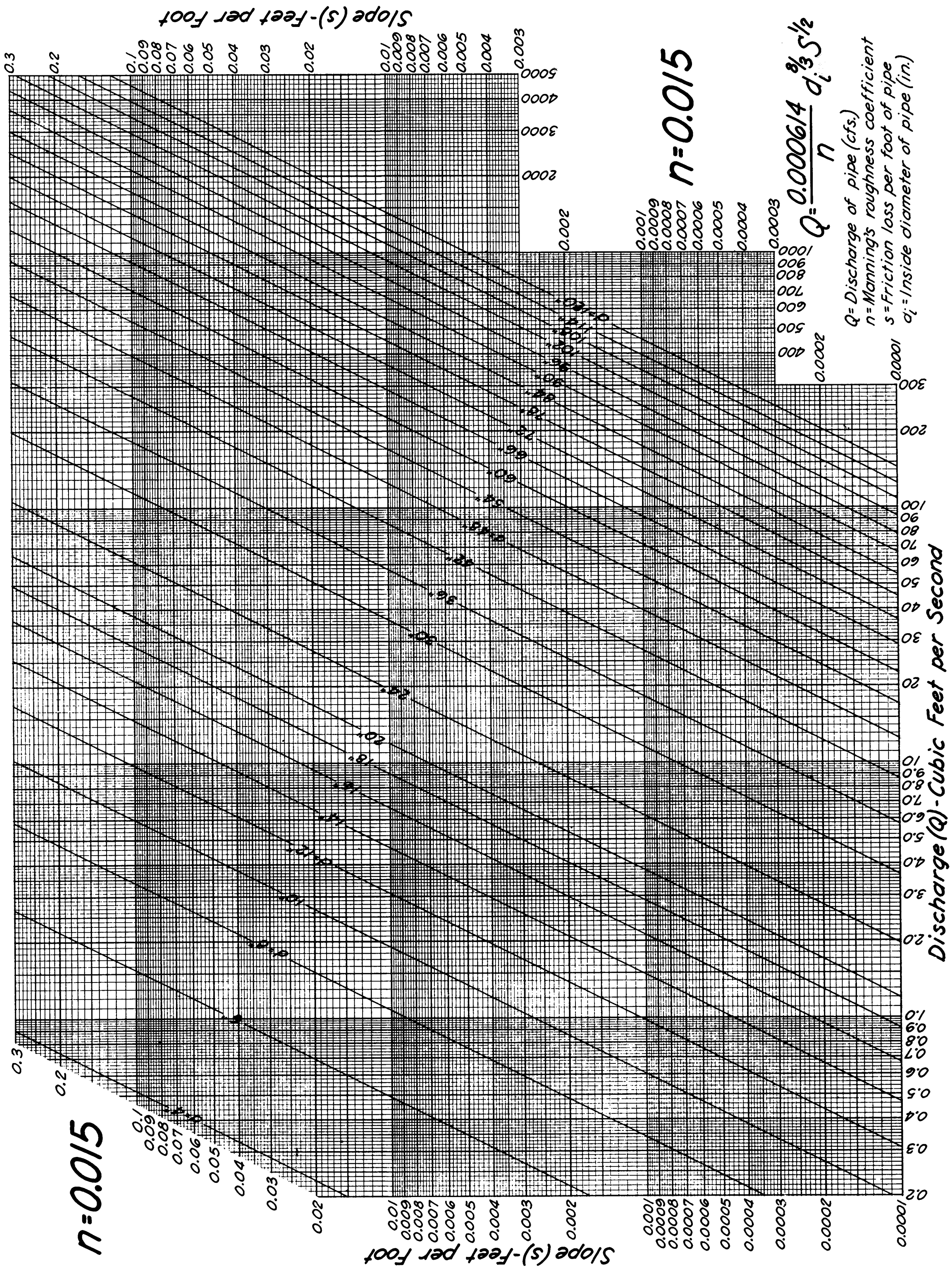
REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-54 SHEET 1 OF 4 DATE 4-27-51

HYDRAULICS: DISCHARGE OF CIRCULAR PIPE FLOWING FULL



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-54 SHEET 2 OF 4 DATE 4-27-51

HYDRAULICS: DISCHARGE OF CIRCULAR PIPE FLOWING FULL



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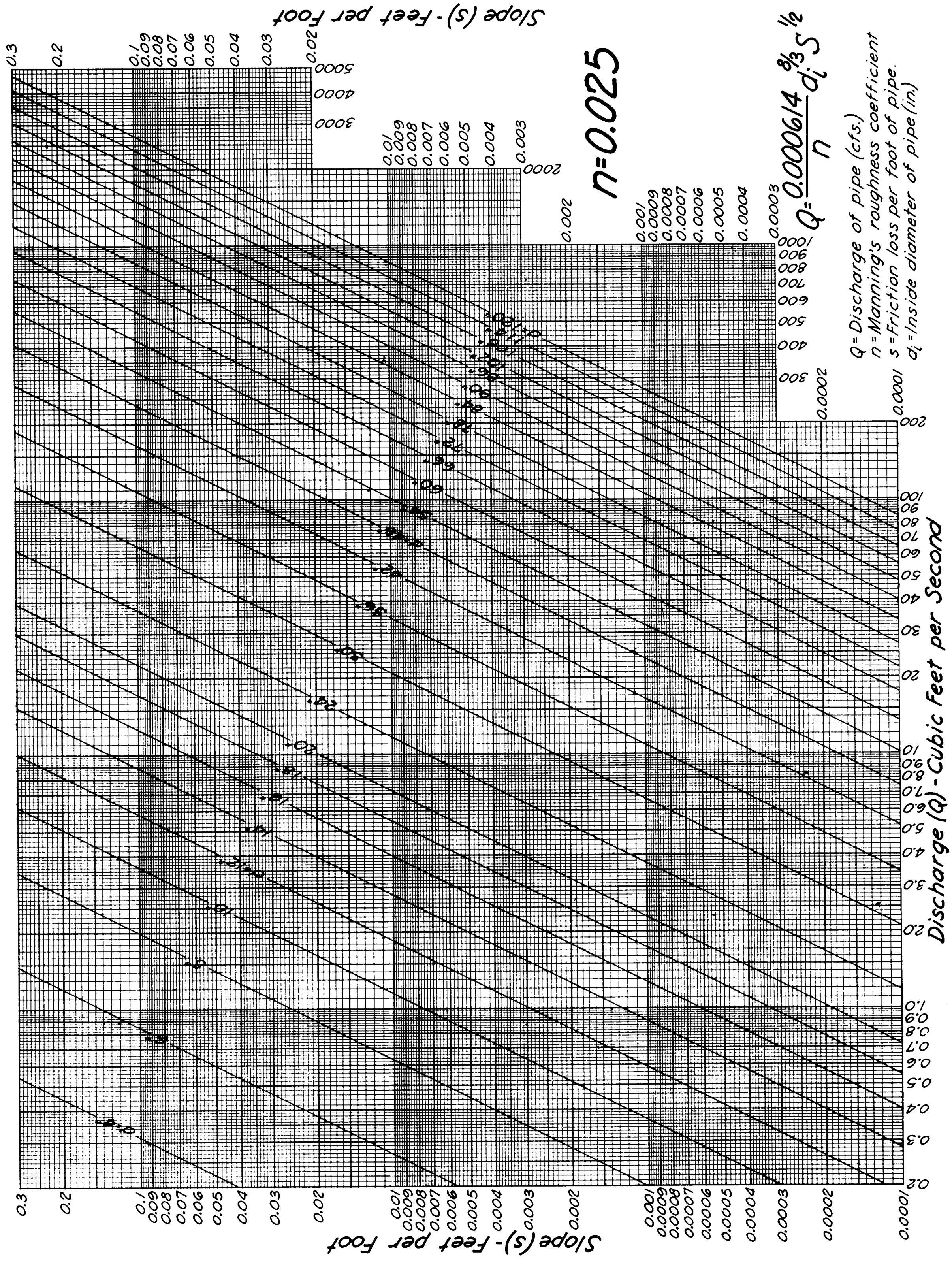
HYDRAULICS: DISCHARGE OF CIRCULAR PIPE FLOWING FULL



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Bennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES-54 SHEET 4 OF 4 DATE 4-27-51

Since Q = av, equation (5.5-14) may be converted to the following formula for discharge in any conduit:

$$Q = 1.318\ a\ C\ r^{0.63}\ s^{0.54} \qquad (5.5\text{-}15)$$

Substitution of a and r in terms of inside diameter of pipe in inches in equation (5.5-15) gives the following general formula for discharge in circular pipes:

$$Q/C = 0.0006273\ d_1^{2.63}\ s^{0.54} \qquad (5.5\text{-}16)$$

Graphical solutions of equation (5.5-16) for standard pipe ranging from 1 to 12 inches in diameter and a wide range in slope may be made by drawing ES-40. The 0.63 and 0.54 powers of numbers for use in the above forms of Hazen-Williams formula may be computed by drawing ES-37.

Values of C for different types of pipe are given in table 5.5-2.

TABLE 5.5-2. VALUES OF HAZEN-WILLIAMS C

Description of Pipe	C
1. Very smooth pipe; straight alignment	140
2. Very smooth pipe; slight curvature	130
3. Cast iron, uncoated - new	130
5 years old	120
10 years old	110
15 years old	100
20 years old	90
30 years old	80
coated - all ages	130
4. Steel pipe, welded, new	130
(Same deterioration with age as cast iron, uncoated)	
For permanent installation use	100
5. Wrought iron or standard galvanized steel - diam. 12 in. up	110
4 to 12 in.	100
4 in. down	80
6. Brass or lead, new	140
7. Concrete, very smooth, excellent joints	140
smooth, good joints	120
rough	110
8. Vitrified	110
9. Smooth wooden or wood stave	120
10. Asbestos, cement	140
11. Corrugated pipe	60

Note: Pipes of small diameter, old age, and very rough inside surface, may give values as low as C = 40

5.4 Other Losses. In addition to the friction head losses there are other losses of energy which occur as the result of turbulence created by changes in velocity and direction of flow. To facilitate their inclusion in Bernoulli's energy equation, such losses are commonly expressed in terms of the mean velocity head at some specific cross section of the pipe.

These losses are sometimes called minor losses which may be a serious misnomer. In long pipe lines, the entrance loss, bend losses, etc., may be a relatively insignificant part of the total loss and in such cases can be ignored without introducing significant error. Such is not the case in many structures such as culverts, drop inlets, and siphons which are relatively short. Safe design practice requires an estimate of such losses. In case the estimate indicates that minor losses amount to 5 percent or more of the total head loss, they should be carefully evaluated and included in the flow calculations.

As velocities increase, careful determination of such minor losses becomes increasingly important; with a mean velocity of 30 feet per second, the neglect of an entrance loss of 0.5 $v^2/2g$ results in an error in head loss of 7 feet, whereas if the mean velocity is 3 feet per second, neglect of such an entrance loss results in an error of only 0.07 feet.

Data on minor losses most commonly required are contained in the following subsections.

5.4.1 Entrance Loss. Loss of head at the entrance of a pipe results from turbulence caused by the contraction of the flow cross section. It is expressed by the following equation:

$$H_o = K_o \frac{v^2}{2g} \qquad (5.5\text{-}17)$$

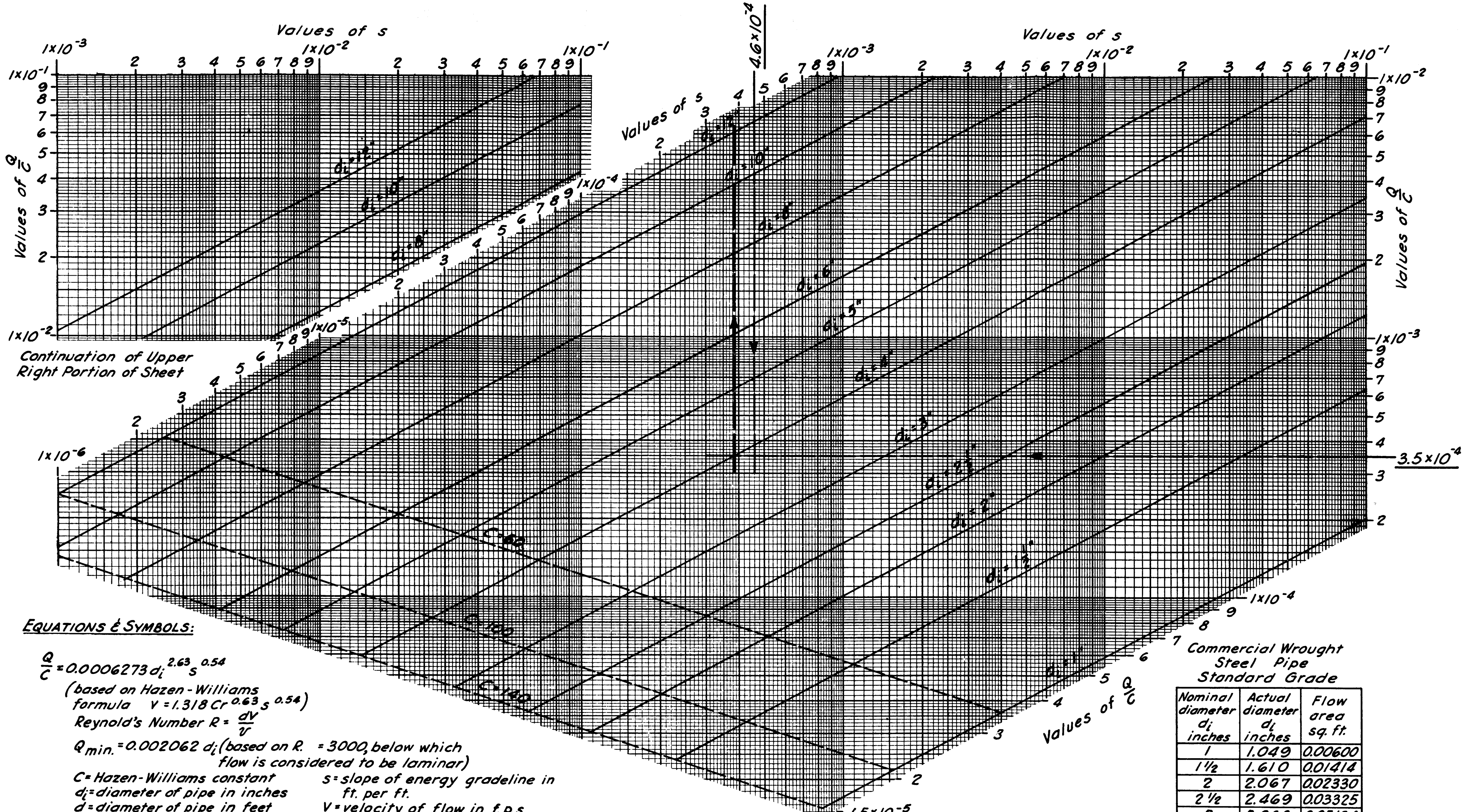
in which

H_o = head loss at the entrance in ft.
K_o = a coefficient dependent on the type of inlet.
v = velocity in pipe in fps.
g = acceleration of gravity in ft. per sec.^{2}

Values of K_o are given in the following table:

Type of Inlet	Value of K_o
Inward projecting	0.78
Sharp cornered	0.50
Bell mouth	0.04

HYDRAULICS: SOLUTION OF HAZEN-WILLIAMS FORMULA FOR ROUND PIPES



<u>EQUATIONS & SYMBOLS:</u>

$\frac{Q}{C} = 0.0006273\, d_i^{2.63} s^{0.54}$

(based on Hazen-Williams formula $V = 1.318\, C r^{0.63} s^{0.54}$)

Reynold's Number $R = \frac{dV}{\nu}$

$Q_{min.} = 0.002062\, d_i$ (based on $R. = 3000$, below which flow is considered to be laminar)

C = Hazen-Williams constant
d_i = diameter of pipe in inches
d = diameter of pipe in feet
Q = discharge in c.f.s.
r = hydraulic radius in feet
s = slope of energy gradeline in ft. per ft.
V = velocity of flow in f.p.s.
ν = kinematic viscosity in ft.^{2} per sec. (assumed to be 1.05×10^{-5} ft.^{2} per sec. for water at 70°F.)

Note: Dashed lines pass through minimum values of Q/C for the given C. For lesser values of Q/C, according to the assumptions of ν and R, there can be no assurance of turbulent flow.

<u>Example</u>: Find size pipe required to carry $Q = 0.042$ cfs with a maximum energy gradient $S = 0.00046$ if $C = 120$. Then $Q/C = \frac{0.042}{120} = 3.5 \times 10^{-4}$. Enter chart, with above values, and find $d_i = 4$ in.. Actual head loss in 4 in. pipe = 3.85×10^{-4} ft. per ft.

Commercial Wrought Steel Pipe
Standard Grade

Nominal diameter d_i inches	Actual diameter d_i inches	Flow area sq. ft.
1	1.049	0.00600
1½	1.610	0.01414
2	2.067	0.02330
2½	2.469	0.03325
3	3.068	0.05134
4	4.026	0.08840
5	5.047	0.1389
6	6.065	0.2006
8	7.981	0.3474
10	10.020	0.5475
12	12.000	0.7854

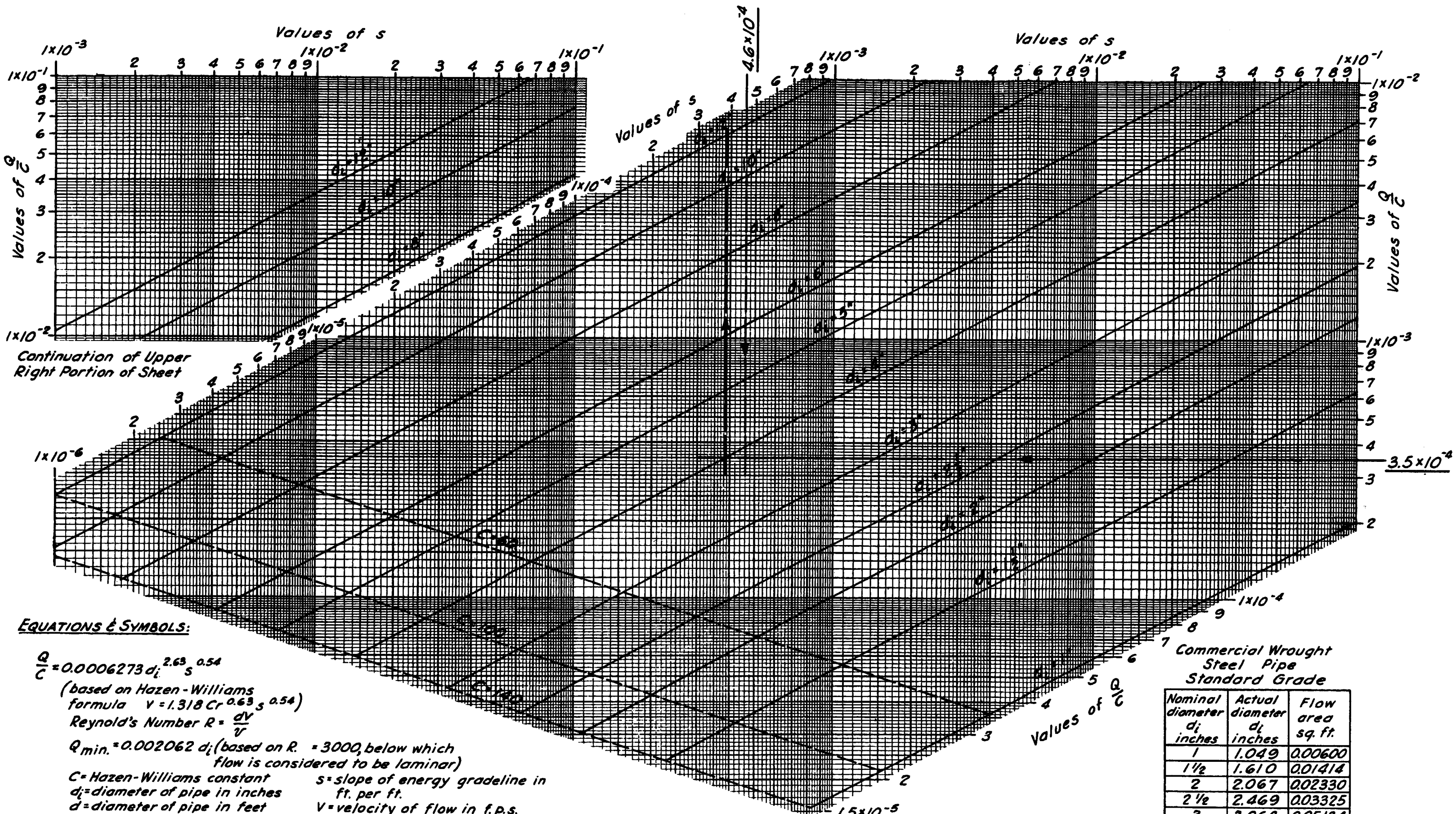
REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
H. H. Bennett, Chief
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-40
SHEET 1 OF 1
DATE 7-21-50

REVISED 3-7-51

HYDRAULICS: SOLUTION OF HAZEN-WILLIAMS FORMULA FOR ROUND PIPES



<u>EQUATIONS & SYMBOLS:</u>

$\frac{Q}{C} = 0.0006273\, d_i^{2.63} s^{0.54}$

(based on Hazen-Williams formula $V = 1.318\, C r^{0.63} s^{0.54}$)

Reynold's Number $R = \frac{dV}{\nu}$

$Q_{min.} = 0.002062\, d_i$ (based on $R = 3000$, below which flow is considered to be laminar)

C = Hazen-Williams constant
d_i = diameter of pipe in inches
d = diameter of pipe in feet
Q = discharge in c.f.s.
r = hydraulic radius in feet
s = slope of energy gradeline in ft. per ft.
V = velocity of flow in f.p.s.
ν = kinematic viscosity in ft.^{2} per sec. (assumed to be 1.05×10^{-5} ft.^{2} per sec. for water at 70°F.)

Note: Dashed lines pass through minimum values of Q/C for the given C. For lesser values of Q/C, according to the assumptions of ν and R, there can be no assurance of turbulent flow.

<u>Example</u>: Find size pipe required to carry $Q = 0.042$ cfs with a maximum energy gradient $S = 0.00046$ if $C = 120$. Then $Q/C = \frac{0.042}{120} = 3.5\times10^{-4}$. Enter chart, with above values, and find $d_i = 4$ in.. Actual head loss in 4 in. pipe $= 3.85\times10^{-4}$ ft. per ft.

Commercial Wrought Steel Pipe Standard Grade

Nominal diameter d_i inches	Actual diameter d_i inches	Flow area sq. ft.
1	1.049	0.00600
1½	1.610	0.01414
2	2.067	0.02330
2½	2.469	0.03325
3	3.068	0.05134
4	4.026	0.08840
5	5.047	0.1389
6	6.065	0.2006
8	7.981	0.3474
10	10.020	0.5475
12	12.000	0.7854

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
H. H. Bennett, Chief
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-40
SHEET 1 OF 1
DATE 7-21-50

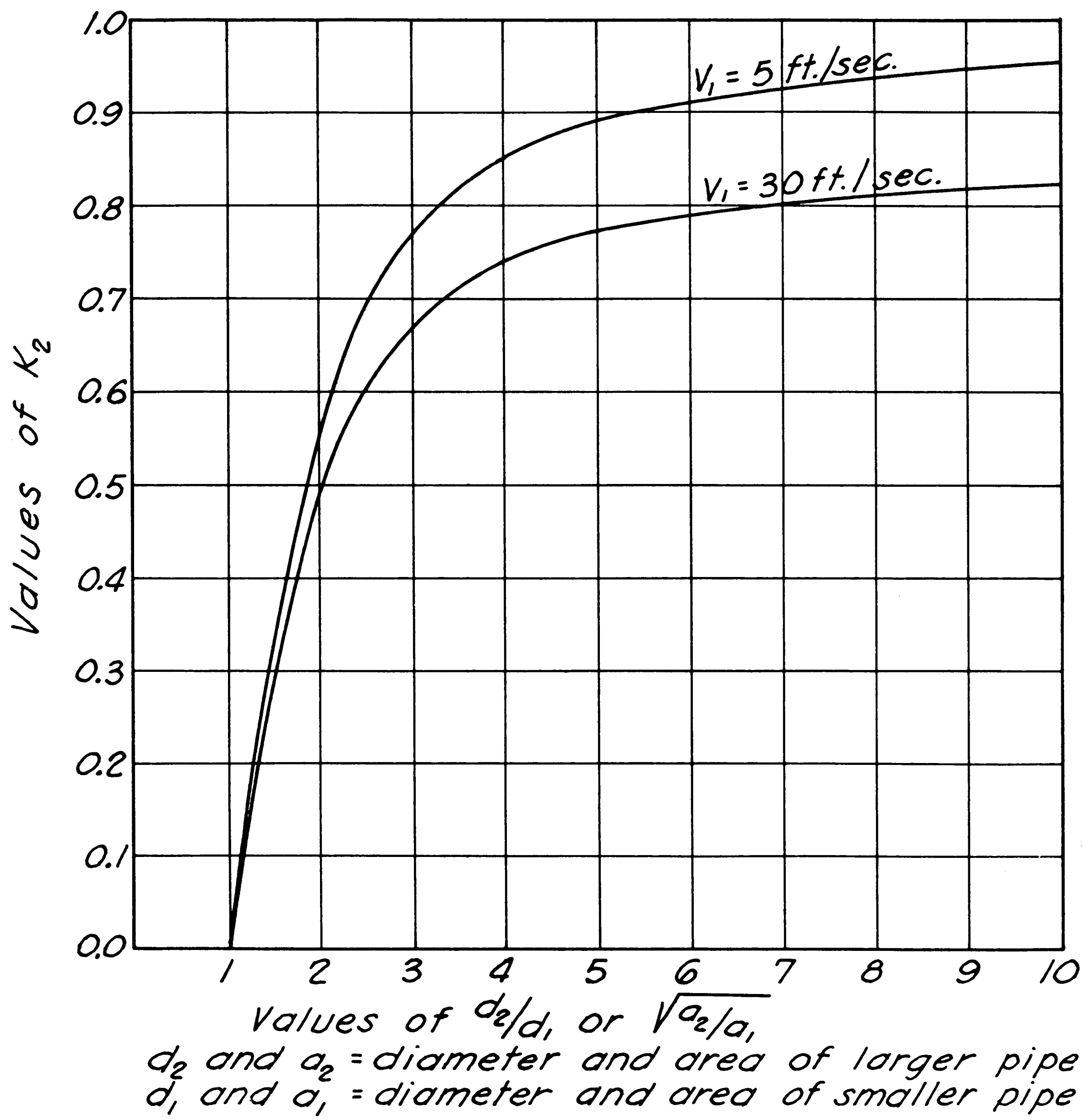
REVISED 3-7-51

5.4.2 Enlargement Loss. Loss of head due to enlargement of the pipe section may be computed by:

$$H_2 = K_2 \frac{v_1^2}{2g} \tag{5.5-18}$$

H_2 = head loss due to enlargement in ft.
K_2 = a coefficient depending on the degree of enlargement of the pipe section.
v_1 = mean velocity in the smaller pipe in fps.

Values of K_2 for sudden enlargement may be taken from fig. 5.5-2. If values corresponding more closely to given velocities are required, refer to King's Handbook, table 73, p. 231. When losses due to gradual enlargement are to be estimated, refer to King's Handbook, table 74, p. 231, for values of K_2 for conical enlargements.



LOSS COEFFICIENT FOR SUDDEN ENLARGEMENT

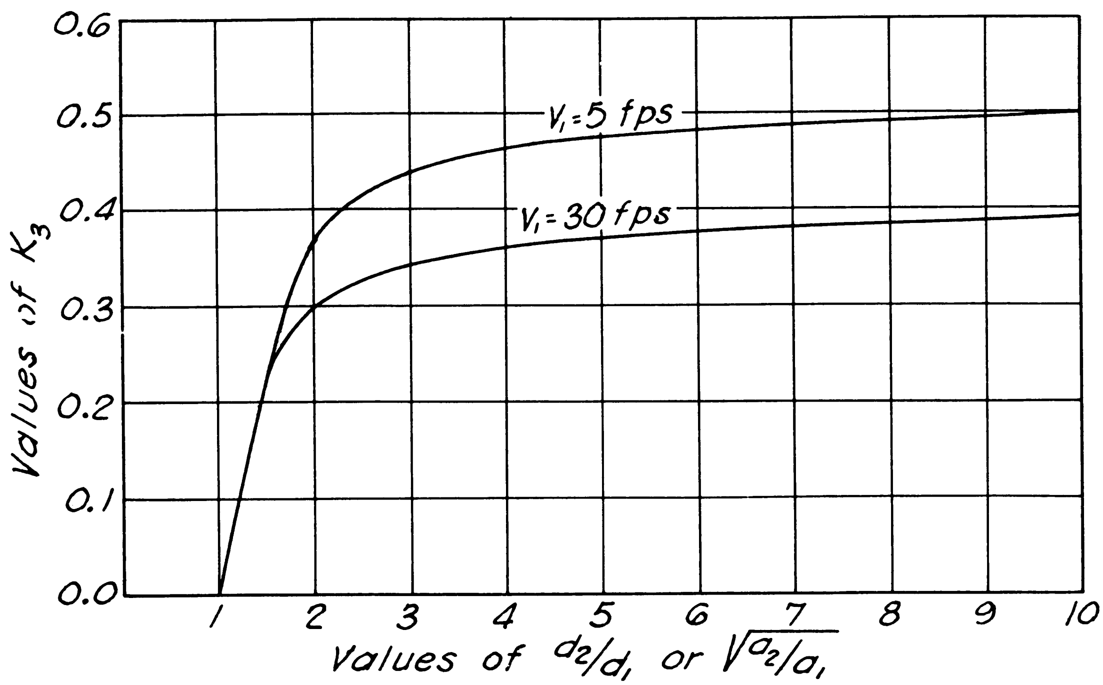
FIG. 5.5-2

5.4.3 Contraction Loss. Loss of head due to sudden contraction of the pipe section may be computed by:

$$H_3 = K_3 \frac{v_1^2}{2g} \qquad (5.5\text{-}19)$$

H_3 = head loss due to sudden contraction in ft.
K_3 = a coefficient depending on the degree of reduction in the pipe section.
v_1 = mean velocity in the smaller pipe in fps.

Values of K_3 may be taken from fig. 5.5-3. Inspection of that figure will show that K_3 varies primarily with the ratio of the larger to the smaller diameter and secondarily with velocity in the smaller pipe. When it is desired to use values of K_3 corresponding more closely to given velocities than may be interpolated from fig. 5.5-3, they may be obtained from King's Handbook, table 76, p. 232. Where one or both pipes have other than a round section, convert cross-sectional area to equivalent diameter to determine the ratio of the larger to the smaller diameter; or use the square root of the ratio of the larger area to the smaller area.



d_2 and a_2 = diameter and area of larger pipe
d_1 and a_1 = diameter and area of smaller pipe

LOSS COEFFICIENT FOR SUDDEN CONTRACTION

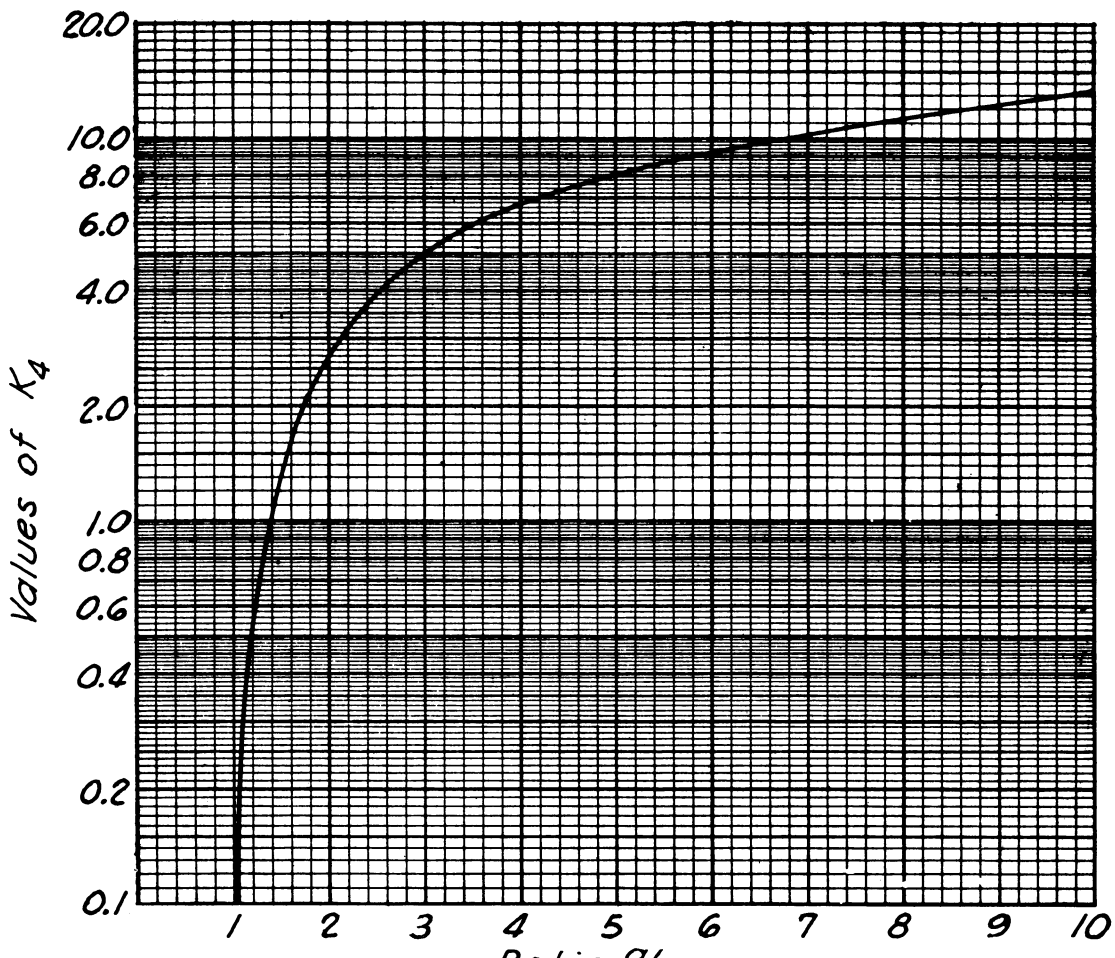
FIG. 5.5-3

5.4.4 Obstruction Loss. Loss of head due to obstruction may be computed by:

$$H_4 = K_4 \frac{v^2}{2g} \tag{5.5-20}$$

H_4 = head loss due to obstruction.
K_4 = a coefficient.
v = mean velocity in pipe.

In practice the most common types of obstructions for which head losses must be determined are valves. Generally reliable values of K_4 for any type of obstruction may be taken from fig. 5.5-4. However, careful judgment should be exercised in selecting K_4 in many cases. Where it is important that reliable determinations of head losses for valves be made, K_4 should be taken from sources of data relating to the specific type or types of valves under consideration.



a = area of conduit; a_o = area at obstruction

LOSS COEFFICIENT FOR PIPE OBSTRUCTION

FIG. 5.5-4

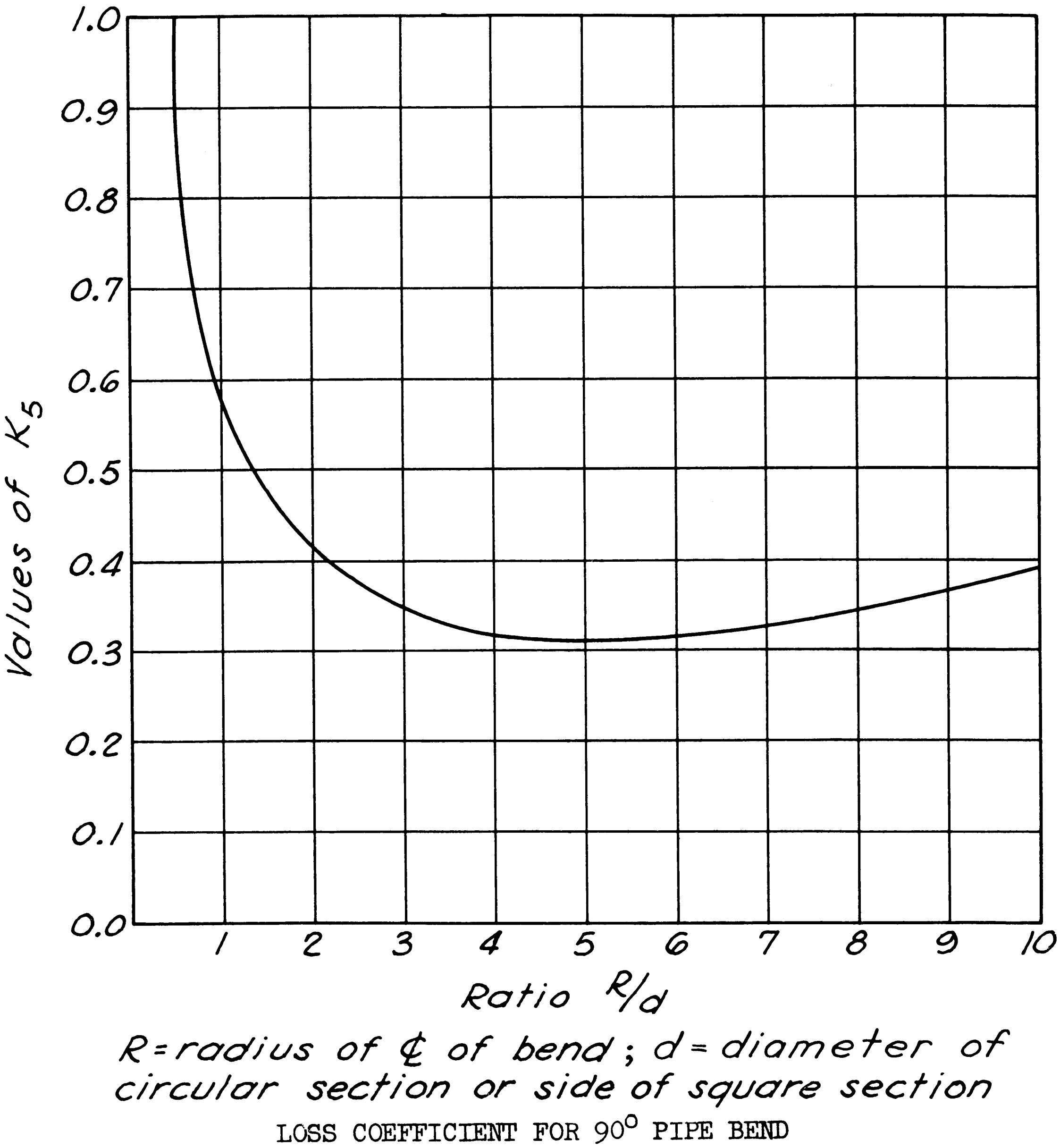
5.4.5 Bend Loss. Loss of head due to bends may be computed by:

$$H_5 = K_5 \frac{v^2}{2g} \qquad (5.5\text{-}21)$$

H_5 = head loss due to bend.
K_5 = a coefficient.
v = mean velocity.

Head loss in a pipe bend is taken as the loss in excess of the friction head. That is, the total loss in a bend is the friction head loss in an equal length of straight pipe plus the head loss due to the bend. The head loss due to the bend is computed separately.

Values of K_5 for 90-degree, curved bends that will be safe for most cases in Service work may be taken from fig. 5.5-5.



LOSS COEFFICIENT FOR 90° PIPE BEND

FIG. 5.5-5

K_5 for 90-degree square elbows, sometimes called miter bends, where there is no rounding of the corners of the intersecting conduits at either outside or inside of the bend should be taken as 1.25 to 1.50. In cases of bends where the deflection is less than 90 degrees, determine K_5 as follows:

$$K_5 \text{ (for bend} < 90^{\circ}) = \left[1 - \left(\frac{90 - \text{deflection in degrees}}{90}\right)^2\right] K_5 \text{ for } 90^{\circ} \text{ bend}$$

It is impractical to reproduce here even a limited number of the many tables, diagrams, and charts available in trade literature that give losses for standard pipe fittings, various types of valves, etc.

5.5 Analysis of Pipe Flow Problems. The basic equations for analyzing pipe flow are the energy equation which expresses Bernoulli's theorem and the continuity equation. Figure 5.5-6 shows the energy gradient and hydraulic gradient for a pipe of uniform section discharging from a reservoir.

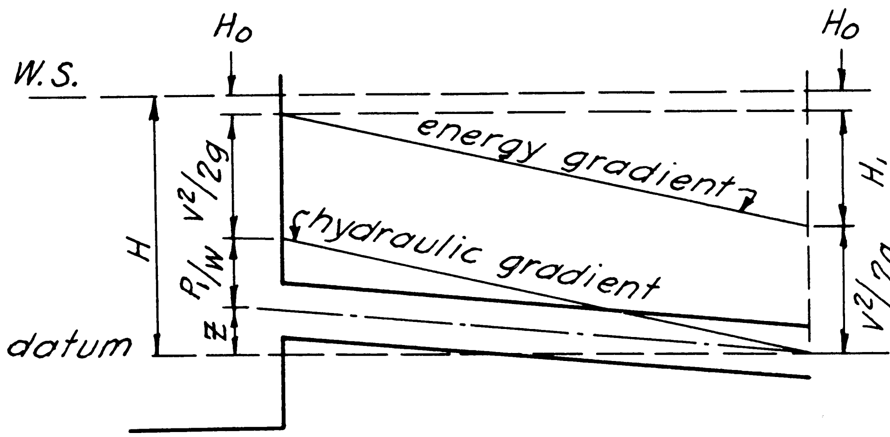


FIG. 5.5-6

The energy equation is:

$$\frac{v^2}{2g} + \frac{p_1}{w} + H_o + z = \frac{v^2}{2g} + H_o + H_1$$

and the friction head,

$$H_1 = (K_c \text{ or } K_p)\ L\ \frac{v^2}{2g} \qquad (5.5\text{-}6)$$

When H_1, d or d_i, and L are given and Q is required, v is computed by formula (5.5-6) and Q is computed by Q = av. When H_1, L and Q are given and d is required, the solution is usually made by trial and error since various types of pipe are available only in certain standard sizes. Select a trial size pipe and compute $v = Q \div a$; compute H_1 and compare with the permissible H_1; repeat trials until a standard size is found which will give the required discharge with a loss of head equal to or less than the permissible H_1.

For the complete solution of general problems of pipe flow, Bernoulli's theorem requires that the total head, H, be represented by velocity head plus all losses. In the simple case illustrated by figure 5.5-6,

$$H = \frac{v^2}{2g} + H_o + H_1$$

In the general case,

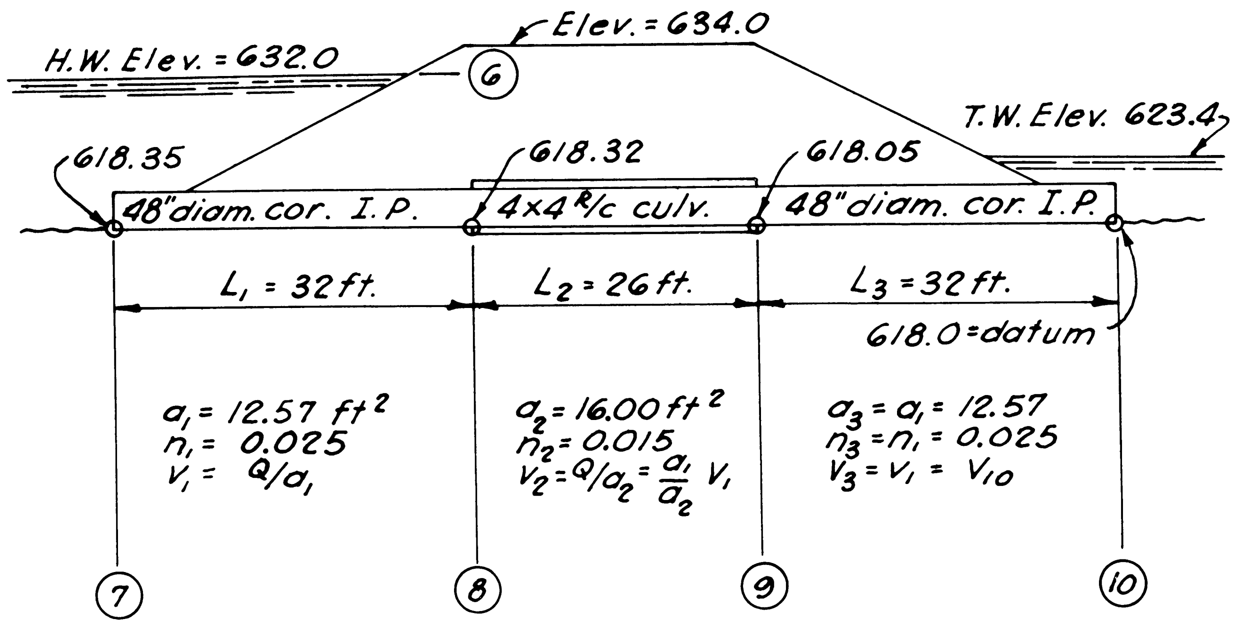
$$H = \frac{v^2}{2g} + H_1 + \text{all other losses.}$$

Methods of evaluating other losses are given in subsection 5.4.

5.5.1 Examples in Pipe Flow.

EXAMPLE 1

Given: A culvert at which the road grade is to be raised and widened and the culvert lengthened. The conditions are shown by the sketch below.



To determine: The discharge capacity of the lengthened culvert.

Solution:

The energy equation between section 6, which is horizontal and coincident with the upstream water surface, and section 10 is:

$$\frac{v_6^2}{2g} + \frac{p_6}{w} + z_6 = \frac{v_{10}^2}{2g} + \frac{p_{10}}{w} + z_{10} + K_{07}\frac{v_1^2}{2g} + K_{p7-8}\,L_1\,\frac{v_1^2}{2g}$$

$$+ K_{28}\frac{v_1^2}{2g} + K_{c8-9}\,L_2\,\frac{v_2^2}{2g} + K_{39}\frac{v_3^2}{2g} + K_{p9-10}\,L_3\,\frac{v_3^2}{2g}$$

Each term of this equation is explained and evaluated or reduced to its simplest form in order:

$v_6^2 \div 2g$; the velocity head at section 6, which by inspection is zero.

$p_6 \div w$; the pressure head at section 6, which at the water surface is zero.

z_6; the elevation head at section 6 or the vertical distance from the assumed datum to the point of measurement of pressure head (see fig. 5.3-1), in this case 632.00 − 618.00 = 14.00 ft.

$v_{10}^2 \div 2g$; the velocity head at section 10, see data on sketch. Since $a_3 = a_1$ and $v_{10} = v_3 = v_1$, this head may be expressed as $v_1^2 \div 2g$.

$p_{10} \div w$; the pressure head at section 10, the elevation of tailwater minus the elevation of the centerline of the pipe, 623.40 − 620.00 = 3.40 ft.

z_{10}; the elevation head at section 10, the elevation of the centerline of the pipe minus the datum elevation, 620.00 − 618.00 = 2.00 ft.

$K_{o7}(v_1^2 \div 2g)$; the entrance loss at section 7. In this case the inlet is between inward projecting and sharp cornered with contraction suppressed around the part of the circumference near the bottom of the inlet and, as chosen by judgment from subsection 5.4.1, $K_o = 0.65$.

$K_{p7-8}\ L_1\ (v_1^2 \div 2g)$; the friction head loss in the corrugated pipe between sections 7 and 8. From table 5.4-1, n = 0.025 for corrugated pipe and from drawing ES-42, $K_p = 0.0182$ and $a_1 = 12.57\ ft^2$ for $d_1 = 48$ in. and n = 0.025.

$K_{28}(v_1^2 \div 2g)$; the head loss due to sudden enlargement at section 8. From the sketch $a_2 = 16.00\ ft^2$ and $a_1 = 12.57\ ft^2$, then $(a_2 \div a_1)^{0.5} = 1.27^{0.5} = 1.13$ and from figure 5.5-2, $K_2 = 0.05$. Note that K_2 is expressed in terms of the velocity head in the smaller pipe.

$K_{c8-9}\ L_2(v_2^2 \div 2g)$; the friction head loss in the concrete culvert between sections 8 and 9. From drawing ES-42, $K_c = 0.00656$ for 4x4 conduit with n = 0.015. In order to have only one unknown in the equation, v_2 is expressed in terms of v_1. Since $Q = a_1 v_1 = a_2 v_2$; $v_2 = v_1(a_1 \div a_2)$ and $v_2^2 = v_1^2(a_1 \div a_2)^2 = v_1^2(12.57 \div 16)^2 = 0.615\ v_1^2$.

$K_{39}(v_3^2 \div 2g)$; the head loss due to sudden contraction at section 9 expressed in terms of velocity head in the smaller pipe. See subsection 5.4.3 and figure 5.5-3. The value of $(a_2 \div a_1)^{0.5}$ at section 9 is the same as at section 8 and is equal to 1.13. From figure 5.5-3, $K_3 = 0.04$. Since $a_3 = a_1$, $v_3 = v_1$.

$K_{p9-10}\ L_3(v_3^2 \div 2g)$; the friction head loss in the corrugated pipe between sections 9 and 10. This loss is equal to and is computed in the same way as $K_{p7-8}\ L_1(v_1^2 \div 2g)$. Note that $L_3 = L_1$ and $v_3 = v_1$.

The equation, with the terms evaluated, is:

$$0 + 0 + 14.00 = \frac{v_1^2}{2g} + 3.40 + 2.00 + 0.65\frac{v_1^2}{2g} + (0.0182 \times 32)\frac{v_1^2}{2g}$$

$$+ 0.05\frac{v_1^2}{2g} + (0.00656 \times 26 \times 0.615)\frac{v_1^2}{2g} + 0.04\frac{v_1^2}{2g}$$

$$+ (0.0182 \times 32)\frac{v_1^2}{2g}$$

$$8.60 = \frac{v_1^2}{2g}(1 + 0.65 + 0.58 + 0.05 + 0.10 + 0.04 + 0.58)$$

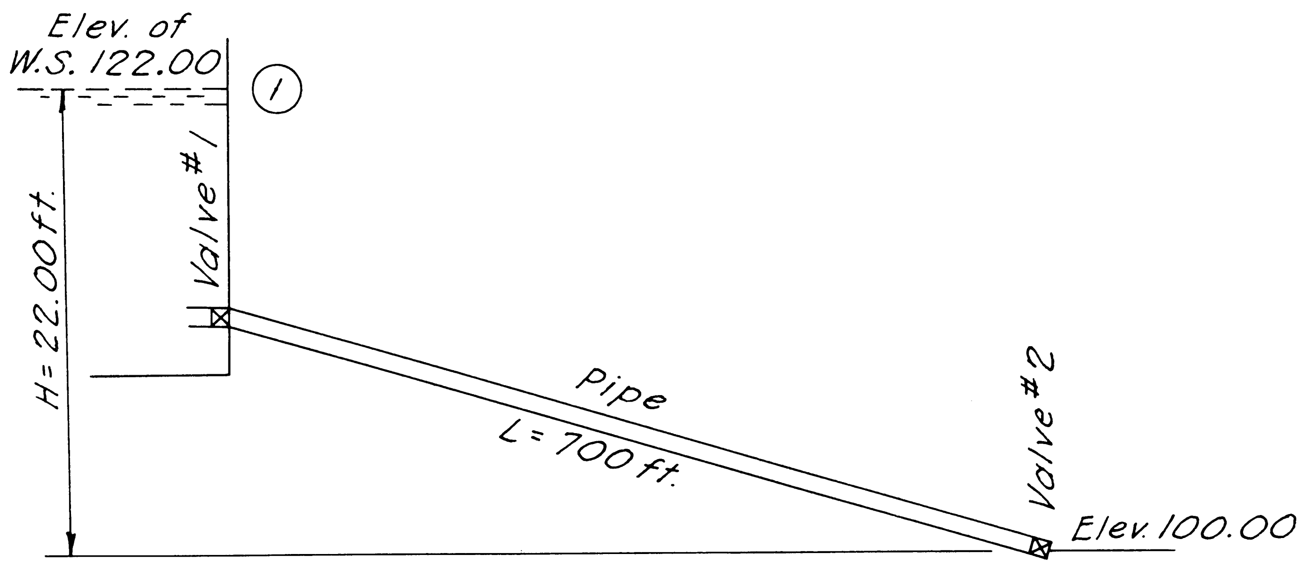
$$v_1^2 = \frac{2g \times 8.60}{3} = \frac{64.4 \times 8.60}{3} = 185$$

$$v_1 = \sqrt{185} = 13.6 \text{ fps}$$

$$Q = a_1 v_1 = 12.57 \times 13.6 = 171 \text{ cfs.}$$

EXAMPLE 2

Given: A galvanized pipe line discharging from a reservoir as shown by the sketch below. The pipe is required to deliver 15 gpm when both valves are full open.



To determine: The size of standard pipe required to deliver a minimum of 15 gpm when valve 1 and valve 2 are full open.

Solution:

1. Converting 15 gpm to cfs: $15 \times \frac{\text{gal}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ ft}^3}{7.48 \text{ gal}} = 0.033$ cfs.

2. By Bernoulli's theorem: the total head is equal to velocity head plus all head losses:

$$H = \frac{v^2}{2g} + H_o + H_{41} + H_{42} + H_1$$

H_o; the entrance loss. The inlet is inward projecting; therefore, from subsection 5.4.1, $K_o = 0.78$ and $H_o = 0.78(v^2 \div 2g)$.

H_{41} and H_{42}; the losses due to obstruction at valves. See subsection 5.4.4. When valves are full open $(a \div a_o) = 1.0$, and from figure 5.5-4, $K_4 = 0.1$. $H_{41} + H_{42} = 0.2\ (v^2 \div 2g)$.

H_1; the friction loss. From table 5.4-1, n = 0.015. By equations (5.5-4) and (5.5-6),

$$H_1 = K_p L \frac{v^2}{2g} \quad \text{and} \quad K_p = \frac{5087\ n^2}{d_1{}^{4/3}}$$

In this case,

$$H_1 = 700 \times \frac{5087 \times (0.015)^2}{d_i^{4/3}} \times \frac{v^2}{2g} = \frac{700 \times 1.14}{d_i^{4/3}} \times \frac{v^2}{2g} = \frac{800}{d_i^{4/3}} \times \frac{v^2}{2g}$$

3. Placing the evaluated terms in the energy equation:

$$22.00 = \frac{v^2}{2g} + 0.78\frac{v^2}{2g} + 0.20\frac{v^2}{2g} + \frac{800}{d_i^{4/3}} \times \frac{v^2}{2g}$$

$$22.00 = \frac{v^2}{2g}\left(1.98 + \frac{800}{d_i^{4/3}}\right)$$

4. Solving for d_i by trial using the continuity equation to compute v for the assumed pipe sizes when Q = 0.033 cfs gives the smallest standard pipe that will deliver 0.033 cfs.

Try d_i = 2 in; $a = 0.023\ ft^2$; $v = \frac{Q}{a} = \frac{0.033}{0.023} = 1.44$ fps

$$\frac{v^2}{2g} = 0.032; \quad 2^{4/3} = 2.52; \quad \frac{800}{2.52} = 317$$

$$0.032(1.98 + 317) = 10.2 \text{ ft.}$$

Try d_i = 1-1/2 in; $a = 0.014\ ft^2$; $v = \frac{0.033}{0.014} = 2.36$ fps

$$\frac{v^2}{2g} = 0.087; \quad 1.5^{4/3} = 1.72; \quad \frac{800}{1.72} = 465$$

$$0.087(1.98 + 465) = 40.7 \text{ ft.}$$

2-inch pipe must be used since a total head of 40.7 ft would be required to produce Q = 0.033 cfs through the 1-1/2 inch pipe.

Discharge for the 2-inch pipe under a head of 22.00 ft is:

$$22.00 = \frac{v^2}{2g}(1.98 + 317) = \frac{v^2}{2g}(318.98)$$

$$v = \sqrt{\frac{64.4 \times 22.00}{318.98}} = 2.10 \text{ fps}$$

$$Q = av = 0.023 \times 2.10 = 0.048 \text{ cfs.}$$

Alternate solution:

1. In step 3 of the above solution it is shown that entrance loss plus obstruction loss at the valves amounts to only $(0.78 + 0.20)(v^2 \div 2g)$ while friction loss is $(800 \div d_i^{4/3})(v^2 \div 2g)$; therefore, losses other than friction are negligible. The slope of the hydraulic gradient may be taken as $s = 22.00 \div 700 = 0.0314 = 3.1 \times 10^{-2}$.

2. From table 5.5-2, C for small sizes of galvanized pipe is found to be 80, then $Q \div C = 0.033 \div 80 = 0.00041 = 4.1 \times 10^{-4}$.

3. On drawing ES-40 the point where $s = 3.1 \times 10^{-2}$ and $Q \div C = 4.1 \times 10^{-4}$ falls between the lines for $d_1 = 2$ in. and $d_1 = 1.5$ in. The larger size, i.e., 2-inch, standard pipe is required.

6. Orifice Flow

6.1 General Formulas. The general formulas for flow through orifices are:

$$v_t = \sqrt{2gh} \qquad (5.6\text{-}1)$$

$$Q = Ca\sqrt{2gh} \qquad (5.6\text{-}2)$$

Q = discharge in cfs.
v_t = theoretical mean velocity through the orifice in fps.
a = cross-sectional area of orifice in ft.^{2}
h = head on center of orifice in ft.
g = acceleration of gravity in ft./sec.^{2}
C = coefficient of discharge.

Theoretical velocities for heads up to 500 ft. are given in King's Handbook, tables 21, 22, and 23, pp. 63, 64, and 65; and the theoretical heads corresponding to velocities up to 50 fps are given in tables 24 and 25, pp. 66 to 68.

6.2 Discharge Under Low Heads. When the head on a vertical orifice is small in comparison with the height of the orifice, the discharge will not be accurately expressed by formula (5.6-2). However, the differences between discharges for a given orifice under low heads determined by formula (5.6-2) and those determined by a formula derived for low head conditions are usually found to be less than 5 percent. Formula (5.6-2) is, therefore, sufficiently accurate for practical determinations.

6.3 Velocity of Approach. A formula for orifice discharge, which includes a correction for velocity of approach, is: (See King's Handbook, p. 48.)

$$Q = Ca\sqrt{2gh}\left(1 + \frac{\alpha C^2 a^2}{2A^2}\right) \qquad (5.6\text{-}3)$$

Notation is the same as given above for formulas (5.6-1) and (5.6-2) with the addition of:

α = (Greek alpha) a kinetic energy correction factor.
A = area of flow in the channel of approach.

The correction for velocity of approach is made by including the term in parentheses. In most cases α is accepted as 1. Inspection of the term in parentheses makes it apparent that the velocity of approach correction is practically unity for values of a/A in the range that will apply to most field structures.

6.4 Submerged Discharge. The discharge of submerged orifices is computed by formula (5.6-2). The head, h, is taken as the difference between water surface elevations immediately above and below the orifice.

6.5 Coefficients of Discharge. The wide range in types of orifices and gates makes it impractical to include tables of coefficients covering an adequate range of conditions. King's Handbook, tables 26 to 36 inclusive, give coefficients for a number of orifices. Manufacturers' publications are normally the best source of reliable coefficients for various types of gates and other appurtenances involving orifice flow.

6.6 Path of Jet. The path of the centerline of a jet for the general conditions shown in fig. 5.6-1 may be determined by:

$$y = x \tan\beta + \frac{gx^2}{2v_o^2 \cos^2\beta} \qquad (5.6\text{-}4)$$

x and y = coordinates of any point on the centerline of the path of the jet.
β = (Greek Beta) angle between the centerline of the pipe and the horizontal.
v_o = velocity of flow at the outlet of the pipe in fps.
g = acceleration of gravity.

When the pipe is horizontal, $\beta = 0^o$, $\tan\beta = 0$, and $\cos^2\beta = 1$, so that formula (5.6-4) reduces to:

$$y = \frac{gx^2}{2v_o^2}, \text{ or} \qquad (5.6\text{-}5)$$

$$x = \sqrt{\frac{2v_o^2\, y}{g}} \qquad (5.6\text{-}6)$$

Formulas (5.6-5) and (5.6-6) define the theoretical path of the centerline of a jet in a vertical plane when the initial velocity is horizontal.

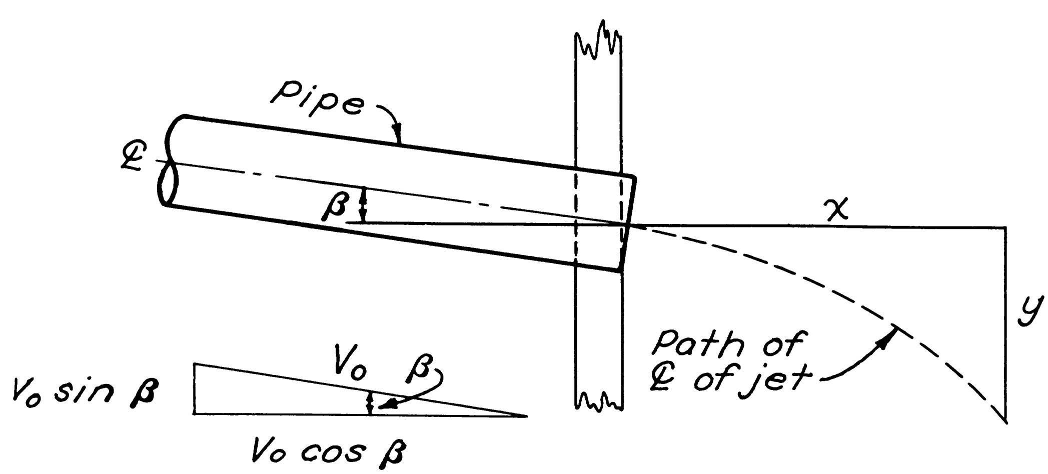


FIG. 5.6-1

7. Weir Flow

7.1 General Formulas. The general formula for the free discharge of a rectangular weir not affected by velocity of approach is:

$$Q = CLH^{3/2} \qquad (5.7\text{-}1)$$

Q = discharge in cfs.
L = length of weir in ft.
H = the head in ft.
C = coefficient of discharge.

Field structures in soil conservation work will only rarely use other than rectangular weirs. When discharge formulas for other types of weirs are required, refer to King's Handbook or to other sources.

7.2 Contractions. In the majority of our field structures, crest and end contractions will be either fully or partially suppressed. In the case of drop spillways, formula (5.7-1) without modification for contraction effect has proven reliable. Discharge determinations for other types of weirs should recognize contraction effect in accordance with conditions.

7.3 Velocity of Approach. When velocity of approach is considered, the discharge formula for rectangular weirs is:

$$Q = CL\left(H + \frac{v_a^2}{2g}\right)^{3/2} \qquad (5.7\text{-}2)$$

Notation is the same as for formula (5.7-1) with the addition of:

v_a = velocity of approach in fps.
g = acceleration of gravity.

The correct v_a to use in formula (5.7-2) is the mean velocity in the approach channel at a distance of about 3H upstream from the weir. The total head for computing weir discharge when H and v_a are known may be taken from drawing ES-43.

7.4 Coefficients of Discharge. It is impractical to include a complete table of discharge coefficients for a wide range of weir types. Since the main weir type used in soil conservation work is the drop spillway, coefficients of discharge for these structures are given in the drop spillway section. King's Handbook gives coefficients for most other types of weirs.

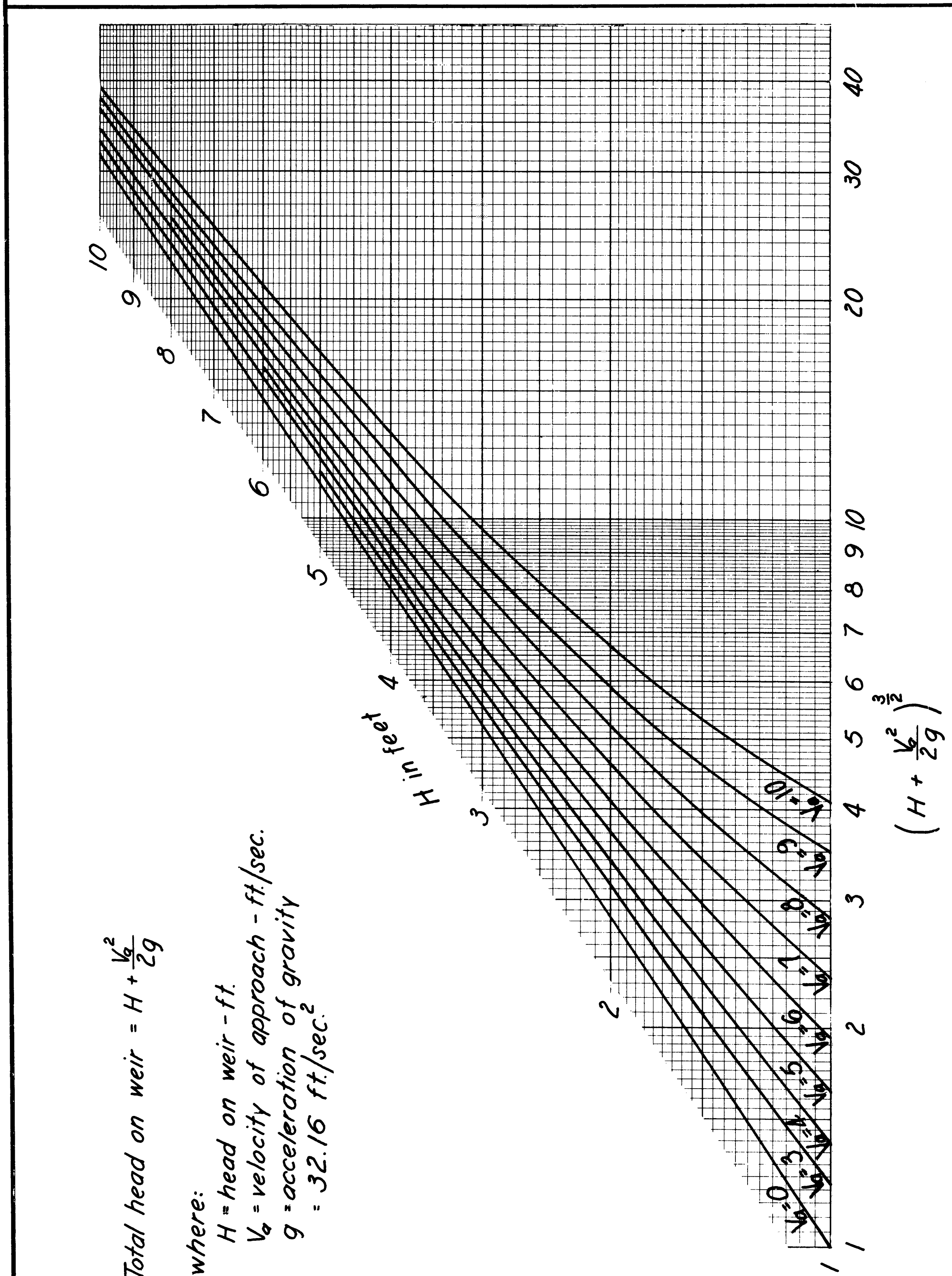
7.5 Submerged Flow. When the water surface elevation just downstream from a weir is higher than the crest elevation, it is described as submerged. It is not feasible to include here selected formulas from the considerable number that have been developed for submerged discharge. The following general results have been shown by investigations of submerged weirs. Let H represent the upstream head on a weir, and h, the downstream head.

The discharges of sharp-crested weirs are subject to greater reductions at lower degrees of submergence than is the case with other types. When the ratio h/H reaches 0.3 to 0.4, the discharge may be reduced by 5 to 10 percent; and for h/H ratios of 0.6 to 0.7, reductions of 20 to 40 percent may be expected.

Broad-crested and ogee weir discharges do not show appreciable reductions until the ratio h/H reaches 2/3. For higher degrees of submergence the discharge is rapidly reduced; and for h/H values of 0.75 to 0.85, reductions of 10 to 30 percent should be recognized.

More complete information on the operation of drop spillways under submerged conditions is given in the drop spillway section.

HYDRAULICS: THREE-HALVES POWERS OF TOTAL HEADS ON WEIRS



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE H. H. Eennett, Chief ENGINEERING STANDARDS UNIT	STANDARD DWG. NO. ES - 43 SHEET 1 OF 1 DATE 7-28-50

8. Flood Routing through Reservoirs

8.1 General. In designing dams it is necessary to determine the height of dam, the required discharge capacity of the spillway, and the required discharge capacity of any conduit through the dam, such as a drop inlet, so that the complete structure will operate satisfactorily and safely in passing the design flood hydrograph. The problem is one of determining how outflow from the reservoir and storage in the reservoir vary with time when inflow to the reservoir is known. The process involves solving the continuity equation for unsteady flow:

$$I = O + S \tag{5.8-1}$$

I = volume of inflow for any time interval.
O = volume of outflow for any time interval.
S = change in volume of storage for any time interval.

Introducing the time factor:

$$\Delta t \left(\frac{i_1 + i_2}{2}\right) = \Delta t \left(\frac{o_1 + o_2}{2}\right) + \Delta S \tag{5.8-2}$$

Δt = any time interval.
i = inflow rate.
o = outflow rate.
ΔS = change in volume of storage during Δt.
1 and 2 = subscripts denoting beginning and ending respectively of Δt.

The treatment of reservoir routing in this subsection is based on the assumption that the water surface in a reservoir is level. This assumption means that both the outflow rate and the storage, as used in equation (5.8-2), depend only on water surface elevation at the dam. In a great majority of the reservoirs considered in Service work this assumption is valid. It is not valid in a reservoir where the backwater effect is such that a significant percentage of the temporary storage occurs as wedge storage between the sloping backwater surface and a horizontal plane extending upstream from the water surface elevation at the dam. Under these conditions, which are most likely to exist with low dams, storage is not a single-valued function of water surface elevation at the dam. When a reliable routing analysis is required for these conditions, stream routing methods, described in the Hydrology Section, should be used.

The solution of the continuity equation, in the various forms in which it may be expressed for reservoir routing, involves integration and the solution may be made by a number of methods. Several of these methods are graphical. Two of these, which are considered to be well adapted to Service work, are described in subsections 8.3 and 8.4.

8.2 Fundamentals of Graphical Methods of Reservoir Routing. An understanding of graphical integration and of certain relationships between hydrographs and mass flow curves will aid in the understanding and use of graphical routing methods.

The hydrograph is a plotting of discharges as ordinates and time as abscissas. The area under a hydrograph between any two points in time, since it is the product of a rate of flow dimension and a time dimension, is equal to the volume of flow for the time period. The accumulated volume of flow from zero time to any given time is the area under the hydrograph up to the given time.

The mass flow curve is a plotting of accumulated volume of flow as ordinates and time as abscissas. At any point, that is, at any time, the slope of the mass flow curve, since it is a volume dimension divided by a time dimension, is equal to the rate of flow. The mass flow curve is the integral of the hydrograph since its ordinates measure accumulated volume at any time.

Graphical integration of the hydrograph to obtain the mass flow curve is illustrated in Part 2 of Method No. 1 below.

8.3 Method No. 1. This method is taken from Vol. XVIII, No. 8, October 1931, Journal of the Boston Society of Civil Engineers. It is devised so as to be a direct solution for the storage and outflow curves for any reservoir in which the water surface is level. The solution requires that the following be given.

1. The hydrograph of inflow to the reservoir. Methods of computing a hydrograph for a given watershed are discussed in the Hydrology Section.
2. The elevation-storage curve for the reservoir.
3. The elevation-discharge curve for the spillway and any conduit through the dam.

The procedures for making the solution are described below as Part I, construction of the storage-discharge curve; Part 2, construction of the mass inflow curve; Part 3, routing the inflow hydrograph. An example is given at the end of this subsection.

Part 1:

Construction of the storage-discharge curve for the reservoir. The storage-discharge curve is a plotting of total discharge versus temporary storage and is derived from the elevation-discharge and elevation-storage curves.

Figure 5.8-1 illustrates the construction of the storage-discharge curve for a reservoir with a drop inlet conduit and an emergency spillway. The following steps are involved in the construction:

1. Plot the elevation-storage curve for the reservoir.

2. Plot the elevation-discharge curves for the spillway and drop inlet. Draw a horizontal line through C, the elevation of the spillway

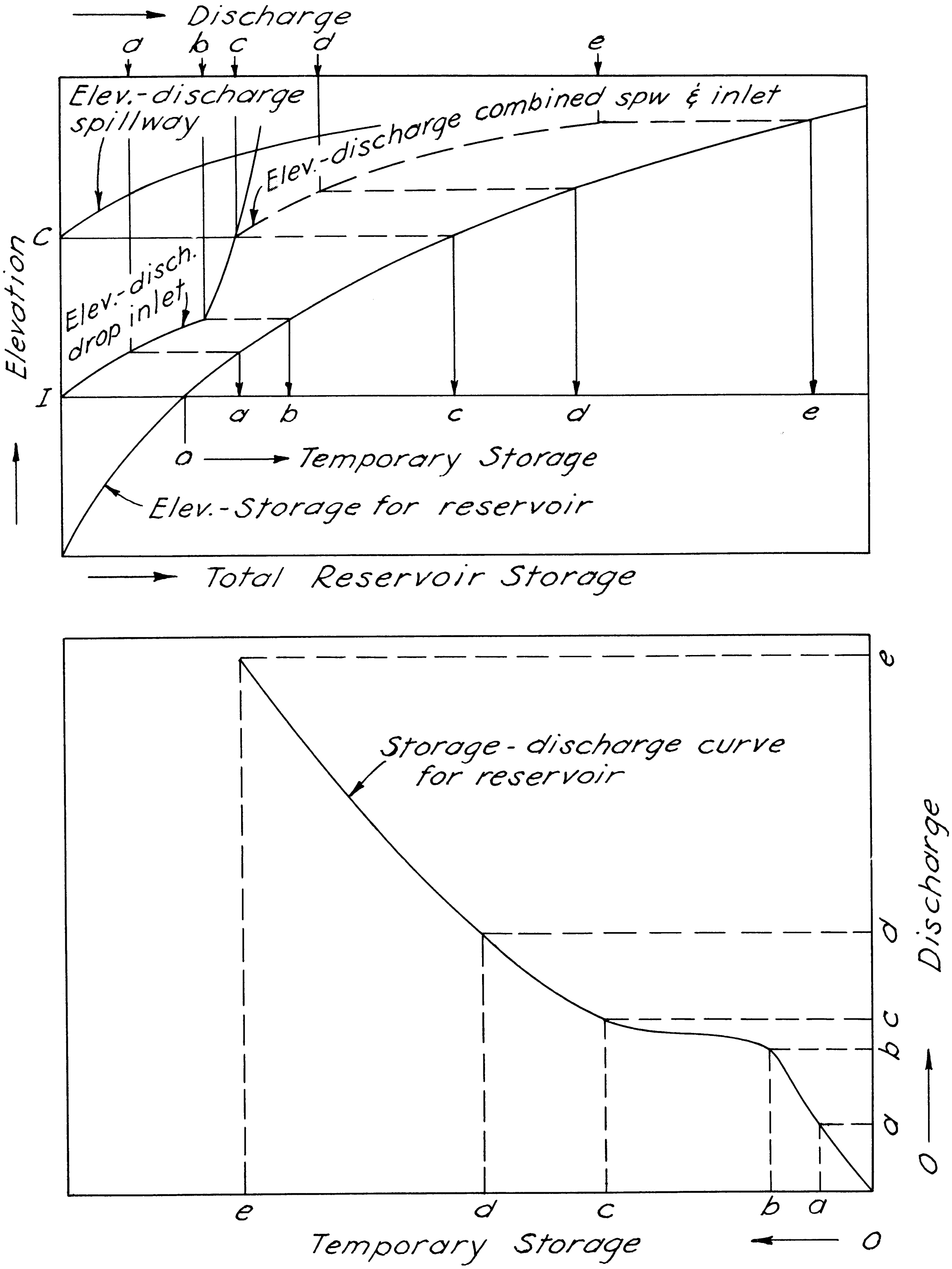
crest, to intersect the elevation-discharge curve for the drop inlet. At elevations above the spillway crest, graphically add the discharges of the drop inlet and spillway to obtain the curve designated as elevation-discharge for the combined spillway and drop inlet.

3. Draw a horizontal line at I, the inlet elevation of the drop inlet. The point at which this line intersects the elevation-storage curve for the reservoir is the point of zero temporary storage. The temporary storage scale is identical to the total reservoir storage scale except that the zero point is shifted to the point of elevation I on the elevation-storage curve.

4. At several elevations draw horizontal lines to intersect both the elevation-storage curve and the elevation-discharge curve for the drop inlet and spillway. From the points where these elevation lines intersect the elevation-discharge curve, project vertically upward to the discharge scale. From the points where these elevation lines intersect the elevation-storage curve, project vertically downward to the temporary storage scale. The points on the discharge and temporary storage scales established in this manner and designated as a-a, b-b, etc., are simultaneous values of storage and discharge. These values define the storage-discharge curve.

5. Plot the simultaneous values of temporary storage and discharge and draw the storage-discharge curve as shown in the lower part of Fig. 5.8-1.

As used in Method No. 1 the storage-discharge curve is plotted as shown on the Routing Work Sheet. The following scale arrangements are required: (1) The discharge scale is vertical and is the scale for inflow and outflow rate; (2) the temporary storage scale is horizontal with values increasing to the left of the origin. The units and scales of temporary storage and volume of inflow and outflow must be identical.



Discharge
a b c d e
Elev.-discharge spillway
Elev.-discharge combined spw & inlet
Elev.-discharge
Elev.-disch. drop inlet
Elevation
C
I
a b c d e
a Temporary Storage
Elev.-Storage for reservoir
Total Reservoir Storage
Storage-discharge curve for reservoir
e
d
b c
b
a
O
Discharge
e d c b a
Temporary Storage
O

FIG. 5.8-1

Part 2:

Construction of the mass inflow curve. In subsection 8.2 the relation of the mass flow curve to the hydrograph was described. The mass inflow curve for a given hydrograph may be (a) constructed directly from the hydrograph by graphical integration, or (b) computed arithmetically and plotted on the Routing Work Sheet. In practically all cases the graphical construction will save time.

(a) The graphical construction of the mass flow curve is shown in fig. 5.8-2.

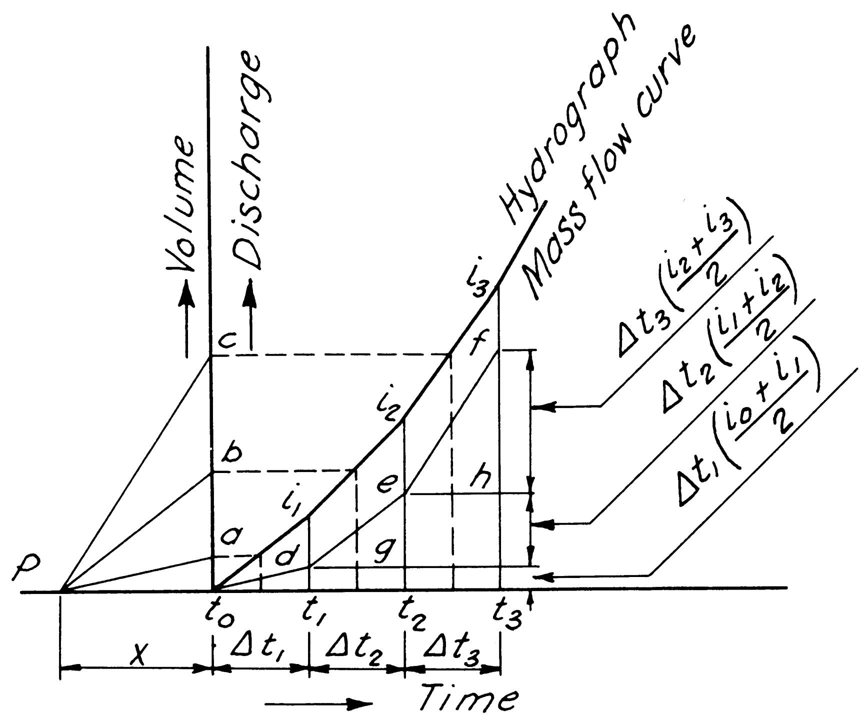


FIG. 5.8-2

Divide the area under the hydrograph into increments by erecting verticals at times t_1, t_2, t_3, etc. Project the mid-ordinates of these incremental areas, which are the mean discharges for the time intervals Δt_1, Δt_2, and Δt_3, leftward to locate points a, b, and c on the vertical axis. Starting at the origin of coordinates, draw successive segments of the mass flow curve: Through the interval Δt_1 draw t_od parallel to Pa; through the interval Δt_2 draw de parallel to Pb; through the interval Δt_3 draw ef parallel to Pc. A complete mass flow curve showing the main construction lines by which it is developed from the hydrograph is shown on fig. 5.8-3. In this construction the time intervals should be so selected that the hydrograph in each interval is approximately a straight line. This procedure may be used to construct the mass flow curve for any hydrograph between any selected times t_o and t_n. The discharge shown by the hydrograph at t_o may be zero or greater than zero. Since summation of flow volume is started at t_o, the mass flow curve originates from zero regardless of discharge at the starting time.

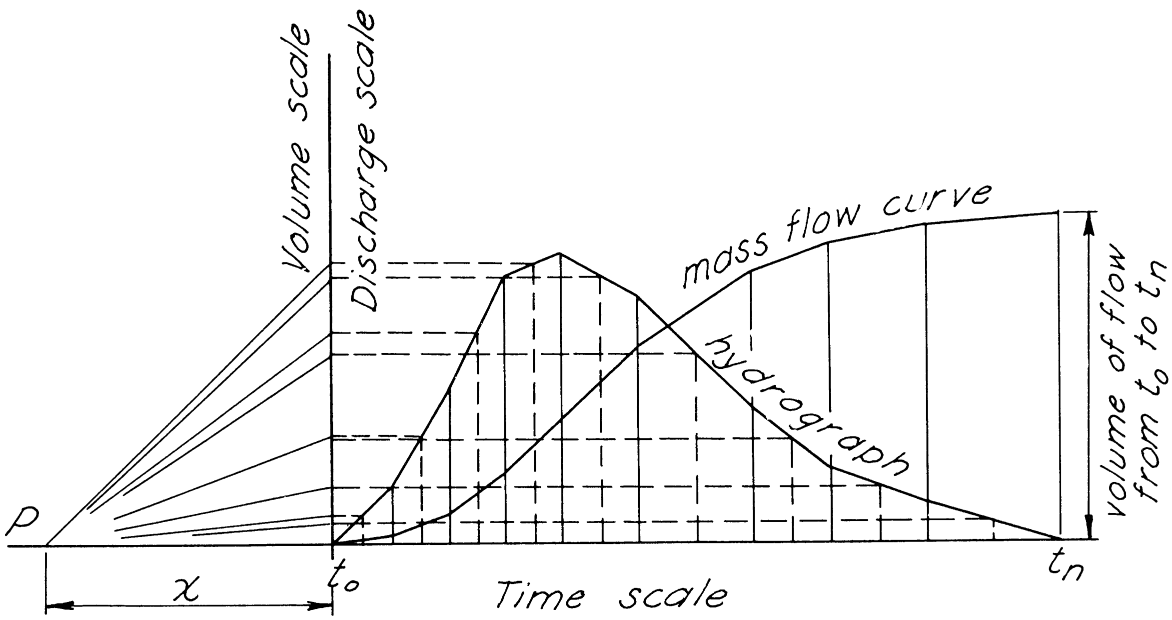


FIG. 5.8-3

The procedure described above accomplishes graphically the multiplication of mid-ordinates of the incremental areas by the distance x and the progressive addition of these products. The products, which are volumes of flow, are expressed by the ordinates t_1d, ge, and hf, in fig. 5.8-2. The distance x must be such that valid scale relations and consistency of units are established between the selected scales and units of time, rate of discharge, and volume of flow. The distance x is computed by:

$$x \text{ in scale-units} = \frac{V_s}{Q_s\ t_s} \tag{5.8-3}$$

V_s = number of ft^3 per scale-unit on the volume scale. Any unit of volume such as ac-ft, cfs-day, cfs-hr, may be used to define the volume scale; however, whatever unit is used must be converted to ft^3 in the above equation.

Q_s = number of cfs per scale-unit on the discharge scale.

t_s = number of seconds per scale-unit on the time scale.

Scale-unit is the major linear unit of the grid of the cross section paper on which the graphical construction is done. On most cross section papers the scale-unit is either 1 inch or 0.5 inch. The requirement here is that values of V_s, Q_s, and t_s be expressed for the same linear scale-unit. Then the distance x will be measured in that unit.

(b) Data for plotting the mass flow curve may be computed arithmetically by performing the operations indicated in the table below. The data for the first three columns are obtained from the hydrograph by dividing the total time into intervals, in each of which the hydrograph is approximately a straight line, then recording the discharges for the times t_o, t_1, t_2, etc. The computations of average discharge rate and volume of flow per time interval are self-explanatory. Values in the last column are progressive summations of the volumes of flow per time interval. The mass flow curve is plotted from the first and last columns.

Time	Time Interval	Discharge rate (inflow)	Av. Discharge rate per time interval	Volume of flow per time interval	Accumulated volume of flow
t_o		i_o			zero
	Δt_1		$(i_o+i_1) \div 2$	$\Delta t_1(i_o+i_1) \div 2 = I_1$	
t_1		i_1			I_1
	Δt_2		$(i_1+i_2) \div 2$	$\Delta t_2(i_1+i_2) \div 2 = I_2$	
t_2		i_2			$I_1 + I_2$
--	---	--	----	-----	---
	Δt_n		$(i_{n-1}+i_n) \div 2$	$\Delta t_n(i_{n-1}+i_n) \div 2 = I_n$	
t_n		i_n			$\sum_1^n I$

Part 3:

Routing the inflow hydrograph through the reservoir to determine the outflow hydrograph is done in a manner similar to that used in constructing the mass flow curve. The arrangement of the work sheet on which the graphical solution is developed is illustrated by the Routing Work Sheet at the end of this subsection.

In a routing operation to determine discharge capacity of a spillway or conduit, it is normally assumed that the reservoir stage is at spillway crest or conduit inlet elevation at the time inflow begins. This assumption establishes the conditions that inflow rate = outflow rate = zero, and temporary storage = zero at t_o.

The solution is made in a series of steps, in each of which an outflow rate is selected and the time at which this selected outflow occurs is determined graphically. Fig. 5.8-4 shows the graphical construction for two successive steps. Detailed operations in the first step are:

(a) Select o_1 and solve for the time t_1 at which o_1 occurs. Draw a horizontal construction line through o_1 on the discharge scale to intersect the temporary storage curve. This intersection establishes the storage S_1 when outflow is o_1 and also $S_1 - S_o$.

(b) The average outflow rate during the interval from t_o to t_1 is $(o_o + o_1) \div 2 = q_1$. Locate q_1 on the discharge scale.

(c) With scale or dividers lay off the distance $S_1 - S_o$ above the mass inflow curve on the ordinate through t_o. From this point draw a line parallel to Pq_1 to intersect the mass inflow curve. The time t_1 is established by this intersection.

(d) The intersection of the construction line through o_1 and the ordinate at t_1 is a point on the outflow hydrograph. Draw a line from this point to the origin of coordinates to define the outflow hydrograph between t_o and t_1.

(e) Draw the line t_ok parallel to Pq_1 to define the mass outflow curve between t_o and t_1.

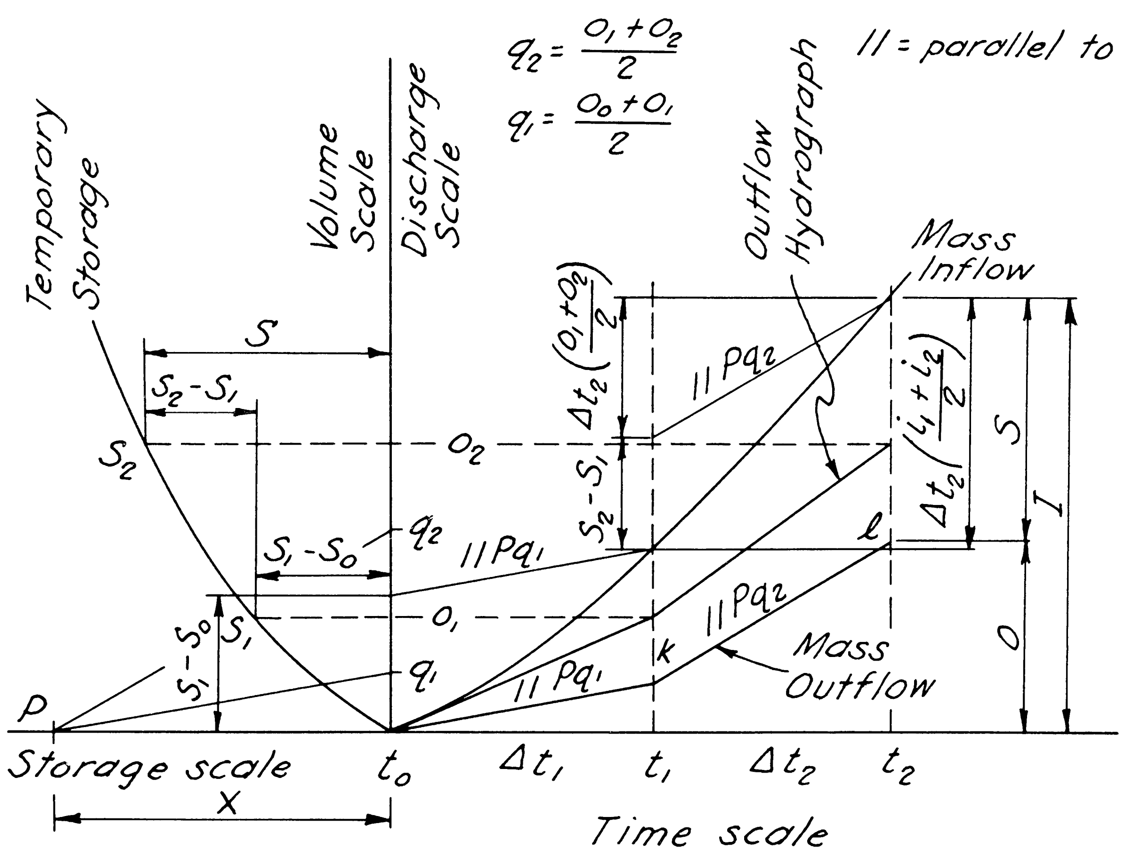


FIG. 5.8-4

The operations described above for the first step are repeated in subsequent steps, and the progressive repetition produces the outflow hydrograph and the mass outflow curve.

Note that the construction for a step graphically solves the continuity equation (5.8-2) for that step. Examine step 2 on fig. 5.8-4. First the value of $S_2 - S_1 = \Delta S$ is laid off above the mass inflow curve. Then the sloping line parallel to Pq_2 is drawn to intersect the mass inflow curve. The volume intercept of this line in the interval Δt_2 is $\Delta t_2(o_1+o_2) \div 2$. The volume intercept of the mass inflow curve in the interval Δt_2 is $\Delta t_2(i_1+i_2) \div 2$; therefore, by graphical construction volume of inflow for the interval has been placed equal to volume of outflow plus change in volume of storage for the interval. Inspection of equation (5.8-2) shows that storage decreases, that is ΔS is negative, when outflow is greater than inflow for a time interval. Therefore, in the graphical construction the ordinate representing change of storage for a step is laid off above the mass inflow curve when inflow is greater than outflow and is laid off below the mass inflow curve when inflow is less than outflow.

A check of the work may be made at the end of each step. On the right of fig. 5.8-4 values of total storage, S, total outflow, O, and total inflow, I, up to time t_2 are shown by the ordinates of the mass inflow and mass outflow curves, with S being represented by the ordinate between these two curves. On the left, storage is also shown by the abscissa S for the outflow rate o_2. At the end of each step dividers or scale may be used to compare the abscissa of the temporary storage curve with the ordinate between the mass inflow and mass outflow curves. If these two linear distances are not equal, the solution is in error.

In each step the outflow rate for which the time of occurrence is to be determined should be so selected that the increase or decrease in outflow during the time interval involved is relatively small. The smaller the value of $(o_2 - o_1)$ for successive intervals, the more accurate the solution. Where both the storage-discharge and mass inflow curves are smooth and uniform, o_2 may be selected to allow higher values of $(o_2 - o_1)$; where either curve changes slope abruptly, o_2 should be selected so as to hold the value of $(o_2 - o_1)$ low.

The maximum outflow rate and the instant at which it occurs are determined by trial based on the following facts: At the instant of maximum outflow, the inflow and outflow hydrographs intersect, that is, inflow equals outflow and the slopes of the mass inflow and mass outflow curves are equal. Furthermore, maximum temporary storage occurs at the instant of maximum outflow.

The distance x in this graphical construction has the same function as described in Part 2 and is computed by equation (5.8-3).

A numerical example of reservoir routing is given on the Routing Work Sheet, Method No. 1. The following suggestions for setting up the work sheet may be found useful:

First, select the scales of discharge and time and plot the inflow hydrograph. The time and discharge scales should be so selected as to give the desired accuracy in plotting and reading values, and they should be such that the rising and falling limbs of the hydrograph are not steeply sloping lines.

Second, select the unit of volume to be used and the volume scale and compute the distance x. Any of the commonly used volume of flow units may be selected. The maximum ordinate of the volume scale may be quickly approximated as follows: (a) divide the inflow hydrograph into 4 or 5 equal time periods such as 0.5 hour, 1 hour, or 5 hour periods; (b) add the discharges at the mid-times of these periods; (c) multiply the sum of these discharges by the period in hours to obtain total volume in cfs-hr. This approximation is illustrated for the hydrograph of the example: (a) The hydrograph is divided into five 1-hour periods; (b) discharges at 30, 90, 150, 210, and 270 minutes are 300 + 1800 + 1300 + 700 + 200 = 4300; (c) approximate total volume = 4300 cfs x 1 hr = 4300 cfs-hrs. Thus, the maximum value of volume of flow to be plotted is 4300 to 4500 cfs-hrs and the volume scale is selected accordingly.

If acre-foot had been selected as the unit of volume, the maximum value of accumulated flow to be plotted would have been:

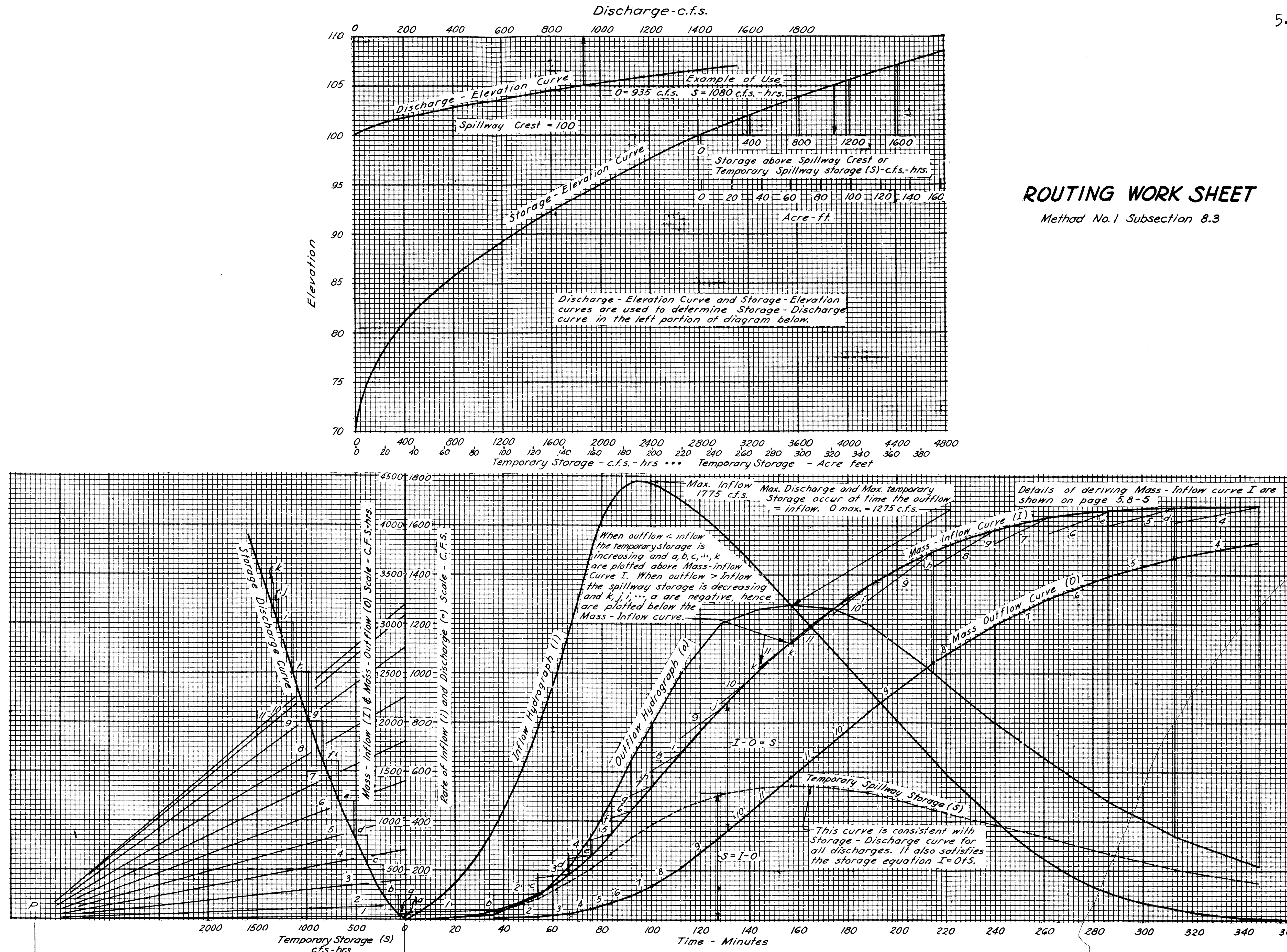
$$4300 \ \frac{\text{ft}^3\text{-hr}}{\text{sec}} \times \frac{1}{43560} \ \frac{\text{ac-ft}}{\text{ft}^3} \times \frac{3600}{1} \ \frac{\text{sec}}{\text{hr}} = 355 \text{ ac ft (approx.)}$$

Compute x by equation (5.8-3).

Third, plot the storage-discharge curve. Note that the temporary storage scale must be identical to the vertical scale of mass inflow and mass outflow.

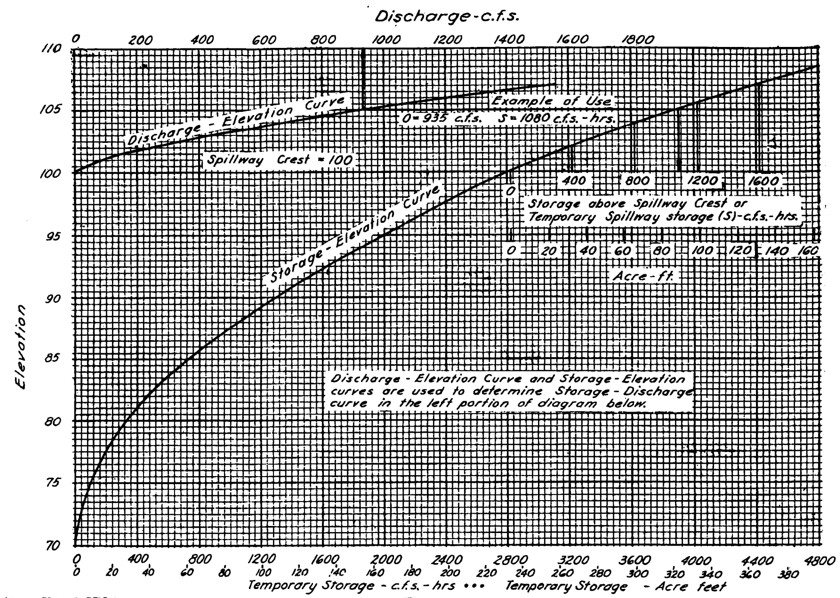
ROUTING WORK SHEET

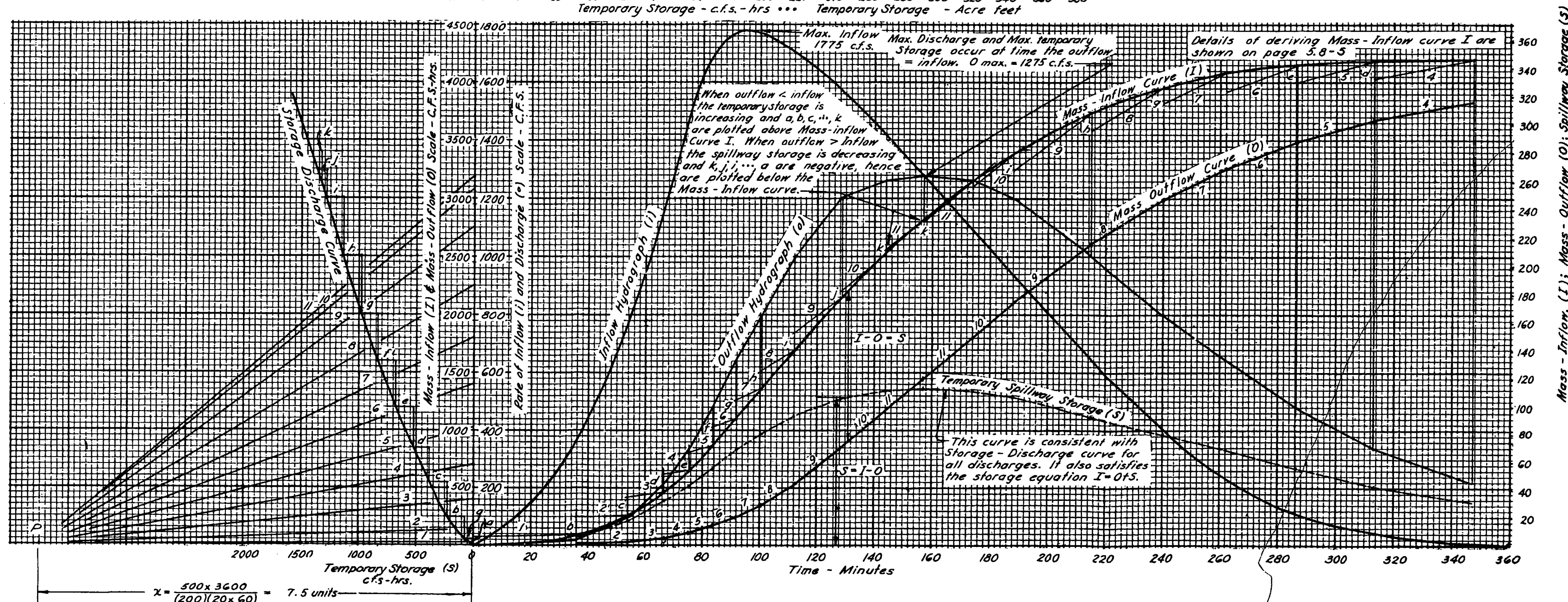
Method No. 1 Subsection 8.3



ROUTING WORK SHEET

Method No. 1 Subsection 8.3





8.4 Method No. 2. This method is taken from U. S. Department of Agriculture, Soil Conservation Service, Washington Mimeograph No. 3823, "Steps in the Graphic Routing of Floods Through Reservoirs". It is a trial and error method. The solution requires that the following be given:

I. The hydrograph of inflow to the reservoir.

II. The elevation-storage curve for the reservoir.

III. The elevation-discharge curve for the spillway and any conduit through the dam.

From these three curves and the storage equation, two additional curves can be derived; they are:

IV. Outflow hydrograph. This curve gives the rate of outflow (discharge over the spillway or spillways) as a function of time.

V. Storage. This curve gives the volume of spillway storage existing in the reservoir as a function of time.

The procedure for derivation of curves IV and V can be outlined as follows:

A. Compute and plot curves numbered I, II, and III as indicated in the example on page 5.8-14. In our work it will be found convenient to plot curve I with the ordinates measuring rate of flow in cfs and the abscissas measuring time in minutes. Curve II should be plotted with ordinates of storage in acre-feet and abscissas as stage above spillway crest in feet. Curve III must have the same scale or ordinates as curve I, and the same scale of abscissas of curves II and III should be chosen so as to make curves II and III fairly steep; this cannot be accomplished for curve III when the spillway is of the pressure conduit or orifice type. The scales for these three curves should be so located that the curves do not overlap.

B. Compute the conversion-time interval, T. The conversion-time interval is the time required for a flow as measured by one unit (say 1 inch) of ordinate on the flow scale to accumulate to the same unit (1 inch) of storage on the storage scale. In computing T, a factor to make the units of the equation consistent must be included.

$$T\ (\text{min}) = \frac{1\ \text{inch of ordinate (ac-ft)} \times 43{,}560 \left(\frac{\text{ft}^3}{\text{ac-ft}}\right)}{1\ \text{inch of ordinate} \left(\frac{\text{ft}^3}{\text{sec}}\right) \times 60 \left(\frac{\text{sec}}{\text{min}}\right)}$$

The two ordinate scales and the time scale should be so chosen that T will plot on the time scale to a length of from 2 to 6 inches.

C. Construct the derived curves numbered IV and V. This procedure will be broken up into the following steps:

1. Select a time interval Δt_1; assume an average rate of outflow for that time interval and plot it as point a_1 at the midpoint of the time interval. The shorter the time interval selected, the more accurate will be the graphical construction; however, it is not ordinarily necessary to select time intervals of less than about 0.025 times the total runoff period; and in some parts of the analysis these time limits may be doubled. The shorter time intervals should be used where there are sharp breaks or rapid changes of slope of any of the five curves.

2. From point b_1, which is on the curve I directly above a_1, measure the distance T horizontally to the right thereby locating point c_1; point b_1 represents the average rate of inflow into the reservoir during the time interval Δt_1.

3. The slope of the line a_1c_1 represents the average rate of change of storage for the time interval Δt_1. In other words, a flow equal to $(b_1 - a_1)$ measured on the flow scale will in time T accumulate an amount of spillway storage equal to $(b_1 - a_1)$ measured on the storage scale.

4. Locate point d_1 with an abscissa of time = 0 and an ordinate of storage = 0.

5. From point d_1 draw a line parallel to line a_1c_1. Locate point e_1 at the midpoint of the time interval on this line.

6. Now check the accuracy of the assumption as to the value of a_1 as follows: From e_1 project horizontally to the left to curve II, then down to curve III, and then horizontally to the right to a vertical line through point a_1. If this last projection intersects the vertical line through a_1 at a_1, then the assumption of the value of a_1 was correct. If it does not intersect at point a_1, then a new trial value of a_1 must be selected and the process repeated until the graphical construction checks the trial value of a_1.

7. After the location of a_1 has been checked, locate point f_1 (which is the same point as d_1 for the next time interval) by drawing a line from d_1 parallel to line a_1c_1 to an intersection with an ordinate through the division point between time intervals Δt_1 and Δt_2.

8. Select a new time interval and repeat the steps 1 to 7 above, and continue this process until the outflow hydrograph has intersected the inflow hydrograph.

RESERVOIR ROUTING EXAMPLE

Method No. 2 Subsection 8.4

Computations for curve III

$Q = 3.0Lh^{3/2}$; $L = 26'\text{-}0''$

$= 3.0 \times 26h^{3/2} = 78h^{3/2}$

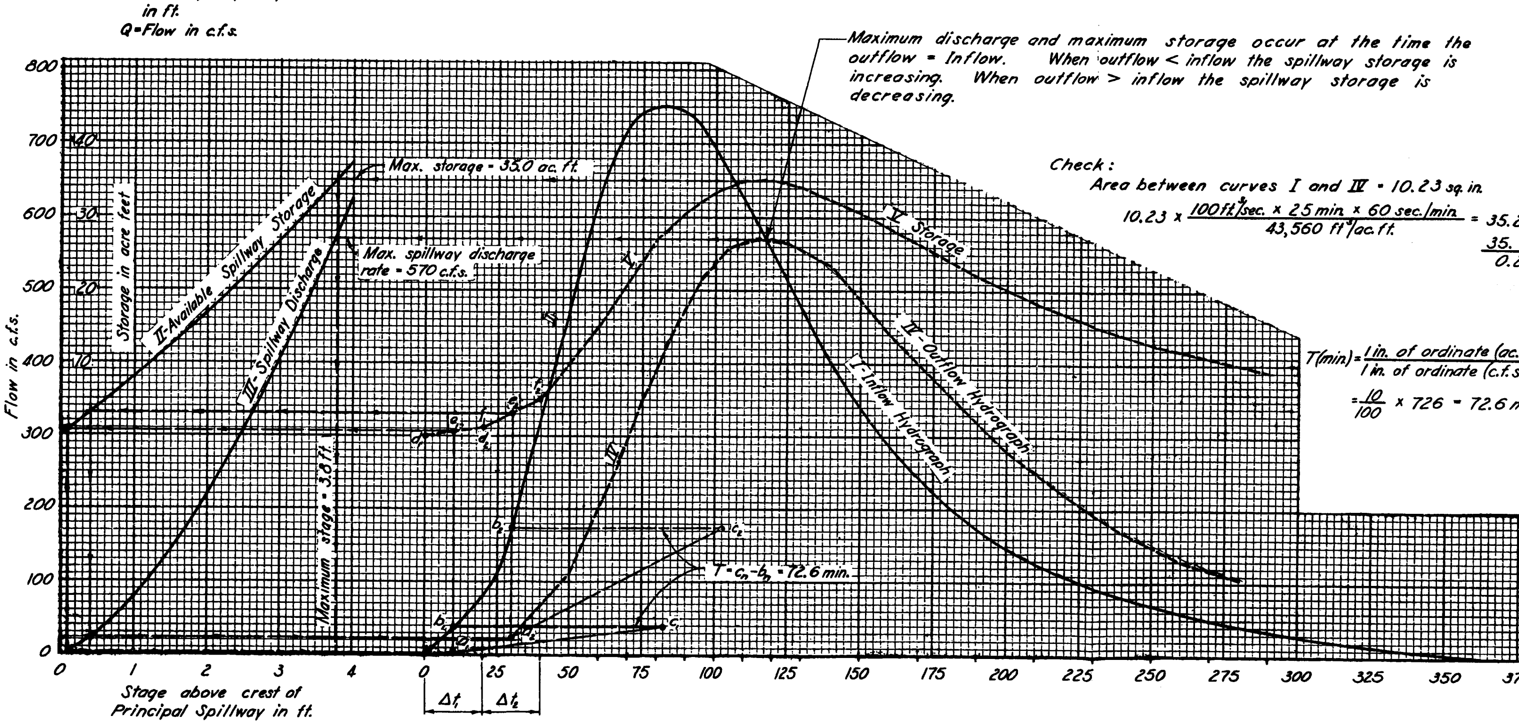
h	$h^{3/2}$	Q
0.50	0.35	27.
1.00	1.00	78.
1.50	1.84	143.
2.00	2.83	221.
2.50	3.95	308.
3.00	5.20	406.
3.50	6.55	511.
4.00	8.00	624.

h = Stage above crest of principal spillway in ft.

Q = Flow in c.f.s.

Computations for curve II

Stage	Area flooded in acres	Available Spillway storage in ac. ft.
0	7.54	0
2.0	9.39	16.93
4.0	11.32	37.64



Check:

Area between curves I and IV = 10.23 sq. in.

$$10.23 \times \frac{100\ ft^3/sec. \times 25\ min. \times 60\ sec./min.}{43{,}560\ ft^3/ac.ft.} = 35.2\ ac.\ ft.$$

$$\frac{35.}{0.2} = \text{error} = 0.57\%$$

$$T(min) = \frac{1\ in.\ of\ ordinate\ (ac.\ ft.)}{1\ in.\ of\ ordinate\ (c.f.s.)} \times 726 = \frac{10}{100} \times 726 = 72.6\ min.$$

RESERVOIR ROUTING EXAMPLE

Method No. 2 Subsection 8.4

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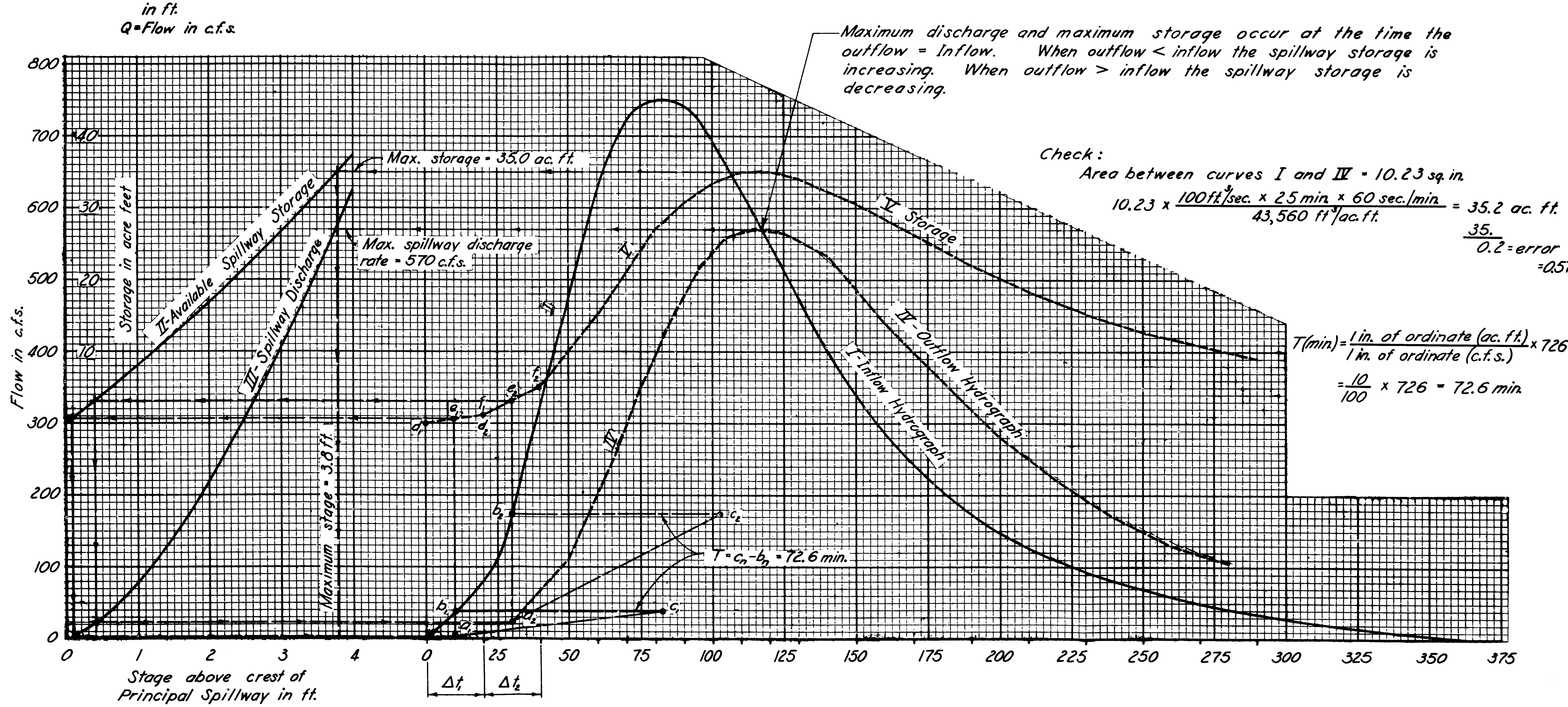
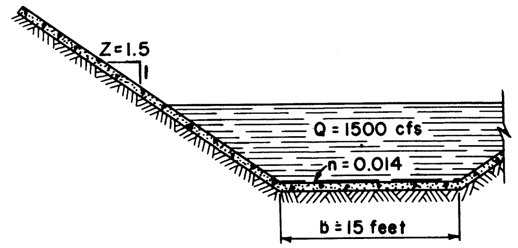
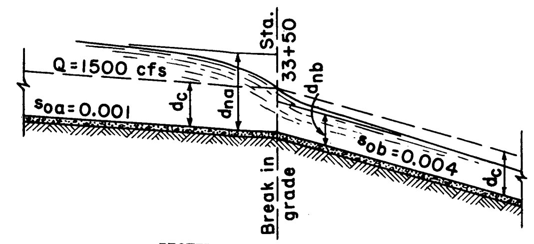


HYDRAULICS: NON-UNIFORM FLOW IN A PRISMATIC CHANNEL WITH POSITIVE BOTTOM SLOPE —— Example 1.

Given: A concrete trapezoidal channel with the values of $z = 1.5$, $b = 15$ ft, $n = 0.014$, $Q = 1500$ cfs. The stationing is in the direction of flow. A break in grade is located at Sta 33 + 50 and elevation of 100.0; the slope upstream from Sta 33+50, s_{oa}, is 0.0010. The slope downstream from Sta 33+50, s_{ob}, is 0.004



TYPICAL SECTION



PROFILE ALONG CENTERLINE

Determine: The water surface profile

Solution: Rewrite Eq. A.8 in the form

$$s_o \frac{d\ell}{dd} = \frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3} s_o^{1/2}}}{\frac{nQ_{n,d}}{b^{8/3} s_o^{1/2}}}\right]^2 - 1} = 2R$$

1. Solve whether the break in grade is a control section

a. Solve for d_c corresponding to $Q = 1500$

$$\frac{Q}{b} = \frac{1500}{15} = 100 \text{ cfs/ft}; \quad \frac{z}{b} = \frac{1.5}{15} = 0.1$$

From ES-24 $\quad d_c = 5.59$ ft

b. Solve for critical slope s_c corresponding to $Q = 1500$ cfs, $d_c = 5.59$ ft

$$\frac{d_c}{b} = \frac{5.59}{15} = 0.3727$$

From ES-55 $\quad \dfrac{nQ}{b^{8/3} s_c^{1/2}} = 0.341$ where $\dfrac{nQ}{b^{8/3}} = \dfrac{(0.014)(1500)}{15^{8/3}} = 0.01533$

$$s_c = \left[\frac{nQ}{b^{8/3}}\right]^2 \frac{1}{(0.341)^2} = \left[\frac{0.01533}{0.341}\right]^2 = 0.002022$$

Hence, Sta 33 + 50 is a control section since $s_{oa} < s_c < s_{ob}$

2. Solve for the normal depth of flow in the upstream and downstream reaches.

a. For the upstream reach

$$\frac{nQ}{b^{8/3} s_{oa}^{1/2}} = \frac{0.01533}{s_{oa}^{1/2}} = \frac{0.01533}{(0.001)^{1/2}} = 0.4850$$

From ES-55 $\quad \dfrac{d_{na}}{b} = 0.449$ and $d_{na} = 15\ (0.449) = 6.73$ ft

b. For the downstream reach

$$\frac{nQ}{b^{8/3} s_{ob}^{1/2}} = \frac{0.01533}{(0.004)^{1/2}} = 0.2425$$

From ES-55 $\quad \dfrac{d_{nb}}{b} = 0.31$ and $d_{nb} = 15\ (0.31) = 4.65$ ft

3. Solve for the water-surface profile and tabulate values in Table 1.

a. column 1 lists arbitrarily selected values of depth between d_c and d_{na} or d_c and d_{nb}
b. column 3 is read from ES-55
c. column 4 is read from ES-24
d. column 5 is the value of $\dfrac{nQ}{b^{8/3} s_o^{1/2}} = 0.4850$ or 0.2425 divided by column 3 (See step 2 above)
e. column 6 is the value $Q/b = 100$ divided by column 4
f. column 7 is read from ES-53 using columns 5 and 6
g. column 10 is obtained by the relation $\ell_2 - \ell_1 = \dfrac{1}{s_o}(R_1 + R_2)(d_2 - d_1)$ - Eq. A.15

The value of R versus depth d is plotted in Fig. 1. The area under the curve between any two depths represents the length of reach between these depths. This plot is useful for ascertaining whether or not the selected depths in column 1 have been chosen at sufficiently close intervals to justify the use of the relation shown by Eq. A.15.

The profile can be readily plotted by using columns 1 and 11.

TABLE 1

Direction	(1) d	(2) $\frac{d}{b}$	(3) $\frac{nQ_{n,d}}{b^{8/3}s_c^{1/2}}$	(4) $\frac{Q_{c,d}}{b}$	(5) $\frac{Q}{Q_{n,d}}$	(6) $\frac{Q}{Q_{c,d}}$	(7) R	(8) $R_1 + R_2$	(9) $d_2 - d_1$	(10) $\ell_2 - \ell_1$	(11) Station
Upstream from break in grade, $s_{oa} = 0.001$	5.59	0.373	0.341	100.0	1.422	1.000	0	- 0.0177	-0.05	0.885	33+50
	5.64	0.376	0.347	101.7	1.398	0.983	- 0.0177	- 0.0591	-0.06	3.55	33+49.1
	5.70	0.380	0.354	103.8	1.370	0.963	- 0.0414	- 0.1246	-0.10	12.46	33+45.6
	5.80	0.387	0.366	107.0	1.325	0.935	- 0.0832	- 0.2212	-0.10	22.12	33+33.1
	5.90	0.393	0.377	110.5	1.286	0.905	- 0.138	- 0.346	-0.10	34.60	33+11.0
	6.00	0.400	0.390	113.8	1.244	0.879	- 0.208	- 0.506	-0.10	50.60	32+76.4
	6.10	0.407	0.402	117.1	1.206	0.854	- 0.298	- 0.725	-0.10	72.50	32+25.8
	6.20	0.413	0.415	120.6	1.169	0.829	- 0.427	- 1.045	-0.10	104.50	31+53.3
	6.30	0.420	0.428	124.1	1.133	0.806	- 0.618	- 1.569	-0.10	156.90	30+48.8
	6.40	0.427	0.442	127.7	1.097	0.783	- 0.951	- 2.441	-0.10	244.10	28+91.9
	6.50	0.433	0.454	131.2	1.068	0.762	- 1.49	- 4.33	-0.10	433.00	26+47.8
	6.60	0.440	0.467	134.9	1.039	0.741	- 2.84	- ∞	-0.13	∞	22+14.8
	6.73	0.449	0.485	140.2	1.000	0.713	- ∞				
Downstream from break in grade, $s_{ob} = 0.004$	5.59	0.373	0.341	100.0	0.711	1.000	0	- 0.0591	-0.09	1.33	33+50
	5.50	0.367	0.331	97.4	0.733	1.027	- 0.0591	- 0.2091	-0.10	5.23	33+51.3
	5.40	0.360	0.320	94.2	0.758	1.062	- 0.150	- 0.406	-0.10	10.15	33+56.6
	5.30	0.353	0.308	91.5	0.787	1.093	- 0.256	- 0.660	-0.10	16.50	33+66.7
	5.20	0.347	0.299	88.5	0.811	1.130	- 0.404	- 1.046	-0.10	26.15	33+83.2
	5.10	0.340	0.288	85.3	0.842	1.172	- 0.642	- 1.653	-0.10	41.33	34+09.4
	5.00	0.333	0.277	82.4	0.875	1.214	- 1.011	- 2.631	-0.10	65.78	34+50.7
	4.90	0.327	0.267	79.8	0.908	1.253	- 1.62	- 4.79	-0.10	119.75	35+16.5
	4.80	0.320	0.257	76.9	0.944	1.300	- 3.17	-14.70	-0.10	367.50	36+36.2
	4.70	0.313	0.247	74.1	0.982	1.350	-11.53	- ∞	-0.05	∞	40+03.7
	4.65	0.310	0.242	72.7	1.000	1.376	- ∞				

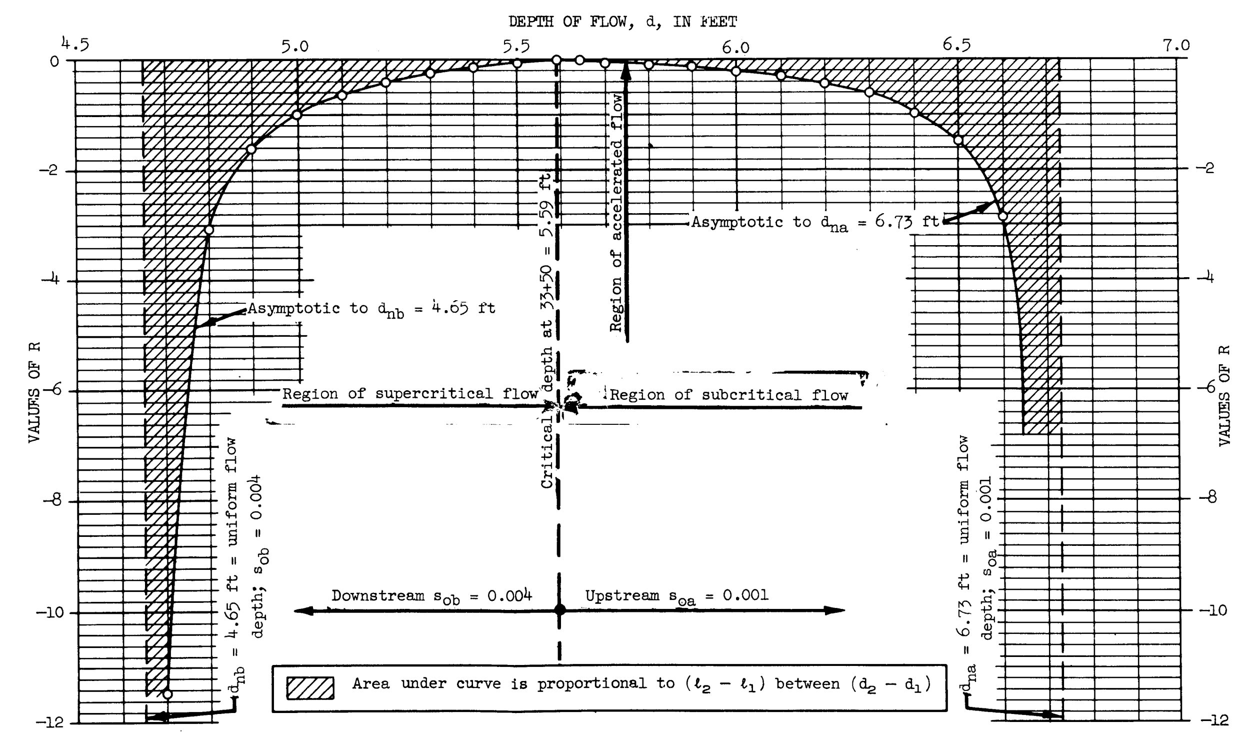


FIGURE 1

HYDRAULICS: NON-UNIFORM FLOW IN A PRISMATIC CHANNEL WITH A LEVEL BOTTOM — Example 2

EXAMPLE 2

Given: A prismatic trapezoidal earth spillway with 3 to 1 side slopes. The bottom of the spillway is level and is 200 ft long and 75 ft wide in the reach between the control section and the reservoir. Manning's coefficient of roughness n, of the spillway within this reach, is estimated to be 0.035. The spillway is expected to convey 1500 cfs.

Determine: A. The water-surface profile in the spillway between the reservoir and the control section.
B. The friction loss through this spillway measured in feet.
C. The elevation of the water surface in the reservoir when the spillway is discharging 1500 cfs.

Solution: Rewrite Eq. A.11 in the form

$$\frac{d\ell}{dd} = \frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3}s_o^{1/2}}}{\frac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}}\right]^2} = R_o$$

where $Q_{n,d}$ is defined as the normal discharge corresponding to a depth d and channel bottom slope of $s_o = 1.0$. Equation A.11 is to be used only for channels with horizontal bottoms. The quantity s_o by definition is unity when working with Eq. A.11 and Manning's formula even though the actual slope of the channel is zero.

A.1. Solve for the depth of flow at the control section Sta 2+00. The depth of flow at the control section is critical depth corresponding to the discharge Q = 1500 cfs.

$$\frac{Q}{b} = \frac{1500}{75} = 20 \text{ cfs/ft} \; ; \qquad \frac{z}{b} = \frac{3}{75} = 0.04$$

From ES-24, $d_c = 2.25$ ft

2. Solve for the value of $\dfrac{nQ}{b^{8/3}s_o^{1/2}}$

$$\frac{nQ}{b^{8/3}s_o^{1/2}} = \frac{(0.035)(1500)}{(75)^{8/3}(1)^{1/2}} = 5.2480 \times 10^{-4}$$

3. Prepare Table 1
(a) Column 1 lists arbitrarily selected depths greater than critical depth, beginning with $d_c = 2.25$ ft. The range of required depths cannot be readily predetermined.
(b) Column 2 lists the selected depths of column 1 divided by the bottom width of the channel.
(c) Column 3 is read from ES-55.
(d) Column 4 is read from ES-24.
(e) Column 5 lists $\dfrac{nQ}{b^{8/3}s_o^{1/2}} = 5.2480 \times 10^{-4}$ (see step 2 above) divided by column 3.
(f) Column 6 lists Q/b = 20 divided by column 4.
(g) Columns 7 and 8 are the squares of columns 5 and 6.
(h) Column 9 lists column 8 minus unity. This is the numerator of the right-hand member of Eq. A.11.
(i) Column 10 lists column 9 divided by column 7. This is equal to the left-hand member of Eq. A.11.
(j) Column 11 lists the sum of the values of R_o at the end sections of each reach.
(k) Column 12 lists the average value of R_o for each reach or column 11 times ½.
(l) Column 13 lists the difference in selected depth of column 1.
(m) Column 14 lists the length of each reach between the selected depths listed in column 1 and is column 12 times column 13.
(n) Column 15 lists the distances from the control section in the upstream direction and is the accumulated total of column 14.

B.1. The specific energy at the control section is

$$H_{ec} = d_c + \frac{v_c^2}{2g}$$

$$a_c = d_c(b + zd_c) = 2.25\,[75 + 3(2.25)] = 183.94 \text{ ft}^2$$

$$v_c = \frac{Q}{a_c} = \frac{1500}{183.94} = 8.155 \text{ ft/sec}$$

$$\frac{v_c^2}{2g} = \frac{(8.155)^2}{64.32} = 1.034 \text{ ft} \qquad H_{ec} = 2.25 + 1.034 = 3.284 \text{ ft}$$

2. The specific energy at Sta 0+00 is

$$H_{eo} = d_o + \frac{v_o^2}{2g}$$

$d_o = 3.77$ ft (by interpolating between d = 3.7 and d = 3.8 and observing that the control section is 200 ft downstream from the reservoir pool.)

$$a_o = d_o(b + zd_o) = 3.77\,[75 + 3(3.77)] = 325.39 \text{ ft}^2$$

$$v_o = \frac{Q}{a_o} = \frac{1500}{325.39} = 4.610 \text{ ft/sec}$$

$$\frac{v_o^2}{2g} = \frac{(4.610)^2}{64.32} = 0.3304 \text{ ft}$$

$$H_{eo} = 3.77 + 0.330 = 4.100 \text{ ft}$$

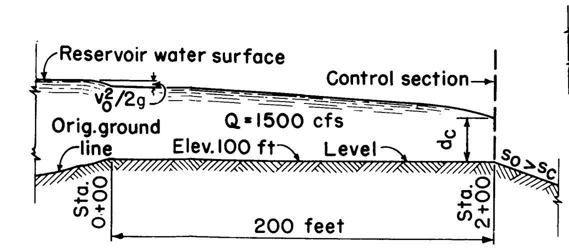
3. Since the spillway bottom is level, the difference in the specific energy head at Sta 0+00 and the control section $(H_{eo} - H_{ec})$ is the friction head loss h_f in this reach.

$$h_f = H_{eo} - H_{ec} = 4.100 - 3.284 = 0.816 \text{ ft}$$

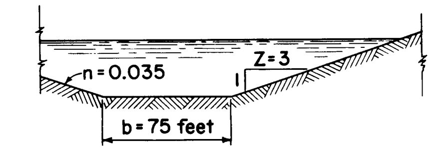
C.1. The elevation of the water surface in the reservoir is equal to the specific energy head at Sta 0+00 plus the elevation of the bottom of channel at Sta 0+00. Elevation of water surface in the reservoir is

$$100.0 + H_{eo} = 104.100 \text{ ft}$$

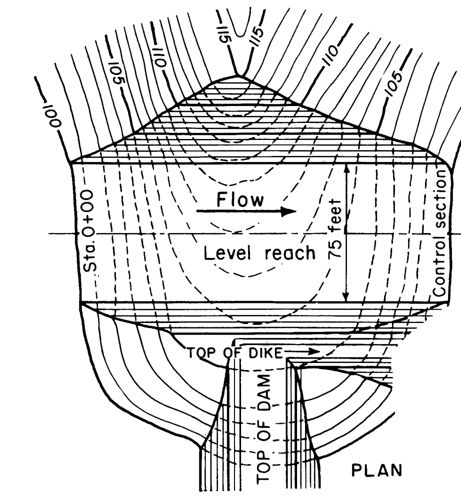
The trapezoidal shape of this spillway does not exist for the full depth of flow in the reach near the entrance of the spillway. By neglecting the effect of this condition, as is done in the example, the water surface elevation in the reservoir required to produce a given discharge through the spillway is slightly greater than the elevation required had this effect been evaluated.



PROFILE ALONG CENTER LINE



TYPICAL SECTION



PLAN

TABLE 1

1 d	2 $\frac{d}{b}$	3 $\frac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}$	4 $\frac{Q_{c,d}}{b}$	5 $\frac{Q}{Q_{n,d}}$	6 $\frac{Q}{Q_{c,d}}$	7 $\left[\frac{Q}{Q_{n,d}}\right]^2$	8 $\left[\frac{Q}{Q_{c,d}}\right]^2$	9 $\left[\frac{Q}{Q_{c,d}}\right]^2 - 1$	10 R_o	11 $R_{o1} + R_{o2}$	12 $\frac{R_{o1} + R_{o2}}{2}$	13 $d_2 - d_1$	14 $\ell_2 - \ell_1$	15 Distance from Control Section
2.25	0.03000	0.00442	20.00	0.11873	1.0000	0.014097	1.00000	0	0	− 5.423	− 2.712	−0.05	0.136	0
2.3	0.03067	0.00459	20.75	0.11434	0.9639	0.013074	0.92910	−0.07090	− 5.423	− 21.328	− 10.664	−0.10	1.066	0.136
2.4	0.03200	0.00492	22.10	0.10667	0.9050	0.011378	0.81903	−0.18097	− 15.905	− 44.317	− 22.158	−0.10	2.216	1.202
2.5	0.03333	0.00527	23.60	0.09958	0.8475	0.0099162	0.71826	−0.28174	− 28.412	− 70.175	− 35.087	−0.10	3.509	3.418
2.6	0.03467	0.00564	25.03	0.09305	0.7990	0.0086583	0.63840	−0.36160	− 41.763	− 99.053	− 49.527	−0.10	4.953	6.927
2.7	0.03600	0.00604	26.55	0.08689	0.7533	0.0075499	0.56746	−0.43254	− 57.290	−130.638	− 65.319	−0.10	6.532	11.880
2.8	0.03733	0.00641	28.05	0.08187	0.7130	0.0067027	0.50837	−0.49163	− 73.348	−164.848	− 82.424	−0.10	8.242	18.412
2.9	0.03867	0.00680	29.65	0.07718	0.6745	0.0059568	0.45495	−0.54505	− 91.500	−203.171	−101.585	−0.10	10.159	26.654
3.0	0.04000	0.00721	31.30	0.07279	0.6390	0.0052984	0.40832	−0.59168	−111.671	−245.401	−122.701	−0.10	12.270	36.813
3.1	0.04133	0.00763	33.00	0.06878	0.6061	0.0047307	0.36736	−0.63264	−133.730	−290.428	−145.214	−0.10	14.521	49.083
3.2	0.04267	0.00805	34.60	0.06519	0.5780	0.0042497	0.33408	−0.66592	−156.698	−338.776	−169.388	−0.10	16.939	63.604
3.3	0.04400	0.00849	36.25	0.06181	0.5517	0.0038205	0.30437	−0.69563	−182.078	−392.743	−196.384	−0.10	19.638	80.543
3.4	0.04533	0.00895	38.10	0.05864	0.5249	0.0034396	0.27552	−0.72448	−210.665	−449.333	−224.679	−0.10	22.468	100.181
3.5	0.04667	0.00937	39.90	0.05601	0.5013	0.0031371	0.25130	−0.74870	−238.668	−510.256	−255.128	−0.10	25.513	122.649
3.6	0.04800	0.00986	41.65	0.05323	0.4802	0.0028334	0.23059	−0.76941	−271.588	−578.583	−289.292	−0.10	28.929	148.162
3.7	0.04933	0.01035	43.60	0.05071	0.4587	0.0025720	0.21041	−0.78959	−306.995	−648.693	−324.347	−0.10	32.435	177.091
3.8	0.05067	0.01080	45.50	0.04859	0.4396	0.0023610	0.19325	−0.80675	−341.698					209.526

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO. ES-83
SHEET 2 OF 10
DATE 7-22-54

HYDRAULICS: NON-UNIFORM FLOW IN A PRISMATIC CHANNEL WITH A NEGATIVE BOTTOM SLOPE — EXAMPLE 3

EXAMPLE 3

Given: A prismatic earth spillway with side slopes of 3 to 1. The spillway has a level bottom upstream from the control section for a distance of 90 ft and an adverse slope of 10 to 1 for an upstream distance of 100 ft. The bottom of the spillway is 75 ft wide. Manning's coefficient of roughness n is estimated as 0.035 feet.

Determine: A. The water-surface profile in the spillway when the discharge is 1500 cfs.

B. The friction loss from Sta 0+00 to Sta 1+90 and the elevation of the water surface in the reservoir when the spillway is discharging 1500 cfs.

C. The friction loss through the spillway from Sta 0+00 to Sta 1+00.

Solution: Rewrite Eq. A.13 in the form

$$|s_o|\,\frac{d\ell}{dd} = \frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3}|s_o|^{1/2}}}{\frac{n\,Q_{n,d}}{b^{8/3}|s_o|^{1/2}}}\right]^2 + 1} = R_n$$

A.1. Solve for the critical depth of flow corresponding to the discharge of 1500 cfs.

$$\frac{Q}{b} = \frac{1500}{75} = 20 \text{ cfs/ft}$$

From ES-24, d_c = 2.25 ft. This is the depth of flow at the control section.

2. Solve for the depth of flow at the break in grade between the 10 to 1 negative slope and the level reach. This was illustrated by Ex. 2 and found to be 3.347 ft.

3. Solve for the value of

$$\frac{nQ}{b^{8/3}|s_o|^{1/2}} = \frac{(0.035)(1500)}{(75)^{8/3}(0.10)^{1/2}} = 1.6596 \times 10^{-3}$$

4. Solve the water-surface profile upstream from the break in grade at Sta 1+00 starting with a depth of 3.347 ft and tabulate values in table 1.

(a) Column 1 lists arbitrarily selected values of depths between the depth of flow at Sta 1+00 and an approximated depth at Sta 0+00. The approximated depth at Sta 0+00 is estimated by assuming the water-surface elevation at Sta 0+00 to be slightly greater than the elevation of the energy gradient at Sta 1+00. The specific energy at Sta 1+00 is

$$H_{e1} = d_1 + \frac{v_1^2}{2g}$$

$$a_1 = d_1(b + zd_1) = 3.347\,[75 + 3(3.347)] = 284.63 \text{ ft}^2$$

$$v_1 = \frac{Q}{a_1} = \frac{1500}{284.63} = 5.27 \text{ ft/sec}$$

$$\frac{v_1^2}{2g} = \frac{(5.27)^2}{64.32} = 0.4318 \text{ ft}$$

$$H_{e_1} = 3.347 + 0.4318 = 3.779 \text{ ft}$$

The elevation of the energy gradient at Sta 1+00 is

$$(100.00 + H_{e1}) = 103.779 \text{ ft}$$

The approximated depth of flow at Sta 0+00 is the difference in the elevation of the energy gradient at Sta 1+00 and bottom of channel at Sta 0+00.

$$103.779 - 90 = 13.779 \text{ ft}$$

Try 14 ft for an upper limit of d in column 1.

(b) Column 3 is read from ES-55.
(c) Column 4 is read from ES-24.
(d) Column 5 is the value of $\frac{nQ}{b^{8/3}|s_o|^{1/2}} = 1.6596 \times 10^{-3}$ (see step 3) divided by column 3.
(e) Column 6 is the value of Q/b = 20 divided by column 4.
(f) Columns 7 and 8 are the squares of columns 5 and 6.
(g) Column 9 lists the values of R_n as given by Eq. A.13 or is

$$R_n = \frac{\text{column 8 minus unity}}{\text{column 7 plus unity}}$$

(h) Column 10 lists the average value of R_n for each reach.
(i) Column 11 lists the difference in depth of flow at the end section of each reach.
(j) Column 12 lists column 11 divided by $|s_o|$.
(k) Column 13 lists the length of each reach and is column 10 times column 12.
(l) Column 14 lists the distance upstream from the break in grade at Sta 1+00 and is the accumulated total of column 13.
(m) Column 15 lists the stationing for the selected depths of flow used in column 1.

B.1. By Ex. 2, the specific energy at Sta 1+90 is H_{ec} = 3.284 ft.

2. The specific energy at Sta 0+00 is

$$H_{eo} = d_o + \frac{v_o^2}{2g} = 13.824 \text{ ft}$$

d_o = 13.81 ft by interpolation between d = 13 ft and d = 14 ft

3. The friction loss from Sta 0+00 to Sta 1+90 is the difference in the elevation of the energy gradient at Sta 0+00 to Sta 1+90. The elevation of the energy gradient at Sta 0+00 is the elevation of the bottom of the spillway plus the specific energy at Sta 0+00.

$$90.00 + H_{eo} = 103.824 \text{ ft}$$

The elevation of the energy gradient at Sta 1+90 is

$$100.00 + H_{ec} = 103.284$$

The friction loss is

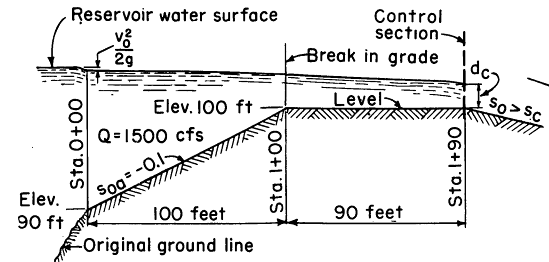
$$103.824 - 103.284 = 0.540 \text{ ft}$$

The elevation of the water surface in the reservoir is equal to the elevation of the energy gradient at Sta 0+00 or 103.824 ft.

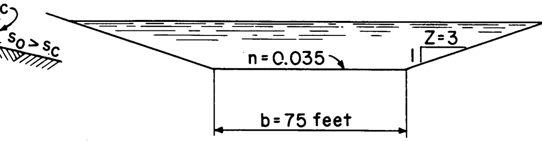
C.1. The friction loss from Sta 0+00 to Sta 1+00 is the difference in the elevation of the energy gradient. By A.4, the elevation of the energy gradient at Sta 1+00 is 103.779 ft. By B.3, the elevation of the energy gradient is 103.824 ft. The friction loss between Sta 0+00 and Sta 1+00 is

$$103.824 - 103.779 = 0.045 \text{ ft}$$

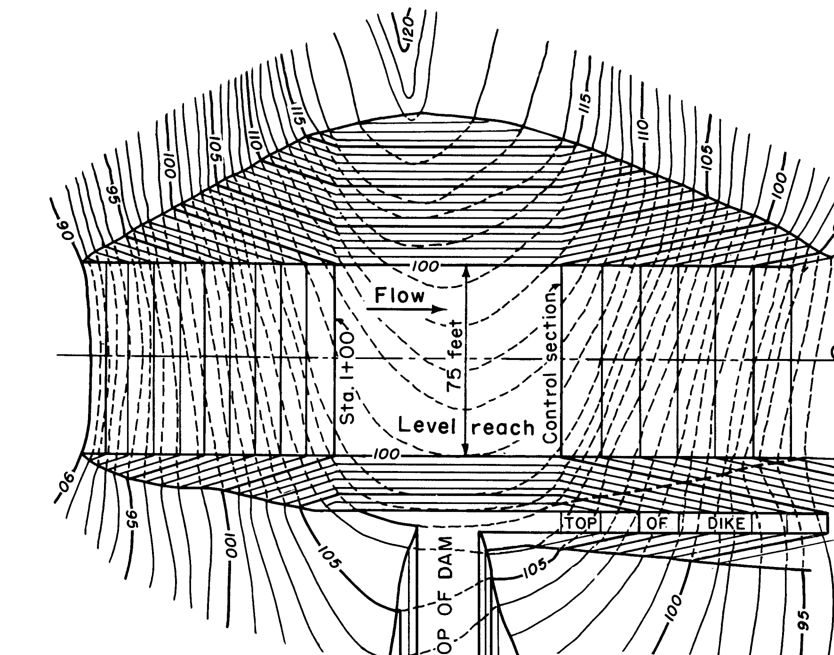
The trapezoidal shape of this spillway does not exist for the full depth of flow in the reach near the entrance of the spillway. By neglecting the effect of this condition, as is done in the example, the water surface elevation in the reservoir required to produce a given discharge through the spillway is slightly greater than the elevation required had this effect been evaluated.



PROFILE ALONG CENTER LINE



TYPICAL SECTION



PLAN

TABLE 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d	$\frac{d}{b}$	$\frac{n\,Q_{n,d}}{b^{8/3}\lvert s_o\rvert^{1/2}}$	$\frac{Q_{c,d}}{b}$	$\frac{Q}{Q_{n,d}}$	$\frac{Q}{Q_{c,d}}$	$\left[\frac{Q}{Q_{n,d}}\right]^2$	$\left[\frac{Q}{Q_{c,d}}\right]^2$	R_n	$\frac{R_{n1}+R_{n2}}{2}$	$d_2 - d_1$	$\frac{(d_2-d_1)}{\lvert s_o\rvert}$	$\ell_2 - \ell_1$	$\Sigma(\ell_2 - \ell_1)$	Station
3.347	0.04463	0.00875	37.10	0.1897	0.5391	0.03599	0.2906	−0.6848	−0.7049	−0.153	−1.53	1.078	0	1+00.00
3.50	0.04667	0.00942	39.80	0.1762	0.5025	0.03105	0.2525	−0.7250	−0.7729	−0.50	−5.00	3.865	1.078	0+98.92
4.00	0.05333	0.01186	49.50	0.1399	0.4040	0.01957	0.1632	−0.8207	−0.8487	−0.50	−5.00	4.244	4.943	0+95.06
4.50	0.06000	0.01456	59.75	0.1140	0.3347	0.01300	0.1120	−0.8766	−0.8940	−0.50	−5.00	4.470	9.187	0+90.81
5.00	0.06667	0.01752	70.50	0.09473	0.2837	0.008974	0.08049	−0.9113	−0.9312	−1.00	−10.0	9.312	13.66	0+86.34
6.00	0.08000	0.02410	94.95	0.06886	0.2106	0.004742	0.04435	−0.9511	−0.9608	−1.00	−10.0	9.608	22.97	0+77.03
7.00	0.09333	0.03175	121.9	0.05227	0.1641	0.002732	0.02693	−0.9704	−0.9757	−1.00	−10.0	9.757	32.58	0+67.42
8.00	0.1067	0.0402	152.1	0.04128	0.1315	0.001704	0.01729	−0.9810	−0.9842	−1.00	−10.0	9.842	42.33	0+57.67
9.00	0.1200	0.04974	185.4	0.03337	0.1079	0.001114	0.01164	−0.9873	−0.9892	−1.00	−10.0	9.892	52.18	0+47.82
10.0	0.1333	0.06025	221.4	0.02755	0.09033	0.0007590	0.008160	−0.9911	−0.9924	−1.00	−10.0	9.924	62.07	0+37.93
11.0	0.1467	0.07210	260.7	0.02302	0.07672	0.0005299	0.005886	−0.9936	−0.9945	−1.00	−10.0	9.945	71.99	0+28.01
12.0	0.1600	0.08460	303.2	0.01962	0.06596	0.0003849	0.004351	−0.9953	−0.9959	−1.00	−10.0	9.959	81.94	0+18.06
13.0	0.1733	0.09799	348.7	0.01694	0.05736	0.0002870	0.003290	−0.9964	−0.9969	−1.00	−10.0	9.969	91.90	0+ 8.10
14.0	0.1867	0.1142	397.5	0.01453	0.05031	0.0002111	0.002531	−0.9973					101.9	

HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL —— Example 4.

THIS EXAMPLE ILLUSTRATES A METHOD TO DETERMINE THE APPROXIMATE DEPTH OF FLOW AT A SECTION

Generally channel cross sections for a natural channel are determined by field surveys at definite stations. The length of the reach between two consecutive sections is therefore fixed or given and a depth of flow is to be determined. In Example 1 the length was determined between two given (actually selected) depths. The relationship between the depth of flow and any hydraulic characteristics of a cross section is the same for every cross section in a prismatic channel. This enables one to choose the length of the reach as the variable to be determined, and the length of the reach between two given depths is determined in Example 1. Since length of reach is given in natural channels the depth of flow is the variable to be determined. This complicates the solution for water-surface profiles because the depth of flow is implicity expressed in the differential equation.

Any method of water-surface profile determination requires that at least one depth of flow corresponding to the discharge, Q, under consideration be given at a station or be determinable. The determination of a depth of flow in a natural channel at a control section presents a trivial problem, for the depth of flow at the section is known to be the critical depth. Some channels for which water-surface profiles are to be determined have no control section, nor is the depth of flow given for any portion of the channel. This example is concerned with the determination of a depth of flow at station 0+00 so water-surface profile computations can be made upstream from station 0+00.

Given: Channel cross sections for a natural channel have been determined by field surveys at stations indicated by Fig. 1. Stationing is in a downstream direction. The roughness coefficient, n, has been estimated to be 0.035 for in-bank flow and 0.06 for out-bank flow. Dikes having 3 to 1 slope have been constructed as shown in Fig. 1. These dikes are sufficiently high to contain the flow of 7000 cfs. The depth of flow at station 17+90 has been determined to be not less than 14 ft nor greater than 18 ft for a discharge of 7000 cfs.

Determine: The approximate depth of flow at station 0+00 when the discharge is 7000 cfs.

Solution: Equation A.18 is used for this solution.

$$\frac{E_1 - E_2}{\left[\frac{1}{a_2^2} + \frac{s_o}{Q^2_{n,d_2}}(\ell_2 - \ell_1)g\right] - \left[\frac{1}{a_1^2} - \frac{s_o}{Q^2_{n,d_1}}(\ell_2 - \ell_1)g\right]} = \frac{Q^2}{2g}$$

1. Prepare tabular form given by Table 1.
   (a) Column 1 lists the stations of cross sections and elevation of channel bottom at these stations.
   (b) Column 2 is an arbitrary selection of flow depths for an estimated range of depth of flow. A selection of depths should be made at each major change in cross section shape and not over 3 or 4 ft interval between consecutive depths.
   (c) Columns 3 and 7 list the flow areas associated with the appropriate n values as obtained from Fig. 1.
   (d) Columns 4 and 8 list the wetted perimeters associated with the appropriate n values as obtained from Fig. 1.
   (e) Columns 5 and 9 list the cross section factor F read from ES-76.
   (f) Columns 6 and 10 are read from ES-77.
   (g) Column 11 is the sum of columns 6 and 10.
   (h) Column 12 is the square of the reciprocal of column 11. This can be read from the double scale of ES-77.

2. Prepare Figures 2 and 3 from Table 1. These values are plotted on log-log coordinate paper. Figure 2 will be used to determine critical slope. Figure 3 will be used later to determine values of $s_o \div Q^2_{n,d}$ for intermediate values of depths of flow.

3. Solve which sections are control sections for the discharge of 7000 cfs.
   (a) Prepare tabular form as given by Table 2.
      (i) Columns 1 and 2 are the same as columns 1 and 2 of Table 1
      (ii) Column 3 lists the top width corresponding to the depth in column 2
      (iii) Column 4 lists the total area corresponding to the depth in column 2
      (iv) Column 5 is read from ES-75
   (b) Prepare Figure 4 on log-log coordinate paper. Values are obtained from Table 2.
   (c) Prepare tabular form as given by Table 3.
      (i) Column 2 will be considered later
      (ii) Column 4 is read from Figure 4 on line Q = 7000 cfs and are critical depths d_c for Q = 7000 cfs
      (iii) Column 5 is read from Figure 2 for critical depths, d_c, in column 4
      (iv) Column 6 lists $s_c^{1/2}$ which is 7000 cfs divided by column 5
      (v) Column 8 lists the slope of channel bottom

No control sections exist for a discharge of 7000 cfs for all bottom slopes, s_o, are less than critical slope, s_c. Thus, flow is subcritical and computations will be carried in an upstream direction.

4. Solve for approximate depth of flow at station 0+00.
   (a) Prepare tabular form given by Table 4.
      (i) Column 2 lists the same range of depths as Table 1 at closer intervals
      (ii) Prepare Figure 5 on log-log coordinate paper from Table 2. Observe that the right hand scale is the square of the reciprocal of the left hand scale in ES-77 and may be used in determining $1 + a^2$ values
      (iii) Column 3 is read from Figure 5
      (iv) Column 4 is read from Figure 3
      (v) Column 5 lists the product of column 4 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of the downstream reach from the station under consideration. The station under consideration is section 1 as is indicated by the subscript d_1 of Q_{n,d_1} in this column heading.
      (vi) Column 6 is the product of column 4 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of the upstream reach from the station under consideration. The station under consideration is section 2 as is indicated by the subscript d_2 of Q_{n,d_2} in this column heading.
      (vii) Column 7 lists column 3 minus column 5
      (viii) Column 8 lists column 3 plus column 6
   (b) It may be desirable to prepare Figures 6 and 7, particularly if values of d have not been selected sufficiently close in column 2.
   (c) Prepare Figure 8.
      (i) Plot U_1^- vs elevation curves
      (ii) Plot U_2^+ vs elevation curves
      (iii) Column 2 of Table 3 gives the slope of straight lines connecting the curves U_1^- and U_2^+. The slope scale for Figure 8 is determined by dividing the ordinate above the reference point by the distance between the scale and the reference point. In this example the reference point has been arbitrarily taken at elevation 1185 and an abscissa of 2.0×10^{-6} from the slope scale. The value of the slope scale at the line having an elevation of 1186 is $(1186 - 1185) \div 2.0 \times 10^{-6} = 5 \times 10^5$.
      (iv) Make the graphical solution for the water-surface profile if the depth of flow at station 17+90 is 14 ft. The beginning point from which a straight line is drawn having the slope given by column 2, Table 3, is at depth of 14 ft or elevation 1184.2 on U_2^+ curve. The elevation of flow at station 13+30 is the intersection of this straight line with the U_1^- curve. The elevation of flow is now known at station 13+30 and the elevation of flow at 9+10 is determined in a similar manner. This procedure is continued to station 0+00. The depth obtained at 0+00 is the depth of flow if the depth of flow at 17+90 is 14 ft.

      A graphical solution for the water-surface profile if the depth of flow at station 17+90 is 18 ft is obtained in a similar manner. Observe the depth of flow at station 0+00 is not less than 13.61 ft nor greater than 14.22 ft for it has been given the depth of flow at 17+90 is not less than 14 ft nor greater than 18 ft. Thus, the depth of flow at station 0+00 has been closely approximated. If a closer approximation is desired it will be necessary to determine water-surface profiles from a station downstream from station 17+90
      (v) Figure 9 shows water-surface profiles determined for various depths at station 17+90.

TABLE 1

1 Station and Bottom Elevation	2 d ft	3 a ft²	4 p ft	5 F	6 $\frac{Q_{n,d}}{s_o^{1/2}}$	7 a ft²	8 p ft	9 F	10 $\frac{Q_{n,d}}{s_o^{1/2}}$	11 $\frac{Q_{n,d}}{s_o^{1/2}}$	12 $\frac{s_o}{Q^2_{n,d}}$
	n = 0.035					n = 0.06				composite n	
17+90	9.6	384	64.9	1867	53,300	15.5	38.2	12.65	211	53,511	3.485×10^{-10}
	11.6	508	64.9	2985	85,300	103.1	50.8	246.0	4,100	89,400	1.250×10^{-10}
1170.2	13.6	632	64.9	4284	122,400	214.7	63.5	720.0	12,000	134,400	5.530×10^{-11}
	15.6	756	64.9	5772	164,900	350.3	76.1	1440.0	24,000	188,900	2.806×10^{-11}
	17.6	880	64.9	7435	212,400	515.0	88.7	2480.0	41,330	253,730	1.553×10^{-11}
13+30	9.4	376	61.2	1872	53,500	15.0	40.2	11.50	1,920	53,692	3.475×10^{-10}
	11.4	492	61.2	2930	83,700	107.0	52.8	254.4	4,240	87,940	1.296×10^{-10}
1171.6	13.4	608	61.2	4180	119,400	223.0	65.5	750.0	12,500	131,900	5.750×10^{-11}
	15.4	724	61.2	5585	159,600	363.0	78.2	1500.0	25,000	184,600	2.932×10^{-11}
	17.4	840	61.2	7155	204,400	527.0	90.8	2530.0	42,180	246,580	1.644×10^{-11}
9+10	10.2	439	69.9	2221	63,500	23.6	52.2	20.6	343	63,843	2.453×10^{-10}
	12.2	571	69.9	3439	98,300	139.6	64.8	346.0	5,767	104,067	9.220×10^{-11}
1173.6	14.2	703	69.9	4865	139,000	279.6	77.4	978.7	16,310	155,310	4.140×10^{-11}
	16.2	835	69.9	6495	185,600	443.6	90.1	1908.0	31,800	217,400	2.118×10^{-11}
4+50	12.4	623	89.3	3380	96,600	21.4	30.6	25.05	418	97,018	1.062×10^{-10}
	14.4	795	89.3	5070	144,900	94.4	43.3	236.0	3,933	148,833	4.525×10^{-11}
1174.8	16.4	967	89.3	7030	200,900	191.4	56.0	646.0	10,770	211,670	2.235×10^{-11}
0+00	13.4	615	83.7	3455	98,700	15.4	28.9	15.04	251	98,951	1.022×10^{-10}
	14.4	694	83.7	4225	120,700	47.4	35.2	85.90	1,432	122,132	6.690×10^{-11}
1175.8	15.4	773	83.7	5060	144,600	85.4	41.6	204.8	3,413	148,013	4.560×10^{-11}
	16.4	852	83.7	5950	170,000	129.4	47.9	374.0	6,233	176,233	3.220×10^{-11}
	17.4	931	83.7	6890	196,900	179.4	54.3	591.5	9,858	206,758	2.350×10^{-11}

TABLE 2

1 Station and Bottom Elevation	2 Depth ft	3 T ft	4 a ft²	5 $Q_{c,d}$ cfs
0+00	13.4	108.2	630	8,610
1175.8	14.4	114.2	741	10,730
	15.4	120.2	858	13,100
	16.4	126.2	981	15,540
	17.4	132.2	1110	18,250
4+50	12.4	116.5	644	8,630
1174.8	14.4	128.5	889	13,280
	16.4	140.5	1158	18,850
9+10	10.2	118.0	463	5,200
1173.6	12.2	130.0	711	9,450
	14.2	142.0	983	14,700
	16.2	154.0	1279	20,910
13+30	9.4	98.0	391	4,410
1171.6	11.4	110.0	599	7,925
	13.4	122.0	831	12,310
	15.4	134.0	1087	17,570
	17.4	146.0	1367	23,800
17+90	9.6	99.8	399	4,535
1170.2	11.6	111.8	611	8,110
	13.6	123.8	847	12,630
	15.6	135.8	1106	17,930
	17.6	147.8	1395	24,400

TABLE 3

1 Q	2 $\frac{Q^2}{2g}$	3 Sta	4 d_c	5 $\frac{Q_{n,d_c}}{s_c^{1/2}}$	6 $s_c^{1/2}$	7 s_c	8 s_o	Remarks
7000	7.618×10^5	0+00	12.53*	81,000	0.08642	0.007468	0.00222	$s_c > s_o$
		4+50	11.50*	78,100	0.08963	0.008034	0.00261	$s_c > s_o$
		9+10	11.14	81,500	0.08589	0.007377	0.00476	$s_c > s_o$
		13+30	10.95	79,600	0.08794	0.007733	0.00304	$s_c > s_o$
		17+90	11.05	78,500	0.08917	0.007951		

*These values were obtained by extrapolation from Figure 4.

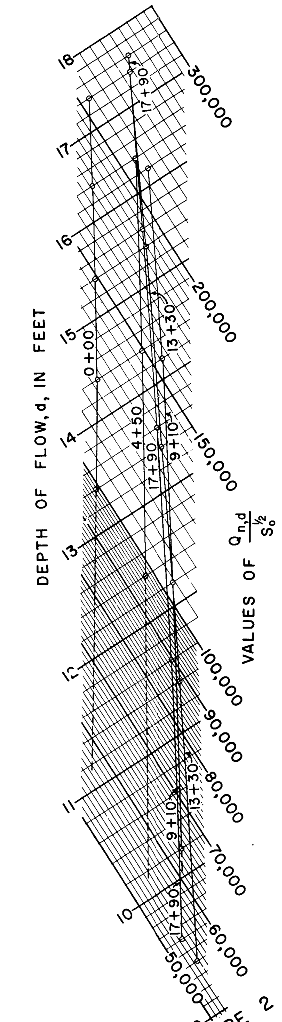


FIGURE 2

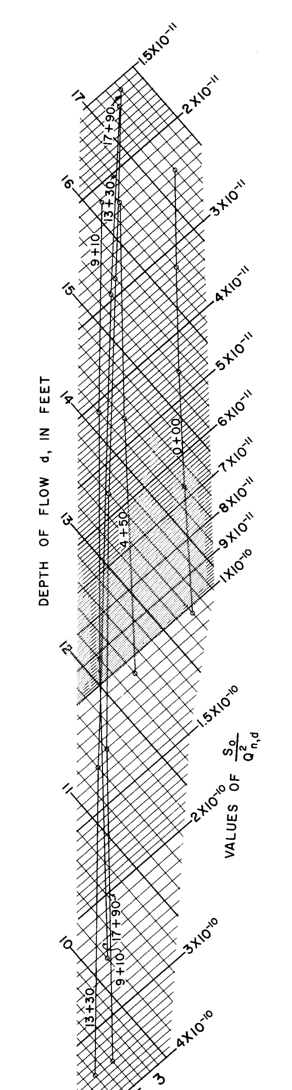


FIGURE 3

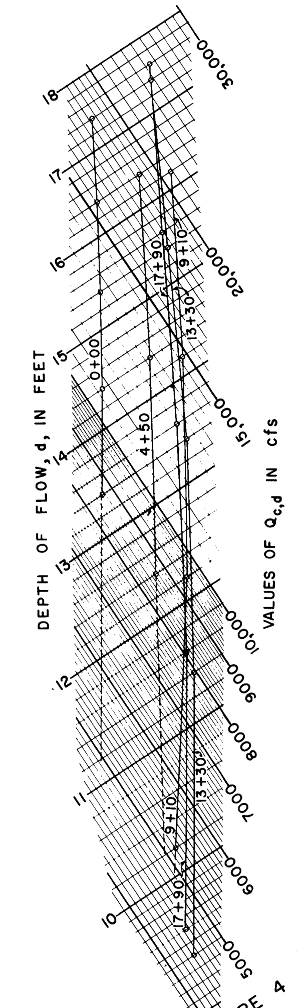


FIGURE 4

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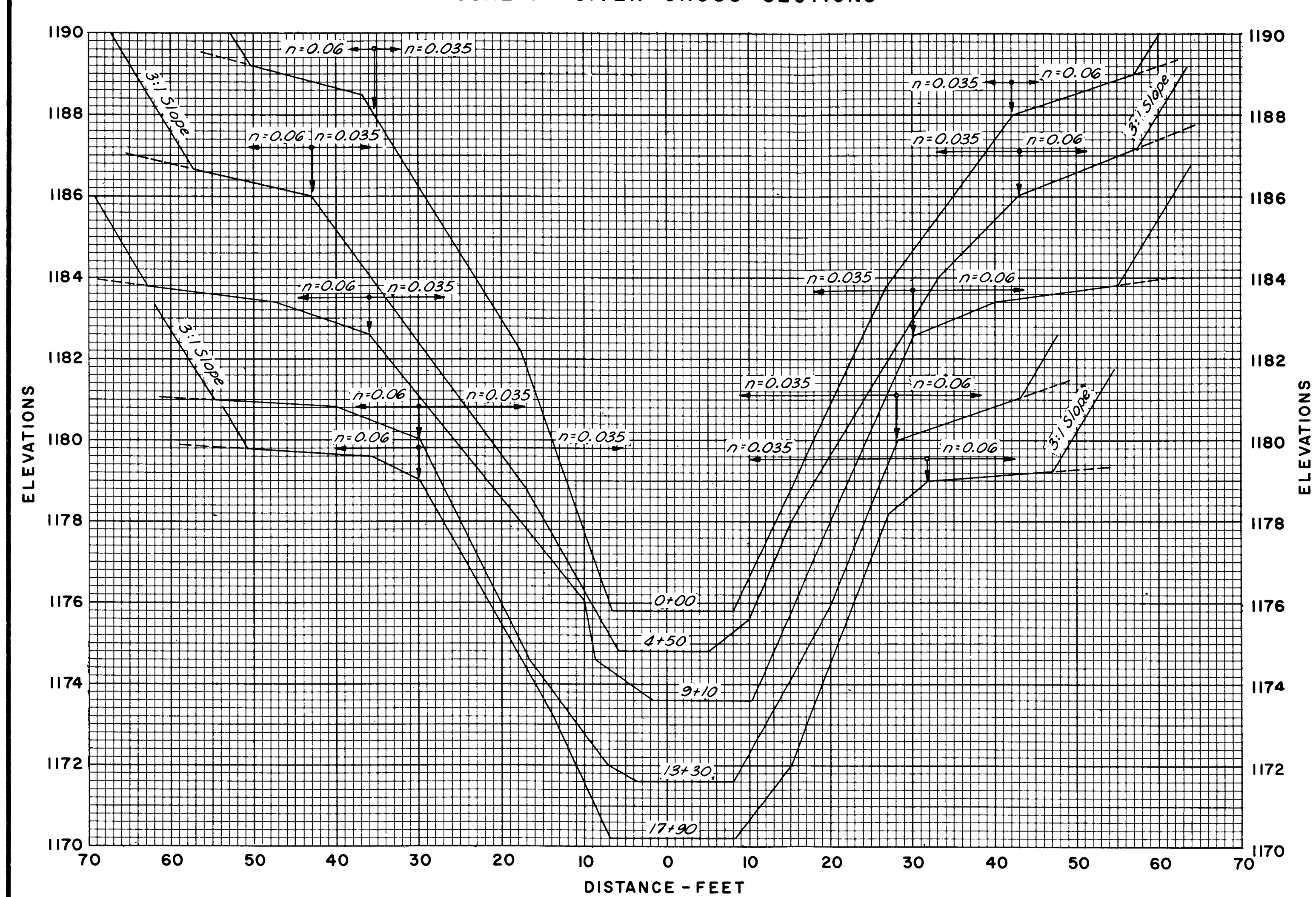
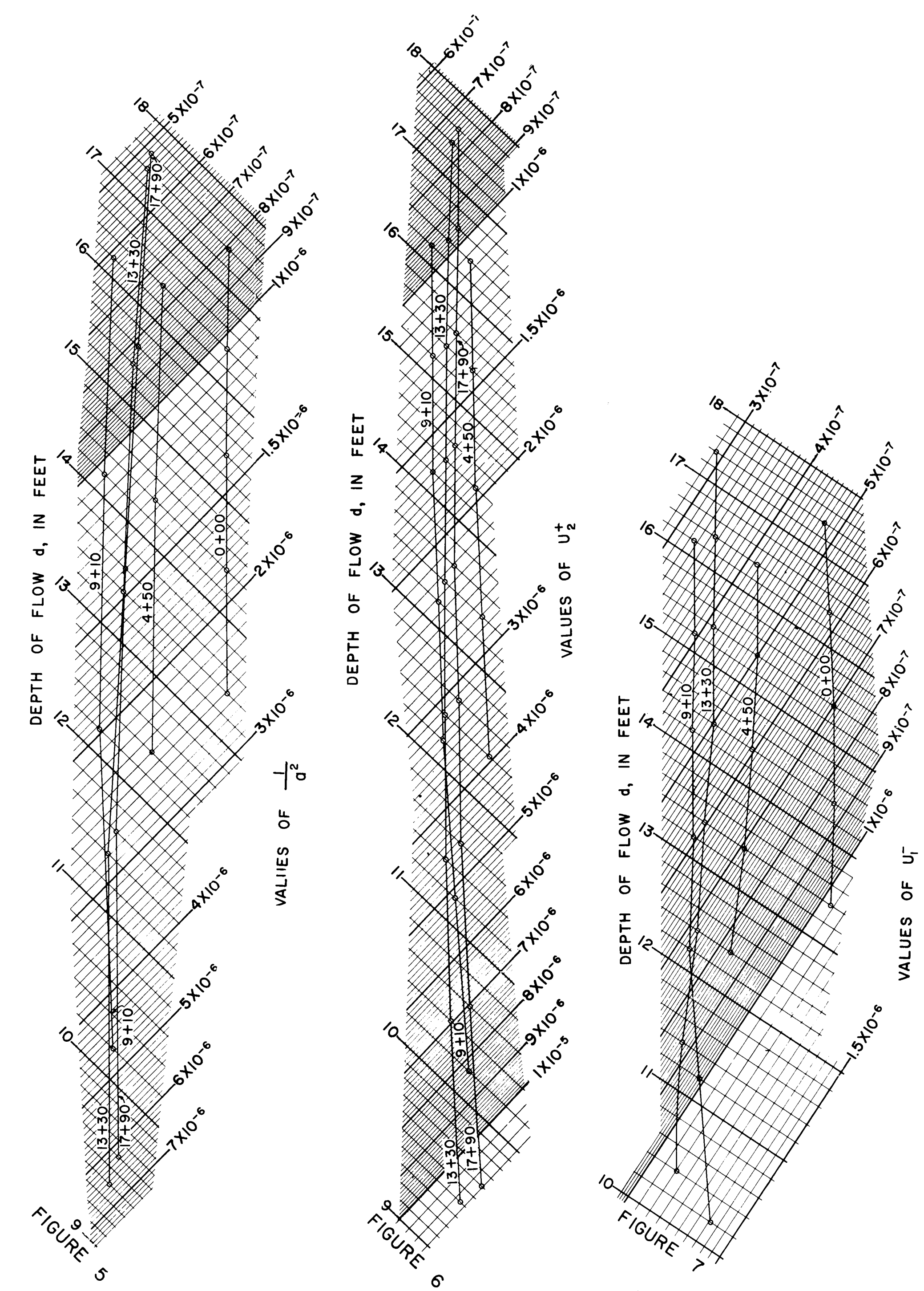
HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL — Example 4.

FIGURE 1 — GIVEN CROSS SECTIONS





HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL — Example 4

TABLE 4

1	2	3	4	5	6	7	8	9
Station and Elevation	d ft	$\frac{1}{a^2}$	$\frac{s_o}{Q^2 n,d}$	$\frac{s_o(\ell_2-\ell_1)g}{Q^2 n,d_1}$	$\frac{s_o(\ell_2-\ell_1)g}{Q^2 n,d_2}$	U_1^-	U_2^+	E
17+90	9.6	6.260×10^{-6}	3.485×10^{-10}		5.156×10^{-6}		11.42×10^{-6}	1179.8
	10.6	4.010×10^{-6}	2.045×10^{-10}		3.025×10^{-6}		7.035×10^{-6}	1180.8
1170.2	11.6	2.678×10^{-6}	1.250×10^{-10}		1.849×10^{-6}		4.527×10^{-6}	1181.8
	12.6	1.915×10^{-6}	8.210×10^{-11}		1.215×10^{-6}		3.130×10^{-6}	1182.8
	13.6	1.393×10^{-6}	5.530×10^{-11}		0.8181×10^{-6}		2.213×10^{-6}	1183.8
	14.6	1.060×10^{-6}	3.920×10^{-11}		0.5799×10^{-6}		1.640×10^{-6}	1184.8
	15.6	0.8170×10^{-6}	2.806×10^{-11}		0.4151×10^{-6}		1.232×10^{-6}	1185.8
	16.6	0.6480×10^{-6}	2.085×10^{-11}		0.3084×10^{-6}		0.9565×10^{-6}	1186.8
	17.6	0.5130×10^{-6}	1.553×10^{-11}		0.2297×10^{-6}		0.7427×10^{-6}	1187.8
13+30	9.4	6.540×10^{-6}	3.475×10^{-10}	5.141×10^{-6}	4.694×10^{-6}	1.399×10^{-6}	11.23×10^{-6}	1181.0
	10.4	4.180×10^{-6}	2.080×10^{-10}	3.077×10^{-6}	2.809×10^{-6}	1.103×10^{-6}	6.990×10^{-6}	1182.0
1171.6	11.4	2.785×10^{-6}	1.296×10^{-10}	1.917×10^{-6}	1.750×10^{-6}	0.8677×10^{-6}	4.536×10^{-6}	1183.0
	12.4	2.000×10^{-6}	8.620×10^{-11}	1.275×10^{-6}	1.164×10^{-6}	0.7248×10^{-6}	3.164×10^{-6}	1184.0
	13.4	1.447×10^{-6}	5.750×10^{-11}	0.8506×10^{-6}	0.7767×10^{-6}	0.5964×10^{-6}	2.224×10^{-6}	1185.0
	14.4	1.105×10^{-6}	4.085×10^{-11}	0.6043×10^{-6}	0.5518×10^{-6}	0.5007×10^{-6}	1.657×10^{-6}	1186.0
	15.4	0.8450×10^{-6}	2.932×10^{-11}	0.4337×10^{-6}	0.3960×10^{-6}	0.4112×10^{-6}	1.241×10^{-6}	1187.0
	16.4	0.6680×10^{-6}	2.185×10^{-11}	0.3232×10^{-6}	0.2951×10^{-6}	0.3448×10^{-6}	0.9631×10^{-6}	1188.0
	17.4	0.5340×10^{-6}	1.644×10^{-11}	0.2432×10^{-6}	0.2221×10^{-6}	0.2908×10^{-6}	0.7561×10^{-6}	1189.0
9+10	10.2	4.670×10^{-6}	2.453×10^{-10}	3.313×10^{-6}	3.629×10^{-6}	1.357×10^{-6}	8.299×10^{-6}	1183.8
	11.2	2.975×10^{-6}	1.475×10^{-10}	1.992×10^{-6}	2.182×10^{-6}	0.9827×10^{-6}	5.157×10^{-6}	1184.8
1173.6	12.2	1.975×10^{-6}	9.220×10^{-11}	1.245×10^{-6}	1.364×10^{-6}	0.7296×10^{-6}	3.339×10^{-6}	1185.8
	13.2	1.415×10^{-6}	6.100×10^{-11}	0.8239×10^{-6}	0.9024×10^{-6}	0.5911×10^{-6}	2.317×10^{-6}	1186.8
	14.2	1.036×10^{-6}	4.140×10^{-11}	0.5592×10^{-6}	0.6125×10^{-6}	0.4768×10^{-6}	1.649×10^{-6}	1187.8
	15.2	0.7910×10^{-6}	2.950×10^{-11}	0.3985×10^{-6}	0.4364×10^{-6}	0.3925×10^{-6}	1.227×10^{-6}	1188.8
	16.2	0.6120×10^{-6}	2.118×10^{-11}	0.2861×10^{-6}	0.3133×10^{-6}	0.3259×10^{-6}	0.9253×10^{-6}	1189.8
4+50	12.4	2.410×10^{-6}	1.062×10^{-10}	1.571×10^{-6}	1.537×10^{-6}	0.8389×10^{-6}	3.947×10^{-6}	1187.2
	13.4	1.725×10^{-6}	6.860×10^{-11}	1.015×10^{-6}	0.9928×10^{-6}	0.7102×10^{-6}	2.718×10^{-6}	1188.2
1174.8	14.4	1.264×10^{-6}	4.525×10^{-11}	0.6694×10^{-6}	0.6549×10^{-6}	0.5946×10^{-6}	1.919×10^{-6}	1189.2
	15.4	0.9670×10^{-6}	3.160×10^{-11}	0.4675×10^{-6}	0.4573×10^{-6}	0.4995×10^{-6}	1.424×10^{-6}	1190.2
	16.4	0.7470×10^{-6}	2.235×10^{-11}	0.3306×10^{-6}	0.3234×10^{-6}	0.4164×10^{-6}	1.071×10^{-6}	1191.2
0+00	13.4	2.518×10^{-6}	1.022×10^{-10}	1.479×10^{-6}		1.0390×10^{-6}		1189.2
	14.4	1.822×10^{-6}	6.690×10^{-11}	0.9682×10^{-6}		0.8538×10^{-6}		1190.2
1175.8	15.4	1.360×10^{-6}	4.560×10^{-11}	0.6599×10^{-6}		0.7001×10^{-6}		1191.2
	16.4	1.038×10^{-6}	3.220×10^{-11}	0.4660×10^{-6}		0.5720×10^{-6}		1192.2
	17.4	0.8100×10^{-6}	2.350×10^{-11}	0.3401×10^{-6}		0.4699×10^{-6}		1193.2

FIGURE 8

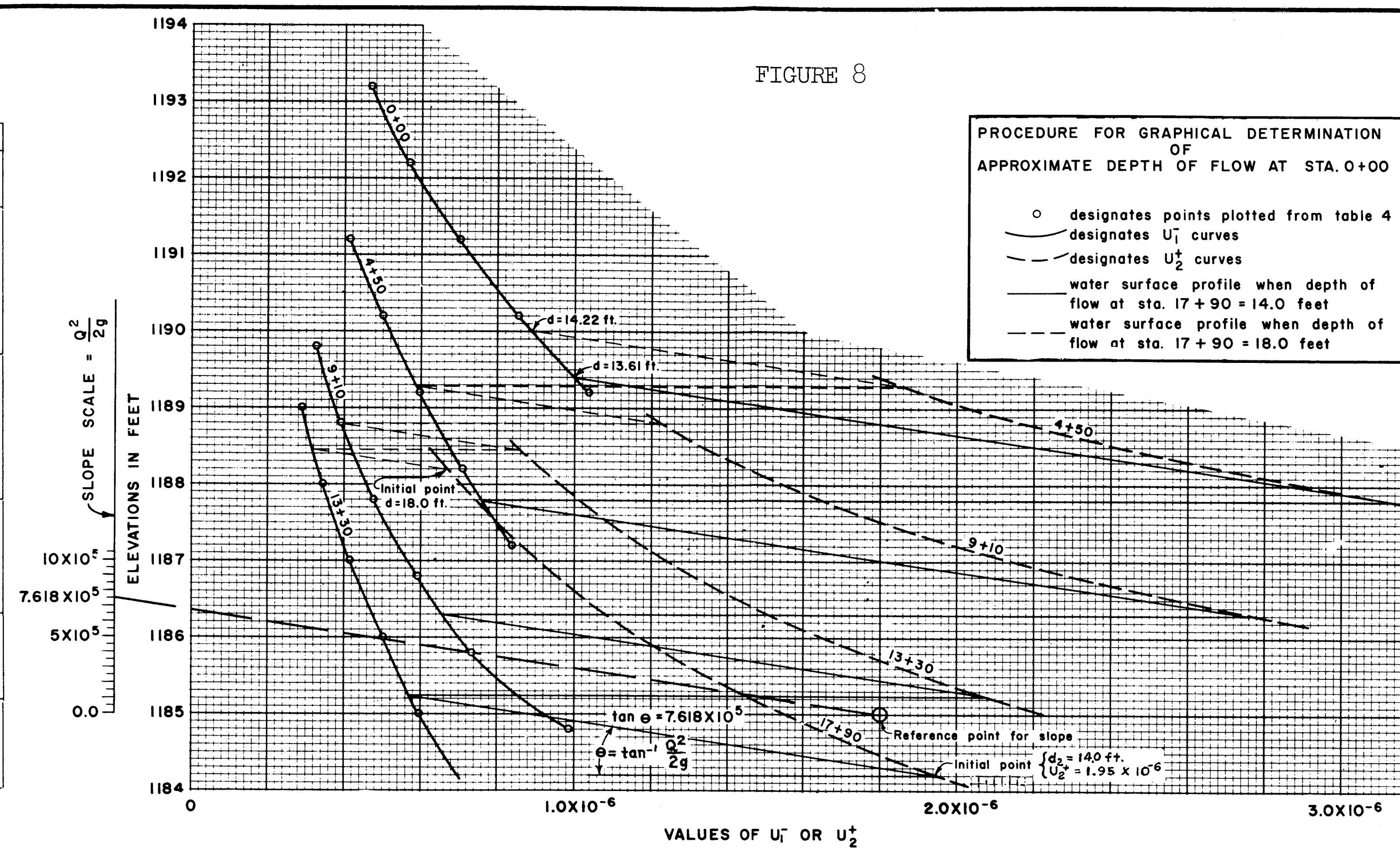
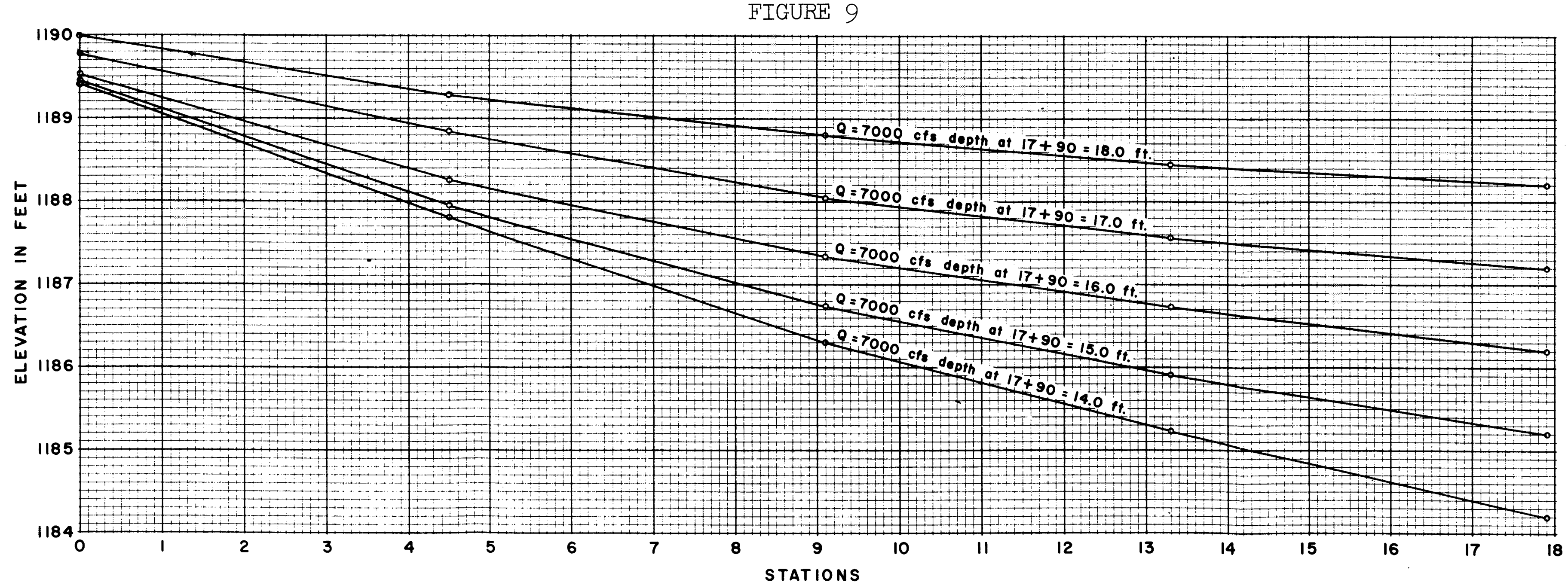


FIGURE 9



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DATE 4-22-54

HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL FOR VARIOUS DISCHARGES
Example 5

EXAMPLE 5

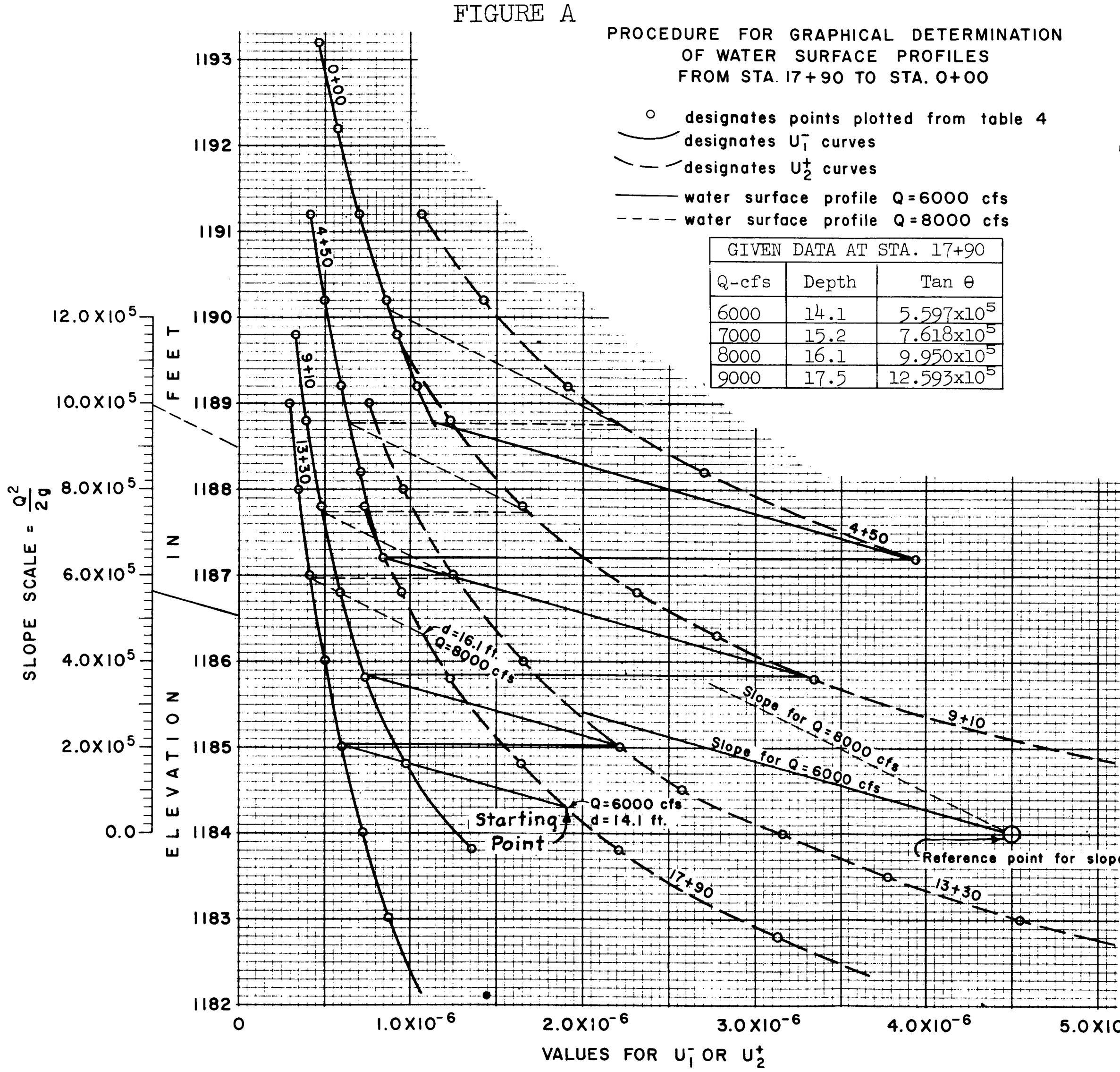
This example deals with the same natural channel as given in Ex. 4 merely to eliminate repetition of an additional set of similar data. Example 5 is distinctly different from Ex. 4 and the problems are in no way related. The approximate depth of flow was determined at Sta 0+00 in Ex. 4. In Ex. 5 the depths at Sta 17+90 have been determined and water-surface profiles are to be determined for various discharges.

Given: Channel cross sections and stationing of a natural channel along with roughness coefficient n as shown by Fig. 1 of Ex. 4. The dikes are sufficiently high to contain flows of 9000 cfs. The depths of flow at Sta 17+90 have been determined for various discharges.

Determine: The water-surface profiles for discharges 6000, 7000, 8000, and 9000 cfs and the discharge vs depth curve at Sta 0+00.

Discharge cfs	Given depth of flow at sta. 17+90 ft
6000	14.1
7000	15.2
8000	16.1
9000	17.5

FIGURE A



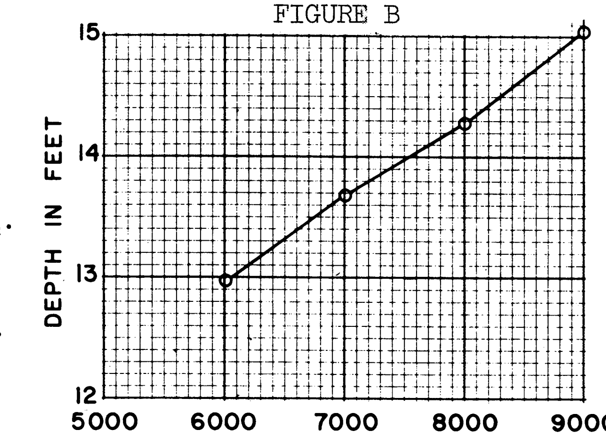
GIVEN DATA AT STA. 17+90		
Q-cfs	Depth	Tan θ
6000	14.1	5.597×10^5
7000	15.2	7.618×10^5
8000	16.1	9.950×10^5
9000	17.5	12.593×10^5

Solution: Equation A.18 is used for this solution. (See Ex. 4)

1. Prepare tabular form given by Table 1, Ex. 4.
2. Prepare Figs. 2 and 3 of Ex. 4.
3. Solve which sections are control sections for discharges of 6000, 7000, 8000, and 9000 cfs. This procedure is described by Ex. 4 and the results are given in Table A. No control sections exist for all discharges considered since all bottom slopes s_o are less than critical slope s_c.
4. Solve for water-surface profiles corresponding to the given discharges.
   a. Prepare tabular form given by Table 4, Ex. 4.
   b. It may be desirable to prepare Figs. 6 and 7 of Ex. 4.
   c. Prepare Fig. A similar to Fig. 8 of Ex. 4.
      (i) Plot U_1^- vs elevation curves.
      (ii) Plot U_2^+ vs elevation curves.
      (iii) Column 2 of Table A gives the slopes of straight lines connecting the curves U_1^- and U_2^+ for the various discharges. This procedure is explained in Ex. 4, item 4c, (iii)
      (iv) Make graphical solution for water surface profiles corresponding to the given discharges, and depths of flow at Sta 17+90. This procedure has been described in Ex. 4.

The graphical solution for the discharges Q = 6000 and 8000 cfs are shown. The discharge-depth curve at Sta 0+00 is given by Fig. B.

FIGURE B



RATING CURVE AT STA. 0+00

TABLE A

1	2	3	4	5	6	7	8	9
Q	$\frac{Q^2}{2g}$	Station	d_c	$\frac{Q_{n,d_c}}{s_c^{1/2}}$	$s_c^{1/2}$	s_c	s_o	Remarks
6000	5.597×10^5	0+00	11.92*	69,000	0.08696	0.007562	.00222	$s_c > s_o$
		4+50	10.90*	67,000	0.08955	0.008019	.00261	$s_c > s_o$
		9+10	10.65	72,000	0.08333	0.006944	.00476	$s_c > s_o$
		13+30	10.40	69,700	0.08608	0.007410	.00304	$s_c > s_o$
		17+90	10.52	68,800	0.08721	0.007606		
7000	7.618×10^5	0+00	12.55*	81,000	0.08642	0.007468	.00222	$s_c > s_o$
		4+50	11.50*	78,100	0.08963	0.008034	.00261	$s_c > s_o$
		9+10	11.14	81,500	0.08589	0.007377	.00476	$s_c > s_o$
		13+30	10.95	79,600	0.08794	0.007733	.00304	$s_c > s_o$
		17+90	11.05	78,500	0.08917	0.007951		
8000	9.950×10^5	0+00	13.10*	92,000	0.08696	0.007562	.00222	$s_c > s_o$
		4+50	12.05*	90,000	0.08889	0.007901	.00261	$s_c > s_o$
		9+10	11.60	91,000	0.08791	0.007728	.00476	$s_c > s_o$
		13+30	11.44	88,900	0.08999	0.008098	.00304	$s_c > s_o$
		17+90	11.55	88,200	0.09070	0.008226		
9000	12.59×10^5	0+00	13.60	103,000	0.08738	0.007635	.00222	$s_c > s_o$
		4+50	12.56	101,000	0.08911	0.007941	.00261	$s_c > s_o$
		9+10	12.02	101,000	0.08911	0.007941	.00476	$s_c > s_o$
		13+30	11.92	98,000	0.09184	0.008435	.00304	$s_c > s_o$
		17+90	12.05	98,750	0.09114	0.008306		

*These values were obtained by extrapolation from Figure 4.

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES- 83
SHEET 7 OF 10
DATE: 5-13-54

HYDRAULICS: NON-UNIFORM FLOW IN A NON-PRISMATIC CHANNEL WITH LEVEL AND ADVERSE BOTTOM SLOPES
Example 6.

EXAMPLE 6

Given: A trapezoidal earth spillway 190 ft long from the reservoir to the control section. Stationing is in a downstream direction and Sta 0+00 is located at the reservoir. The bottom of the spillway at Sta 0+00 is 95 ft wide and converges uniformly to a width of 75 ft at Sta 1+00. The bottom width of 75 ft is constant from Sta 1+00 to Sta 1+90. The bottom slope of the spillway between Sta 0+00 and Sta 1+00 is adverse and is -3% between Sta 0+00 and Sta 0+80 and -20% between Sta 0+80 and Sta 1+00. The bottom of the spillway is level between Sta 1+00 and Sta 1+90. Manning's coefficient of roughness n is estimated to be 0.035.

Determine: A. The water-surface profiles for discharges of 600, 800, 1000, 1200, 1500, and 1800 cfs in this spillway.
B. Curve showing discharge vs elevation of water surface in the reservoir.
C. Friction loss in spillway between Sta 0+00 and Sta 1+90 for Q = 1500 cfs.

Solution: Equation A.18 is used for this solution.

$$\frac{E_1 - E_2}{\left[\frac{1}{a_2^2} + \frac{s_o}{Q_{n,d_2}^2}(\ell_2 - \ell_1)g\right] - \left[\frac{1}{a_1^2} - \frac{s_o}{Q_{n,d_1}^2}(\ell_2 - \ell_1)g\right]} = \frac{Q^2}{2g}$$

Stations 0+00, 0+50, 0+80, 0+90, 1+00, 1+50, 1+70, 1+80, and 1+90 have been arbitrarily selected as those stations at which depths of flow are to be determined.

A.1. Prepare tabular form given by Table 1.
(a) Column 1 lists the selected stations at which depths of flow are to be determined and the elevations of the channel bottom at these stations.
(b) Column 2 is an arbitrary selection of flow depths for an estimated range of depth of flow.
(c) Column 3 lists the bottom widths of the channel at the selected stations in column 1.
(d) Column 4 lists the cross-sectional areas for the depths in column 2.
(e) Column 5 lists the wetted perimeters for the depths in column 2.
(f) Column 6 is read from ES-76.
(g) Column 7 is the square of the reciprocal of column 4. This can be read from the double scale of ES-77.
(h) Column 8 is read from ES-77.
(i) Column 9 lists the product of Column 8 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of reach downstream from the station under consideration. The station under consideration is section 1 as is indicated by the subscript d_1 of Q_{n,d_1} in this column heading.
(j) Column 10 lists the product of column 8 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of reach upstream from the station under consideration. The station under consideration is section 2 as is indicated by the subscript d_2 of Q_{n,d_2}. in this column heading.
(k) Column 11 lists values of U_1^- when the section under consideration is the upstream section or section 1. Column 11 lists column 7 minus column 9. (See Eq. A.20)
(l) Column 12 lists values of U_2^+ when the section under consideration is the downstream section of the reach or section 2. Column 12 lists column 7 plus column 10. (See Eq. A.19)
(m) Column 13 lists the elevations of the water surfaces corresponding to the depths of column 2 or the depths of column 2 added to the bottom elevations of column 1.

2. Prepare Figs. 1 and 2 by plotting column 2 vs columns 11 and 12 respectively on log-log paper.

3. Solve for the values of $Q^2/2g$ as shown by Table 2 for the discharges under consideration.

4. Solve for water-surface profiles corresponding to the given discharges.
(a) Prepare Figs. 3a and 3b.
(i) Plot U_1^- vs elevation curves for various values of d shown in Fig. 1.
(ii) Plot U_2^+ vs elevation curves for various values of d shown in Fig. 2.
(iii) Column 2 of Table 2 gives the slopes of straight lines connecting the U_1^- and U_2^+ curves for the various discharges. This procedure is explained in Ex. 4, item 4c, (iii).
(iv) Make graphical solution for water-surface profiles corresponding to the given discharges at Sta 0+00. This procedure has been described in Ex. 4, item 4c, (iv).

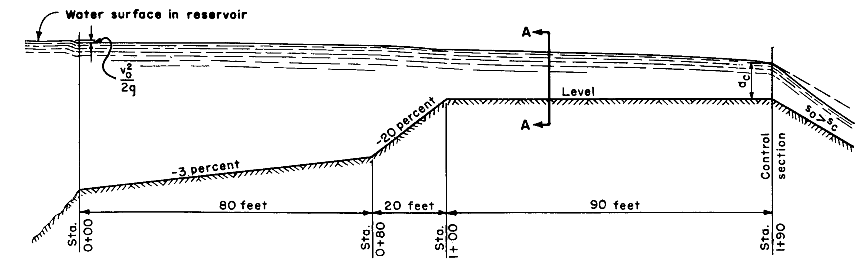
The graphical solutions for discharges 800 and 1500 have been shown on Figs. 3a and 3b respectively. Two figures, 3a and 3b, are drawn instead of a single figure because of the desirability of a scale adjustment. The water-surface profiles for the various discharges are given by Fig. 4. The rapid drop in elevation in water-surface profiles between Sta 0+80 and Sta 1+00 is the result of a change from potential head to kinetic head.

Observe that the depth of flow at Sta 1+00 for a discharge Q = 1500 cfs is 3.37 ft. Example 2 for this same channel and discharge shows a depth of flow of 3.347 ft at a section 90 ft upstream from the break in grade. The principal cause for the difference in the answers for the depth at the section 90 ft upstream from the control section by the two methods is the number of sections used upstream from the control section in computing this depth of flow. The result d = 3.347 ft, obtained in Ex. 2, is by far the more accurate because eleven intermediate sections were used in computing this depth, while in Ex. 6 only three intermediate sections were used in computing the depth d = 3.37 ft.

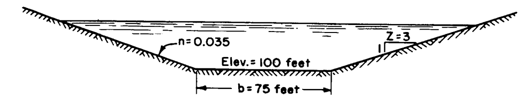
B.1. The depths of flow at Sta 0+00 for various discharges are given by Table 2. These values were read from Figs. 3a and 3b. The water-surface elevation in the reservoir is greater than the water-surface elevation at Sta 0+00 by the quantity $v_o^2/2g$ where v_o is the velocity in ft/sec at Sta 0+00. The curve showing discharge vs elevation of water surface in the reservoir is shown by Fig. 5.

C.1. The friction loss between Sta 0+00 and Sta 1+90 is the difference in elevation of the energy gradient. The elevation of the energy gradient at Sta 0+00 is 103.831 ft when Q = 1500 cfs. See Table 2. The elevation of the energy gradient at Sta 1+90 is 103.284 ft. See Ex. 2. The friction loss is 103.831 − 103.284 = 0.547 ft.

The trapezoidal shape of this spillway does not exist for the full depth of flow in the reach near the entrance of the spillway. By neglecting the effect of this condition, as is done in the example, the water-surface elevation in the reservoir required to produce a given discharge through the spillway is slightly greater than the elevation required had this effect been evaluated.



PROFILE ALONG CENTER LINE



SECTION A—A

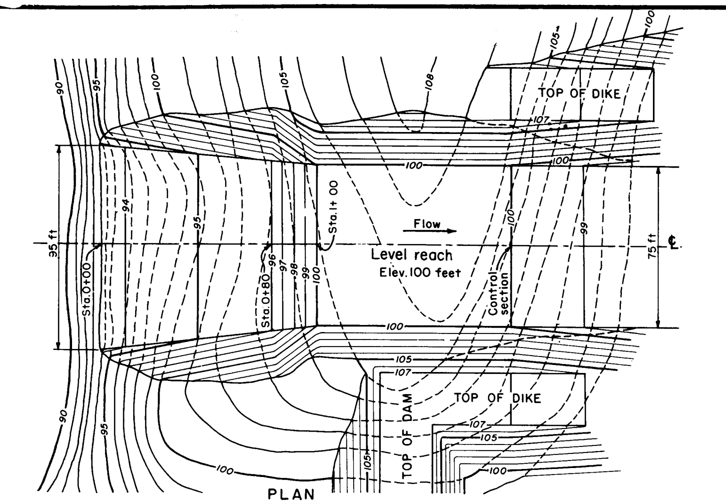


TABLE 1

1	2	3	4	5	6	7	8	9	10	11	12	13
Station and Bottom Elevation	d ft	b ft	a ft²	P ft	F	$\frac{1}{a^2}$	$\frac{s_o}{Q_{n,d}^2}$	$\frac{s_o(\ell_2-\ell_1)g}{Q_{n,d_1}^2}$	$\frac{s_o(\ell_2-\ell_1)g}{Q_{n,d_2}^2}$	U_1^-	U_2^+	E
1+90	1.25	75.0	98.44	82.91	164.1	103.500×10^{-6}	4.570×10^{-8}		14.6971×10^{-6}		118.197×10^{-6}	101.25
	2.25	75.0	183.94	89.23	440.5	29.600×10^{-6}	6.330×10^{-9}		2.0358×10^{-6}		31.636×10^{-6}	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760×10^{-6}	2.345×10^{-9}		0.7542×10^{-6}		16.514×10^{-6}	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305×10^{-6}	5.820×10^{-10}		0.1872×10^{-6}		6.492×10^{-6}	104.50
1+80	1.25	75.0	98.44	82.91	164.1	103.500×10^{-6}	4.570×10^{-8}	14.6971×10^{-6}	14.6971×10^{-6}	88.803×10^{-6}	118.197×10^{-6}	101.25
	2.25	75.0	183.94	89.23	440.5	29.600×10^{-6}	6.330×10^{-9}	2.0357×10^{-6}	2.0358×10^{-6}	27.564×10^{-6}	31.636×10^{-6}	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760×10^{-6}	2.345×10^{-9}	0.7542×10^{-6}	0.7542×10^{-6}	15.006×10^{-6}	16.514×10^{-6}	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305×10^{-6}	5.820×10^{-10}	0.1872×10^{-6}	0.1872×10^{-6}	6.118×10^{-6}	6.492×10^{-6}	104.50
1+70	1.25	75.0	98.44	82.91	164.1	103.500×10^{-6}	4.570×10^{-8}	14.6971×10^{-6}	29.3942×10^{-6}	88.803×10^{-6}	132.894×10^{-6}	101.25
	2.25	75.0	183.94	89.23	440.5	29.600×10^{-6}	6.330×10^{-9}	2.0357×10^{-6}	4.0715×10^{-6}	27.564×10^{-6}	33.672×10^{-6}	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760×10^{-6}	2.345×10^{-9}	0.7542×10^{-6}	1.5083×10^{-6}	15.006×10^{-6}	17.268×10^{-6}	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305×10^{-6}	5.820×10^{-10}	0.1872×10^{-6}	0.3743×10^{-6}	6.118×10^{-6}	6.679×10^{-6}	104.50
1+50	1.25	75.0	98.44	82.91	164.1	103.500×10^{-6}	4.570×10^{-8}	29.3942×10^{-6}	73.4856×10^{-6}	74.106×10^{-6}	176.986×10^{-6}	101.25
	2.25	75.0	183.94	89.23	440.5	29.600×10^{-6}	6.330×10^{-9}	4.0715×10^{-6}	10.1786×10^{-6}	25.528×10^{-6}	39.779×10^{-6}	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760×10^{-6}	2.345×10^{-9}	1.5083×10^{-6}	3.7708×10^{-6}	14.252×10^{-6}	19.531×10^{-6}	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305×10^{-6}	5.820×10^{-10}	0.3743×10^{-6}	0.9359×10^{-6}	5.931×10^{-6}	7.241×10^{-6}	104.50
1+00	1.25	75.0	98.44	82.91	164.1	103.500×10^{-6}	4.570×10^{-8}	73.4856×10^{-6}	14.6971×10^{-6}	30.014×10^{-6}	118.197×10^{-6}	101.25
	2.25	75.0	183.94	89.23	440.5	29.600×10^{-6}	6.330×10^{-9}	10.1786×10^{-6}	2.0358×10^{-6}	19.421×10^{-6}	31.636×10^{-6}	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760×10^{-6}	2.345×10^{-9}	3.7708×10^{-6}	0.7542×10^{-6}	11.989×10^{-6}	16.514×10^{-6}	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305×10^{-6}	5.820×10^{-10}	0.9359×10^{-6}	0.1872×10^{-6}	5.369×10^{-6}	6.492×10^{-6}	104.50
0+90	3.00	77.0	258.00	95.97	740.0	15.023×10^{-6}	2.235×10^{-9}	0.7188×10^{-6}	0.7188×10^{-6}	14.304×10^{-6}	15.742×10^{-6}	101.00
	4.50	77.0	407.25	105.46	1480.0	6.029×10^{-6}	5.620×10^{-10}	0.1807×10^{-6}	0.1807×10^{-6}	5.848×10^{-6}	6.210×10^{-6}	102.50
98.00	6.50	77.0	627.25	118.11	2825.0	2.542×10^{-6}	1.536×10^{-10}	0.04939×10^{-6}	0.04939×10^{-6}	2.493×10^{-6}	2.591×10^{-6}	104.50
0+80	4.00	79.0	364.00	104.29	1245.0	7.547×10^{-6}	7.895×10^{-10}	0.2539×10^{-6}	0.7616×10^{-6}	7.293×10^{-6}	8.309×10^{-6}	100.00
	6.50	79.0	640.25	120.11	2908.0	2.440×10^{-6}	1.447×10^{-10}	0.04653×10^{-6}	0.1396×10^{-6}	2.393×10^{-6}	2.580×10^{-6}	102.50
96.00	8.50	79.0	888.25	132.76	4675.0	1.267×10^{-6}	5.590×10^{-11}	0.01797×10^{-6}	0.05393×10^{-6}	1.249×10^{-6}	1.321×10^{-6}	104.50
0+50	6.00	85.0	618.00	122.95	2690.0	2.618×10^{-6}	1.693×10^{-10}	0.1633×10^{-6}	0.2722×10^{-6}	2.455×10^{-6}	2.890×10^{-6}	101.10
	8.50	85.0	939.25	138.76	4985.0	1.134×10^{-6}	4.925×10^{-11}	0.04752×10^{-6}	0.07919×10^{-6}	1.087×10^{-6}	1.213×10^{-6}	103.60
95.10	11.00	85.0	1298.00	154.57	7980.0	0.5935×10^{-6}	1.940×10^{-11}	0.01872×10^{-6}	0.03120×10^{-6}	0.5748×10^{-6}	0.6247×10^{-6}	106.10
0+00	6.00	95.0	678.00	132.95	2980.0	2.175×10^{-6}	1.380×10^{-10}	0.2219×10^{-6}		1.953×10^{-6}		99.60
	8.50	95.0	1024.25	148.76	5520.0	0.9532×10^{-6}	4.005×10^{-11}	0.06440×10^{-6}		0.8888×10^{-6}		102.10
93.60	11.00	95.0	1408.00	164.57	8730.0	0.5044×10^{-6}	1.609×10^{-11}	0.02587×10^{-6}		0.4785×10^{-6}		104.60

TABLE 2

Q cfs	$\frac{Q^2}{2g}$ = tan θ	Water-Surface Elevation at Sta 0+00	$\frac{v_o^2}{2g}$	Water-Surface Elevation in the Reservoir
600	0.0560×10^5	102.330	0.005000	102.335
800	0.0995×10^5	102.710	0.008012	102.718
1000	0.156×10^5	103.055	0.01143	103.066
1200	0.224×10^5	103.403	0.01505	103.418
1500	0.350×10^5	103.810	0.02126	103.831
1800	0.504×10^5	104.230	0.02769	104.258

FIGURE 5

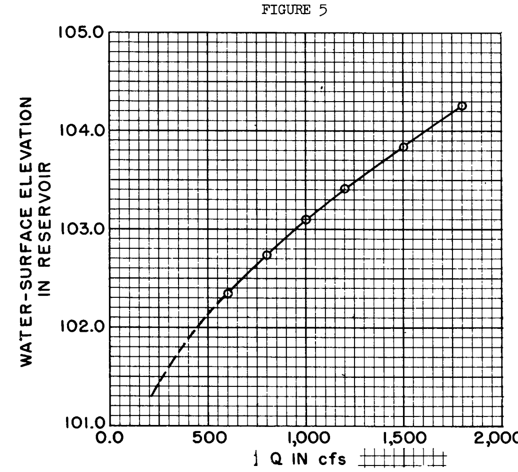
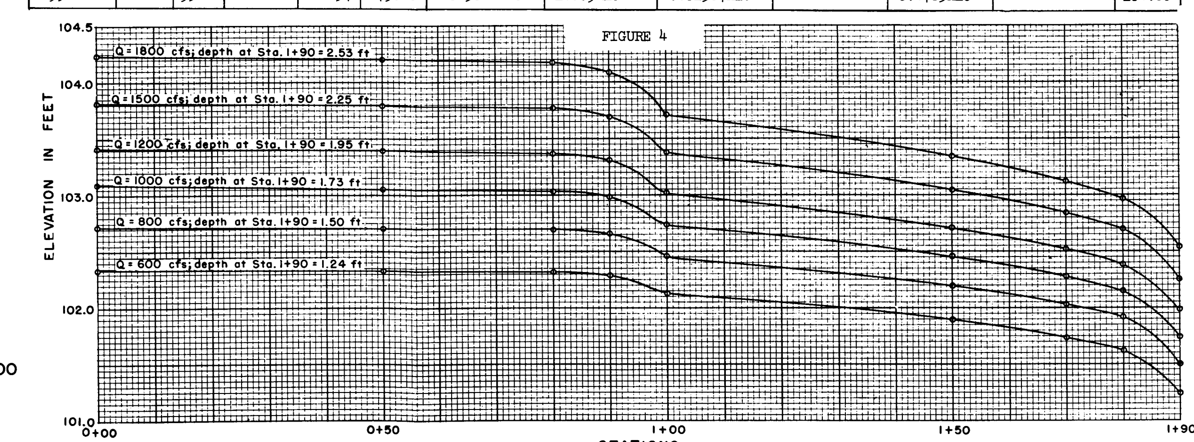


FIGURE 4



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION- DESIGN SECTION

STANDARD DWG. NO.
ES-83
SHEET 8 OF 10
DATE 8-6-54

HYDRAULICS: NON-UNIFORM FLOW IN A NON-PRISMATIC CHANNEL WITH LEVEL AND ADVERSE BOTTOM SLOPES
Example 6.

FIGURE 1

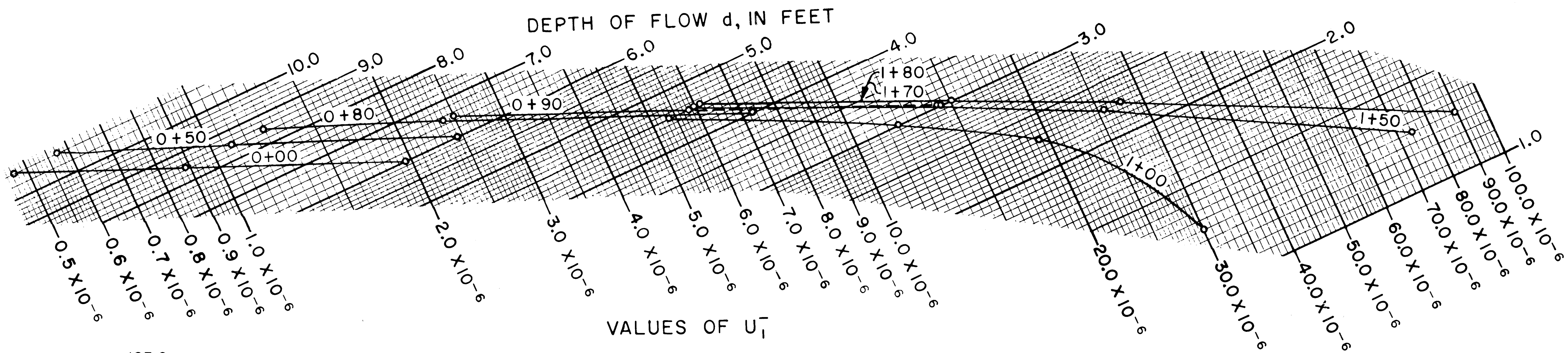
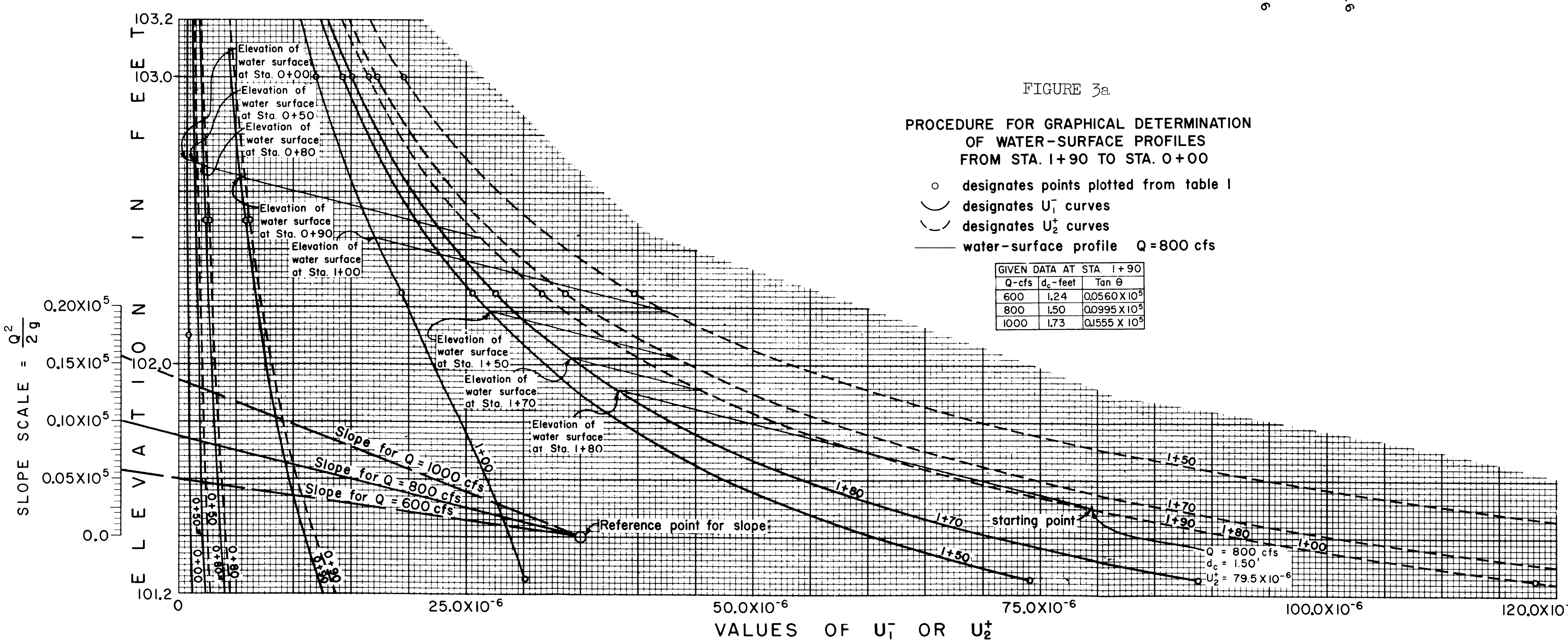


FIGURE 3a

PROCEDURE FOR GRAPHICAL DETERMINATION OF WATER-SURFACE PROFILES FROM STA. 1+90 TO STA. 0+00

- ○ designates points plotted from table I
- designates U_1^- curves
- designates U_2^+ curves
- water-surface profile Q = 800 cfs

GIVEN DATA AT STA 1+90		
Q-cfs	d_c-feet	Tan θ
600	1.24	0.0560×10^5
800	1.50	0.0995×10^5
1000	1.73	0.1555×10^5



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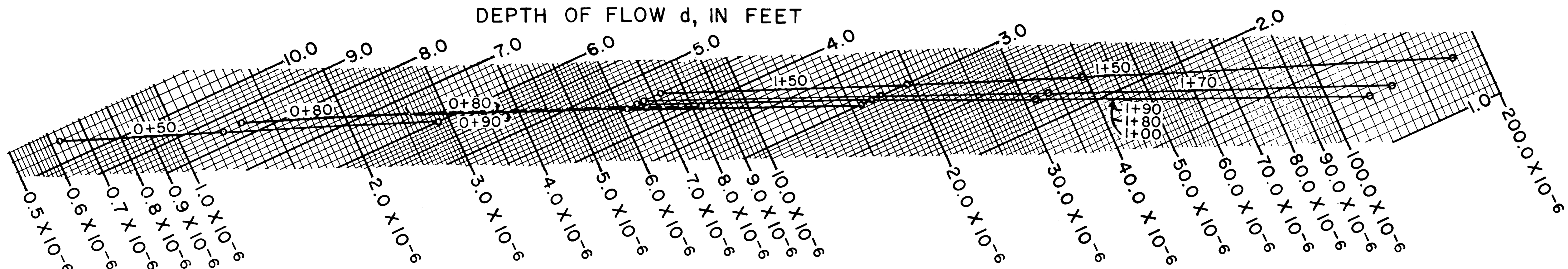
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO.
ES-83
SHEET 9 OF 10
DATE 8-6-54

HYDRAULICS: NON-UNIFORM FLOW IN A NON-PRISMATIC CHANNEL WITH LEVEL AND ADVERSE BOTTOM SLOPES —— Example 6.

FIGURE 2



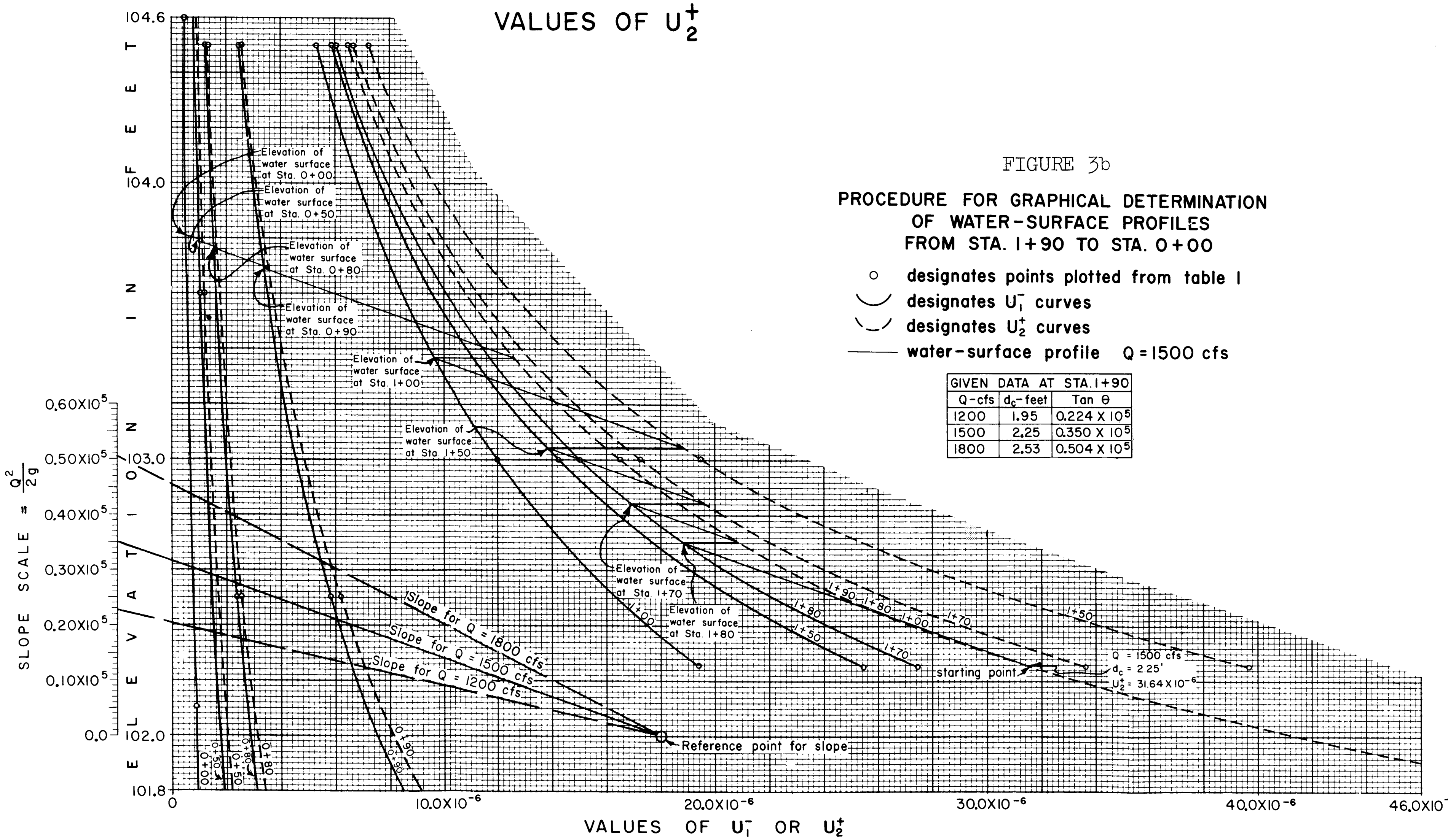


FIGURE 3b

PROCEDURE FOR GRAPHICAL DETERMINATION OF WATER-SURFACE PROFILES FROM STA. 1+90 TO STA. 0+00

- ○ designates points plotted from table 1
- designates U_1^- curves
- designates U_2^+ curves
- —— water-surface profile Q = 1500 cfs

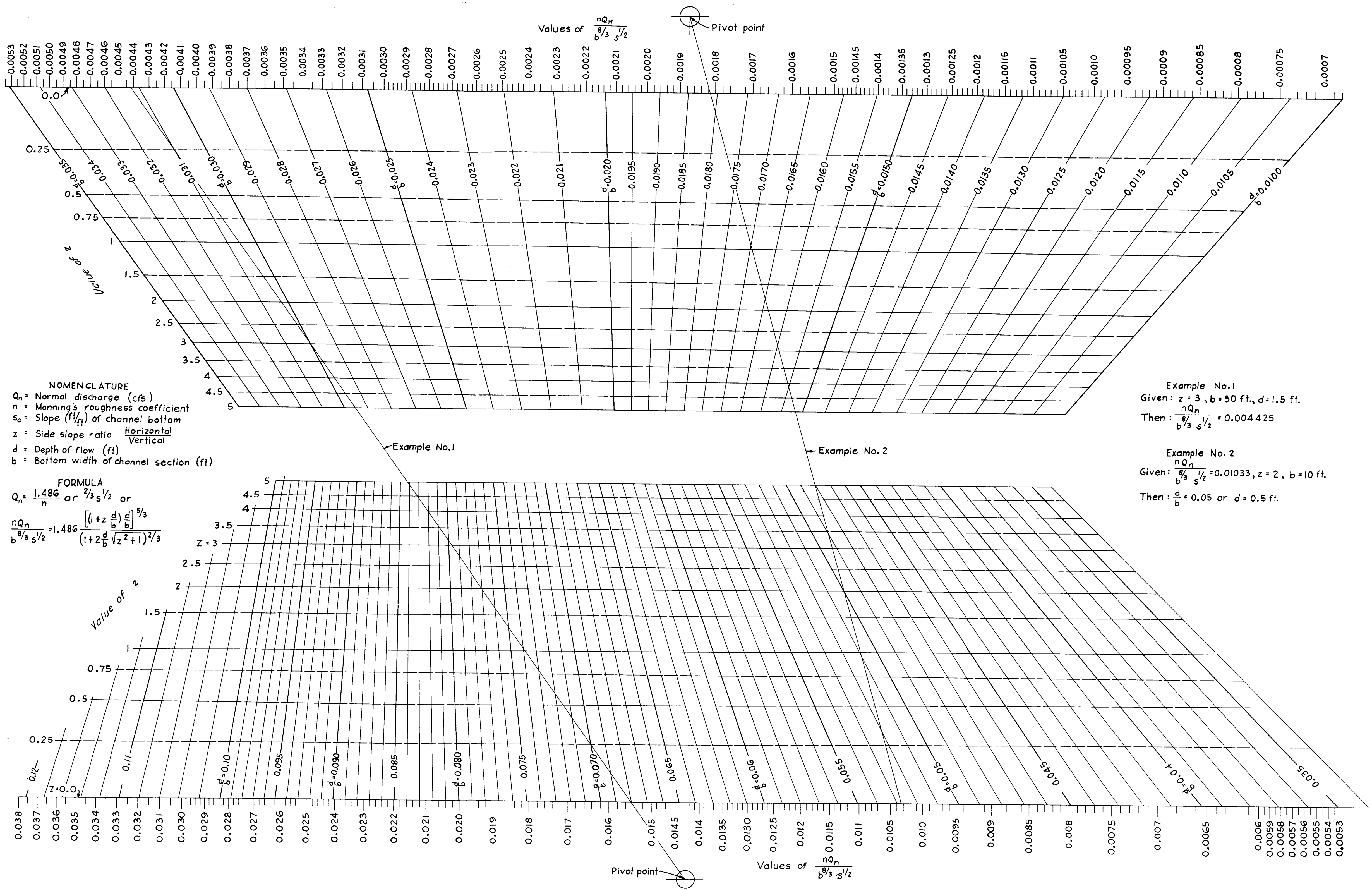
GIVEN DATA AT STA. 1+90		
Q-cfs	d_c-feet	Tan θ
1200	1.95	0.224×10^5
1500	2.25	0.350×10^5
1800	2.53	0.504×10^5

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO. ES-83
SHEET 10 OF 10
DATE 8-9-54

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
S_o = Slope (ft/ft) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2}$ or

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1+z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1+2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 1

Given: z = 3, b = 50 ft., d = 1.5 ft.

Then: $\frac{nQ_n}{b^{8/3} s^{1/2}}$ = 0.004425

Example No. 2

Given: $\frac{nQ_n}{b^{8/3} s^{1/2}}$ = 0.01033, z = 2, b = 10 ft.

Then: $\frac{d}{b}$ = 0.05 or d = 0.5 ft.

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

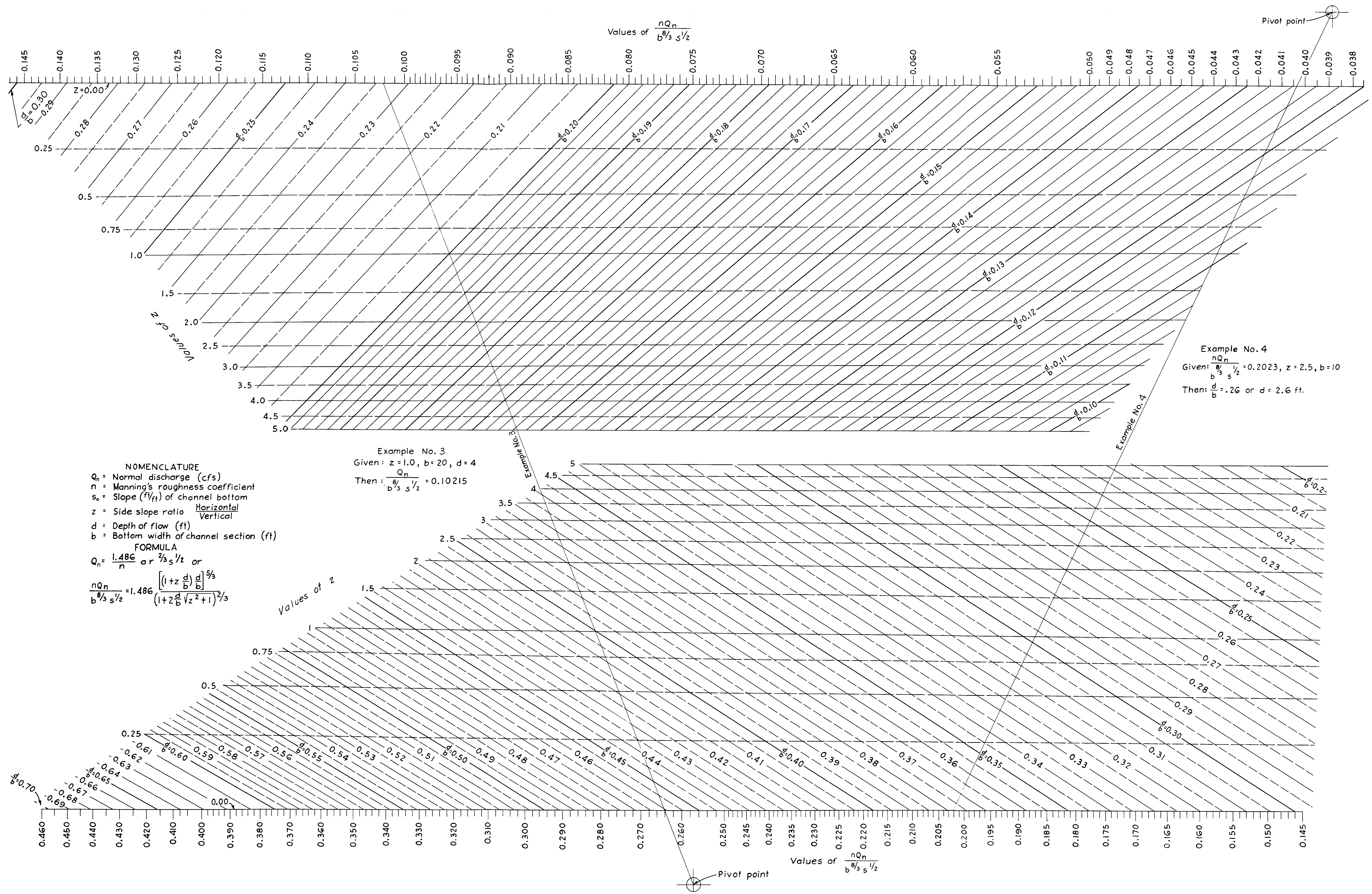
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55

SHEET 1 OF 4

DATE 4-30-51

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
s_o = Slope (ft/ft) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2}$ or

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1+z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1+2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 3
Given: z = 1.0, b = 20, d = 4
Then: $\frac{Q_n}{b^{8/3} s^{1/2}}$ = 0.10215

Example No. 4
Given: $\frac{nQ_n}{b^{8/3} s^{1/2}}$ = 0.2023, z = 2.5, b = 10
Then: $\frac{d}{b}$ = .26 or d = 2.6 ft.

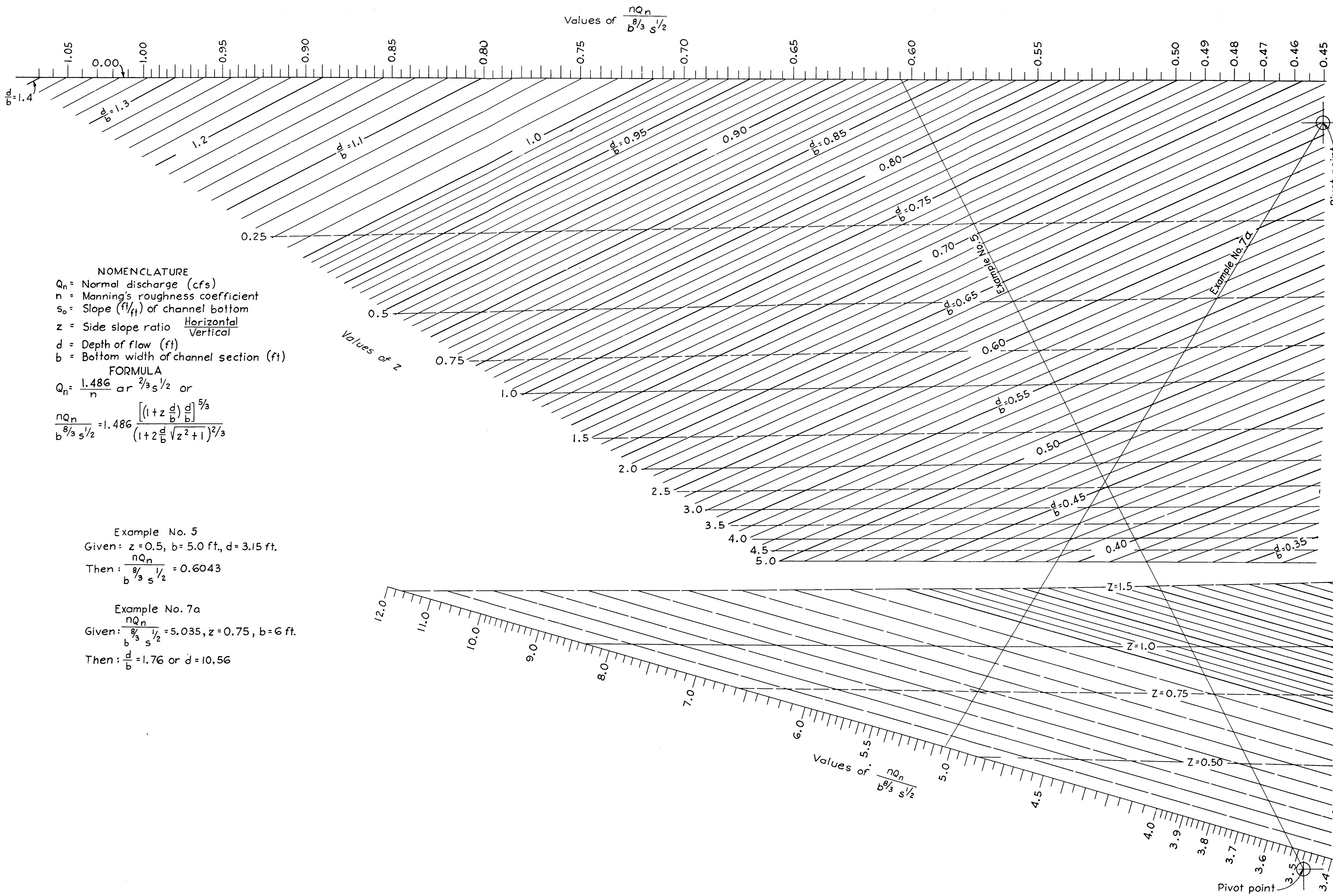
REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55
SHEET 2 OF 4
DATE 4-30-51

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)

n = Manning's roughness coefficient

S_o = Slope (ft/ft) of channel bottom

z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$

d = Depth of flow (ft)

b = Bottom width of channel section (ft)

FORMULA

$$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2} \text{ or}$$

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1 + z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1 + 2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 5

Given: $z = 0.5$, $b = 5.0$ ft., $d = 3.15$ ft.

Then: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 0.6043$

Example No. 7a

Given: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 5.035$, $z = 0.75$, $b = 6$ ft.

Then: $\frac{d}{b} = 1.76$ or $d = 10.56$

REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55

SHEET 3 OF 4

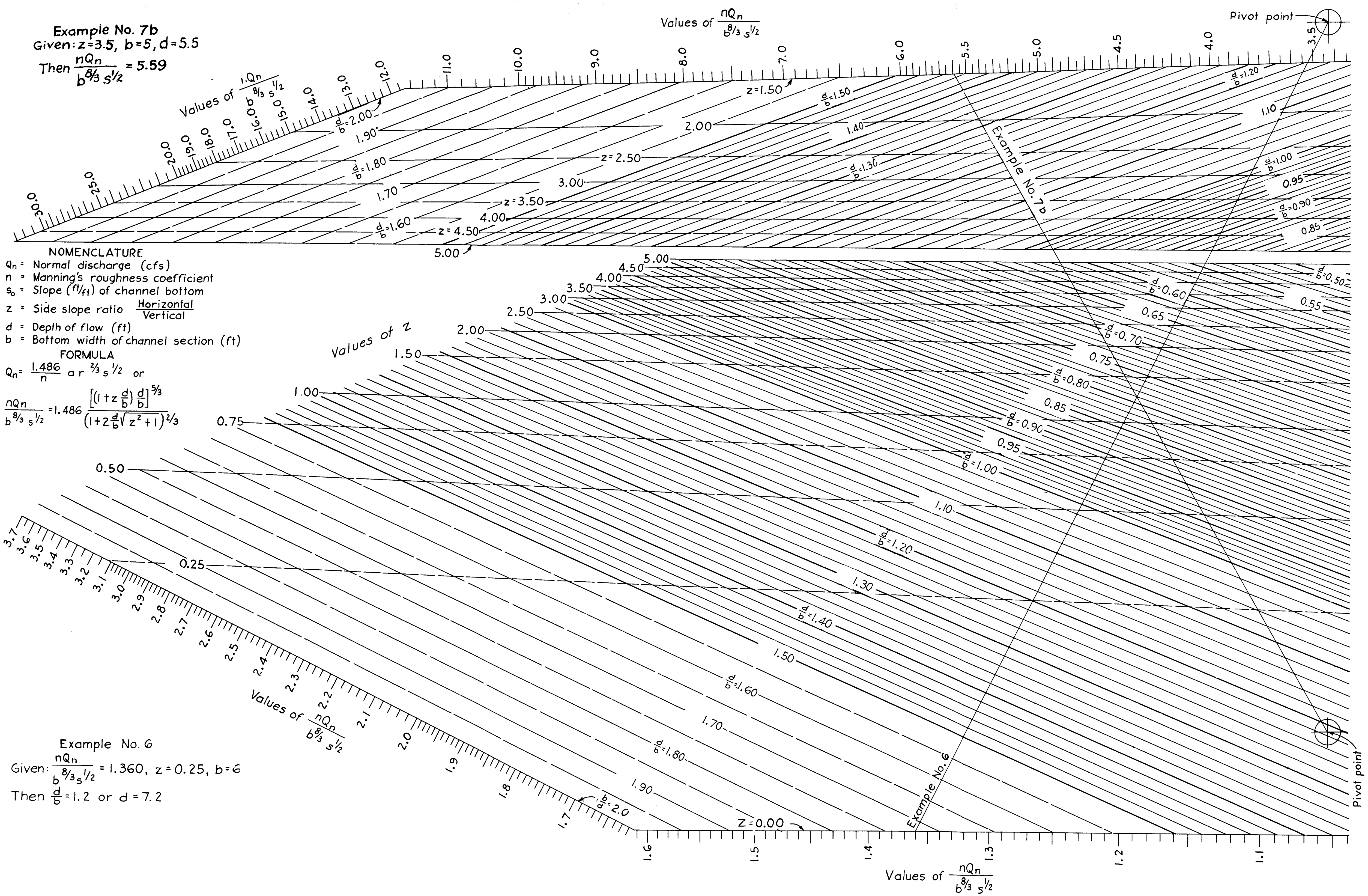
DATE 4-30-51

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS

Example No. 7b

Given: $z = 3.5$, $b = 5$, $d = 5.5$

Then $\frac{nQ_n}{b^{8/3} s^{1/2}} = 5.59$



NOMENCLATURE

Q_n = Normal discharge (cfs)

n = Manning's roughness coefficient

s_o = Slope (ft/ft) of channel bottom

z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$

d = Depth of flow (ft)

b = Bottom width of channel section (ft)

FORMULA

$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2}$ or

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1 + z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1 + 2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 6

Given: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.360$, $z = 0.25$, $b = 6$

Then $\frac{d}{b} = 1.2$ or $d = 7.2$

REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE

ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55

SHEET 4 OF 4

DATE 4-30-51

HYDRAULICS: CHART FOR DETERMINING WATER SURFACE PROFILES FOR POSITIVE VALUE OF s_o ; STEP METHOD

NOMENCLATURE

Q = Steady discharge (cfs) for which the surface profile is to be determined.
$Q_{n,d}$ = Normal discharge (cfs) at depth d
$Q_{c,d}$ = Critical discharge (cfs) at depth d
S_o = Bottom slope (ft./ft.) of channel
ℓ = Length (ft.) of channel
d = Depth (ft.) of water
R = Chart value

Subscripts 1 and 2 denote specific channel sections. Section 2 is downstream from Section 1 when $\ell_2 - \ell_1 > 0$

FORMULAS

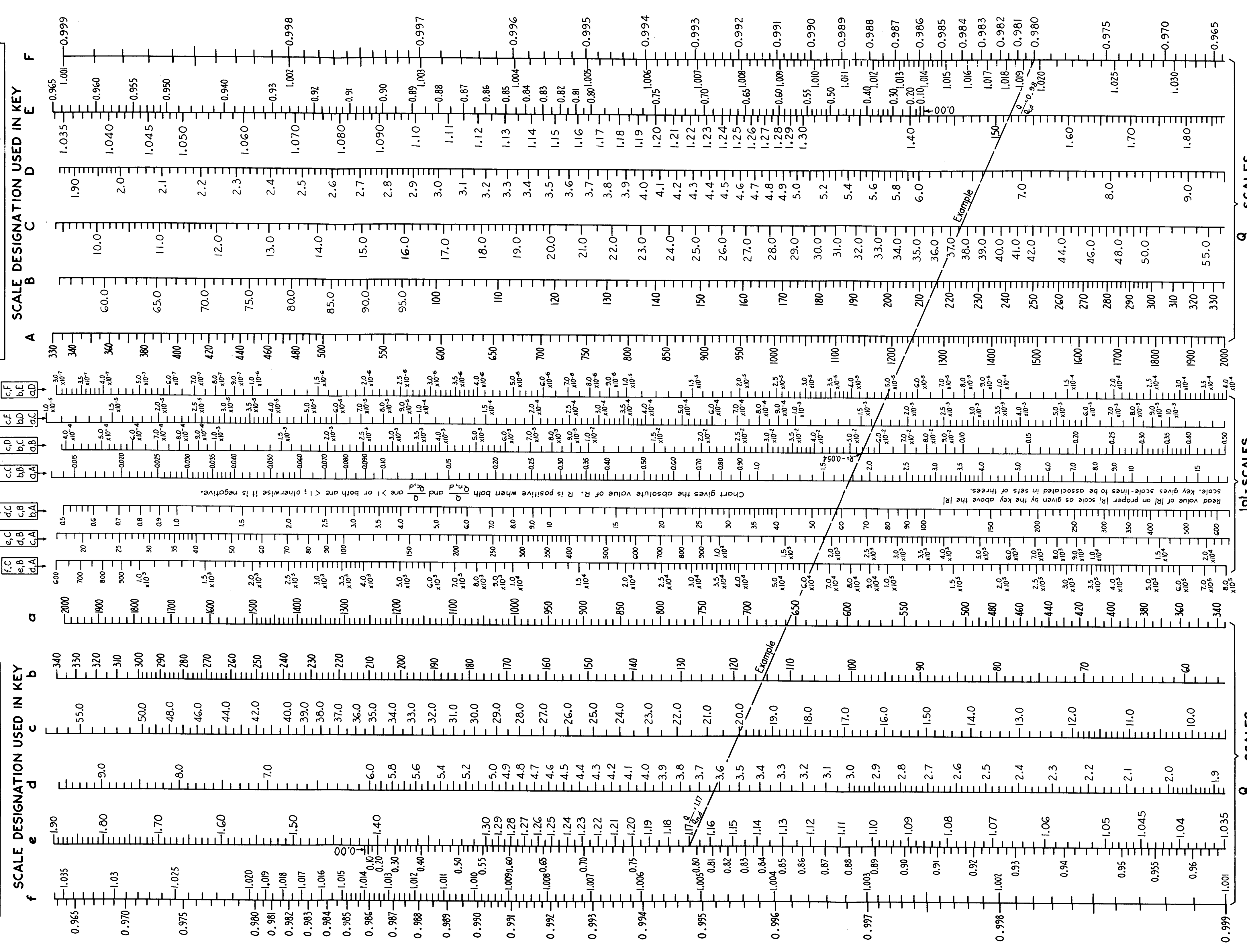
Chart solves $\dfrac{d\ell}{dd} = \dfrac{1}{S_o}\left(\dfrac{Q_{n,d}}{Q_{c,d}}\right)^2\left(\dfrac{Q_{c,d}^2 - Q^2}{Q_{n,d}^2 - Q^2}\right) = \dfrac{2R}{S_o}$

a. For $\dfrac{Q}{Q_{n,d}} = 1.0$ $\begin{cases} R = +\infty \text{ when } \dfrac{Q}{Q_{c,d}} > 1 \\ R = -\infty \text{ when } \dfrac{Q}{Q_{c,d}} < 1 \end{cases}$

b. For $\dfrac{Q}{Q_{c,d}} = 1.0$ $R = 0$ for all $\dfrac{Q}{Q_{n,d}} \neq 1.0$

c. For $\dfrac{Q}{Q_{c,d}} = \dfrac{Q}{Q_{n,d}} = 1.0$ R = [illegible]

For sufficiently small values of $(d_2 - d_1)$;
$\ell_2 - \ell_1 = \dfrac{1}{S_o}(d_2 - d_1)(R_1 + R_2)$ [Approx.]



REFERENCE

This nomogram was developed by Paul D. Doubt of the Engineering Standards Unit.

Revised 6-12-52

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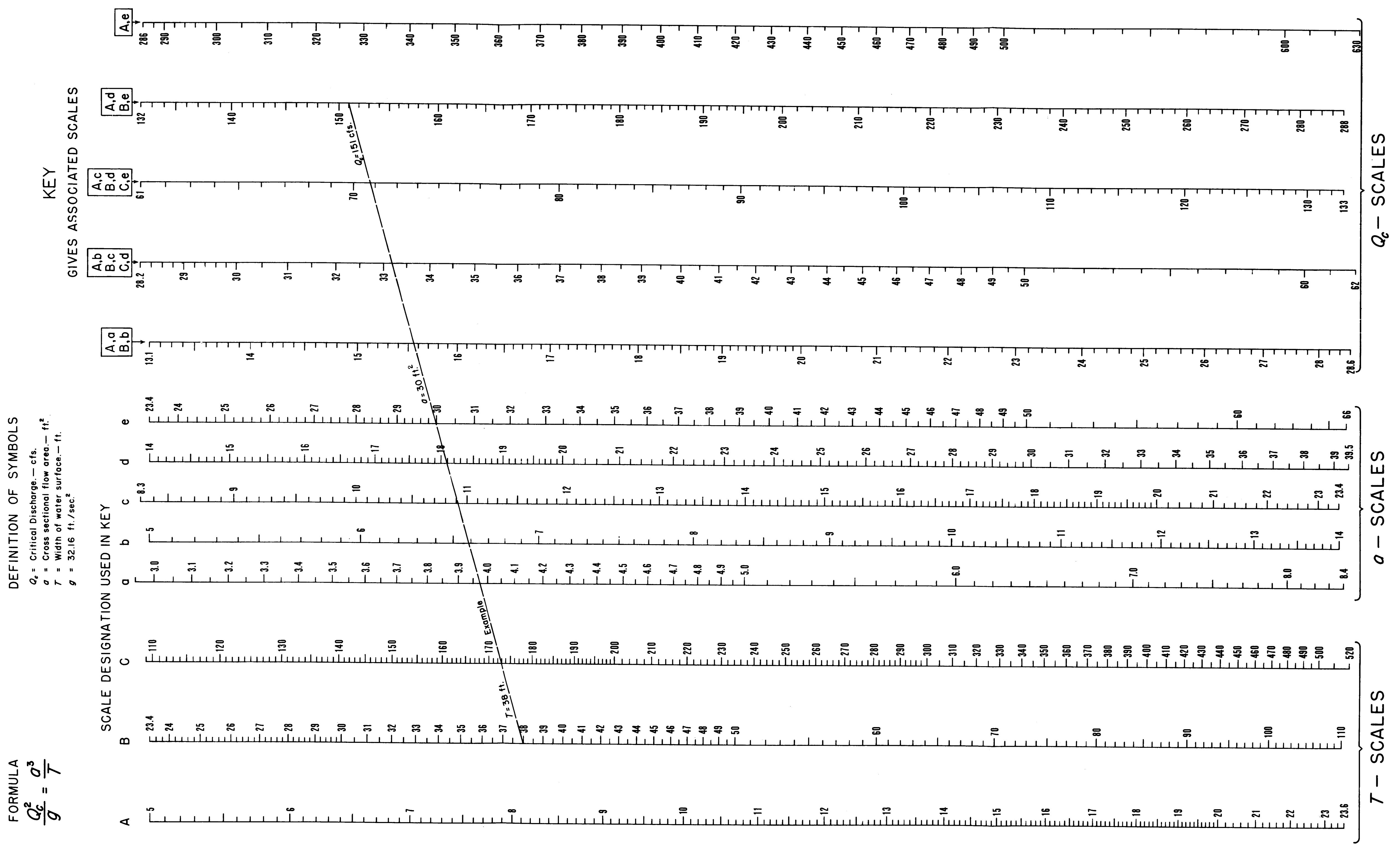
SOIL CONSERVATION SERVICE
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-53
SHEET 1 OF 1
DATE 1-5-51

HYDRAULICS: SOLUTION OF GENERAL EQUATION FOR CRITICAL FLOW

T = 5 to 520
a = 3 to 66
Q_c = 13 to 630

Note that area values are repeated to insure solution on chart within stated range.



REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

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SOIL CONSERVATION SERVICE
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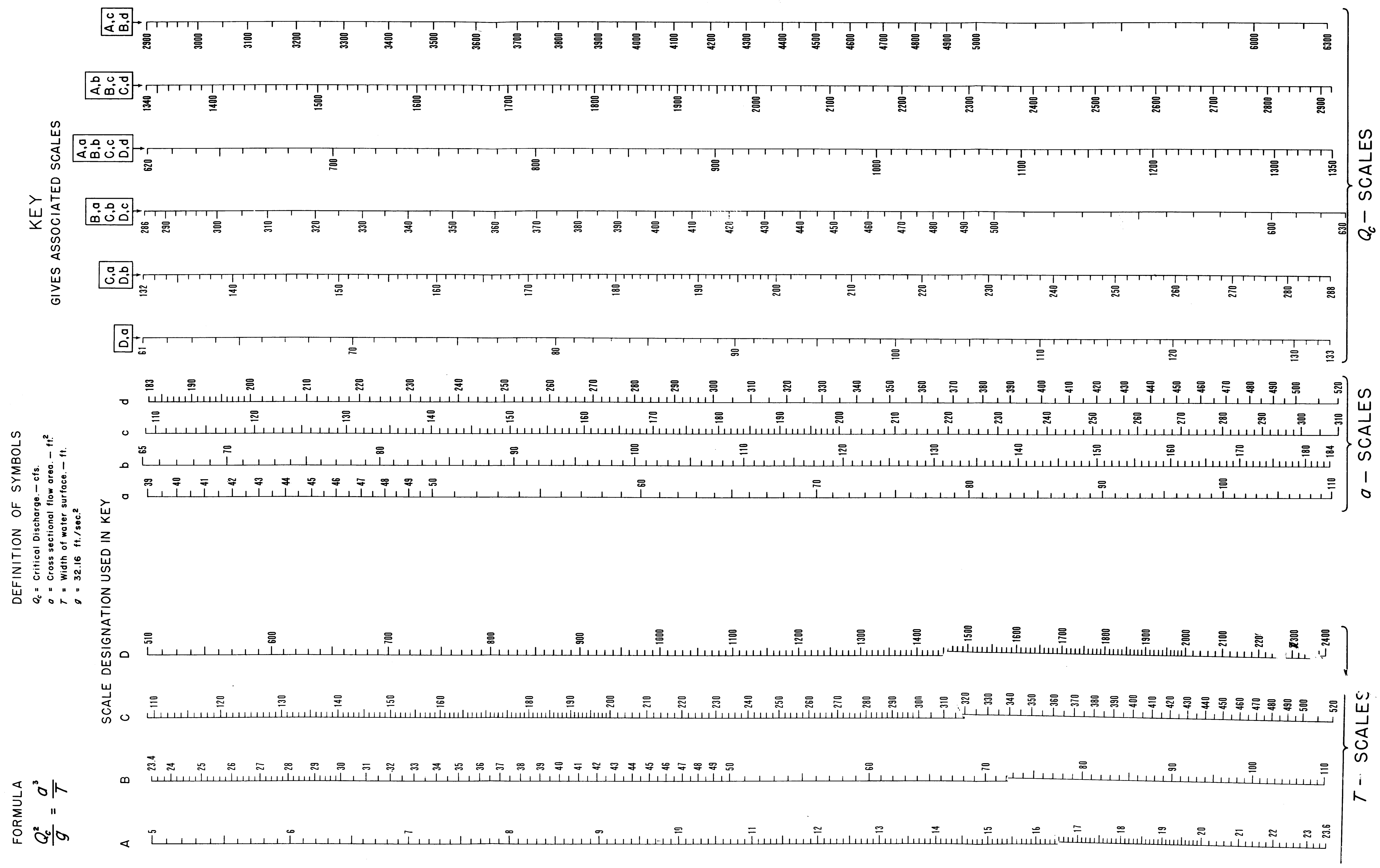
STANDARD DWG. NO. ES-75
SHEET 1 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF GENERAL EQUATION FOR CRITICAL FLOW

T = 5 to 2400
a = 39 to 520
Q_c = 61 to 6300

Note that area values are repeated to insure solution on chart within stated ranges.



REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

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STANDARD DWG. NO. ES-75
SHEET 2 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF GENERAL EQUATION FOR CRITICAL FLOW

T = 23.4 to 2400
a = 305 to 11,200
Q_c = 1340 to 139,000

Note that area values are repeated to insure solution on chart within stated range.

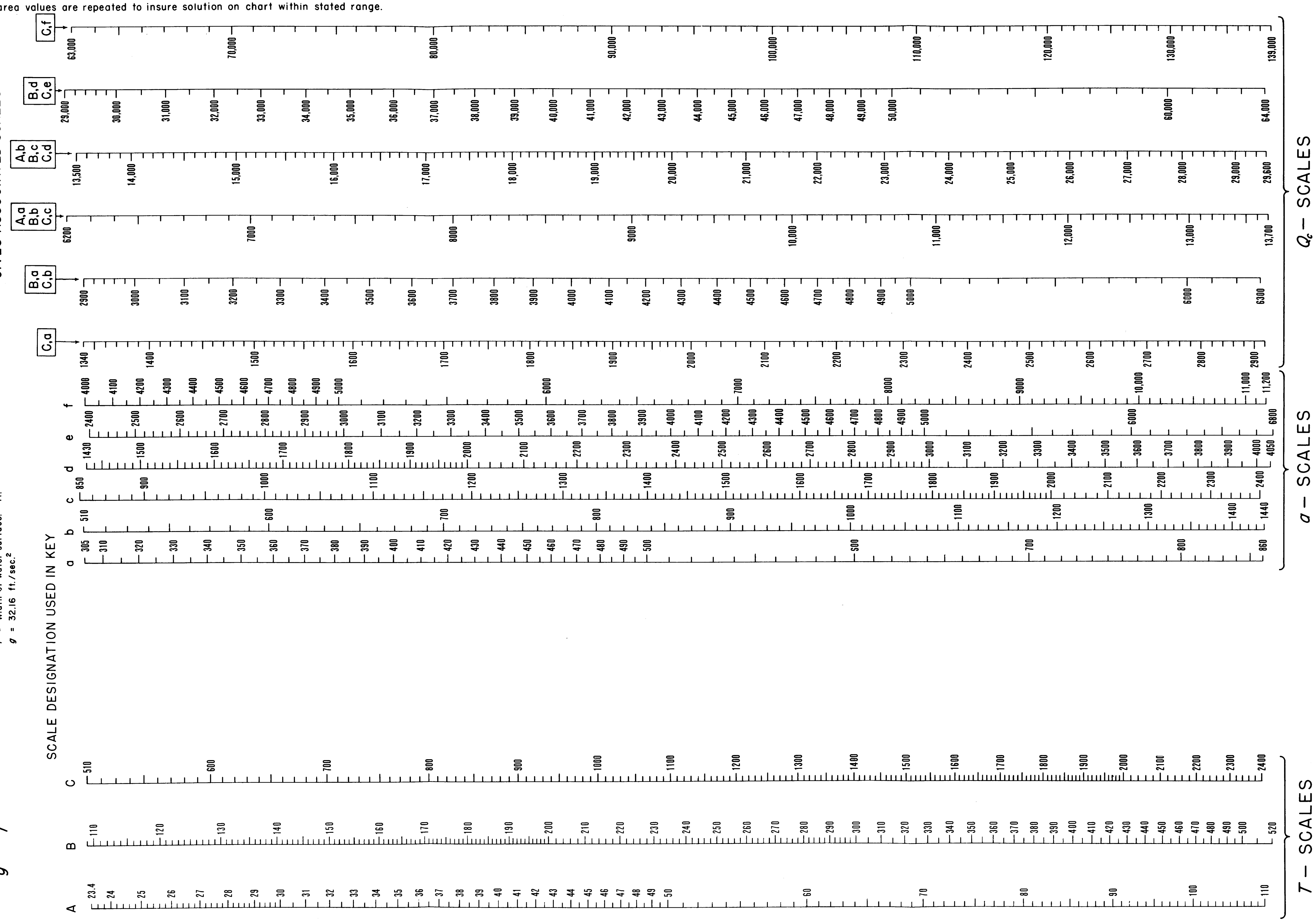
KEY
GIVES ASSOCIATED SCALES

DEFINITION OF SYMBOLS
Q_c = Critical Discharge. — cfs.
a = Cross sectional flow area. — ft.^{2}
T = Width of water surface. — ft.
g = 32.16 ft./sec.^{2}

FORMULA

$$\frac{Q_c^2}{g} = \frac{a^3}{T}$$

SCALE DESIGNATION USED IN KEY



REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

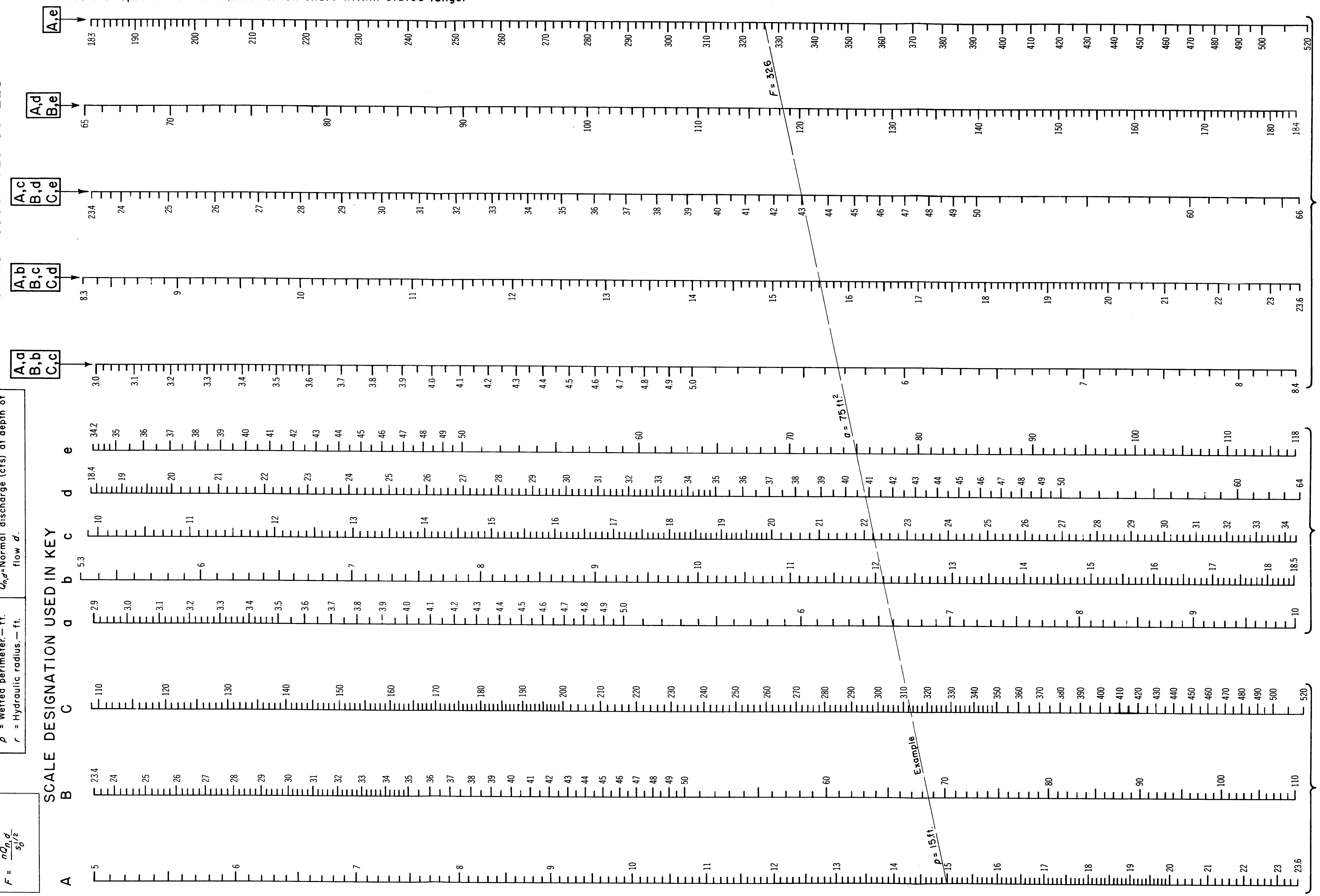
U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-75
SHEET 3 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF CROSS SECTION FACTOR $F = \frac{nQ_{n,d}}{s_o^{1/2}} = 1.486\,a r^{2/3}$

p = 5 to 520
a = 2.9 to 118
F = 3 to 520

Note that area values are repeated to insure solution on chart within stated range.



DESIGNATION OF SYMBOLS

F = Cross-section factor.
n = Coefficient of roughness.
p = Wetted perimeter. – ft.
r = Hydraulic radius. – ft.
s_o = Bottom slope of channel.
a = Cross-sectional area of flow. – ft.²
$Q_{n,d}$ = Normal discharge (cfs) at depth of flow d.

FORMULA

$F = 1.486\,a r^{2/3}$
$F = 1.486\,\frac{a^{5/3}}{p^{2/3}}$
$F = \frac{nQ_{n,d}}{s_o^{1/2}}$

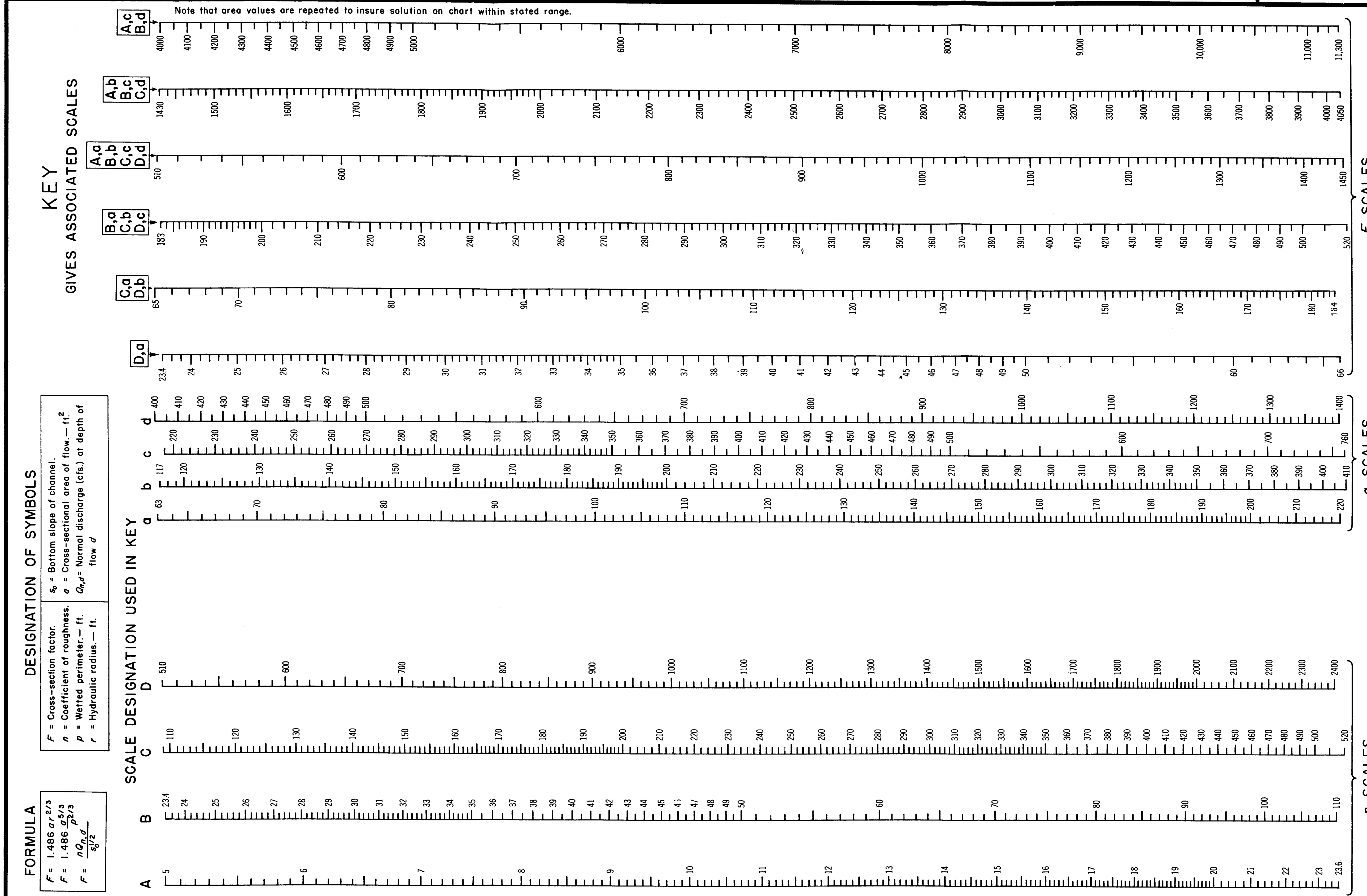
REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES 76
SHEET 1 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF CROSS SECTION FACTOR $F = \frac{nQ_{n,d}}{S_o^{1/2}} = 1.486\,a r^{2/3}$

p = 5 to 2400
a = 63 to 1400
F = 23.4 to 11,300



REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

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ENGINEERING DIVISION - DESIGN SECTION

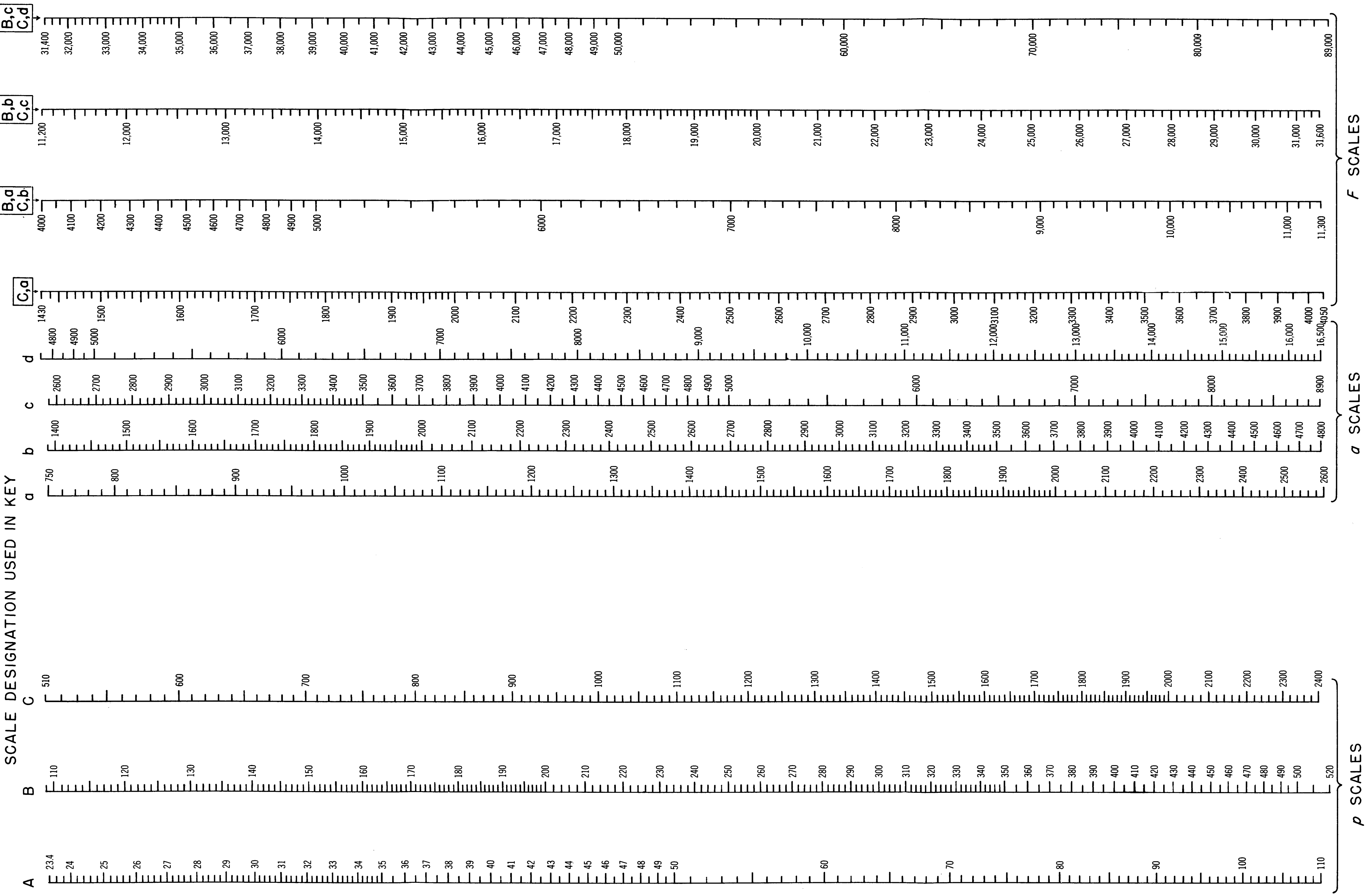
STANDARD DWG. NO.
ES 76
SHEET 2 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF CROSS SECTION FACTOR $F = \frac{nQ_{n,d}}{S_o^{1/2}} = 1.486\,ar^{2/3}$

p = 23.4 to 2400
a = 750 to 16,500
F = 1430 to 89,000

Note that area values are repeated to insure solution on chart within stated range.

KEY
GIVES ASSOCIATED SCALES



SCALE DESIGNATION USED IN KEY

DESIGNATION OF SYMBOLS

F = Cross-section factor.
n = Coefficient of roughness.
p = Wetted perimeter.– ft.
r = Hydraulic radius.– ft.
S_o = Bottom slope of channel.
a = Cross-sectional area of flow.– ft.^{2}
$Q_{n,d}$ = Normal discharge (cfs.) at depth of flow d

FORMULA

$F = 1.486\,ar^{2/3}$
$F = 1.486\,\frac{a^{5/3}}{p^{2/3}}$
$F = \frac{nQ_{n,d}}{S_o^{1/2}}$

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
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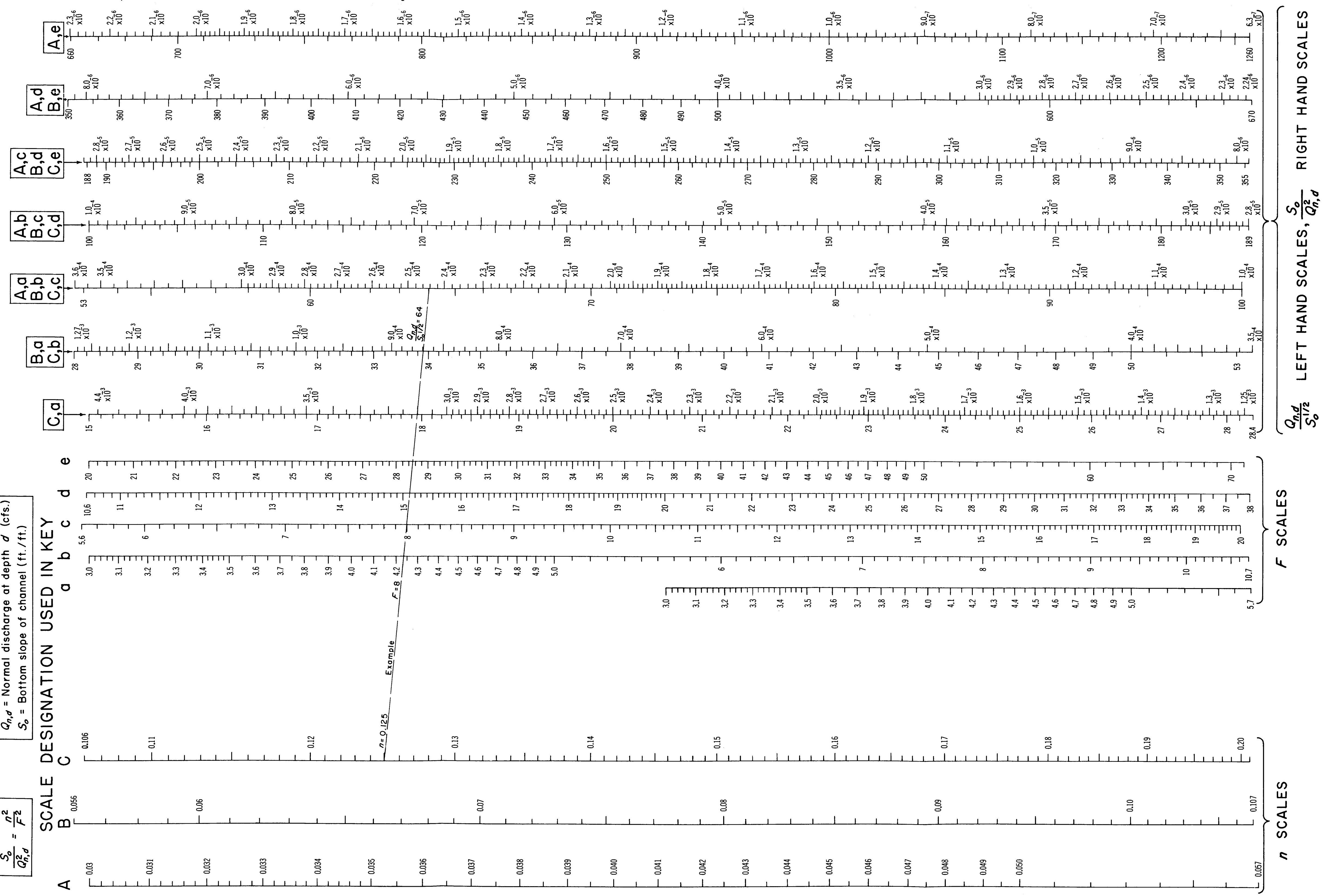
STANDARD DWG. NO.
ES 76
SHEET 3 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} a r^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 F = 3.0 to 70

$\frac{Q_{n,d}}{S_o^{1/2}}$ = 15 to 1260 $\frac{S_o}{Q_{n,d}^2}$ = 6.3×10^{-7} to 4.4×10^{-3}

Note that factor F values are repeated to insure solution on chart within stated range.



FORMULA

$$\frac{Q_{n,d}}{S_o^{1/2}} = \frac{F}{n}$$

$$\frac{S_o}{Q_{n,d}^2} = \frac{n^2}{F^2}$$

DESIGNATION OF SYMBOLS

F = Cross-section factor (see ES 76)
n = Roughness coefficient
$Q_{n,d}$ = Normal discharge at depth d (cfs.)
S_o = Bottom slope of channel (ft./ft.)

REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
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STANDARD DWG. NO.
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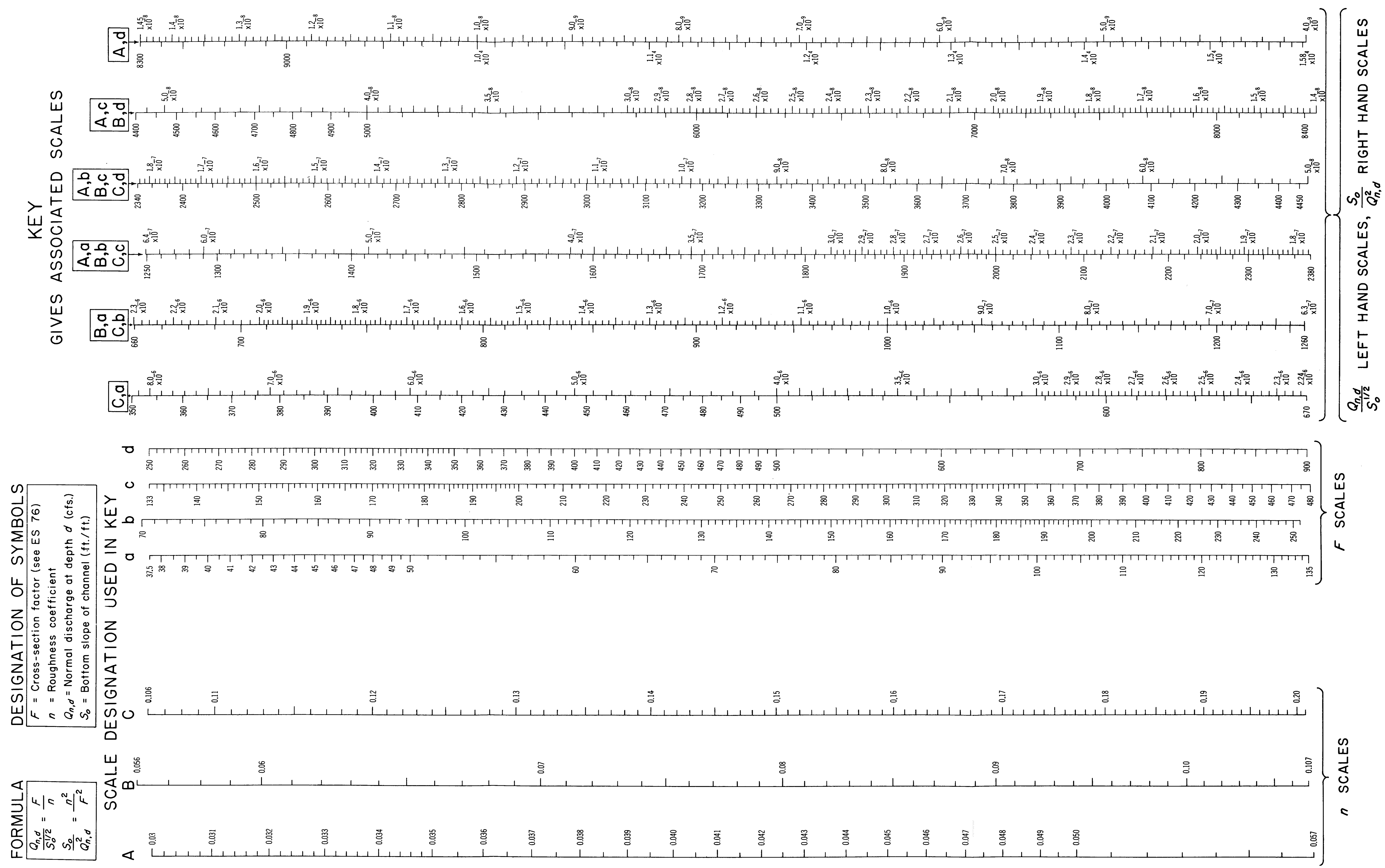
HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} a r^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 F = 38 to 900

$\frac{Q_{n,d}}{S_o^{1/2}}$ = 350 to 1.58×10^4 $\frac{S_o}{Q_{n,d}^2}$ = 4.0×10^{-9} to 8.0×10^{-6}

Note that factor F values are repeated to insure solution on chart within stated range.



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This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
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ENGINEERING DIVISION - DESIGN SECTION

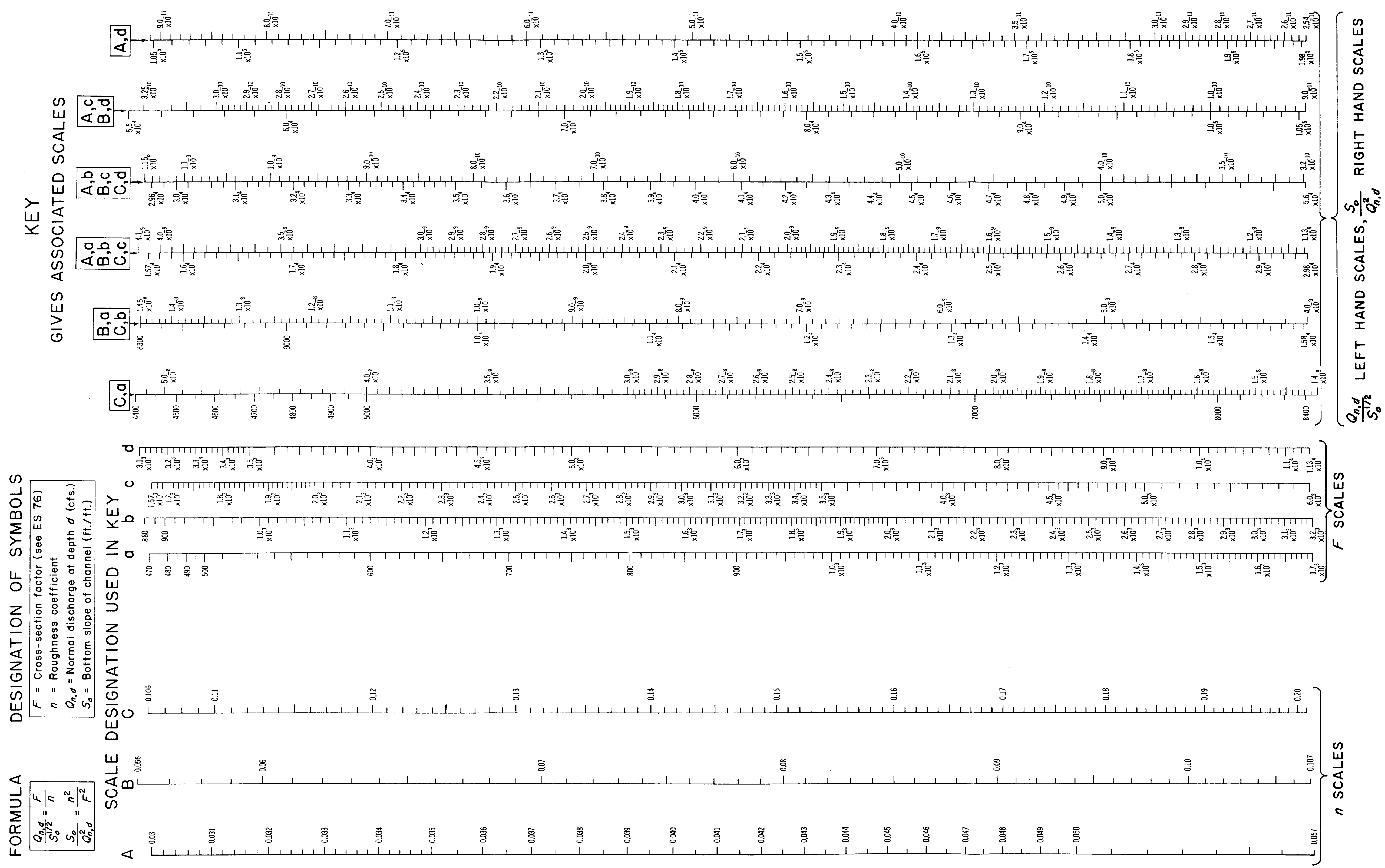
STANDARD DWG. NO.
ES-77
SHEET 2 OF 4
DATE JULY 1953

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} a r^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 $\quad F$ = 470 to 1.1×10^4

$\frac{Q_{n,d}}{S_o^{1/2}}$ = 4400 to 1.98×10^5 $\quad \frac{S_o}{Q_{n,d}^2} = 2.54 \times 10^{-11}$ to 5.0×10^{-8}

Note that factor F values are repeated to insure solution on chart within stated range.



REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

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STANDARD DWG. NO. ES-77

SHEET 3 OF 4

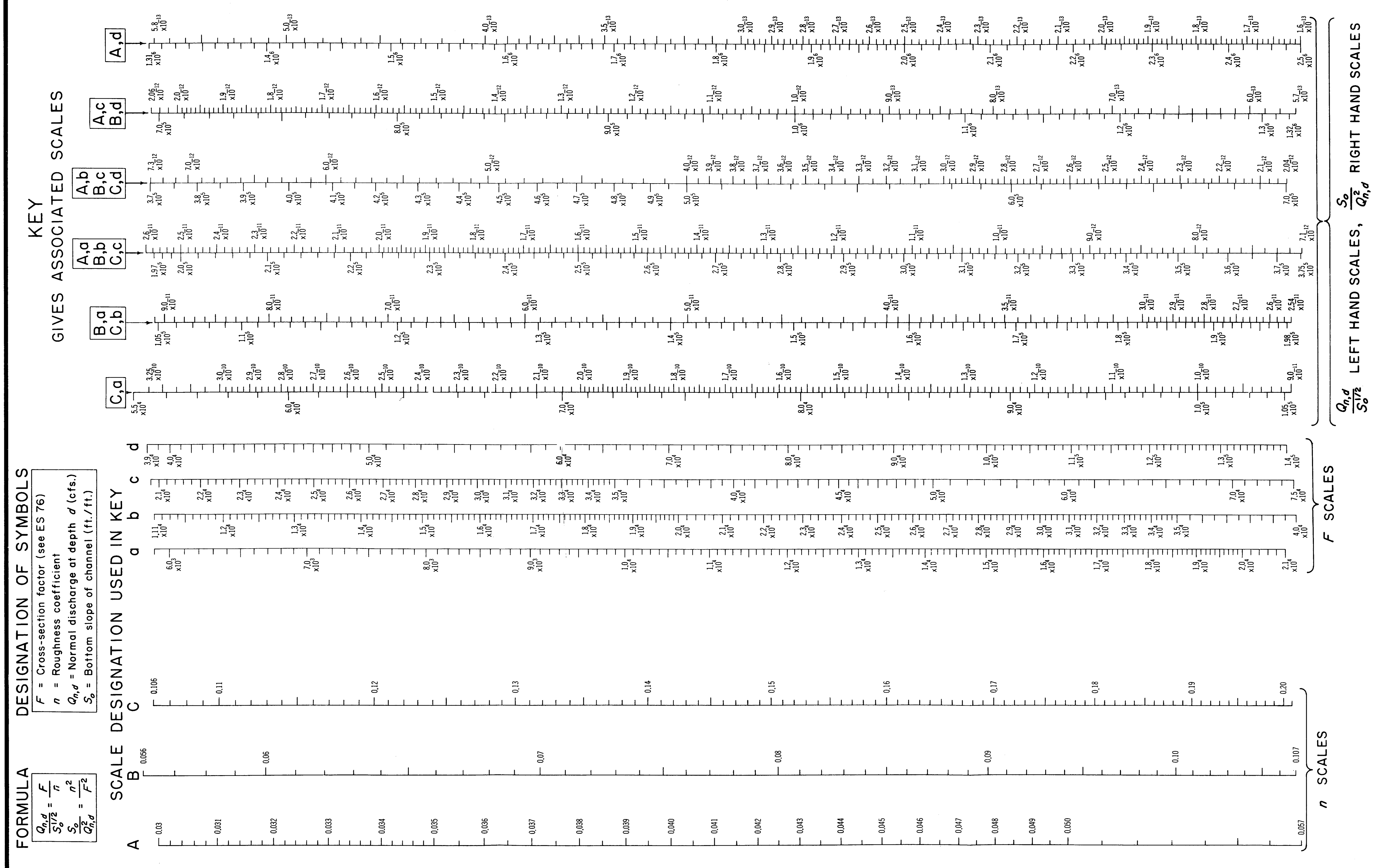
DATE JULY 1953

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n}\, ar^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 $\quad F = 6.0\times10^3$ to 1.4×10^5

$\frac{Q_{n,d}}{S_o^{1/2}} = 5.5\times10^4$ to 2.5×10^6 $\quad \frac{S_o}{Q_{n,d}^2} = 1.6\times10^{-13}$ to 3.25×10^{-10}

Note that factor F values are repeated to insure solution on chart within stated range.



REFERENCE

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9. Model Investigations

9.1 Purposes. The main purposes for which model studies of hydraulic structures are made may be listed as follows:

(a) To obtain basic knowledge of some phase of hydraulics.
(b) To establish design criteria for types of structures or parts of structures subject to standardization.
(c) To determine the proper dimensions and operating characteristics of individual structures.

This subsection is primarily concerned with the last of these purposes.

9.2 Types of Structures For Which Model Studies May Be Required. It is not possible to state categorically the types of structures and related conditions that will require model tests as a basis for design. A given type of structure operating under special conditions and meeting exacting requirements may, for adequate design, demand a thorough model study; whereas the same type of structure under standard conditions and meeting no particularly restrictive requirements would not need a model study. Mainly, the decision rests on whether the factors necessary for sound design and dependable operation can be determined with sufficient reliability by recognized methods of analysis. Other factors, summarized below, will also require consideration.

It is recommended that model studies be considered in the following cases:

(a) Spillways which, because of site conditions: (1) are unsymmetrical in plan; (2) have poor approach channel alignment; (3) have poor entrance conditions; (4) will have to meet some type of exacting requirement not readily subject to analysis.

(b) Channel curves and confluences and reaches of channels in which the cross section is either expanding or contracting, where the discharges involved are in the supercritical range.

(c) Stilling basin structures operating under unusual conditions or exacting requirements that have not been satisfactorily represented by experience or model tests on the same or similar type structure.

9.3 Elements to be Considered in Determining Whether a Model Investigation should be Undertaken. In addition to the type of structure, other elements of the situation should be considered in determining whether a model study should be undertaken. Briefly, the most significant elements are:

(a) Cost of the structure and the possibility of making a saving in total cost through a thorough analysis and design. Model studies will, in many cases, provide an understanding of the hydraulic functioning of a structure that cannot be obtained by analysis, and this knowledge may make important, over-all savings in cost possible.

(b) The degree of hazard involved. The types and amounts of loss that might result from failure of the structure should be examined, at least in qualitative terms. The degree to which faulty or improper operation of the structure may reduce its ability to meet the objectives for which it is constructed should also be considered.

(c) The need for establishing and maintaining public confidence in the Service. It is important that the Service build for itself a reputation for competence in the field of conservation engineering.

9.4 Field Data for Model Studies. Since model studies are made to determine how a structure will function in prototype, they should be supported by essentially the same types and amounts of field data as would be required for a sound design job by the usual methods. In a given case the project design engineer would complete all necessary field surveys and office analyses to fully establish the basic requirements to be met by the structure. A tentative design would then be made by the design engineer and/or the hydraulic engineer at the laboratory where the model tests are to be made. The following survey data should be supplied in support of this and succeeding phases of the investigation:

(a) Profiles, topographic maps, cross sections, and soils data in the vicinity of the structure site.

(b) The maximum discharge to be carried by the structure and such information as is pertinent to determining the proper operation at discharges less than the maximum.

(c) A table or curve giving tailwater elevation in relation to discharge when the structure is a type whose operating characteristics are affected by tailwater conditions.

PREFACE

SUPPLEMENT A

HYDRAULICS

This supplement is intended to augment the material of subsection 4.7 of the Hydraulics Section of the Engineering Handbook.

The objective in the preparation of Supplement A is two-fold: (a) Present improved methods for the computation of water-surface profiles which also require less effort and time, (b) Furnish working aids for the computations.

The ability to solve water-surface profiles in artificial or natural channels is required in many phases of the work of the Soil Conservation Service. Typical problems requiring such computations are: (1) The determination of tailwater depth needed in the hydraulic design of spillways in which dissipation of kinetic energy is necessary. (2) The design of artificial channels to determine their size or capacity. (3) The determination of areas of inundation for various discharges in natural drainage ways for flood damage evaluation. (4) The design of gradient control systems. (5) The design of drainage and irrigation systems.

It is expected that material improvement and saving of time in the computation of water-surface profiles can be made in the future.

Mr. Paul D. Doubt has developed major improvements in the methodology of computing water-surface profiles in the preparation of Supplement A that can effect significant savings in time spent on such work. Messrs. Richard M. Matthews and Rulon M. Jensen have helped materially with the calculations and in the preparation of the charts and examples. Miss Joan Donovan typed the manuscript. This work was done under the general direction of Mr. M. M. Culp, Head, Design Section.

July 6, 1954

SUPPLEMENT A TO SUBSECTION 4 OPEN CHANNEL FLOW

WATER SURFACE PROFILES

PART 1

INTRODUCTION AND DERIVATIONS

A number of writers have presented various methods for solving the surface profile of steady flow in open channels. Some methods are derived by integrating the differential equation of varied flow for special limiting cases or by means of certain approximations. These methods restrict their solutions to prismatic channels. Other methods make use of the slope which the differential equation defines. The slope, $dd/d\ell$, for a reach is evaluated either as an average of the slopes at the end sections or as the slope existing at some intermediate section in the reach. It is proper to use any such process of averaging for the arithmetical error may be kept as arbitrarily small as desired by choosing a sufficiently small increment of the reach.

This supplement presents methods and working aids by which water-surface profiles for steady flow in open channels may be determined as a direct solution for either prismatic or natural channels. Two methods of water-surface-profile determinations are given along with examples. The first method is associated with prismatic channels. The varied or nonuniform-flow differential equation has been rewritten in terms of two channel parameters and the discharge Q for which the profile is being determined in prismatic channels. Because of this, a number of water-surface profiles for various discharges Q may be solved for a given channel with a minimum amount of additional work. The two parameters designated as $Q_{c,d}$ and $Q_{n,d}$ are the critical and normal discharges, respectively, for the depth of flow in the channel. These two parameters are also used to determine flow condition or profile type. Thus, use is made of the parameters in two distinct ways. Charts are included to enable quick evaluation of these parameters. The second method is associated with natural channels.

Equation of varied flow. The customary basic assumptions generally postulated in the theory of varied flow are reiterated briefly in this derivation. The restrictive character of some assumptions can be relieved by approximations or corrections.

The assumptions are:

1. Steady flow conditions exist; that is, the discharge at each section remains constant and equal for the time interval under consideration.

2. Manning's formula defines the slope of the energy gradient.

3. Flat slope channels are assumed; that is, channel slopes are sufficiently flat so the following approximations are valid or acceptable.

   a. The depth of flow, which is generally measured normal to the bottom slope, may be measured vertically.

   b. Hydrostatic pressure distribution exists throughout the depth of flow. This statement requires that little vertical curvilinear movement exists.

c. The slope of the water surface is sufficiently flat so that the depth of flow measured in a vertical direction is a close approximation of the hydrostatic head.

d. No air entrainment occurs.

4. The velocity distribution in each section is nearly equal to the average velocity v such that the quantity $v^2/2g$ may be a sufficiently close expression of the kinetic energy head.

a. This requires that the cross profile of the water surface be nearly level.

5. No significant unaccountable losses occur, such as those due to bends, hydraulic jumps, and abrupt transitions.

Although the derivation of the varied flow equation may be found in many references, it will be repeated here because of the numerous forms in which it appears

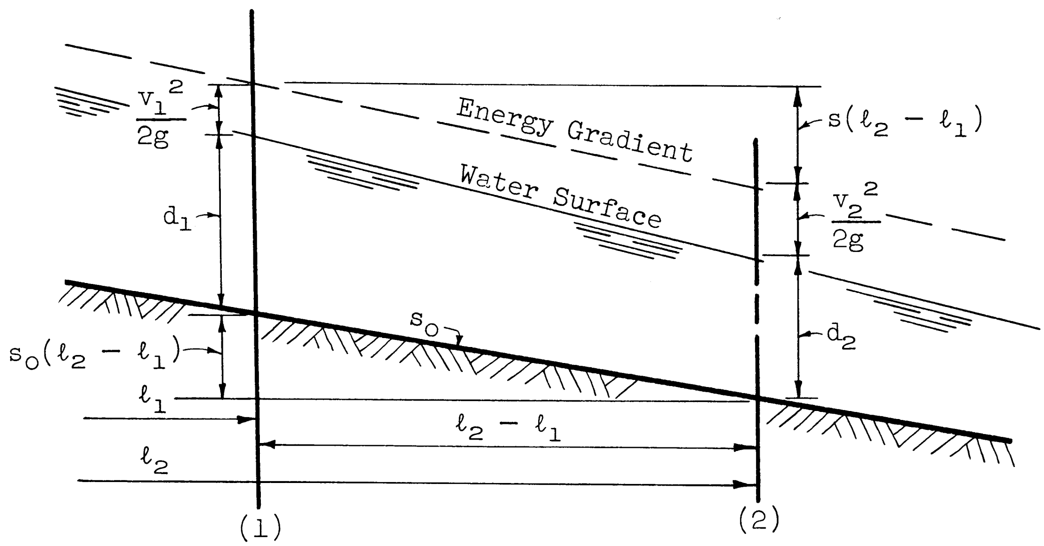

FIGURE A.1

$$d_1 + \frac{v_1^2}{2g} + s_o\ (\ell_2 - \ell_1) = d_2 + \frac{v_2^2}{2g} + s\ (\ell_2 - \ell_1)$$

Rearranging

$$(d_2 - d_1) + \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = (s_o - s)(\ell_2 - \ell_1) \qquad \text{A.1}$$

Now as $(\ell_2 - \ell_1) \equiv \Delta\ell \rightarrow 0$; then $(d_2 - d_1) \equiv \Delta d \rightarrow 0$ and

$$\left[\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right] \equiv \Delta\frac{v^2}{2g} \rightarrow 0$$

Hence, writing in differential form after dividing Eq. A.1 by $\Delta\ell$

$$\frac{dd}{d\ell} + \frac{d}{d\ell}\left[\frac{v^2}{2g}\right] = s_o - s \qquad A.2$$

This is a common form in which the varied flow equation is presented. The rate of change of depth of flow with respect to length of reach, $dd/d\ell$, can be written as a function of s_o, $Q_{c,d}$, $Q_{n,d}$, and Q.

<u>Differential equation of varied flow for positive slopes</u>. Proceeding to write the terms of Eq. A.2 in terms of the five quantities mentioned, the second term will be considered first. The term, $\frac{d}{d\ell}\left[\frac{v^2}{2g}\right]$, which expresses the rate of change in velocity head can be written by observing

$$v = \frac{Q}{a}$$

$$\frac{d}{d\ell}\left[\frac{v^2}{2g}\right] = \frac{Q^2}{2g}\frac{d}{d\ell}\left[\frac{1}{a^2}\right]$$

But

$$\frac{d}{d\ell}\left[\frac{1}{a^2}\right] = -2\frac{1}{a^3}\frac{da}{d\ell} = -2\frac{T}{a^3}\frac{dd}{d\ell}$$

for $da = T\,dd$

Therefore

$$\frac{d}{d\ell}\left[\frac{v^2}{2g}\right] = -\frac{Q^2}{g}\frac{T}{a^3}\frac{dd}{d\ell}$$

Let

$$Q^2_{c,d} = g\frac{a^3}{T} \qquad A.3$$

Then

$$\frac{d}{d\ell}\left[\frac{v^2}{2g}\right] = -\left[\frac{Q}{Q_{c,d}}\right]^2\frac{dd}{d\ell} \qquad A.4$$

The slope of the energy gradient, s, (or the rate of energy loss due to friction per unit length of channel) is also to be rewritten. By Manning's formula

$$s = \left[\frac{nQ}{1.486\ ar^{2/3}}\right]^2 \qquad A.5$$

Further, at the same depth of flow used in evaluating Eq. A.5 there corresponds a normal discharge $Q_{n,d}$ for a channel having a positive slope s_o and a roughness coefficient n. This discharge is written

$$Q_{n,d} = \frac{1.486\ ar^{2/3}}{n}\,s_o^{1/2} \qquad A.6$$

eliminating the factor $\frac{1.486}{n}\,ar^{2/3}$ in Eqs. A.5 and A.6 obtain

$$s = s_o\left[\frac{Q}{Q_{n,d}}\right]^2 \qquad A.7$$

Substituting the terms expressed by Eqs. A.4 and A.7 into the varied flow Eq. A.2 and rearranging

$$s_o \frac{d\ell}{dd} = \frac{\left[\frac{Q}{Q_{c,d}}\right]^2 - 1}{\left[\frac{Q}{Q_{n,d}}\right]^2 - 1} \qquad \text{A.8}$$

is the differential equation of varied flow for positive-slope channels which will be considered further.

Differential equation of varied flow for channels having horizontal bottoms. The differential equations for channels having horizontal or negative (adverse) bottom slopes can be written in terms of Q, $Q_{c,d}$, and $Q_{n,d}$ if the quantity $Q_{n,d}$ is defined. The normal discharge, as defined for positive-slope channels, does not exist for channels having horizontal bottoms. This can be readily ascertained for s_o is zero in Eq. A.6 and $Q_{n,d}$ is zero. To eliminate this seemingly unfortunate situation, the quantity $Q_{n,d}$ for channels having horizontal bottoms is arbitrarily defined to be the same as though the channel has a positive slope of unity. The slope of the energy gradient is still given by Eq. A.5, but at the same depth of flow used in evaluating Eq. A.5 there corresponds a normal discharge, $Q_{n,d}$, for the same channel having a positive slope of unity and of roughness coefficient n. This discharge is written

$$Q_{n,d} = \frac{1.486\ ar^{2/3}}{n} \qquad \text{A.9}$$

Eliminating the factor $\frac{1.486\ ar^{2/3}}{n}$ in Eqs. A.5 and A.9 obtain

$$s = \left[\frac{Q}{Q_{n,d}}\right]^2 \qquad \text{A.10}$$

Then by Eq. A.2 in which s_o is zero and Eqs. A.4 and A.10, the varied flow equation for channels with horizontal bottoms becomes

$$\frac{d\ell}{dd} = \frac{\left[\frac{Q}{Q_{c,d}}\right]^2 - 1}{\left[\frac{Q}{Q_{n,d}}\right]^2} \qquad \text{A.11}$$

Differential equation of varied flow for channels having negative slopes. For channels having negative slopes, the normal discharge $Q_{n,d}$ becomes imaginary according to an evaluation of Manning's formula and the definition of normal discharge for positive slopes.

$$Q_{n,d} = \frac{1.486}{n}\ ar^{2/3}\ s_o^{1/2} \qquad \text{A.6}$$

This is readily ascertained for s_o is negative in Eq. A.6. To eliminate any element of confusion which might arise in the use of imaginary numbers, the normal discharge $Q_{n,d}$ for negative slopes will be given a definition different from that used for channels with positive slopes. This change in definition will necessarily cause a change in the varied flow differential equation from that given by Eq. A.8. On taking the definition of the normal discharge of the same channel with a positive slope $|s_o|$ the evaluation of the slope of the energy gradient, on using the same argument as before, becomes

$$s = - s_o \left[\frac{Q}{Q_{n,d}}\right]^2 \qquad \text{A.12}$$

The varied flow differential equation for channels with negative slope is, by substituting Eqs. A.4 and A.12 into Eq. A.2,

$$|s_o| \ \frac{d\ell}{dd} = \frac{\left[\frac{Q}{Q_{c,d}}\right]^2 - 1}{\left[\frac{Q}{Q_{n,d}}\right]^2 + 1} \qquad \text{A.13}$$

The symbol $|s_o|$ is the absolute value (or the positive value) of the negative slope $|s_o|$ and $Q_{n,d}$ is evaluated as a real number for the channel having the positive slope $|s_o|$ at the depth of flow d.

PART 2

FLOW CONDITIONS

During the process of the solution of Eq. A.8, it will be necessary to know or be able to recognize flow conditions. Therefore, before further considering this equation, the criteria which will define flow conditions will be considered. Flow conditions have three aspects: (1) flow is subcritical, critical, or supercritical; (2) flow is subnormal, normal, or supernormal; (3) flow is retarded, constant velocity (i.e., uniform), or accelerated. As will be seen, these three aspects are not independent of one another for, in general, any two will establish the third. Flow conditions are evaluated at a particular section having a depth of flow d and a given discharge Q. The flow conditions will become apparent during the solution for a surface profile upon observing the values of the terms $Q/Q_{c,d}$; $Q/Q_{n,d}$; and $dd/d\ell$. The criteria which will define flow conditions will be discussed in the following three paragraphs. The relationship between the three aspects will be considered in the fourth paragraph. A summary of the discussion is given in table A.1, page A.8.

The critical discharge at depth d may be determined for a given channel section by the relation Eq. A.3. The value of the ratio $Q/Q_{c,d}$ can be used as a criterion to determine whether a discharge of Q flowing at a depth d is subcritical, critical, or supercritical flow. By definition, the flow of a discharge Q at the depth d is subcritical, critical, or supercritical according to whether Q is less than, equal to, or greater than the critical discharge $Q_{c,d}$ corresponding to the depth d. Hence, the general criterion may be written: the flow of a discharge Q at a depth d will be subcritical, critical, or supercritical according to whether $Q/Q_{c,d}$ is less than, equal to, or greater than unity. It is important to remember that the discharge Q at a flow depth d is compared to the datum $Q_{c,d}$ in determining the prefix to the word critical. Frequently, the depth of flow d for a discharge Q is compared to the datum d_c corresponding to the discharge Q to determine this flow condition. An interchange of prefixes is obtained by this comparison, for when d is less than d_c the flow is supercritical and when d is greater than d_c flow is subcritical. The criterion can also be worded: the depth of flow d will be less than, equal to, or greater than critical depth of flow for Q according to whether $Q/Q_{c,d}$ is greater than, equal to, or less than unity.

The normal discharge at depth d for a given channel section of infinitesimal length, having a bottom slope s_o and roughness coefficient n, may be obtained by Eq. A.6. The value of the ratio $Q/Q_{n,d}$ can be used as a criterion to determine whether a discharge Q flowing at depth d is subnormal, normal, or supernormal flow. The flow of a discharge Q at a depth d is subnormal, normal, or supernormal according to whether Q is less than, equal to, or greater than the normal discharge $Q_{n,d}$ corresponding to the depth d. The criterion may be written: The flow of a discharge Q at a depth d will be subnormal, normal, or supernormal according to whether $Q/Q_{n,d}$ is less than, equal to, or greater than unity. Here again, it is important to remember the discharge Q flowing at the depth d is compared to the datum $Q_{n,d}$ in determining the prefix to the word normal. It is desirable to observe that the depth of flow d for a discharge Q may be compared to the datum d_n, the normal depth of flow corresponding to the discharge Q, to determine this flow condition. Once more, an interchange of prefixes is obtained by this comparison, for the flow is supernormal when d is less than d_n and when d is greater than d_n the flow is subnormal. Rewriting the criterion: the depth of flow d for a discharge Q is less than, equal to, or greater than the normal depth d_n corresponding to Q according to whether the ratio $Q/Q_{n,d}$ is greater than equal to, or less than unity.

According to the sign convention used in deriving Eq. A.8, the value $dd/d\ell$ (the instantaneous rate of change of depth with respect to the length of channel at a section) is positive when the depth of flow is increasing in a downstream direction. This imposes that flow be retarded. The criterion becomes: a discharge Q flowing at a depth d is retarded, uniform, or accelerated flow according to whether $dd/d\ell$ is positive, zero, or negative.

Equation A.8 shows that $dd/d\ell$ is positive if the ratios $Q/Q_{c,d}$ and $Q/Q_{n,d}$ are:

(a) both greater than unity, or
(b) both less than unity.

In other words, as has been shown by the three preceding paragraphs, flow is retarded for the following two sets of flow conditions:

(a') flow is supercritical and supernormal
(b') flow is subcritical and subnormal

Again Eq. A.8 reveals that $dd/d\ell$ is negative if

(c) the ratio $Q/Q_{c,d}$ is greater than unity and $Q/Q_{n,d}$ is less than unity, or
(d) the ratio $Q/Q_{c,d}$ is less than unity and $Q/Q_{n,d}$ is greater than unity.

Therefore, flow is accelerated when

(c') flow is supercritical and subnormal, or
(d') flow is subcritical and supernormal.

Certain special cases of flow conditions occur when $Q/Q_{n,d} = 1$. If $Q/Q_{n,d} = 1$ and $Q/Q_{c,d} \neq 1$ then $dd/d\ell = 0$ and flow is at normal depth or uniform flow occurs. The uniform flow is

(1) subcritical if $Q/Q_{c,d} < 1$

(2) supercritical if $Q/Q_{c,d} > 1$.

Another set of special cases is obtained when $Q/Q_{c,d} = 1$ and $Q/Q_{n,d} \neq 1$. If $Q/Q_{c,d} = 1$ and $Q/Q_{n,d} > 1$ then $dd/d\ell = +\infty$ and

(3) flow is at critical depth and is being retarded through a hydraulic jump, i.e., flow is passing from a supercritical condition to a subcritical condition and is supernormal.

If $Q/Q_{c,d} = 1$ and $Q/Q_{n,d} < 1$ then $dd/d\ell = -\infty$ and

(4) flow is at critical depth and is being accelerated through a control section, i.e., the bottom slope of the channel increases from a slope less than critical to a slope equal to or greater than critical.

When $Q/Q_{c,d} = 1$ and $Q/Q_{n,d} = 1$

(5) the very special unstable case of uniform critical flow is obtained. In this case, $dd/d\ell$ is of the indeterminate form, 0/0.

It can be observed that any two aspects of flow conditions will not necessarily determine the third, nor are the three aspects independent for the special cases of flow condition.

A knowledge of flow conditions is required before computations can be performed properly and reliably. For instance, computations for profiles of subcritical flows are made in an upstream direction and computations for supercritical flows are made in a downstream direction. There are definite physical reasons for this statement. It has been shown that when flow is supercritical the depth of flow at a given section is unaffected by any channel elements, or change of channel elements, downstream from the section so long as the flow remains supercritical at the section under consideration. Therefore, the depth of flow at the given section is dependent only on the flow characteristics upstream from the section at which supercritical flow exists. Hence, to consider the effect of these upstream flow characteristics in deducing the value of depth of flow at a section, profile computations for supercritical flows must be carried in a downstream direction to this section.

Similarly, the depth of flow at a given section which is subcritical is not influenced by any of the channel elements or change of channel elements upstream from the section as long as the flow remains subcritical at the section. The depth of flow at the section is dependent only on the flow characteristics of the channel downstream from the section. The effects of these flow characteristics on the subcritical depth at the section can only be used by considering them in solving for the depth at the section. Thus, surface profile computations must be carried in an upstream direction in regions of subcritical flow.

When computations are made in the incorrect directions, solutions for the depth of flow diverge from the correct depth. Conversely, when computations are made in the correct direction the calculated depth of flow tends to converge to the correct depth even though the initial depth is incorrect. This fact is used in determining a depth of flow. See ES-83, Ex. 4, page A.19.

TABLE A.1

Equation A.8

Summary of Criteria and Relationship between flow conditions; positive slopes

When $\frac{Q}{Q_{c,d}}$ is → / When $\frac{Q}{Q_{n,d}}$ is ↓	greater than unity then flow is supercritical $d < d_c$ and ↓	equal to unity then flow is critical $d = d_c$ and ↓	less than unity then flow is subcritical $d > d_c$ and ↓
greater than unity then flow is supernormal $d < d_n$ and →	(a) $\frac{dd}{d\ell} > 0$ flow is retarded $d_1 < d_2$	(3) $\frac{dd}{d\ell} = +\infty$ flow is retarded (hydraulic jump)	(d) $\frac{dd}{d\ell} < 0$ flow is accelerated $d_1 > d_2$
equal to unity then flow is normal $d = d_n$ and →	(2) $\frac{dd}{d\ell} = 0$ flow is uniform $d_1 = d_2$	(5) $\frac{dd}{d\ell} = \frac{0}{0}$ = indeterminate very special unstable case of uniform critical flow. $d_1 = d_n = d_c = d_2$	(1) $\frac{dd}{d\ell} = 0$ flow is uniform $d_1 = d_2$
less than unity then flow is subnormal $d > d_n$ and →	(c) $\frac{dd}{d\ell} < 0$ flow is accelerated $d_1 > d_2$	(4) $\frac{dd}{d\ell} = -\infty$ flow is accelerated (control section)	(b) $\frac{dd}{d\ell} > 0$ flow is retarded $d_1 < d_2$

Because of these facts, the value of the ratio $Q/Q_{c,d}$ will be used to determine the direction in which computations are to be carried. Computations are to be carried in a downstream or upstream direction according to whether $Q/Q_{c,d}$ is greater or less than unity. If $Q/Q_{c,d}$ is unity, either a control section exists at some break in grade or a hydraulic jump exists or flow is uniform and critical.

PART 3

DETERMINATION OF WATER SURFACE PROFILE

The objective in the majority of water-surface-profile problems is the determination of the surface profile for a given discharge Q. Two methods are presented. The first and simpler method presented is applicable only to prismatic channels and is illustrated by Exs. 1, 2, and 3, ES-83, pages A.16 to A.18 inclusive. The second method, illustrated by Exs. 4, 5, and 6, is applicable to either prismatic or natural channels and is particularly useful in the determination of a number of surface profiles for various discharges in a given channel. Both methods give direct results, thus eliminating all trial and error procedures.

The type of problem solvable by the simpler method. The simpler method determines the length of reach between two depths of flow for a given discharge Q in a given prismatic channel. The water-surface profile solvable by this method has the following data given:

1. The discharge Q, for which the water-surface profile is to be determined.

2. The size and shape of the prismatic channel.

3. The slopes of the channel bottom and locations at which changes in grades occur.

4. Manning's roughness coefficient n.

5. Depth of flow at the downstream section of the channel, if flow is subcritical, or at the upstream section of the channel, if flow is supercritical. If the depth of flow is not given, it must be determinable. One or both of two procedures can be used to determine the depth when the depth of flow is not given. One procedure is to determine a control section. This is illustrated by Ex. 1, ES-83, page A.16. The second procedure is illustrated by Ex. 4, ES-83, pages A.19 through A.21.

The first step in solving a water-surface profile is to determine which of those breaks in grade of the channel bottom are control sections for the discharge Q, the surface profile of which is being determined. The depth of flow at a control section is the critical depth d_c, corresponding to the discharge Q. Any method of water-surface-profile determination requires that one depth of flow be given or be determinable. See sheet 4, ES-38, for definition of control sections.

The second step for the solution of a water-surface profile in prismatic channels is generally the determination of the normal depth of flow for each bottom slope of the channel for the discharge Q.

The third step is the determination of the water-surface profile itself. The direction in which the computations are to be made (see pages A.7 and A.9) is known. Equation A.8 may be rewritten in the form

$$\frac{d\ell}{dd} = \frac{1}{s_o}\,\frac{\left[\dfrac{Q}{Q_{c,d}}\right]^2 - 1}{\left[\dfrac{Q}{Q_{n,d}}\right]^2 - 1} \equiv \frac{1}{s_o}\,\frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\dfrac{nQ}{b^{8/3}s_o^{1/2}}}{\dfrac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}}\right]^2 - 1} = \frac{2R}{s_o} \qquad \text{A.14}$$

It is good practice to tabulate data in a manner similar to that shown by the table in ES-83, page A.16. Various values of depths are arbitrarily selected between the known depth and the normal depth of flow for the discharge Q. Observe that the identities

$$\left[\frac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right] \equiv \frac{Q}{Q_{c,d}} \quad \text{and} \quad \left[\frac{\dfrac{nQ}{b^{8/3}s_o^{1/2}}}{\dfrac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}}\right] \equiv \frac{Q}{Q_{n,d}}$$

are true. The numerator of the left hand member of each identity is found in performing the second step and the denominators can be read from ES-24 and ES-55, pages A.26 to A.29 for the selected depths d of flow. The value of R may be determined by ES-53, page A.30, or calculated by substituting into Eq. A.8.

For various values of d, a plot of corresponding values of $2R/s_o$ can be made. Such a plot is given by Fig. A.2, page A.11. In general, when $d = d_c$ the value of $2R/s_o$ is zero and for values of $d \rightarrow d_n$ the value of $2R/s_o \longrightarrow \pm\infty$, where d_n is the normal depth corresponding to the discharge Q. The value of $2R/s_o$ is positive when flow is retarded and negative when flow is accelerated. The value of $2R/s_o$ is also the value of $d\ell/dd$. The area under the curve in Fig. A.2, page A.11, between the depths d_1 and d_2 is the integral

$$A = \int_{d_1}^{d_2} \frac{d\ell}{dd}\, dd = \ell_2 - \ell_1$$

where $\ell_2 - \ell_1$ is the length of the channel between sections 1 and 2. Since $d\ell/dd$ is a function of d, which is not readily integrated, it will be easier to evaluate the area under the curve by numerical calculations. A close approximation of this area is the area of a parallelogram having parallel sides of lengths $2R_1/s_o$ and $2R_2/s_o$ and width of $(d_2 - d_1)$. The area of this parallelogram is

$$A = \frac{1}{2}\left[\frac{2R_1}{s_o} + \frac{2R_2}{s_o}\right](d_2 - d_1) \equiv \frac{1}{s_o}(R_1 + R_2)(d_2 - d_1) \qquad \text{A.15}$$

which has been shown to be approximately equal to $\ell_2 - \ell_1$, the length of the channel between sections 1 and 2. The approximation is improved considerably by selecting smaller values of increments of depths $(d_2 - d_1)$.

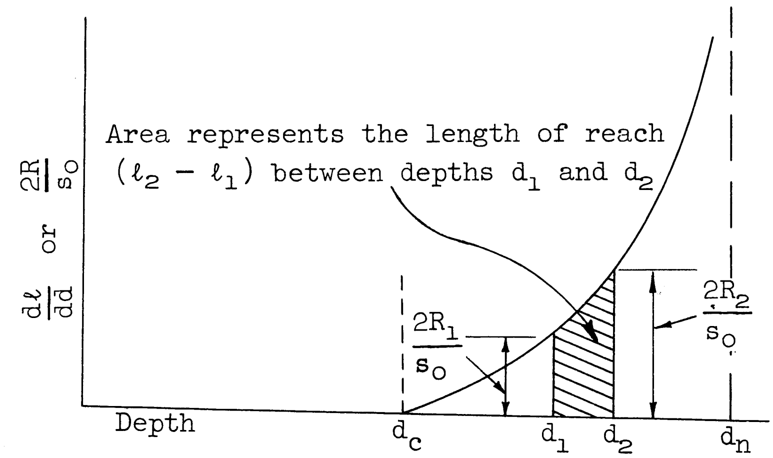

FIGURE A.2

In the derivation of the varied-flow equation, the origin of ℓ was considered as some point upstream from sections 1 and 2 and ℓ was positive in the direction of flow; further, section 1 was considered to be upstream from section 2. Hence, in all calculations, if section 1 is upstream from section 2, the quantity $(\ell_2 - \ell_1)$ must be positive irrespective of the direction in which computations are carried.

The evaluation of water-surface profiles for prismatic channels having horizontal and adverse bottom slopes may be solved in a similar way using Eqs. A.11 and A.13. See Exs. 2 and 3, pages A.17 and A.18.

Introduction to second method. The method of solving for a water-surface profile by the second method is distinctly different from that presented by the foregoing method. Water-surface profiles for prismatic channels would seldom be determined by this method, since the first method presented is simpler and more direct. This method would be advantageous for a given prismatic channel if a number of water-surface profiles are required for various discharges Q.

The following method is basically a re-evaluation and extension of Francis F. Escoffier's[1] method, in which changes in velocity heads were neglected. This method does not neglect velocity heads. A considerable amount

[1]Escoffier, Francis F., Graphical Calculation of Backwater Eliminates Solution by Trial, Engineering News-Record, June 27, 1946.

of computations may be eliminated by using certain relationships between the depth of flow and channel characteristics as shown by Bakhmeteff[1], and Von Seggern[2]. These relationships show a nearly linear variation of depth of flow d and the characteristics $s_o/Q^2_{n.d}$ and $Q_{c.d}$ when log-log plots are made. This is illustrated by Figs. 2, 3, and 4 of Ex. 4, ES-83, page A.19.

Non-prismatic channels, such as natural waterways, have cross sections given at definite stations. Thus, definite lengths of reaches are given and the depth of flow is the variable to be determined. Dictating that the depth be the variable to be determined complicates the solution for water-surface profiles because the depth is expressed implicitly in the equation.

Certain close approximations and assumptions will be made in the development of this method. Only by increasing the number of cross sections and correct evaluation of Manning's roughness coefficient n, rather than algebraic manipulation of formulas, is the error in predicting a water-surface profile kept small. The error introduced by the given approximations will, without exception, be smaller than that introduced by incorrect evaluation of n and an inaccurate and insufficient number of cross sections.

Once the water-surface profile has been solved for a given discharge in a particular channel, only a small amount of additional work is required to compute water-surface profiles for a large number of other discharges.

The type of problem solvable by the second method will have the following data given:

1. The discharge Q, for which the water-surface profile is determined.

2. The cross sections of the channel.

3. The distance between the given cross sections.

4. The slopes of the channel bottom or the elevations of the bottom of the given cross sections.

5. Manning's roughness coefficient n for various flow areas of the cross sections.

6. The depth of flow at some section or a control section. Generally, this depth will not be given but can be approximated closely. A method of making this approximation is given.

The first step in solving a water-surface profile, as in the foregoing method, is the determination of which breaks in grade in the channel bottom are control sections. See page 4, ES-38. When no depth of flow is given and the channel has no control section, the depth of flow can be closely approximated for subcritical flows in the following manner. Determine a

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[1]Bakhmeteff, Boris A., Hydraulics of Open Channels, McGraw-Hill, Equation 70, page 84.

[2]Von Seggern, M. E., Integrating the Equation of Non-uniform Flow, ASCE Transactions, V. 115, (1950), page 71.

probable maximum and minimum depth at a sufficiently great distance downstream from the portion of the channel for which the water-surface profile is to be solved. Compute two water-surface profiles, in an upstream direction, commencing the profiles at these maximum and minimum depths. These two water-surface profiles converge to nearly the same depth of flow at the downstream section of the reach of channel for which the profiles are to be determined when the maximum and minimum depths have been chosen at a sufficiently great distance downstream. See Ex. 4, ES-83, page A.19.

Development of equation. By Bernoulli's theorem, see Fig. A.3.

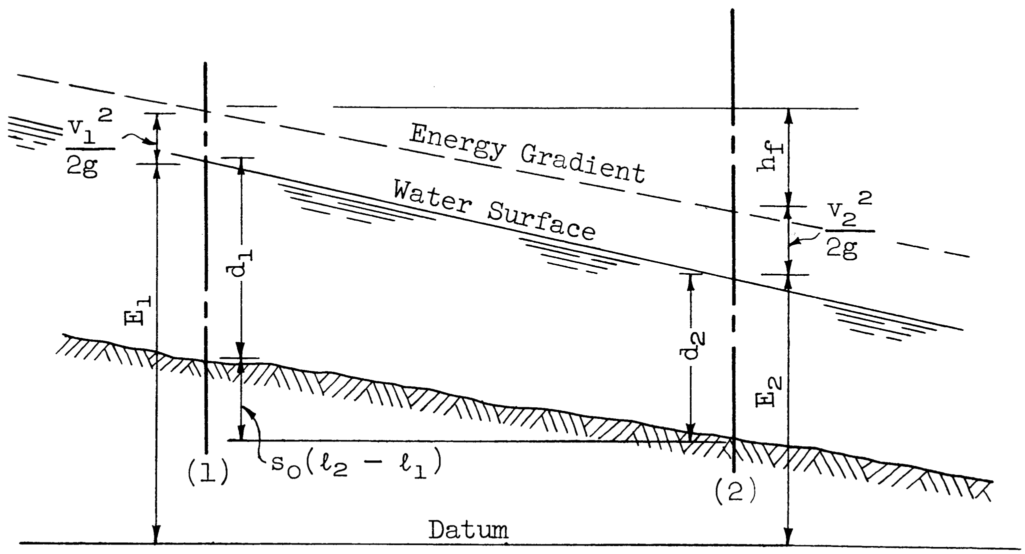


FIGURE A.3

$$E_1 + \frac{v_1^2}{2g} = E_2 + \frac{v_2^2}{2g} + h_f \qquad \text{A.16}$$

where E = elevation of water surface in feet
h_f = friction head loss between sections 1 and 2

Rewriting

$$E_1 - E_2 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + h_f$$

As shown by Eq. A.7, the rate of friction-head loss at any section 1, s_1, is

$$s_1 = s_o \left[\frac{Q}{Q_{n,d_1}}\right]^2$$

Assuming the rate of friction-head loss s for the reach $(\ell_2 - \ell_1)$ as the average of the friction-head loss at section 1 and 2, or

$$h_f = \frac{s_o}{2}\left\{\left[\frac{Q}{Q_{n,d_1}}\right]^2 + \left[\frac{Q}{Q_{n,d_2}}\right]^2\right\}(\ell_2 - \ell_1) \qquad \text{A.17}$$

Observing that

$$v = \frac{Q}{a}$$

obtain on substituting into Eqs. A.16 and A.17

$$E_1 - E_2 = \left[\frac{Q^2}{2ga_2^2} - \frac{Q^2}{2ga_1^2}\right] + \frac{Q^2(\ell_2 - \ell_1)g}{2g}\left[\frac{s_o}{Q_{n,d_1}^2} + \frac{s_o}{Q_{n,d_2}^2}\right]$$

Rearranging

$$\frac{E_1 - E_2}{\left[\frac{1}{a_2^2} + \frac{s_o}{Q_{n,d_2}^2}(\ell_2 - \ell_1)g\right] - \left[\frac{1}{a_1^2} - \frac{s_o}{Q_{n,d_1}^2}(\ell_2 - \ell_1)g\right]} = \frac{Q^2}{2g} \qquad \text{A.18}$$

on letting

$$U_2^+ = \frac{1}{a_2^2} + (\ell_2 - \ell_1)g\,\frac{s_o}{Q_{n,d_2}^2} \qquad \text{A.19}$$

and

$$U_1^- = \frac{1}{a_1^2} - (\ell_2 - \ell_1)g\,\frac{s_o}{Q_{n,d_1}^2} \qquad \text{A.20}$$

Equation A.18 becomes

$$\frac{E_1 - E_2}{U_2^+ - U_1^-} = \frac{Q^2}{2g} \qquad \text{A.21}$$

This formula is the form used to determine water-surface profiles. Assume d_2 is given for the discharge along with the first 5 listed quantities of page A.12. If $d_2 > d_c$, then water-surface-profile calculations are made in an upstream direction and it is required that d_1 be determined. The terms E_1 and U_1^- are functions of d_1, and E_2 and U_2^+ are functions of d_2. Thus, every term in Eq. A.21 is known except E_1 and U_1^-. The terms E_1 and U_1^- would also be known if d_1 were known. Select various values of d_1 and plot the U_1^- curve, using ordinates of E_1 and abscissas of U_1^-. Plot the point P_2 having the coordinates U_2^+, E_2 for the given value of d_2. (See Fig. A.4, page A.15)

Draw a straight line having a slope of $-\frac{Q^2}{2g}$ from P_2 to intersect the curve U_1^- at point P_1. By this construction, the point P_1 has the relationship

$$\frac{E_1 - E_2}{U_2^+ - U_1^-} = \tan\theta = \frac{Q^2}{2g}$$

But this is also the relationship which must be satisfied to determine the water-surface profile, see Eq. A.21. Therefore, this is a graphical solution for the water-surface profile and E_1 at the point P_1 is the water-surface elevation at section 1. Moreover, this construction constitutes a direct graphical solution of the depth of flow d_1. This graphical solution is of a form which requires little additional work to determine the water-surface profile for any discharge once the values of U_2^+ and U_1^- have been computed.

The procedure for computing a water-surface profile in natural channels is illustrated in Exs. 4, 5, and 6 of ES-83, pages A.19 through A.25. Much computational work can be eliminated in determining the values of $Q_{c,d}$, $\frac{s_o}{Q_{n,d}^2}$, $\frac{1}{a^2}$, U_2^+, and U_1^- for intermediate values of depths by observing the nearly straight-line relationship which exists on a log-log plot when the side slopes of the banks remain nearly constant. See ES-83, Figs. 3, 4, 5, 6, and 7, pages A.19 and A.20. Coordinates for these plots should be determined at depths of flows at which the cross-sectional bank slope abruptly changes. If no appreciable change in the slope of banks occurs in the region of depths under consideration, these values should be evaluated at depth intervals of 3 to 4 feet. The plots of Figs. 2 and 3, page A.19, are used to determine which sections are control sections, and whether flow is subcritical or supercritical.

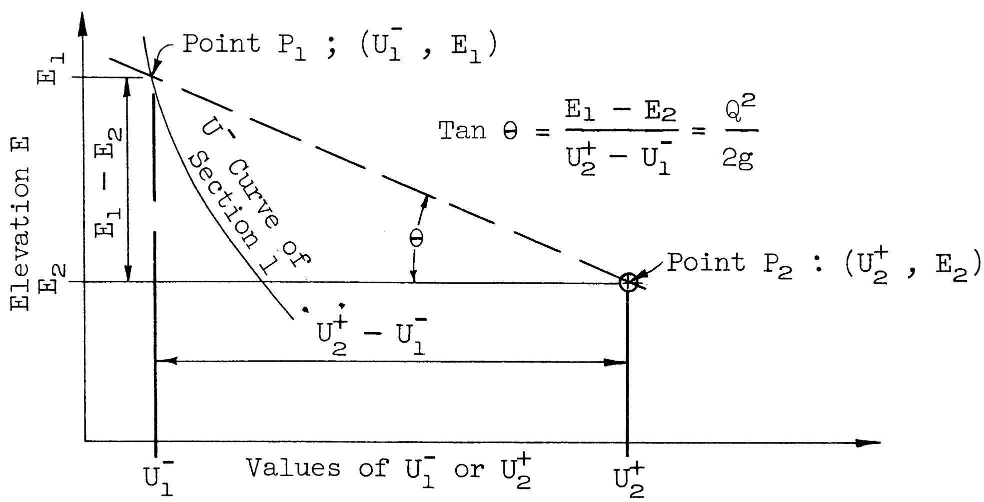
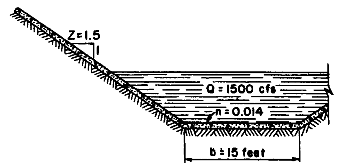


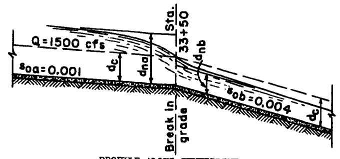
FIGURE A.4

HYDRAULICS: NON-UNIFORM FLOW IN A PRISMATIC CHANNEL WITH POSITIVE BOTTOM SLOPE —— Example 1.

Given: A concrete trapezoidal channel with the values of $z = 1.5$, $b = 15$ ft, $n = 0.014$, $Q = 1500$ cfs. The stationing is in the direction of flow. A break in grade is located at Sta 33 + 50 and elevation of 100.0; the slope upstream from Sta 33+50, s_{oa}, is 0.0010. The slope downstream from Sta 33+50, s_{ob}, is 0.004



TYPICAL SECTION



PROFILE ALONG CENTERLINE

Determine: The water surface profile

Solution: Rewrite Eq. A.8 in the form

$$s_o \frac{d\ell}{dd} = \frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3}\, s_o^{1/2}}}{\frac{nQ_{n,d}}{b^{8/3}\, s_o^{1/2}}}\right]^2 - 1} = 2R$$

1. Solve whether the break in grade is a control section
   a. Solve for d_c corresponding to $Q = 1500$

$$\frac{Q}{b} = \frac{1500}{15} = 100 \text{ cfs/ft}; \quad \frac{z}{b} = \frac{1.5}{15} = 0.1$$

From ES-24 $\quad d_c = 5.59$ ft

   b. Solve for critical slope s_c corresponding to $Q = 1500$ cfs, $d_c = 5.59$ ft

$$\frac{d_c}{b} = \frac{5.59}{15} = 0.3727$$

From ES-55 $\quad \dfrac{nQ}{b^{8/3}\, s_c^{1/2}} = 0.341$ where $\dfrac{nQ}{b^{8/3}} = \dfrac{(0.014)(1500)}{15^{8/3}} = 0.01533$

$$s_c = \left[\frac{nQ}{b^{8/3}}\right]^2 \frac{1}{(0.341)^2} = \left[\frac{0.01533}{0.341}\right]^2 = 0.002022$$

Hence, Sta 33 + 50 is a control section since $s_{oa} < s_c < s_{ob}$

2. Solve for the normal depth of flow in the upstream and downstream reaches.
   a. For the upstream reach

$$\frac{nQ}{b^{8/3}\, s_{oa}^{1/2}} = \frac{0.01533}{s_{oa}^{1/2}} = \frac{0.01533}{(0.001)^{1/2}} = 0.4850$$

From ES-55 $\quad \dfrac{d_{na}}{b} = 0.449$ and $d_{na} = 15\,(0.449) = 6.73$ ft

   b. For the downstream reach

$$\frac{nQ}{b^{8/3}\, s_{ob}^{1/2}} = \frac{0.01533}{(0.004)^{1/2}} = 0.2425$$

From ES-55 $\quad \dfrac{d_{nb}}{b} = 0.31$ and $d_{nb} = 15\,(0.31) = 4.65$ ft

3. Solve for the water-surface profile and tabulate values in Table 1.
   a. column 1 lists arbitrarily selected values of depth between d_c and d_{na} or d_c and d_{nb}
   b. column 3 is read from ES-55
   c. column 4 is read from ES-24
   d. column 5 is the value of $\dfrac{nQ}{b^{8/3}\, s_o^{1/2}} = 0.4850$ or 0.2425 divided by column 3 (See step 2 above)
   e. column 6 is the value $Q/b = 100$ divided by column 4
   f. column 7 is read from ES-53 using columns 5 and 6
   g. column 10 is obtained by the relation $\ell_2 - \ell_1 = \frac{1}{s_o}(R_1 + R_2)(d_2 - d_1)$ · Eq. A.15

The value of R versus depth d is plotted in Fig. 1. The area under the curve between any two depths represents the length of reach between these depths. This plot is useful for ascertaining whether or not the selected depths in column 1 have been chosen at sufficiently close intervals to justify the use of the relation shown by Eq. A.15.

The profile can be readily plotted by using columns 1 and 11.

TABLE 1

Direction	(1) d	(2) $\frac{d}{b}$	(3) $\frac{nQ_{n,d}}{b^{8/3} s_c^{1/2}}$	(4) $\frac{Q_{c,d}}{b}$	(5) $\frac{Q}{Q_{n,d}}$	(6) $\frac{Q}{Q_{c,d}}$	(7) R	(8) $R_1 + R_2$	(9) $d_2 - d_1$	(10) $\ell_2 - \ell_1$	(11) Station
Upstream from break in grade, ← s_{oa} = 0.001	5.59	0.373	0.341	100.0	1.422	1.000	0	- 0.0177	-0.05	0.885	33+50
	5.64	0.376	0.347	101.7	1.398	0.983	- 0.0177	- 0.0591	-0.06	3.55	33+49.1
	5.70	0.380	0.354	103.8	1.370	0.963	- 0.0414	- 0.1246	-0.10	12.46	33+45.6
	5.80	0.387	0.366	107.0	1.325	0.935	- 0.0832	- 0.2212	-0.10	22.12	33+33.1
	5.90	0.393	0.377	110.5	1.286	0.905	- 0.138	- 0.346	-0.10	34.60	33+11.0
	6.00	0.400	0.390	113.8	1.244	0.879	- 0.208	- 0.506	-0.10	50.60	32+76.4
	6.10	0.407	0.402	117.1	1.206	0.854	- 0.298	- 0.725	-0.10	72.50	32+25.8
	6.20	0.413	0.415	120.6	1.169	0.829	- 0.427	- 1.045	-0.10	104.50	31+53.3
	6.30	0.420	0.428	124.1	1.133	0.806	- 0.618	- 1.569	-0.10	156.90	30+48.8
	6.40	0.427	0.442	127.7	1.097	0.783	- 0.951	- 2.441	-0.10	244.10	28+91.9
	6.50	0.433	0.454	131.2	1.068	0.762	- 1.49	- 4.33	-0.10	433.00	26+47.8
	6.60	0.440	0.467	134.9	1.039	0.741	- 2.84	- ∞	-0.13	∞	22+14.8
	6.73	0.449	0.485	140.2	1.000	0.713	- ∞				
Downstream from break in grade, ← s_{ob} = 0.004	5.59	0.373	0.341	100.0	0.711	1.000	0	- 0.0591	-0.09	1.33	33+50
	5.50	0.367	0.331	97.4	0.733	1.027	- 0.0591	- 0.2091	-0.10	5.23	33+51.3
	5.40	0.360	0.320	94.2	0.758	1.062	- 0.150	- 0.406	-0.10	10.15	33+56.6
	5.30	0.353	0.308	91.5	0.787	1.093	- 0.256	- 0.660	-0.10	16.50	33+66.7
	5.20	0.347	0.299	88.5	0.811	1.130	- 0.404	- 1.046	-0.10	26.15	33+83.2
	5.10	0.340	0.288	85.3	0.842	1.172	- 0.642	- 1.653	-0.10	41.33	34+09.4
	5.00	0.333	0.277	82.4	0.875	1.214	- 1.011	- 2.631	-0.10	65.78	34+50.7
	4.90	0.327	0.267	79.8	0.908	1.253	- 1.62	- 4.79	-0.10	119.75	35+16.5
	4.80	0.320	0.257	76.9	0.944	1.300	- 3.17	-14.70	-0.10	367.50	36+36.2
	4.70	0.313	0.247	74.1	0.982	1.350	-11.53	- ∞	-0.05	∞	40+03.7
	4.65	0.310	0.242	72.7	1.000	1.376	- ∞				

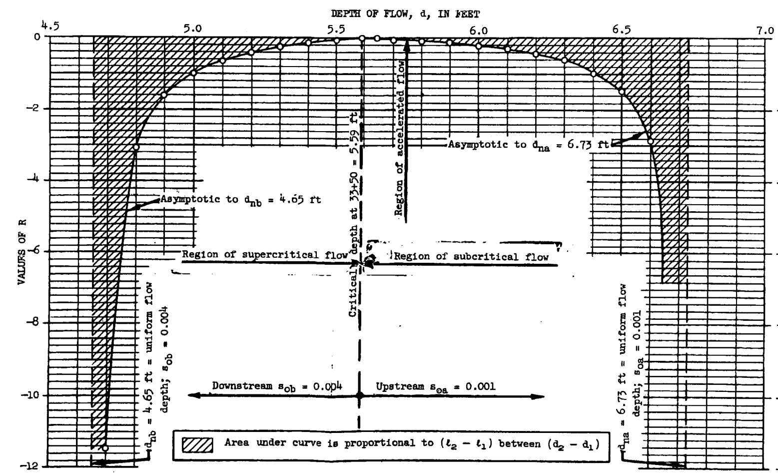


FIGURE 1

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO.
ES – 83
SHEET 1 OF 10
DATE 8-6-54

HYDRAULICS: NON-UNIFORM FLOW IN A PRISMATIC CHANNEL WITH A LEVEL BOTTOM — Example 2

EXAMPLE 2

Given: A prismatic trapezoidal earth spillway with 3 to 1 side slopes. The bottom of the spillway is level and is 200 ft long and 75 ft wide in the reach between the control section and the reservoir. Manning's coefficient of roughness n, of the spillway within this reach, is estimated to be 0.035. The spillway is expected to convey 1500 cfs.

Determine: A. The water-surface profile in the spillway between the reservoir and the control section.
B. The friction loss through this spillway measured in feet.
C. The elevation of the water surface in the reservoir when the spillway is discharging 1500 cfs.

Solution: Rewrite Eq. A.11 in the form

$$\frac{d\ell}{dd} = \frac{\left[\dfrac{\frac{Q}{b}}{\frac{Q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3}s_o^{1/2}}}{\frac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}}\right]^2} = R_o$$

where $Q_{n,d}$ is defined as the normal discharge corresponding to a depth d and channel bottom slope of $s_o = 1.0$. Equation A.11 is to be used only for channels with horizontal bottoms. The quantity s_o by definition is unity when working with Eq. A.11 and Manning's formula even though the actual slope of the channel is zero.

A.1. Solve for the depth of flow at the control section Sta 2+00. The depth of flow at the control section is critical depth corresponding to the discharge Q = 1500 cfs.

$$\frac{Q}{b} = \frac{1500}{75} = 20 \text{ cfs/ft} ; \quad \frac{z}{b} = \frac{3}{75} = 0.04$$

From ES-24, $d_c = 2.25$ ft

2. Solve for the value of $\dfrac{nQ}{b^{8/3}s_o^{1/2}}$

$$\frac{nQ}{b^{8/3}s_o^{1/2}} = \frac{(0.035)(1500)}{(75)^{8/3}(1)^{1/2}} = 5.2480 \times 10^{-4}$$

3. Prepare Table 1
(a) Column 1 lists arbitrarily selected depths greater than critical depth, beginning with $d_c = 2.25$ ft. The range of required depths cannot be readily predetermined.
(b) Column 2 lists the selected depths of column 1 divided by the bottom width of the channel.
(c) Column 3 is read from ES-55.
(d) Column 4 is read from ES-24.
(e) Column 5 lists $\dfrac{nQ}{b^{8/3}s_o^{1/2}} = 5.2480 \times 10^{-4}$ (see step 2 above) divided by column 3.
(f) Column 6 lists Q/b = 20 divided by column 4.
(g) Columns 7 and 8 are the squares of columns 5 and 6.
(h) Column 9 lists column 8 minus unity. This is the numerator of the right-hand member of Eq. A.11.
(i) Column 10 lists column 9 divided by column 7. This is equal to the left-hand member of Eq. A.11.
(j) Column 11 lists the sum of the values of R_o at the end sections of each reach.
(k) Column 12 lists the average value of R_o for each reach or column 11 times ½.
(l) Column 13 lists the difference in selected depth of column 1.
(m) Column 14 lists the length of each reach between the selected depths listed in column 1 and is column 12 times column 13.
(n) Column 15 lists the distances from the control section in the upstream direction and is the accumulated total of column 14.

B.1. The specific energy at the control section is

$$H_{ec} = d_c + \frac{v_c^2}{2g}$$

$$a_c = d_c(b + zd_c) = 2.25\,[75 + 3(2.25)] = 183.94 \text{ ft}^2$$

$$v_c = \frac{Q}{a_c} = \frac{1500}{183.94} = 8.155 \text{ ft/sec}$$

$$\frac{v_c^2}{2g} = \frac{(8.155)^2}{64.32} = 1.034 \text{ ft} \qquad H_{ec} = 2.25 + 1.034 = 3.284 \text{ ft}$$

2. The specific energy at Sta 0+00 is

$$H_{eo} = d_o + \frac{v_o^2}{2g}$$

$d_o = 3.77$ ft (by interpolating between d = 3.7 and d = 3.8 and observing that the control section is 200 ft downstream from the reservoir pool.)

$$a_o = d_o(b + zd_o) = 3.77\,[75 + 3(3.77)] = 325.39 \text{ ft}^2$$

$$v_o = \frac{Q}{a_o} = \frac{1500}{325.39} = 4.610 \text{ ft/sec}$$

$$\frac{v_o^2}{2g} = \frac{(4.610)^2}{64.32} = 0.3304 \text{ ft}$$

$$H_{eo} = 3.77 + 0.330 = 4.100 \text{ ft}$$

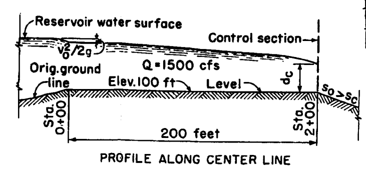
3. Since the spillway bottom is level, the difference in the specific energy head at Sta 0+00 and the control section ($H_{eo} - H_{ec}$) is the friction head loss h_f in this reach.

$$h_f = H_{eo} - H_{ec} = 4.100 - 3.284 = 0.816 \text{ ft}$$

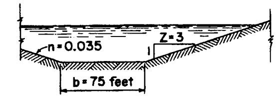
C.1. The elevation of the water surface in the reservoir is equal to the specific energy head at Sta 0+00 plus the elevation of the bottom of channel at Sta 0+00. Elevation of water surface in the reservoir is

$$100.0 + H_{eo} = 104.100 \text{ ft}$$

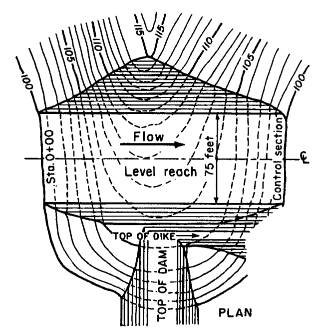
The trapezoidal shape of this spillway does not exist for the full depth of flow in the reach near the entrance of the spillway. By neglecting the effect of this condition, as is done in the example, the water surface elevation in the reservoir required to produce a given discharge through the spillway is slightly greater than the elevation required had this effect been evaluated.



PROFILE ALONG CENTER LINE



TYPICAL SECTION



PLAN

TABLE 1

1 d	2 $\frac{d}{b}$	3 $\frac{n\,Q_{n,d}}{b^{8/3}s_o^{1/2}}$	4 $\frac{Q_{c,d}}{b}$	5 $\frac{Q}{Q_{n,d}}$	6 $\frac{Q}{Q_{c,d}}$	7 $\left[\frac{Q}{Q_{n,d}}\right]^2$	8 $\left[\frac{Q}{Q_{c,d}}\right]^2$	9 $\left[\frac{Q}{Q_{c,d}}\right]^2 - 1$	10 R_o	11 $R_{o1} + R_{o2}$	12 $\frac{R_{o1} + R_{o2}}{2}$	13 $d_2 - d_1$	14 $\ell_2 - \ell_1$	15 Distance from Control Section
2.25	0.03000	0.00442	20.00	0.11873	1.0000	0.014097	1.00000	0	0					0
2.3	0.03067	0.00459	20.75	0.11434	0.9639	0.013074	0.92910	-0.07090	- 5.423	- 5.423	- 2.712	-0.05	0.136	0.136
2.4	0.03200	0.00492	22.10	0.10667	0.9050	0.011378	0.81903	-0.18097	- 15.905	- 21.328	- 10.664	-0.10	1.066	1.202
2.5	0.03333	0.00527	23.60	0.09958	0.8475	0.0099162	0.71826	-0.28174	- 28.412	- 44.317	- 22.158	-0.10	2.216	3.418
2.6	0.03467	0.00564	25.03	0.09305	0.7990	0.0086583	0.63840	-0.36160	- 41.763	- 70.175	- 35.087	-0.10	3.509	6.927
2.7	0.03600	0.00604	26.55	0.08689	0.7533	0.0075499	0.56746	-0.43254	- 57.290	- 99.053	- 49.527	-0.10	4.953	11.880
2.8	0.03733	0.00641	28.05	0.08187	0.7130	0.0067027	0.50837	-0.49163	- 73.348	-130.638	- 65.319	-0.10	6.532	18.412
2.9	0.03867	0.00680	29.65	0.07718	0.6745	0.0059568	0.45495	-0.54505	- 91.500	-164.848	- 82.424	-0.10	8.242	26.654
3.0	0.04000	0.00721	31.30	0.07279	0.6390	0.0052984	0.40832	-0.59168	-111.671	-203.171	-101.585	-0.10	10.159	36.813
3.1	0.04133	0.00763	33.00	0.06878	0.6061	0.0047307	0.36736	-0.63264	-133.730	-245.401	-122.701	-0.10	12.270	49.083
3.2	0.04267	0.00805	34.60	0.06519	0.5780	0.0042497	0.33408	-0.66592	-156.698	-290.428	-145.214	-0.10	14.521	63.604
3.3	0.04400	0.00849	36.25	0.06181	0.5517	0.0038205	0.30437	-0.69563	-182.078	-338.776	-169.388	-0.10	16.939	80.543
3.4	0.04533	0.00895	38.10	0.05864	0.5249	0.0034396	0.27552	-0.72448	-210.665	-392.743	-196.384	-0.10	19.638	100.181
3.5	0.04667	0.00937	39.90	0.05601	0.5013	0.0031371	0.25130	-0.74870	-238.668	-449.333	-224.679	-0.10	22.468	122.649
3.6	0.04800	0.00986	41.65	0.05323	0.4802	0.0028334	0.23059	-0.76941	-271.588	-510.256	-255.128	-0.10	25.513	148.162
3.7	0.04933	0.01035	43.60	0.05071	0.4587	0.0025720	0.21041	-0.78959	-306.995	-578.583	-289.292	-0.10	28.929	177.091
3.8	0.05067	0.01080	45.50	0.04859	0.4396	0.0023610	0.19325	-0.80675	-341.698	-648.693	-324.347	-0.10	32.435	209.526

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO.
ES-83
SHEET 2 OF 10
DATE 7-22-54

HYDRAULICS: NON-UNIFORM FLOW IN A PRISMATIC CHANNEL WITH A NEGATIVE BOTTOM SLOPE — EXAMPLE 3

EXAMPLE 3

Given: A prismatic earth spillway with side slopes of 3 to 1. The spillway has a level bottom upstream from the control section for a distance of 90 ft and an adverse slope of 10 to 1 for an upstream distance of 100 ft. The bottom of the spillway is 75 ft wide. Manning's coefficient of roughness n is estimated as 0.035 feet.

Determine: A. The water-surface profile in the spillway when the discharge is 1500 cfs.
B. The friction loss from Sta 0+00 to Sta 1+90 and the elevation of the water surface in the reservoir when the spillway is discharging 1500 cfs.
C. The friction loss through the spillway from Sta 0+00 to Sta 1+00.

Solution: Rewrite Eq. A.13 in the form

$$|s_o|\frac{d\ell}{dd} = \frac{\left[\dfrac{\frac{Q}{b}}{\frac{q_{c,d}}{b}}\right]^2 - 1}{\left[\dfrac{\frac{nQ}{b^{8/3}|s_o|^{1/2}}}{\frac{n\,q_{n,d}}{b^{8/3}|s_o|^{1/2}}}\right]^2 + 1} = R_n$$

A.1. Solve for the critical depth of flow corresponding to the discharge of 1500 cfs.

$$\frac{Q}{b} = \frac{1500}{75} = 20 \text{ cfs/ft}$$

From ES-24, d_c = 2.25 ft. This is the depth of flow at the control section.

2. Solve for the depth of flow at the break in grade between the 10 to 1 negative slope and the level reach. This was illustrated by Ex. 2 and found to be 3.347 ft.

3. Solve for the value of

$$\frac{nQ}{b^{8/3}|s_o|^{1/2}} = \frac{(0.035)(1500)}{(75)^{8/3}(0.10)^{1/2}} = 1.6596 \times 10^{-3}$$

4. Solve the water-surface profile upstream from the break in grade at Sta 1+00 starting with a depth of 3.347 ft and tabulate values in table 1.

(a) Column 1 lists arbitrarily selected values of depths between the depth of flow at Sta 1+00 and an approximated depth at Sta 0+00. The approximated depth at Sta 0+00 is estimated by assuming the water-surface elevation at Sta 0+00 to be slightly greater than the elevation of the energy gradient at Sta 1+00. The specific energy at Sta 1+00 is

$$H_{e1} = d_1 + \frac{v_1^2}{2g}$$

$$a_1 = d_1(b + zd) = 3.347\,[75 + 3(3.347)] = 284.63 \text{ ft}^2$$

$$v_1 = \frac{Q}{a_1} = \frac{1500}{284.63} = 5.27 \text{ ft/sec}$$

$$\frac{v_1^2}{2g} = \frac{(5.27)^2}{64.32} = 0.4318 \text{ ft}$$

$$H_{e1} = 3.347 + 0.4318 = 3.779 \text{ ft}$$

The elevation of the energy gradient at Sta 1+00 is

$$(100.00 + H_{e1}) = 103.779 \text{ ft}$$

The approximated depth of flow at Sta 0+00 is the difference in the elevation of the energy gradient at Sta 1+00 and bottom of channel at Sta 0+00.

$$103.779 - 90 = 13.779 \text{ ft}$$

Try 14 ft for an upper limit of d in column 1.

(b) Column 3 is read from ES-55.
(c) Column 4 is read from ES-24.
(d) Column 5 is the value of $\frac{nQ}{b^{8/3}|s_o|^{1/2}} = 1.6596 \times 10^{-3}$ (see step 3) divided by column 3.
(e) Column 6 is the value of Q/b = 20 divided by column 4.
(f) Columns 7 and 8 are the squares of columns 5 and 6.
(g) Column 9 lists the values of R_n as given by Eq. A.13 or is

$$R_n = \frac{\text{column 8 minus unity}}{\text{column 7 plus unity}}$$

(h) Column 10 lists the average value of R_n for each reach.
(i) Column 11 lists the difference in depth of flow at the end section of each reach.
(j) Column 12 lists column 11 divided by $|s_o|$.
(k) Column 13 lists the length of each reach and is column 10 times column 12.
(l) Column 14 lists the distance upstream from the break in grade at Sta 1+00 and is the accumulated total of column 13.
(m) Column 15 lists the stationing for the selected depths of flow used in column 1.

B.1. By Ex. 2, the specific energy at Sta 1+90 is H_{ec} = 3.284 ft.

2. The specific energy at Sta 0+00 is

$$H_{eo} = d_o + \frac{v_o^2}{2g} = 13.824 \text{ ft}$$

d_o = 13.81 ft by interpolation between d = 13 ft and d = 14 ft

3. The friction loss from Sta 0+00 to Sta 1+90 is the difference in the elevation of the energy gradient at Sta 0+00 to Sta 1+90. The elevation of the energy gradient at Sta 0+00 is the elevation of the bottom of the spillway plus the specific energy at Sta 0+00.

$$90.00 + H_{eo} = 103.824 \text{ ft}$$

The elevation of the energy gradient at Sta 1+90 is

$$100.00 + H_{ec} = 103.284$$

The friction loss is

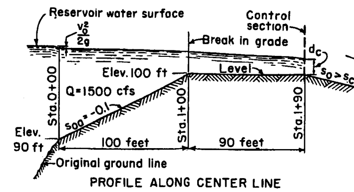
$$103.824 - 103.284 = 0.540 \text{ ft}$$

The elevation of the water surface in the reservoir is equal to the elevation of the energy gradient at Sta 0+00 or 103.824 ft.

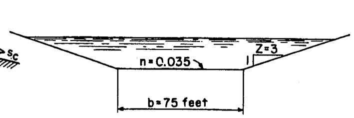
C.1. The friction loss from Sta 0+00 to Sta 1+00 is the difference in the elevation of the energy gradient. By A.4, the elevation of the energy gradient at Sta 1+00 is 103.779 ft. By B.3, the elevation of the energy gradient is 103.824 ft. The friction loss between Sta 0+00 and Sta 1+00 is

$$103.824 - 103.779 = 0.045 \text{ ft}$$

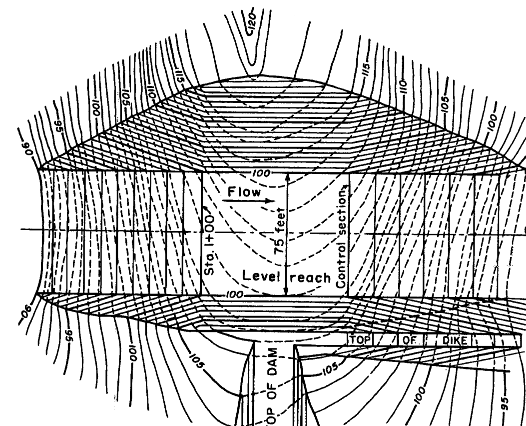
The trapezoidal shape of this spillway does not exist for the full depth of flow in the reach near the entrance of the spillway. By neglecting the effect of this condition, as is done in the example, the water surface elevation in the reservoir required to produce a given discharge through the spillway is slightly greater than the elevation required had this effect been evaluated.



PROFILE ALONG CENTER LINE



TYPICAL SECTION



PLAN

TABLE 1

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d	$\frac{d}{b}$	$\frac{n\,q_{n,d}}{b^{8/3}\|s_o\|^{1/2}}$	$\frac{q_{c,d}}{b}$	$\frac{Q}{q_{n,d}}$	$\frac{Q}{q_{c,d}}$	$\left[\frac{Q}{q_{n,d}}\right]^2$	$\left[\frac{Q}{q_{c,d}}\right]^2$	R_n	$\frac{R_{n1}+R_{n2}}{2}$	$d_2 - d_1$	$\frac{(d_2-d_1)}{\|s_o\|}$	$\ell_2 - \ell_1$	$\Sigma(\ell_2 - \ell_1)$	Station
3.347	0.04463	0.00875	37.10	0.1897	0.5391	0.03599	0.2906	-0.6848	-0.7049	-0.153	- 1.53	1.078	0	1+00.00
3.50	0.04667	0.00942	39.80	0.1762	0.5025	0.03105	0.2525	-0.7250	-0.7729	-0.50	- 5.00	3.865	1.078	0+98.92
4.00	0.05333	0.01186	49.50	0.1399	0.4040	0.01957	0.1632	-0.8207	-0.8487	-0.50	- 5.00	4.244	4.943	0+95.06
4.50	0.06000	0.01456	59.75	0.1140	0.3347	0.01300	0.1120	-0.8766	-0.8940	-0.50	- 5.00	4.470	9.187	0+90.81
5.00	0.06667	0.01752	70.50	0.09473	0.2837	0.008974	0.08049	-0.9113	-0.9312	-1.00	-10.0	9.312	13.66	0+86.34
6.00	0.08000	0.02410	94.95	0.06886	0.2106	0.004742	0.04435	-0.9511	-0.9608	-1.00	-10.0	9.608	22.97	0+77.03
7.00	0.09333	0.03175	121.9	0.05227	0.1641	0.002732	0.02693	-0.9704	-0.9757	-1.00	-10.0	9.757	32.58	0+67.42
8.00	0.1067	0.0402	152.1	0.04128	0.1315	0.001704	0.01729	-0.9810	-0.9842	-1.00	-10.0	9.842	42.33	0+57.67
9.00	0.1200	0.04974	185.4	0.03337	0.1079	0.001114	0.01164	-0.9873	-0.9892	-1.00	-10.0	9.892	52.18	0+47.82
10.0	0.1333	0.06025	221.4	0.02755	0.09033	0.0007590	0.008160	-0.9911	-0.9924	-1.00	-10.0	9.924	62.07	0+37.93
11.0	0.1467	0.07210	260.7	0.02302	0.07672	0.0005299	0.005886	-0.9936	-0.9945	-1.00	-10.0	9.945	71.99	0+28.01
12.0	0.1600	0.08460	303.2	0.01962	0.06596	0.0003849	0.004351	-0.9953	-0.9959	-1.00	-10.0	9.959	81.94	0+18.06
13.0	0.1733	0.09799	348.7	0.01694	0.05736	0.0002870	0.003290	-0.9964	-0.9969	-1.00	-10.0	9.969	91.90	0+ 8.10
14.0	0.1867	0.1142	397.5	0.01453	0.05031	0.0002111	0.002531	-0.9973					101.9	

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION- DESIGN SECTION

STANDARD DWG. NO.
ES-83
SHEET 3 OF 10
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HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL — Example 4.

THIS EXAMPLE ILLUSTRATES A METHOD TO DETERMINE THE APPROXIMATE DEPTH OF FLOW AT A SECTION

Generally channel cross sections for a natural channel are determined by field surveys at definite stations. The length of the reach between two consecutive sections is therefore fixed or given and a depth of flow is to be determined. In Example 1 the length was determined between two given (actually, selected) depths. The relationship between the depth of flow and any hydraulic characteristics of a cross section is the same for every cross section in a prismatic channel. This enables one to choose the length of the reach as the variable to be determined, and the length of the reach between two given depths is determined in Example 1. Since length of reach is given in natural channels the depth of flow is the variable to be determined. This complicates the solution for water-surface profiles because the depth of flow is implicity expressed in the differential equation.

Any method of water-surface profile determination requires that at least one depth of flow corresponding to the discharge, Q, under consideration be given at a station or be determinable. The determination of a depth of flow in a natural channel at a control section presents a trivial problem, for the depth of flow at the section is known to be the critical depth. Some channels for which water-surface profiles are to be determined have no control section, nor is the depth of flow given for any portion of the channel. This example is concerned with the determination of a depth of flow at station 0+00 so water-surface profile computations can be made upstream from station 0+00.

Given: Channel cross sections for a natural channel have been determined by field surveys at stations indicated by Fig. 1. Stationing is in a downstream direction. The roughness coefficient, n, has been estimated to be 0.035 for in-bank flow and 0.06 for out-bank flow. Dikes having 3 to 1 slope have been constructed as shown in Fig. 1. These dikes are sufficiently high to contain the flow of 7000 cfs. The depth of flow at station 17+90 has been determined to be not less than 14 ft nor greater than 18 ft for a discharge of 7000 cfs.

Determine: The approximate depth of flow at station 0+00 when the discharge is 7000 cfs.

Solution: Equation A.18 is used for this solution.

$$\frac{E_1 - E_2}{\left[\frac{1}{a_2^2} + \frac{s_o}{Q^2_{n,d_2}}(\ell_2 - \ell_1)g\right] - \left[\frac{1}{a_1^2} - \frac{s_o}{Q^2_{n,d_1}}(\ell_2 - \ell_1)g\right]} = \frac{Q^2}{2g}$$

1. Prepare tabular form given by Table 1.
   (a) Column 1 lists the stations of cross sections and elevation of channel bottom at these stations.
   (b) Column 2 is an arbitrary selection of flow depths for an estimated range of depth of flow. A selection of depths should be made at each major change in cross section shape and not over 3 or 4 ft interval between consecutive depths.
   (c) Columns 3 and 7 list the flow areas associated with the appropriate n values as obtained from Fig. 1.
   (d) Columns 4 and 8 list the wetted perimeters associated with the appropriate n values as obtained from Fig. 1.
   (e) Columns 5 and 9 list the cross section factor F read from ES-76.
   (f) Columns 6 and 10 are read from ES-77.
   (g) Column 11 is the sum of columns 6 and 10.
   (h) Column 12 is the square of the reciprocal of column 11. This can be read from the double scale of ES-77.

2. Prepare Figures 2 and 3 from Table 1. These values are plotted on log-log coordinate paper. Figure 2 will be used to determine critical slope. Figure 3 will be used later to determine values of $s_o \div Q^2_{n,d}$ for intermediate values of depths of flow.

3. Solve which sections are control sections for the discharge of 7000 cfs.
   (a) Prepare tabular form as given by Table 2.
      (i) Columns 1 and 2 are the same as columns 1 and 2 of Table 1
      (ii) Column 3 lists the top width corresponding to the depth in column 2
      (iii) Column 4 lists the total area corresponding to the depth in column 2
      (iv) Column 5 is read from ES-75
   (b) Prepare Figure 4 on log-log coordinate paper. Values are obtained from Table 2.
   (c) Prepare tabular form as given by Table 3.
      (i) Column 2 will be considered later
      (ii) Column 4 is read from Figure 4 on line Q = 7000 cfs and are critical depths d_c for Q = 7000 cfs
      (iii) Column 5 is read from Figure 2 for critical depths, d_c, in column 4
      (iv) Column 6 lists $s_c^{1/2}$ which is 7000 cfs divided by column 5
      (v) Column 8 lists the slope of channel bottom

No control sections exist for a discharge of 7000 cfs for all bottom slopes, s_o, are less than critical slope, s_c. Thus, flow is subcritical and computations will be carried in an upstream direction.

4. Solve for approximate depth of flow at station 0+00.
   (a) Prepare tabular form given by Table 4.
      (i) Column 2 lists the same range of depths as Table 1 at closer intervals
      (ii) Prepare Figure 5 on log-log coordinate paper from Table 2. Observe that the right hand scale is the square of the reciprocal of the left hand scale in ES-77 and may be used in determining $1 \div a^2$ values
      (iii) Column 3 is read from Figure 5
      (iv) Column 4 is read from Figure 3
      (v) Column 5 lists the product of column 4 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of the downstream reach from the station under consideration. The station under consideration is section 1 as is indicated by the subscript d_1 of Q_{n,d_1} in this column heading.
      (vi) Column 6 is the product of column 4 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of the upstream reach from the station under consideration. The station under consideration is section 2 as is indicated by the subscript d_2 of Q_{n,d_2} in this column heading.
      (vii) Column 7 lists column 3 minus column 5
      (viii) Column 8 lists column 3 plus column 6
   (b) It may be desirable to prepare Figures 6 and 7, particularly if values of d have not been selected sufficiently close in column 2.
   (c) Prepare Figure 8.
      (i) Plot U_1^- vs elevation curves
      (ii) Plot U_2^+ vs elevation curves
      (iii) Column 2 of Table 3 gives the slope of straight lines connecting the curves U_1^- and U_2^+. The slope scale for Figure 8 is determined by dividing the ordinate above the reference point by the distance between the scale and the reference point. In this example the reference point has been arbitrarily taken at elevation 1185 and an abscissa of 2.0×10^{-8} from the slope scale. The value of the slope scale at the line having an elevation of 1186 is $(1186 - 1185) \div 2.0 \times 10^{-6} = 5 \times 10^5$.
      (iv) Make the graphical solution for the water-surface profile if the depth of flow at station 17+90 is 14 ft. The beginning point from which a straight line is drawn having the slope given by column 2, Table 3, is at depth of 14 ft or elevation 1184.2 on U_2^+ curve. The elevation of flow at station 13+30 is the intersection of this straight line with the U_1^- curve. The elevation of flow is now known at station 13+30 and the elevation of flow at 9+10 is determined in a similar manner. This procedure is continued to station 0+00. The depth obtained at 0+00 is the depth of flow if the depth of flow at 17+90 is 14 ft.

      A graphical solution for the water-surface profile if the depth of flow at station 17+90 is 18 ft is obtained in a similar manner. Observe the depth of flow at station 0+00 is not less than 13.61 ft nor greater than 14.22 ft for it has been given the depth of flow at 17+90 is not less than 14 ft nor greater than 18 ft. Thus, the depth of flow at station 0+00 has been closely approximated. If a closer approximation is desired it will be necessary to determine water-surface profiles from a station downstream from station 17+90
      (v) Figure 9 shows water-surface profiles determined for various depths at station 17+90.

TABLE 1

1	2	3	4	5	6	7	8	9	10	11	12
Station and Bottom Elevation	d ft	a ft²	p ft	F	$\frac{Q_{n,d}}{s_o^{1/2}}$	a ft²	p ft	F	$\frac{Q_{n,d}}{s_o^{1/2}}$	$\frac{Q_{n,d}}{s_o^{1/2}}$	$\frac{s_o}{Q^2_{n,d}}$
	n = 0.035					n = 0.06				composite n	
17+90	9.6	384	64.9	1867	53,300	15.5	38.2	12.65	211	53,511	3.485×10^{-10}
	11.6	508	64.9	2985	85,300	103.1	50.8	246.0	4,100	89,400	1.250×10^{-10}
1170.2	13.6	632	64.9	4284	122,400	214.7	63.5	720.0	12,000	134,400	5.530×10^{-11}
	15.6	756	64.9	5772	164,900	350.3	76.1	1440.0	24,000	188,900	2.806×10^{-11}
	17.6	880	64.9	7435	212,400	515.0	88.7	2480.0	41,330	253,730	1.553×10^{-11}
13+30	9.4	376	61.2	1872	53,500	15.0	40.2	11.50	1,920	53,692	3.475×10^{-10}
	11.4	492	61.2	2930	83,700	107.0	52.8	254.4	4,240	87,940	1.296×10^{-10}
1171.6	13.4	608	61.2	4180	119,400	223.0	65.5	750.0	12,500	131,900	5.750×10^{-11}
	15.4	724	61.2	5585	159,600	363.0	78.2	1500.0	25,000	184,600	2.932×10^{-11}
	17.4	840	61.2	7155	204,400	527.0	90.8	2530.0	42,180	246,580	1.644×10^{-11}
9+10	10.2	439	69.9	2221	63,500	23.6	52.2	20.6	343	63,843	2.453×10^{-10}
	12.2	571	69.9	3439	98,300	139.6	64.8	346.0	5,767	104,067	9.220×10^{-11}
1173.6	14.2	703	69.9	4865	139,000	279.6	77.4	978.7	16,310	155,310	4.140×10^{-11}
	16.2	835	69.9	6495	185,600	443.6	90.1	1908.0	31,800	217,400	2.118×10^{-11}
4+50	12.4	623	89.3	3380	96,600	21.4	30.6	25.05	418	97,018	1.062×10^{-10}
	14.4	795	89.3	5070	144,900	94.4	43.3	236.0	3,933	148,833	4.525×10^{-11}
1174.8	16.4	967	89.3	7030	200,900	191.4	56.0	646.0	10,770	211,670	2.235×10^{-11}
0+00	13.4	615	83.7	3455	98,700	15.4	28.9	15.04	251	98,951	1.022×10^{-10}
	14.4	694	83.7	4225	120,700	47.4	35.2	85.90	1,432	122,132	6.690×10^{-11}
1175.8	15.4	773	83.7	5060	144,600	85.4	41.6	204.8	3,413	148,013	4.560×10^{-11}
	16.4	852	83.7	5950	170,000	129.4	47.9	374.0	6,233	176,233	3.220×10^{-11}
	17.4	931	83.7	6890	196,900	179.4	54.3	591.5	9,858	206,758	2.350×10^{-11}

TABLE 2

1	2	3	4	5
Station and Bottom Elevation	Depth ft	T ft	a ft²	$Q_{c,d}$ cfs
0+00 1175.8	13.4	108.2	630	8,610
	14.4	114.2	741	10,730
	15.4	120.2	858	13,100
	16.4	126.2	981	15,540
	17.4	132.2	1110	18,250
4+50 1174.8	12.4	116.5	644	8,630
	14.4	128.5	889	13,280
	16.4	140.5	1158	18,850
9+10 1173.6	10.2	118.0	463	5,200
	12.2	130.0	711	9,450
	14.2	142.0	983	14,700
	16.2	154.0	1279	20,910
13+30 1171.6	9.4	98.0	391	4,410
	11.4	110.0	599	7,925
	13.4	122.0	831	12,310
	15.4	134.0	1087	17,570
	17.4	146.0	1367	23,800
17+90 1170.2	9.6	99.8	399	4,535
	11.6	111.8	611	8,110
	13.6	123.8	847	12,630
	15.6	135.8	1106	17,930
	17.6	147.8	1395	24,400

TABLE 3

1	2	3	4	5	6	7	8	
Q	$\frac{Q^2}{2g}$	Sta	d_c	$\frac{Q_{n,d_c}}{s_c^{1/2}}$	$s_c^{1/2}$	s_c	s_o	Remarks
7000	7.618×10^5	0+00	12.53*	81,000	0.08642	0.007468	0.00222	$s_c > s_o$
		4+50	11.50*	78,100	0.08963	0.008034	0.00261	$s_c > s_o$
		9+10	11.14	81,500	0.08589	0.007377	0.00476	$s_c > s_o$
		13+30	10.95	79,600	0.08794	0.007733	0.00304	$s_c > s_o$
		17+90	11.05	78,500	0.08917	0.007951		

*These values were obtained by extrapolation from Figure 4.

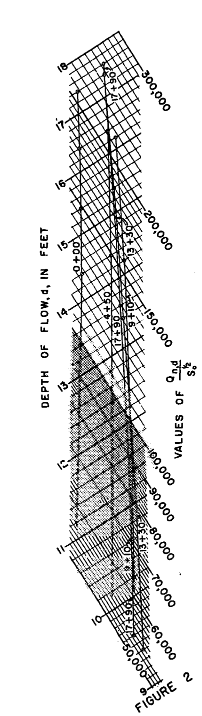

FIGURE 2

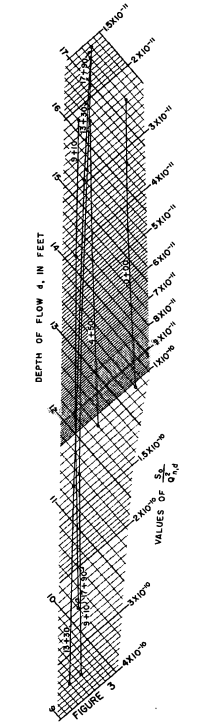

FIGURE 3

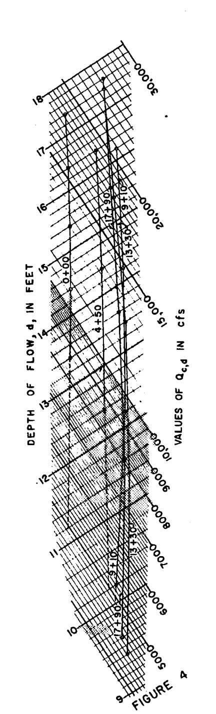

FIGURE 4

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
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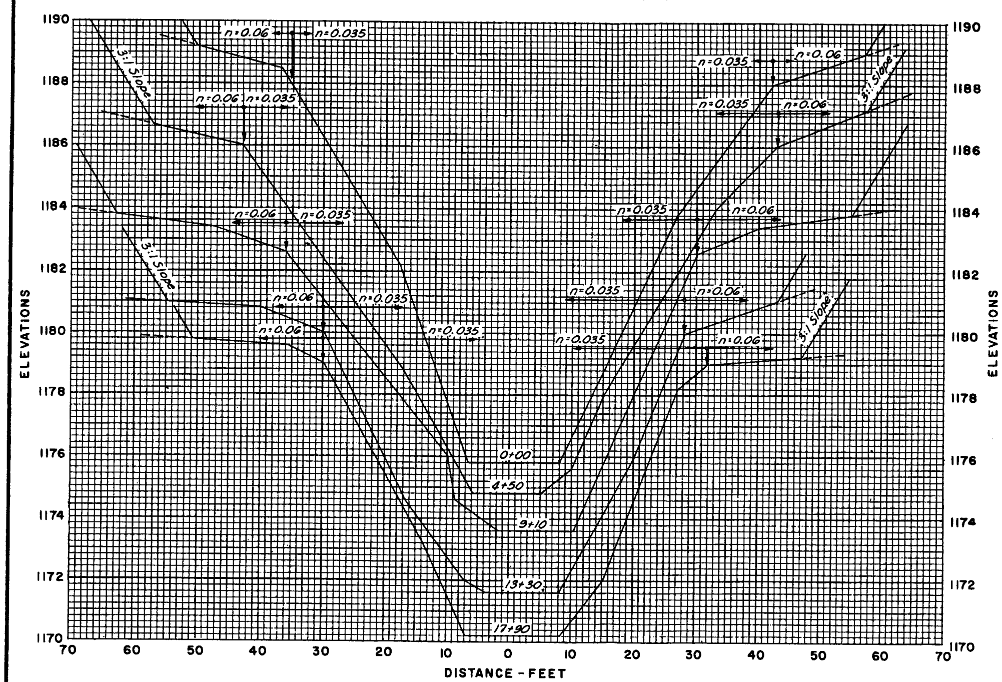
HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL — Example 4.

FIGURE 1 — GIVEN CROSS SECTIONS



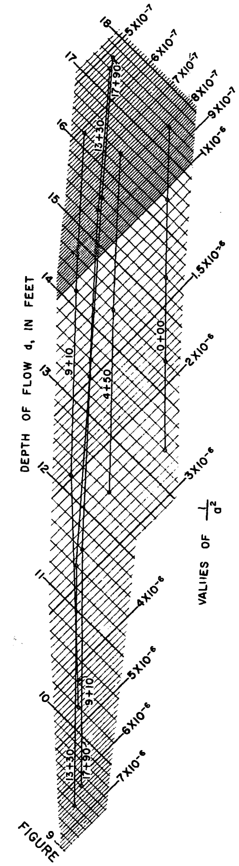


FIGURE 5

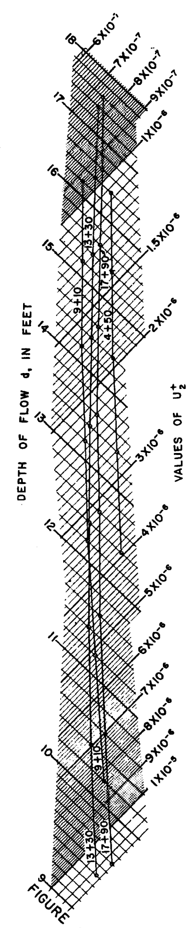


FIGURE 6

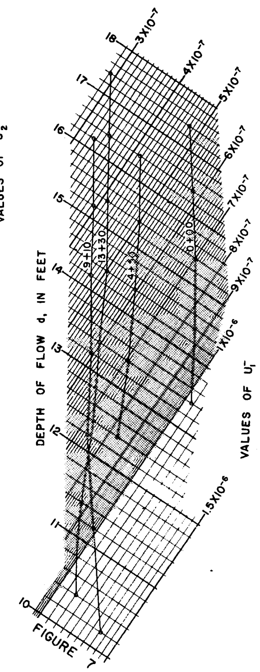


FIGURE 7

U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 83 SHEET 5 OF 10 DATE 4-22-54

HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL — Example 4

TABLE 4

1	2	3	4	5	6	7	8	9
Station and Elevation	d ft	$\frac{1}{a^2}$	$\frac{s_o}{Q^2 n,d}$	$\frac{s_o(\ell_2-\ell_1)g}{Q^2 n,d_1}$	$\frac{s_o(\ell_2-\ell_1)g}{Q^2 n,d_2}$	U_1^-	U_2^+	E
17+90	9.6	6.260×10^{-6}	3.485×10^{-10}		5.156×10^{-6}		11.42×10^{-6}	1179.8
	10.6	4.010×10^{-6}	2.045×10^{-10}		3.025×10^{-6}		7.035×10^{-6}	1180.8
1170.2	11.6	2.678×10^{-6}	1.250×10^{-10}		1.849×10^{-6}		4.527×10^{-6}	1181.8
	12.6	1.915×10^{-6}	8.210×10^{-11}		1.215×10^{-6}		3.130×10^{-6}	1182.8
	13.6	1.393×10^{-6}	5.530×10^{-11}		0.8181×10^{-6}		2.213×10^{-6}	1183.8
	14.6	1.060×10^{-6}	3.920×10^{-11}		0.5799×10^{-6}		1.640×10^{-6}	1184.8
	15.6	0.8170×10^{-6}	2.806×10^{-11}		0.4151×10^{-6}		1.232×10^{-6}	1185.8
	16.6	0.6480×10^{-6}	2.085×10^{-11}		0.3084×10^{-6}		0.9565×10^{-6}	1186.8
	17.6	0.5130×10^{-6}	1.553×10^{-11}		0.2297×10^{-6}		0.7427×10^{-6}	1187.8
13+30	9.4	6.540×10^{-6}	3.475×10^{-10}	5.141×10^{-6}	4.694×10^{-6}	1.399×10^{-6}	11.23×10^{-6}	1181.0
	10.4	4.180×10^{-6}	2.080×10^{-10}	3.077×10^{-6}	2.809×10^{-6}	1.103×10^{-6}	6.990×10^{-6}	1182.0
1171.6	11.4	2.785×10^{-6}	1.296×10^{-10}	1.917×10^{-6}	1.750×10^{-6}	0.8677×10^{-6}	4.536×10^{-6}	1183.0
	12.4	2.000×10^{-6}	8.620×10^{-11}	1.275×10^{-6}	1.164×10^{-6}	0.7248×10^{-6}	3.164×10^{-6}	1184.0
	13.4	1.447×10^{-6}	5.750×10^{-11}	0.8506×10^{-6}	0.7767×10^{-6}	0.5964×10^{-6}	2.224×10^{-6}	1185.0
	14.4	1.105×10^{-6}	4.085×10^{-11}	0.6043×10^{-6}	0.5518×10^{-6}	0.5007×10^{-6}	1.657×10^{-6}	1186.0
	15.4	0.8450×10^{-6}	2.932×10^{-11}	0.4337×10^{-6}	0.3960×10^{-6}	0.4112×10^{-6}	1.241×10^{-6}	1187.0
	16.4	0.6680×10^{-6}	2.185×10^{-11}	0.3232×10^{-6}	0.2951×10^{-6}	0.3448×10^{-6}	0.9631×10^{-6}	1188.0
	17.4	0.5340×10^{-6}	1.644×10^{-11}	0.2432×10^{-6}	0.2221×10^{-6}	0.2908×10^{-6}	0.7561×10^{-6}	1189.0
9+10	10.2	4.670×10^{-6}	2.453×10^{-10}	3.313×10^{-6}	3.629×10^{-6}	1.357×10^{-6}	8.299×10^{-6}	1183.8
	11.2	2.975×10^{-6}	1.475×10^{-10}	1.992×10^{-6}	2.182×10^{-6}	0.9827×10^{-6}	5.157×10^{-6}	1184.8
1173.6	12.2	1.975×10^{-6}	9.220×10^{-11}	1.245×10^{-6}	1.364×10^{-6}	0.7296×10^{-6}	3.339×10^{-6}	1185.8
	13.2	1.415×10^{-6}	6.100×10^{-11}	0.8239×10^{-6}	0.9024×10^{-6}	0.5911×10^{-6}	2.317×10^{-6}	1186.8
	14.2	1.036×10^{-6}	4.140×10^{-11}	0.5592×10^{-6}	0.6125×10^{-6}	0.4768×10^{-6}	1.649×10^{-6}	1187.8
	15.2	0.7910×10^{-6}	2.950×10^{-11}	0.3985×10^{-6}	0.4364×10^{-6}	0.3925×10^{-6}	1.227×10^{-6}	1188.8
	16.2	0.6120×10^{-6}	2.118×10^{-11}	0.2861×10^{-6}	0.3133×10^{-6}	0.3259×10^{-6}	0.9253×10^{-6}	1189.8
4+50	12.4	2.410×10^{-6}	1.062×10^{-10}	1.571×10^{-6}	1.537×10^{-6}	0.8389×10^{-6}	3.947×10^{-6}	1187.2
	13.4	1.725×10^{-6}	6.860×10^{-11}	1.015×10^{-6}	0.9928×10^{-6}	0.7102×10^{-6}	2.718×10^{-6}	1188.2
1174.8	14.4	1.264×10^{-6}	4.525×10^{-11}	0.6694×10^{-6}	0.6549×10^{-6}	0.5946×10^{-6}	1.919×10^{-6}	1189.2
	15.4	0.9670×10^{-6}	3.160×10^{-11}	0.4675×10^{-6}	0.4573×10^{-6}	0.4995×10^{-6}	1.424×10^{-6}	1190.2
	16.4	0.7470×10^{-6}	2.235×10^{-11}	0.3306×10^{-6}	0.3234×10^{-6}	0.4164×10^{-6}	1.071×10^{-6}	1191.2
0+00	13.4	2.518×10^{-6}	1.022×10^{-10}	1.479×10^{-6}		1.0390×10^{-6}		1189.2
	14.4	1.822×10^{-6}	6.690×10^{-11}	0.9682×10^{-6}		0.8538×10^{-6}		1190.2
1175.8	15.4	1.360×10^{-6}	4.560×10^{-11}	0.6599×10^{-6}		0.7001×10^{-6}		1191.2
	16.4	1.038×10^{-6}	3.220×10^{-11}	0.4660×10^{-6}		0.5720×10^{-6}		1192.2
	17.4	0.8100×10^{-6}	2.350×10^{-11}	0.3401×10^{-6}		0.4699×10^{-6}		1193.2

FIGURE 8

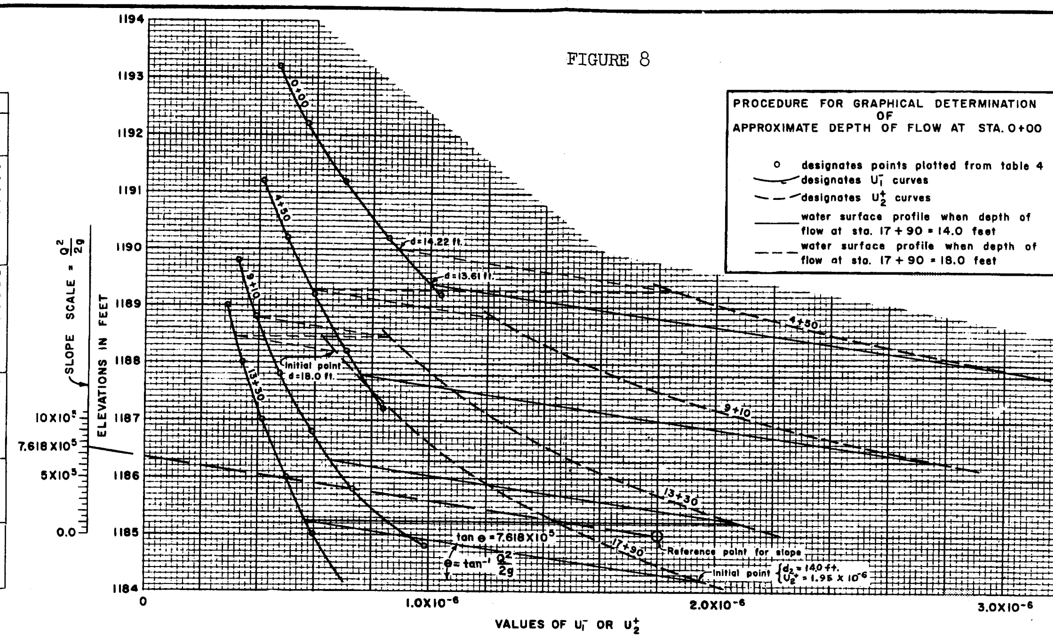
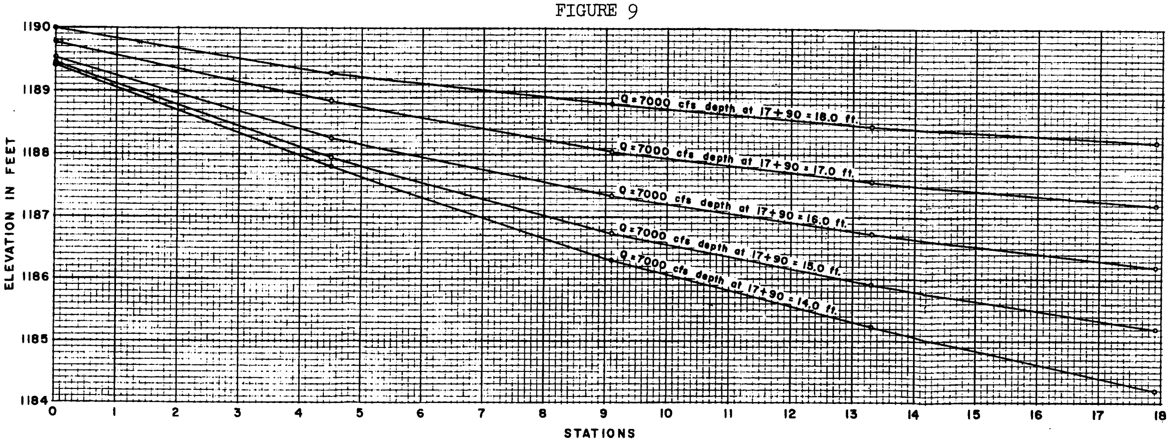


FIGURE 9



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-83
SHEET 6 OF 10
DATE 4-22-54

HYDRAULICS: NON-UNIFORM FLOW IN A NATURAL CHANNEL FOR VARIOUS DISCHARGES
Example 5

EXAMPLE 5

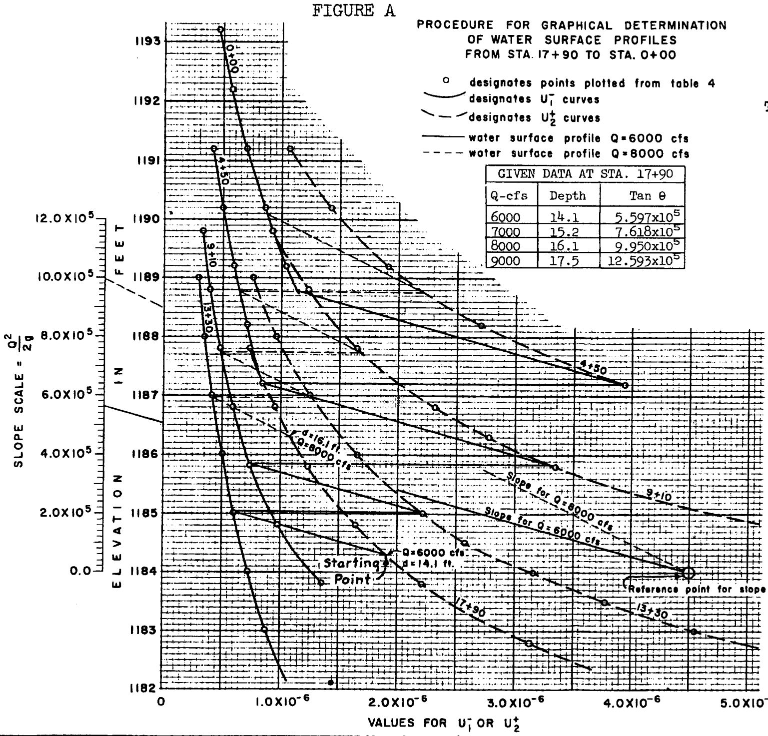
This example deals with the same natural channel as given in Ex. 4 merely to eliminate repetition of an additional set of similar data. Example 5 is distinctly different from Ex. 4 and the problems are in no way related. The approximate depth of flow was determined at Sta 0+00 in Ex. 4. In Ex. 5 the depths at Sta 17+90 have been determined and water-surface profiles are to be determined for various discharges.

Given: Channel cross sections and stationing of a natural channel along with roughness coefficient n as shown by Fig. 1 of Ex. 4. The dikes are sufficiently high to contain flows of 9000 cfs. The depths of flow at Sta 17+90 have been determined for various discharges.

Determine: The water-surface profiles for discharges 6000, 7000, 8000, and 9000 cfs and the discharge vs depth curve at Sta 0+00.

Discharge cfs	Given depth of flow at sta. 17+90 ft
6000	14.1
7000	15.2
8000	16.1
9000	17.5

FIGURE A



PROCEDURE FOR GRAPHICAL DETERMINATION OF WATER SURFACE PROFILES FROM STA. 17+90 TO STA. 0+00

- o designates points plotted from table 4
- ——— designates U_1^- curves
- — — designates U_2^+ curves
- ——— water surface profile Q=6000 cfs
- - - - water surface profile Q=8000 cfs

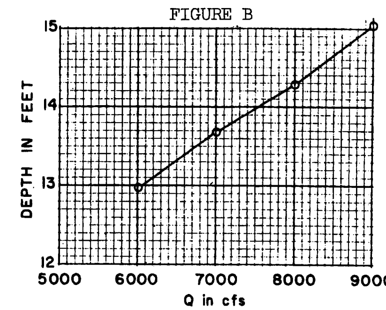
GIVEN DATA AT STA. 17+90		
Q-cfs	Depth	Tan θ
6000	14.1	5.597x10^5
7000	15.2	7.618x10^5
8000	16.1	9.950x10^5
9000	17.5	12.593x10^5

Solution: Equation A.18 is used for this solution. (See Ex. 4)

1. Prepare tabular form given by Table 1, Ex. 4.
2. Prepare Figs. 2 and 3 of Ex. 4.
3. Solve which sections are control sections for discharges of 6000, 7000, 8000, and 9000 cfs. This procedure is described by Ex. 4 and the results are given in Table A. No control sections exist for all discharges considered since all bottom slopes s_o are less than critical slope s_c.
4. Solve for water-surface profiles corresponding to the given discharges.
   a. Prepare tabular form given by Table 4, Ex. 4.
   b. It may be desirable to prepare Figs. 6 and 7 of Ex. 4.
   c. Prepare Fig. A similar to Fig. 8 of Ex. 4.
      (i) Plot U_1^- vs elevation curves.
      (ii) Plot U_2^+ vs elevation curves.
      (iii) Column 2 of Table A gives the slopes of straight lines connecting the curves U_1^- and U_2^+ for the various discharges. This procedure is explained in Ex. 4, item 4c, (iii)
      (iv) Make graphical solution for water surface profiles corresponding to the given discharges. and depths of flow at Sta 17+90. This procedure has been described in Ex. 4.

The graphical solution for the discharges Q = 6000 and 8000 cfs are shown. The discharge-depth curve at Sta 0+00 is given by Fig. B.

FIGURE B



RATING CURVE AT STA. 0+00

TABLE A

1	2	3	4	5	6	7	8	9
Q	$\frac{Q^2}{2g}$	Station	d_c	$\frac{Q_n, d_c}{s_c^{1/2}}$	$s_c^{1/2}$	s_c	s_o	Remarks
6000	$5.597x10^5$	0+00	11.92*	69,000	0.08696	0.007562	.00222	$s_c > s_o$
		4+50	10.90*	67,000	0.08955	0.008019	.00261	$s_c > s_o$
		9+10	10.65	72,000	0.08333	0.006944	.00476	$s_c > s_o$
		13+30	10.40	69,700	0.08608	0.007410	.00304	$s_c > s_o$
		17+90	10.52	68,800	0.08721	0.007606		
7000	$7.618x10^5$	0+00	12.55*	81,000	0.08642	0.007468	.00222	$s_c > s_o$
		4+50	11.50*	78,100	0.08963	0.008034	.00261	$s_c > s_o$
		9+10	11.14	81,500	0.08589	0.007377	.00476	$s_c > s_o$
		13+30	10.95	79,600	0.08794	0.007733	.00304	$s_c > s_o$
		17+90	11.05	78,500	0.08917	0.007951		
8000	$9.950x10^5$	0+00	13.10*	92,000	0.08696	0.007562	.00222	$s_c > s_o$
		4+50	12.05*	90,000	0.08889	0.007901	.00261	$s_c > s_o$
		9+10	11.60	91,000	0.08791	0.007728	.00476	$s_c > s_o$
		13+30	11.44	88,900	0.08999	0.008098	.00304	$s_c > s_o$
		17+90	11.55	88,200	0.09070	0.008226		
9000	$12.59x10^5$	0+00	13.60	103,000	0.08738	0.007635	.00222	$s_c > s_o$
		4+50	12.56	101,000	0.08911	0.007941	.00261	$s_c > s_o$
		9+10	12.02	101,000	0.08911	0.007941	.00476	$s_c > s_o$
		13+30	11.92	98,000	0.09184	0.008435	.00304	$s_c > s_o$
		17+90	12.05	98,750	0.09114	0.008306		

*These values were obtained by extrapolation from Figure 4.

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION -DESIGN SECTION

STANDARD DWG. NO.
ES- 83
SHEET 7 OF 10
DATE: 5-13-54

HYDRAULICS: NON-UNIFORM FLOW IN A NON-PRISMATIC CHANNEL WITH LEVEL AND ADVERSE BOTTOM SLOPES
Example 6.

EXAMPLE 6

Given: A trapezoidal earth spillway 190 ft long from the reservoir to the control section. Stationing is in a downstream direction and Sta 0+00 is located at the reservoir. The bottom of the spillway at Sta 0+00 is 95 ft wide and converges uniformly to a width of 75 ft at Sta 1+00. The bottom width of 75 ft is constant from Sta 1+00 to Sta 1+90. The bottom slope of the spillway between Sta 0+00 and Sta 1+00 is adverse and is -3% between Sta 0+00 and Sta 0+80 and -20% between Sta 0+80 and Sta 1+00. The bottom of the spillway is level between Sta 1+00 and Sta 1+90. Manning's coefficient of roughness n is estimated to be 0.035.

Determine: A. The water-surface profiles for discharges of 600, 800, 1000, 1200, 1500, and 1800 cfs in this spillway.
B. Curve showing discharge vs elevation of water surface in the reservoir.
C. Friction loss in spillway between Sta 0+00 and Sta 1+90 for Q = 1500 cfs.

Solution: Equation A.18 is used for this solution.

$$\frac{E_1 - E_2}{\left[\frac{1}{a_2^2} + \frac{s_o}{Q_{n,d_2}^2}(\ell_2 - \ell_1)g\right] - \left[\frac{1}{a_1^2} - \frac{s_o}{Q_{n,d_1}^2}(\ell_2 - \ell_1)g\right]} = \frac{Q^2}{2g}$$

Stations 0+00, 0+50, 0+80, 0+90, 1+00, 1+50, 1+70, 1+80, and 1+90 have been arbitrarily selected as those stations at which depths of flow are to be determined.

A.1. Prepare tabular form given by Table 1.
- (a) Column 1 lists the selected stations at which depths of flow are to be determined and the elevations of the channel bottom at these stations.
- (b) Column 2 is an arbitrary selection of flow depths for an estimated range of depth of flow.
- (c) Column 3 lists the bottom widths of the channel at the selected stations in column 1.
- (d) Column 4 lists the cross-sectional areas for the depths in column 2.
- (e) Column 5 lists the wetted perimeters for the depths in column 2.
- (f) Column 6 is read from ES-76.
- (g) Column 7 is the square of the reciprocal of column 4. This can be read from the double scale of ES-77.
- (h) Column 8 is read from ES-77.
- (i) Column 9 lists the product of Column 8 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of reach downstream from the station under consideration. The station under consideration is section 1 as is indicated by the subscript d_1 of Q_{n,d_1} in this column heading.
- (j) Column 10 lists the product of column 8 and $(\ell_2 - \ell_1)g$ where $(\ell_2 - \ell_1)$ is the length of reach upstream from the station under consideration. The station under consideration is section 2 as is indicated by the subscript d_2 of Q_{n,d_2} in this column heading.
- (k) Column 11 lists values of U_1^- when the section under consideration is the upstream section or section 1. Column 11 lists column 7 minus column 9. (See Eq. A.20)
- (l) Column 12 lists values of U_2^+ when the section under consideration is the downstream section of the reach or section 2. Column 12 lists column 7 plus column 10. (See Eq. A.19)
- (m) Column 13 lists the elevations of the water surfaces corresponding to the depths of column 2 or the depths of column 2 added to the bottom elevations of column 1.

2. Prepare Figs. 1 and 2 by plotting column 2 vs columns 11 and 12 respectively on log-log paper.
3. Solve for the values of $Q^2/2g$ as shown by Table 2 for the discharges under consideration.
4. Solve for water-surface profiles corresponding to the given discharges.
   - (a) Prepare Figs. 3a and 3b.
     - (i) Plot U_1^- vs elevation curves for various values of d shown in Fig. 1.
     - (ii) Plot U_2^+ vs elevation curves for various values of d shown in Fig. 2.
     - (iii) Column 2 of Table 2 gives the slopes of straight lines connecting the U_1^- and U_2^+ curves for the various discharges. This procedure is explained in Ex. 4, item 4c, (iii).
     - (iv) Make graphical solution for water-surface profiles corresponding to the given discharges at Sta 0+00. This procedure has been described in Ex. 4, item 4c, (iv).

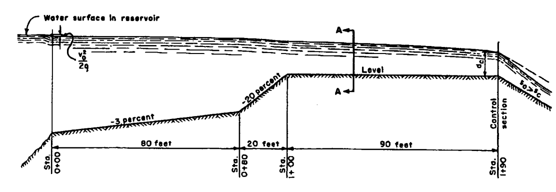
The graphical solutions for discharges 800 and 1500 have been shown on Figs. 3a and 3b respectively. Two figures, 3a and 3b, are drawn instead of a single figure because of the desirability of a scale adjustment. The water-surface profiles for the various discharges are given by Fig. 4. The rapid drop in elevation in water-surface profiles between Sta 0+80 and Sta 1+00 is the result of a change from potential head to kinetic head.

Observe that the depth of flow at Sta 1+00 for a discharge Q = 1500 cfs is 3.37 ft. Example 2 for this same channel and discharge shows a depth of flow of 3.347 ft at a section 90 ft upstream from the break in grade. The principal cause for the difference in the answers for the depth at the section 90 ft upstream from the control section by the two methods is the number of sections used upstream from the control section in computing this depth of flow. The result d = 3.347 ft, obtained in Ex. 2, is by far the more accurate because eleven intermediate sections were used in computing this depth, while in Ex. 6 only three intermediate sections were used in computing the depth d = 3.37 ft.

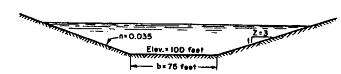
B.1. The depths of flow at Sta 0+00 for various discharges are given by Table 2. These values were read from Figs. 3a and 3b. The water-surface elevation in the reservoir is greater than the water-surface elevation at Sta 0+00 by the quantity $v_o^2/2g$ where v_o is the velocity in ft/sec at Sta 0+00. The curve showing discharge vs elevation of water surface in the reservoir is shown by Fig. 5.

C.1. The friction loss between Sta 0+00 and Sta 1+90 is the difference in elevation of the energy gradient. The elevation of the energy gradient at Sta 0+00 is 103.831 ft when Q = 1500 cfs. See Table 2. The elevation of the energy gradient at Sta 1+90 is 103.284 ft. See Ex. 2. The friction loss is 103.831 − 103.284 = 0.547 ft.

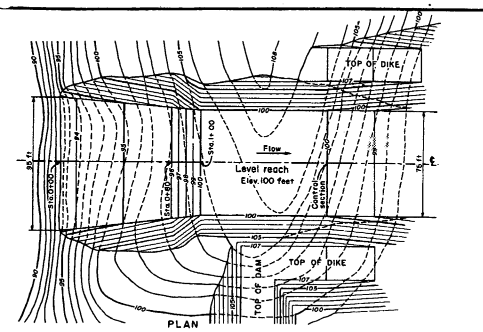
The trapezoidal shape of this spillway does not exist for the full depth of flow in the reach near the entrance of the spillway. By neglecting the effect of this condition, as is done in the example, the water-surface elevation in the reservoir required to produce a given discharge through the spillway is slightly greater than the elevation required had this effect been evaluated.



PROFILE ALONG CENTER LINE



SECTION A—A



PLAN

TABLE 2

Q cfs	$\frac{Q^2}{2g}$ = tan θ	Water-Surface Elevation at Sta 0+00	$\frac{v_o^2}{2g}$	Water-Surface Elevation in the Reservoir
600	0.0560x10[5]	102.330	0.005000	102.335
800	0.0995x10[5]	102.710	0.008012	102.718
1000	0.156x10[5]	103.055	0.01143	103.066
1200	0.224x10[5]	103.403	0.01505	103.418
1500	0.350x10[5]	103.810	0.02126	103.831
1800	0.504x10[5]	104.230	0.02769	104.258

FIGURE 5

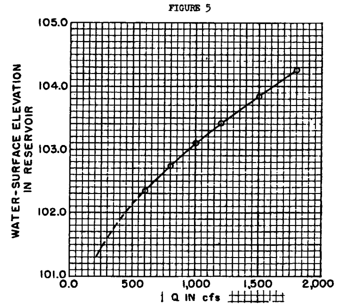
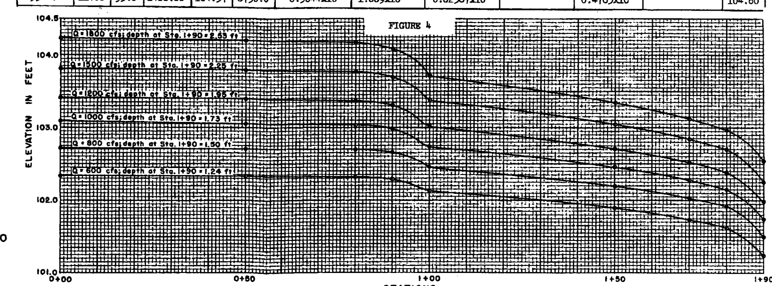


TABLE 1

1	2	3	4	5	6	7	8	9	10	11	12	13
Station and Bottom Elevation	d ft	b ft	a ft²	P ft	F	$\frac{1}{a^2}$	$\frac{s_o}{Q_{n,d}^2}$	$\frac{s_o(\ell_2-\ell_1)g}{Q_{n,d_1}^2}$	$\frac{s_o(\ell_2-\ell_1)g}{Q_{n,d_2}^2}$	U_1^-	U_2^+	E
1+90	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸		14.6971x10⁻⁶		118.197x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹		2.0358x10⁻⁶		31.636x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹		0.7542x10⁻⁶		16.514x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰		0.1872x10⁻⁶		6.492x10⁻⁶	104.50
1+80	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸	14.6971x10⁻⁶	14.6971x10⁻⁶	88.803x10⁻⁶	118.197x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹	2.0357x10⁻⁶	2.0358x10⁻⁶	27.564x10⁻⁶	31.636x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹	0.7542x10⁻⁶	0.7542x10⁻⁶	15.006x10⁻⁶	16.514x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰	0.1872x10⁻⁶	0.1872x10⁻⁶	6.118x10⁻⁶	6.492x10⁻⁶	104.50
1+70	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸	14.6971x10⁻⁶	29.3942x10⁻⁶	88.803x10⁻⁶	132.894x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹	2.0357x10⁻⁶	4.0715x10⁻⁶	27.564x10⁻⁶	33.672x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹	0.7542x10⁻⁶	1.5083x10⁻⁶	15.006x10⁻⁶	17.268x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰	0.1872x10⁻⁶	0.3743x10⁻⁶	6.118x10⁻⁶	6.679x10⁻⁶	104.50
1+50	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸	29.3942x10⁻⁶	73.4856x10⁻⁶	74.106x10⁻⁶	176.986x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹	4.0715x10⁻⁶	10.1786x10⁻⁶	25.528x10⁻⁶	39.779x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹	1.5083x10⁻⁶	3.7708x10⁻⁶	14.252x10⁻⁶	19.531x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰	0.3743x10⁻⁶	0.9359x10⁻⁶	5.931x10⁻⁶	7.241x10⁻⁶	104.50
1+00	1.25	75.0	98.44	82.91	164.1	103.500x10⁻⁶	4.570x10⁻⁸	73.4856x10⁻⁶	14.6971x10⁻⁶	30.014x10⁻⁶	118.197x10⁻⁶	101.25
	2.25	75.0	183.94	89.23	440.5	29.600x10⁻⁶	6.330x10⁻⁹	10.1786x10⁻⁶	2.0358x10⁻⁶	19.421x10⁻⁶	31.636x10⁻⁶	102.25
100.00	3.00	75.0	252.00	93.97	722.0	15.760x10⁻⁶	2.345x10⁻⁹	3.7708x10⁻⁶	0.7542x10⁻⁶	11.989x10⁻⁶	16.514x10⁻⁶	103.00
	4.50	75.0	398.25	103.46	1453.0	6.305x10⁻⁶	5.820x10⁻¹⁰	0.9359x10⁻⁶	0.1872x10⁻⁶	5.369x10⁻⁶	6.492x10⁻⁶	104.50
0+90	3.00	77.0	258.00	95.97	740.0	15.023x10⁻⁶	2.235x10⁻⁹	0.7188x10⁻⁶	0.7188x10⁻⁶	14.304x10⁻⁶	15.742x10⁻⁶	101.00
	4.50	77.0	407.25	105.46	1480.0	6.029x10⁻⁶	5.620x10⁻¹⁰	0.1807x10⁻⁶	0.1807x10⁻⁶	5.848x10⁻⁶	6.210x10⁻⁶	102.50
98.00	6.50	77.0	627.25	118.11	2825.0	2.542x10⁻⁶	1.536x10⁻¹⁰	0.04939x10⁻⁶	0.04939x10⁻⁶	2.493x10⁻⁶	2.591x10⁻⁶	104.50
0+80	4.00	79.0	364.00	104.29	1245.0	7.547x10⁻⁶	7.895x10⁻¹⁰	0.2539x10⁻⁶	0.7616x10⁻⁶	7.293x10⁻⁶	8.309x10⁻⁶	100.00
	6.50	79.0	640.25	120.11	2908.0	2.440x10⁻⁶	1.447x10⁻¹⁰	0.04653x10⁻⁶	0.1396x10⁻⁶	2.393x10⁻⁶	2.580x10⁻⁶	102.50
96.00	8.50	79.0	888.25	132.76	4675.0	1.267x10⁻⁶	5.590x10⁻¹¹	0.01797x10⁻⁶	0.05393x10⁻⁶	1.249x10⁻⁶	1.321x10⁻⁶	104.50
0+50	6.00	85.0	618.00	122.95	2690.0	2.618x10⁻⁶	1.693x10⁻¹⁰	0.1633x10⁻⁶	0.2722x10⁻⁶	2.455x10⁻⁶	2.890x10⁻⁶	101.10
	8.50	85.0	939.25	138.76	4985.0	1.134x10⁻⁶	4.925x10⁻¹¹	0.04752x10⁻⁶	0.07919x10⁻⁶	1.087x10⁻⁶	1.213x10⁻⁶	103.60
95.10	11.00	85.0	1298.00	154.57	7980.0	0.5935x10⁻⁶	1.940x10⁻¹¹	0.01872x10⁻⁶	0.03120x10⁻⁶	0.5748x10⁻⁶	0.6247x10⁻⁶	106.10
0+00	6.00	95.0	678.00	132.95	2980.0	2.175x10⁻⁶	1.380x10⁻¹⁰	0.2219x10⁻⁶		1.953x10⁻⁶		99.60
	8.50	95.0	1024.25	148.76	5520.0	0.9532x10⁻⁶	4.005x10⁻¹¹	0.06440x10⁻⁶		0.8888x10⁻⁶		102.10
93.60	11.00	95.0	1408.00	164.57	8730.0	0.5044x10⁻⁶	1.609x10⁻¹¹	0.02587x10⁻⁶		0.4785x10⁻⁶		104.60

FIGURE 4



REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION- DESIGN SECTION

STANDARD DWG. NO.
ES-83
SHEET 8 OF 10
DATE 8-6-54

HYDRAULICS: NON-UNIFORM FLOW IN A NON-PRISMATIC CHANNEL WITH LEVEL AND ADVERSE BOTTOM SLOPES
Example 6.

FIGURE 1

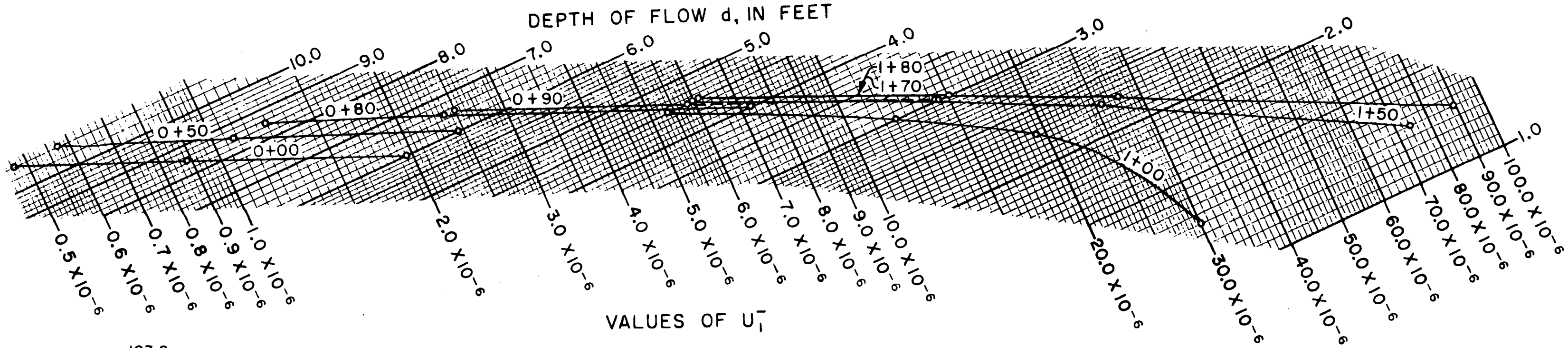
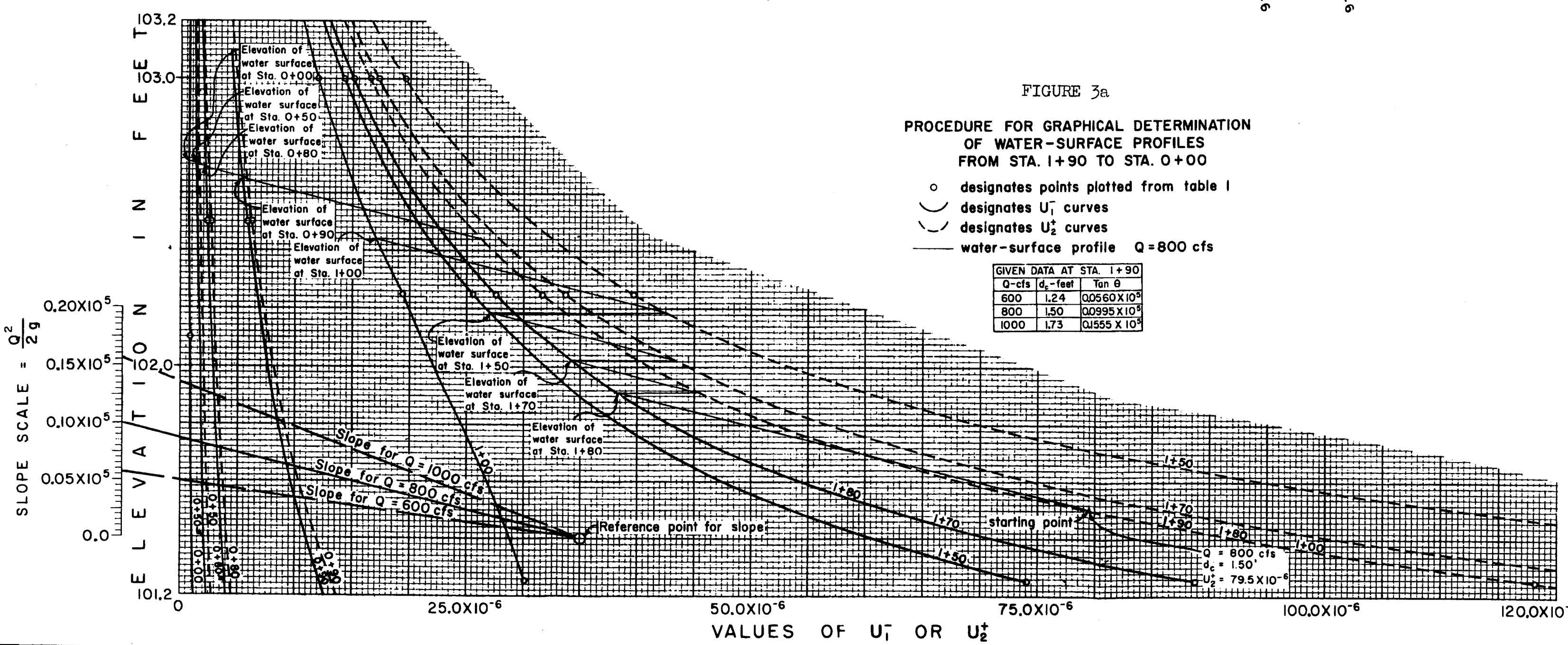


FIGURE 3a

PROCEDURE FOR GRAPHICAL DETERMINATION OF WATER-SURFACE PROFILES FROM STA. 1+90 TO STA. 0+00

- ○ designates points plotted from table 1
- ◡ designates U_1^- curves
- ◡ designates U_2^+ curves
- —— water-surface profile Q = 800 cfs

GIVEN DATA AT STA. 1+90		
Q-cfs	d_c-feet	Tan θ
600	1.24	0.0560×10^5
800	1.50	0.0995×10^5
1000	1.73	0.1555×10^5

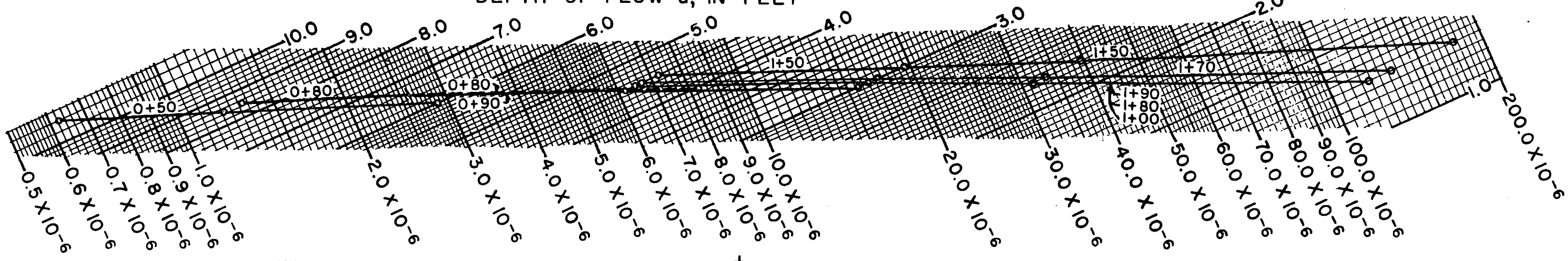


REFERENCE

U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION-DESIGN SECTION	STANDARD DWG. NO. ES-83 SHEET 9 OF 10 DATE 8-6-54

HYDRAULICS: NON-UNIFORM FLOW IN A NON-PRISMATIC CHANNEL WITH LEVEL AND ADVERSE BOTTOM SLOPES —— Example 6.

FIGURE 2



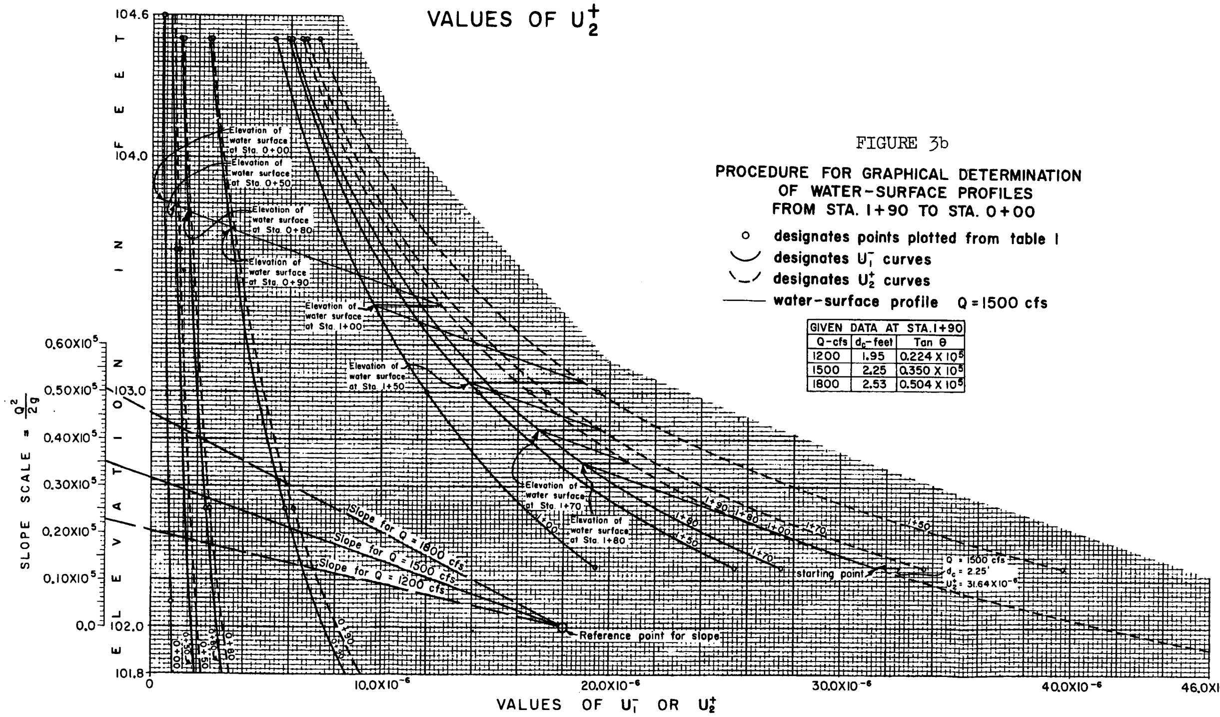


FIGURE 3b

PROCEDURE FOR GRAPHICAL DETERMINATION OF WATER-SURFACE PROFILES FROM STA. 1+90 TO STA. 0+00

○ designates points plotted from table 1

designates U_1^- curves

designates U_2^+ curves

water-surface profile Q = 1500 cfs

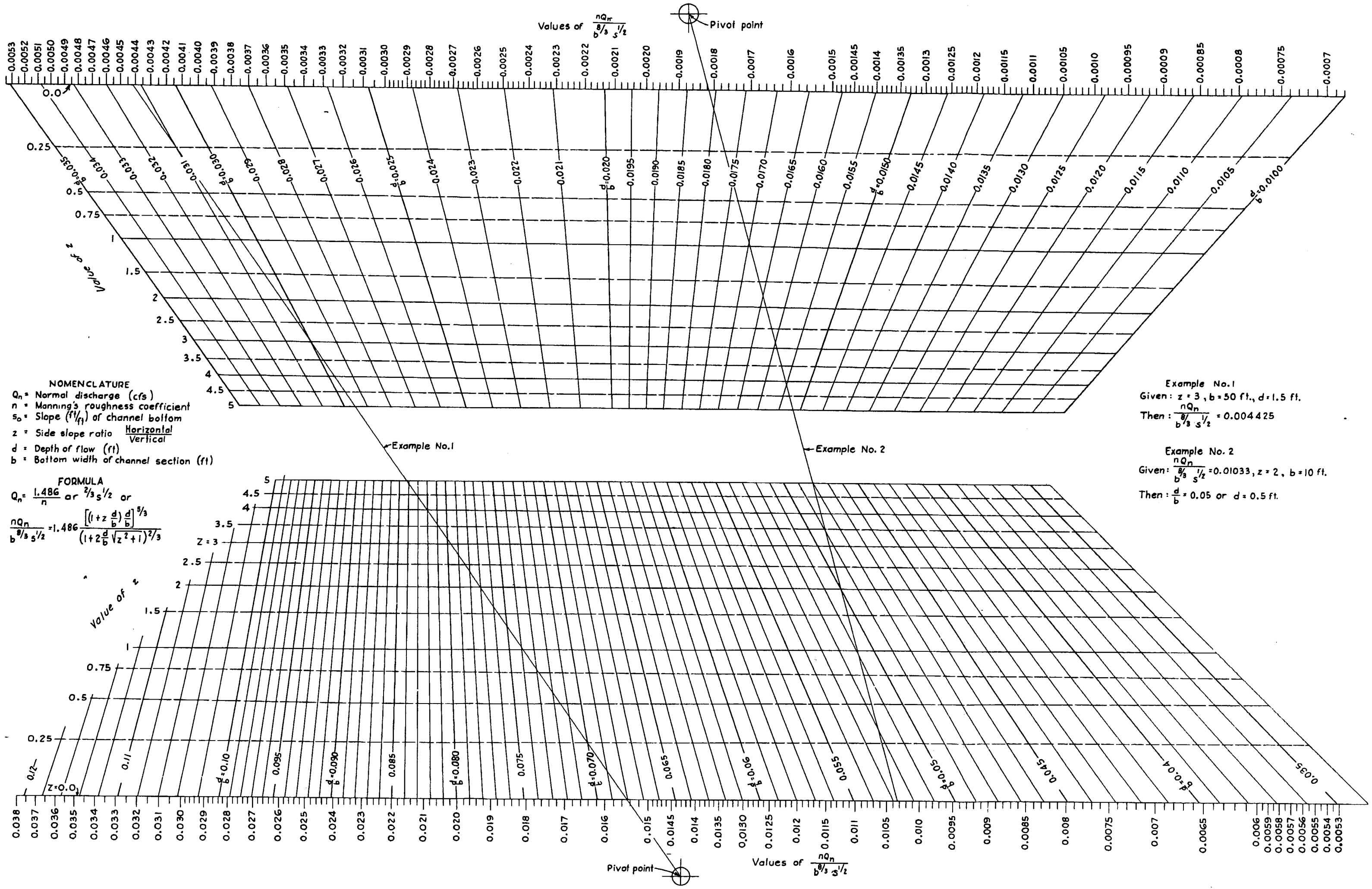
GIVEN DATA AT STA. 1+90		
Q-cfs	d_c-feet	Tan θ
1200	1.95	0.224×10^5
1500	2.25	0.350×10^5
1800	2.53	0.504×10^5

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN SECTION

STANDARD DWG. NO. ES-83
SHEET 10 OF 10
DATE 8-9-54

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
s_o = Slope (ft/ft) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2}$ or

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[(1+z\frac{d}{b})\frac{d}{b}\right]^{5/3}}{\left(1+2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 1

Given: z = 3, b = 50 ft., d = 1.5 ft.

Then: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 0.004425$

Example No. 2

Given: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 0.01033$, z = 2, b = 10 ft.

Then: $\frac{d}{b} = 0.05$ or d = 0.5 ft.

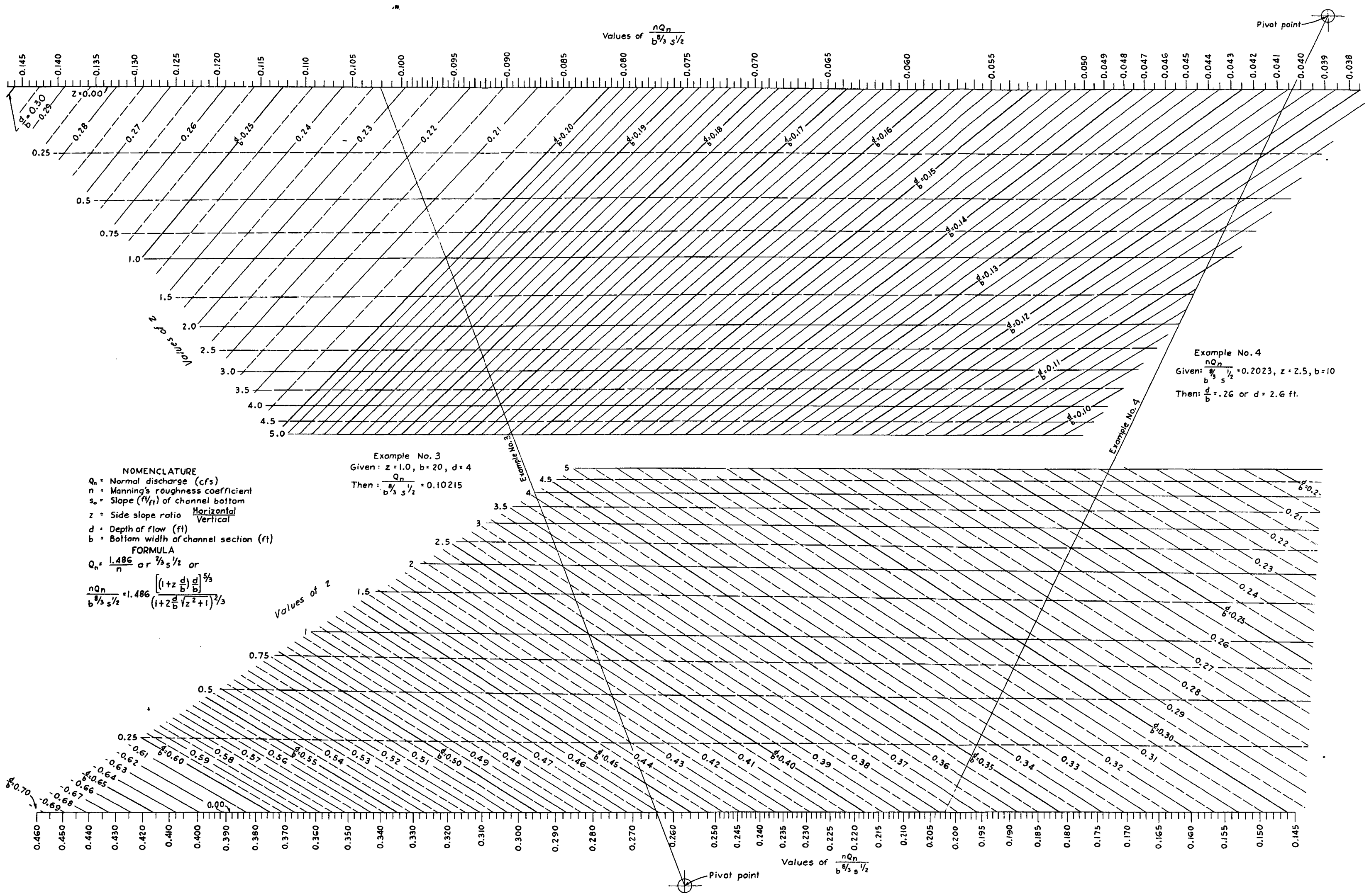
REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES - 55
SHEET 1 OF 4
DATE 4-30-51

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
s_o = Slope (ft/ft) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$Q_n = \frac{1.486}{n}$ or $^{2/3} s^{1/2}$ or

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1+z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1+2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 3
Given: z = 1.0, b = 20, d = 4
Then: $\frac{Q_n}{b^{8/3} s^{1/2}} = 0.10215$

Example No. 4
Given: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 0.2023$, z = 2.5, b = 10
Then: $\frac{d}{b} = .26$ or d = 2.6 ft.

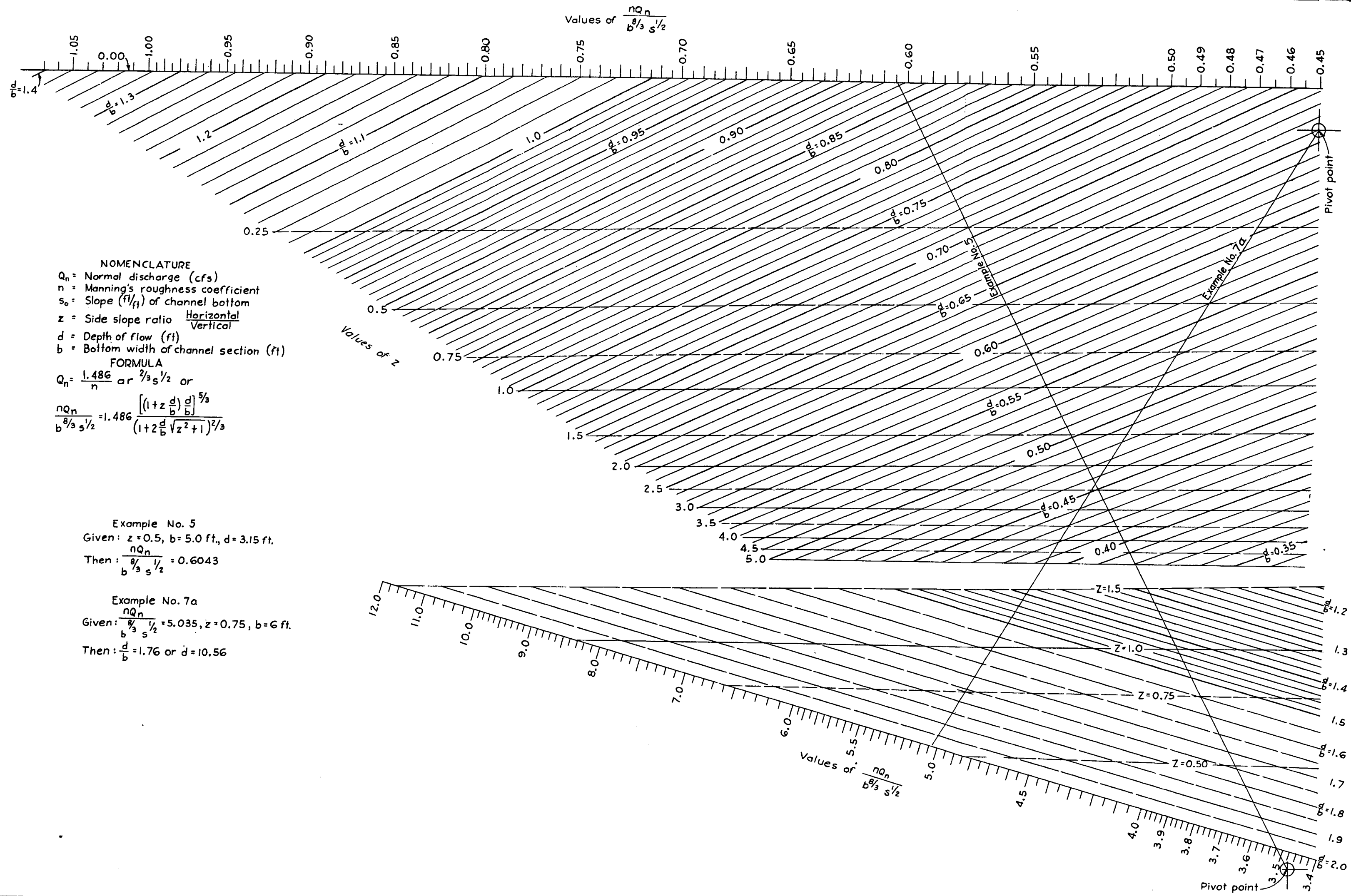
REFERENCE
This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO.
ES-55
SHEET 2 OF 4
DATE 4-30-51

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
s_o = Slope (ft/ft) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$Q_n = \frac{1.486}{n}$ ar $^{2/3} s^{1/2}$ or

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1+z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1+2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 5

Given: z = 0.5, b = 5.0 ft., d = 3.15 ft.

Then: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 0.6043$

Example No. 7a

Given: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 5.035$, z = 0.75, b = 6 ft.

Then: $\frac{d}{b} = 1.76$ or d = 10.56

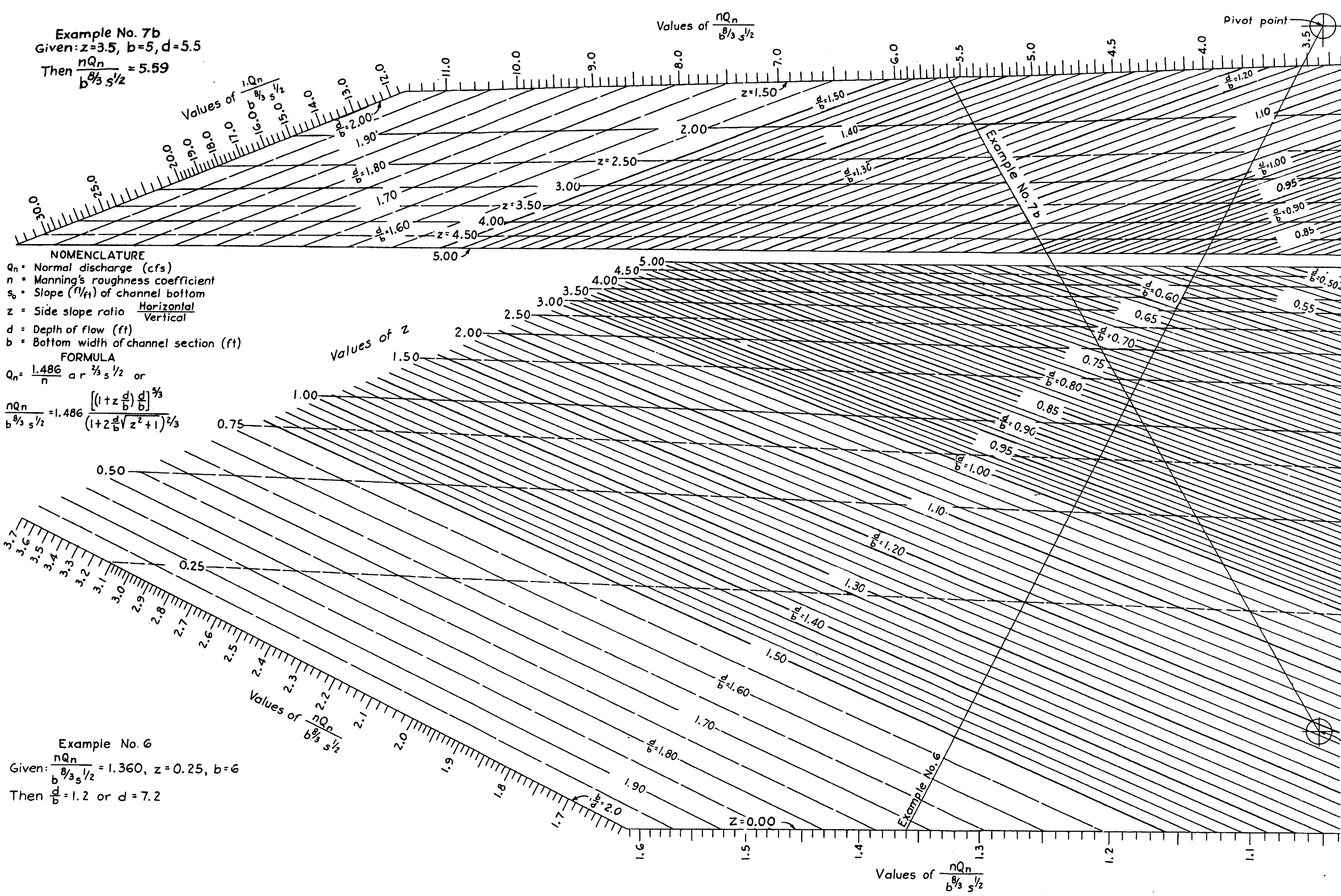
REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55
SHEET 3 OF 4
DATE 4-30-51

HYDRAULICS: UNIFORM DEPTHS AND DISCHARGES IN TRAPEZOIDAL AND RECTANGULAR CHANNELS



Example No. 7b
Given: z = 3.5, b = 5, d = 5.5
Then $\frac{nQ_n}{b^{8/3} s^{1/2}} = 5.59$

NOMENCLATURE

Q_n = Normal discharge (cfs)
n = Manning's roughness coefficient
s_o = Slope (ft/ft) of channel bottom
z = Side slope ratio $\frac{\text{Horizontal}}{\text{Vertical}}$
d = Depth of flow (ft)
b = Bottom width of channel section (ft)

FORMULA

$Q_n = \frac{1.486}{n} a r^{2/3} s^{1/2}$ or

$$\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.486 \frac{\left[\left(1 + z\frac{d}{b}\right)\frac{d}{b}\right]^{5/3}}{\left(1 + 2\frac{d}{b}\sqrt{z^2+1}\right)^{2/3}}$$

Example No. 6
Given: $\frac{nQ_n}{b^{8/3} s^{1/2}} = 1.360$, z = 0.25, b = 6
Then $\frac{d}{b} = 1.2$ or d = 7.2

REFERENCE
This nomogram was developed by Paul D. Doubt of the Design Section.

Revised 8-17-53

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-55
SHEET 4 OF 4
DATE 4-30-51

HYDRAULICS: CHART FOR DETERMINING WATER SURFACE PROFILES FOR POSITIVE VALUE OF s_o ; STEP METHOD

NOMENCLATURE

- Q = Steady discharge (cfs) for which the surface profile is to be determined.
- $Q_{n,d}$ = Normal discharge (cfs) at depth d
- $Q_{c,d}$ = Critical discharge (cfs) at depth d
- S_o = Bottom slope (ft./ft.) of channel
- l = Length (ft.) of channel
- d = Depth (ft.) of water
- R = Chart value

Subscripts 1 and 2 denote specific channel sections. Section 2 is downstream from Section 1 when $l_2 - l_1 > 0$

FORMULAS

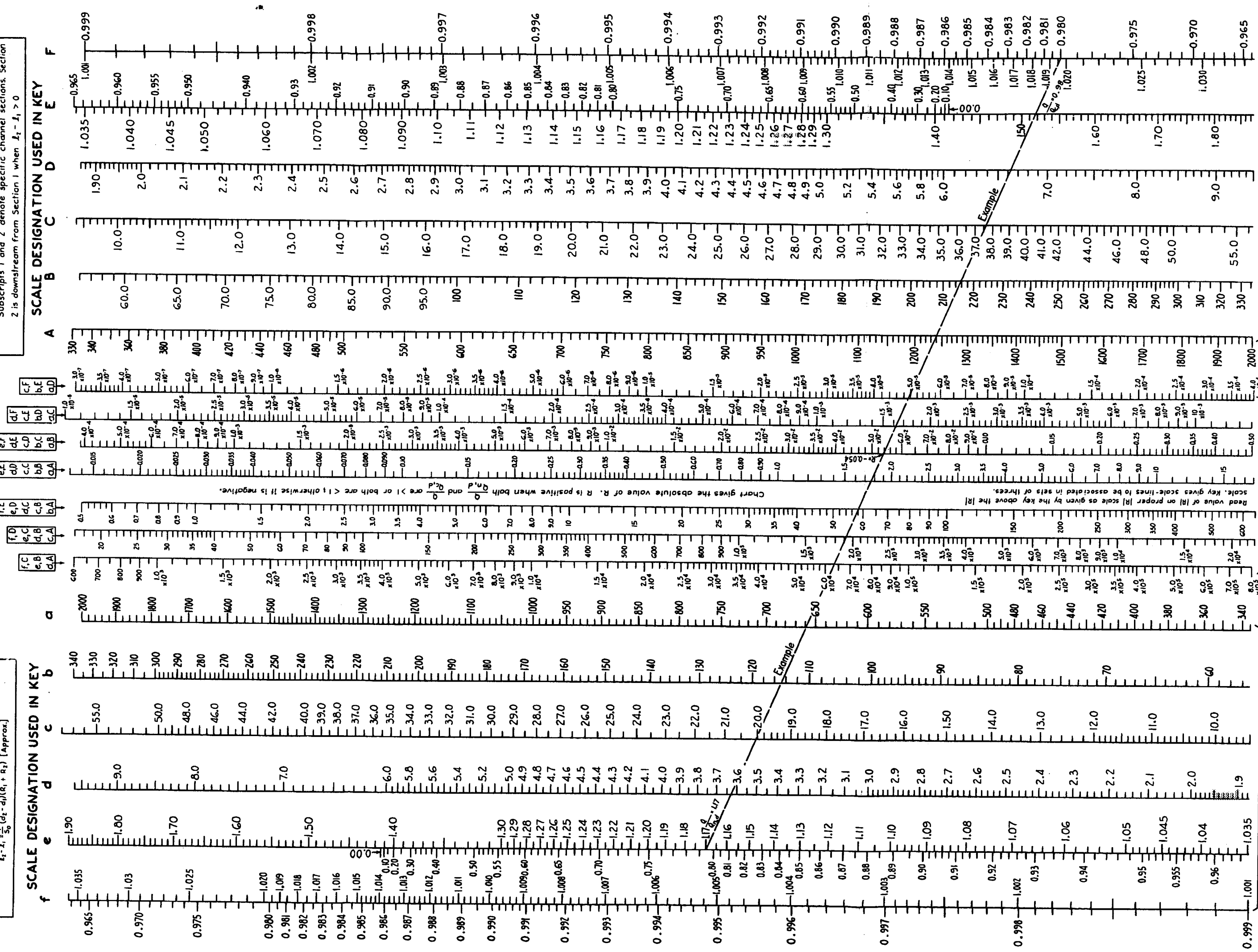
Chart solves $\frac{dl}{dd} = \frac{1}{S_o}\left(\frac{Q_{n,d}}{Q_{c,d}}\right)^2 \left(\frac{Q_{c,d}^2 - Q^2}{Q_{n,d}^2 - Q^2}\right) = \frac{2R}{S_o}$

a. For $\frac{Q}{Q_{n,d}} = 1.0$ $\begin{cases} R = +\infty \text{ when } \frac{Q}{Q_{c,d}} > 1 \\ R = -\infty \text{ when } \frac{Q}{Q_{c,d}} < 1 \end{cases}$

b. For $\frac{Q}{Q_{c,d}} = 1.0$ $R = 0$ for all $\frac{Q}{Q_{n,d}} \neq 1.0$

c. For $\frac{Q}{Q_{c,d}} = \frac{Q}{Q_{n,d}} = 1.0$ $R = 1$.

For sufficiently small values of $(d_2 - d_1)$; $l_2 - l_1 = \frac{1}{S_o}(d_2 - d_1)(R_1 + R_2)$ [Approx.]



REFERENCE

This nomogram was developed by Paul D. Doubt of the Engineering Standards Unit.

Revised 6-12-52

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING STANDARDS UNIT

STANDARD DWG. NO. ES-53
SHEET 1 OF 1
DATE 1-5-51

A.31

HYDRAULICS: SOLUTION OF GENERAL EQUATION FOR CRITICAL FLOW

T = 5 to 520
a = 3 to 66
Q_c = 13 to 630

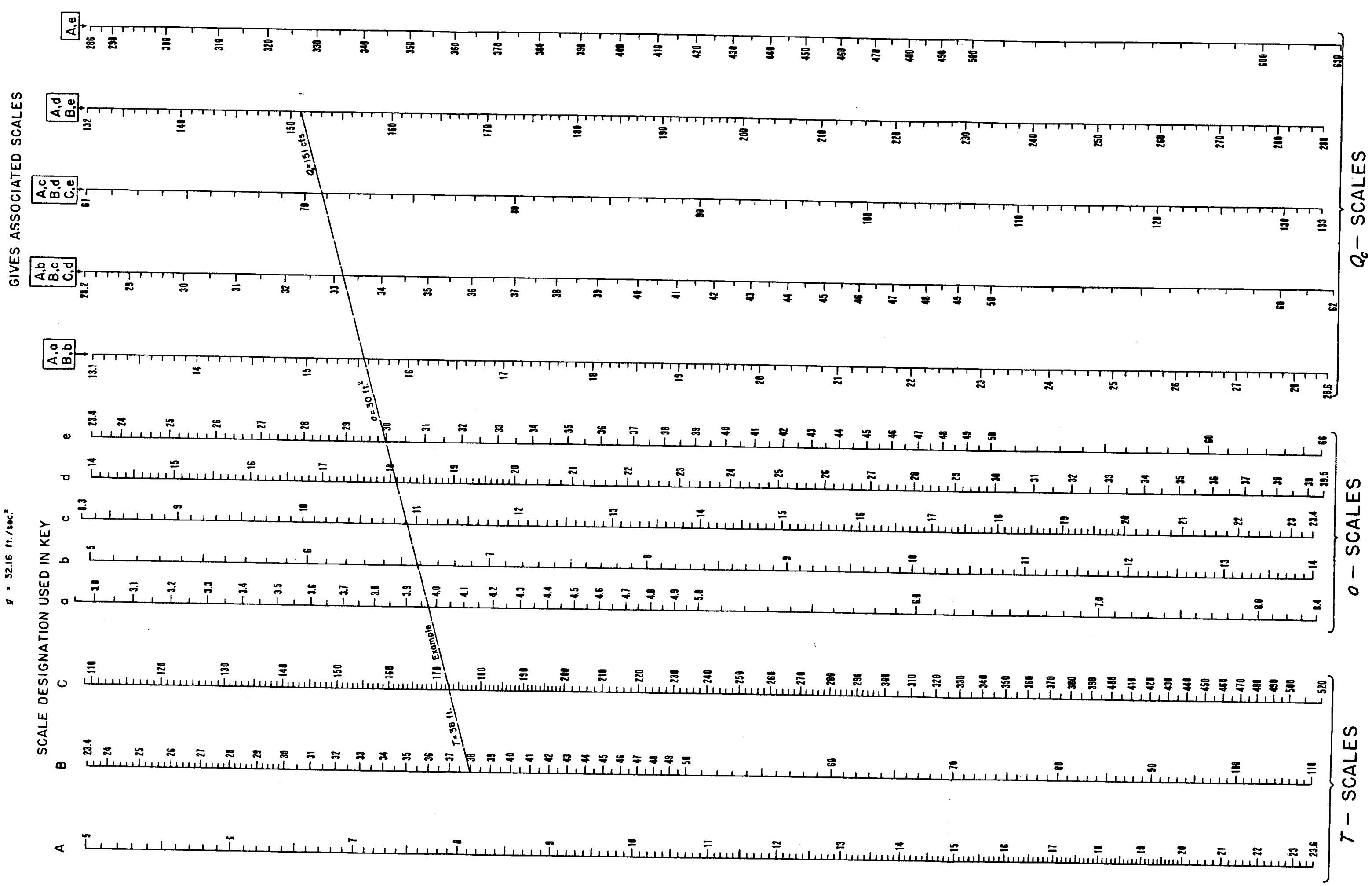
Note that area values are repeated to insure solution on chart within stated range.

FORMULA

$$\frac{Q_c^2}{g} = \frac{a^3}{T}$$

DEFINITION OF SYMBOLS

Q_c = Critical Discharge. — cfs.
a = Cross sectional flow area. — ft.^{2}
T = Width of water surface. — ft.
g = 32.16 ft./sec.^{2}



REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-75
SHEET 1 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF GENERAL EQUATION FOR CRITICAL FLOW

T = 5 to 2400
a = 39 to 520
Q_c = 61 to 6300

Note that area values are repeated to insure solution on chart within stated ranges.

KEY
GIVES ASSOCIATED SCALES

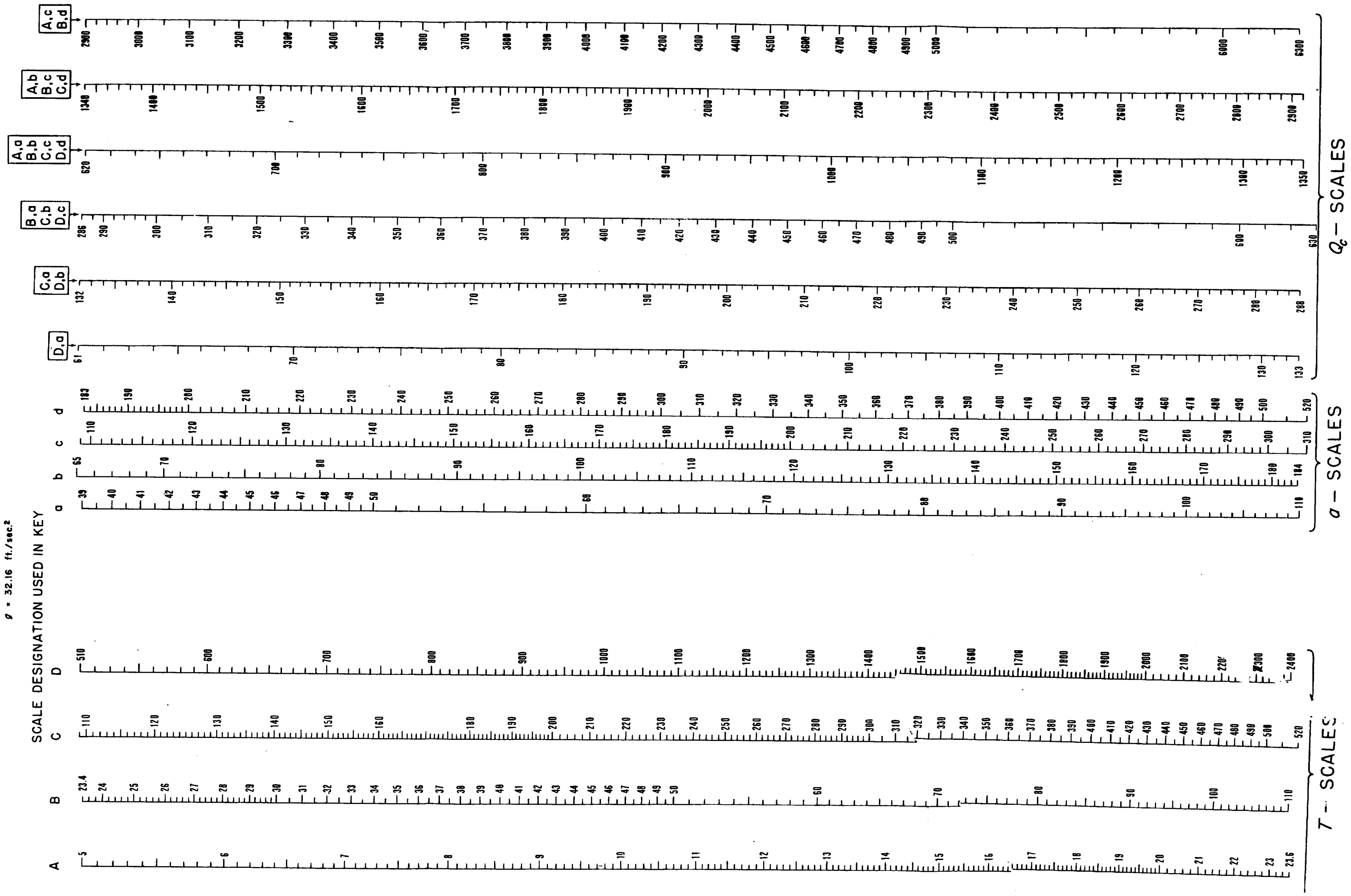
DEFINITION OF SYMBOLS

Q_c = Critical Discharge. — cfs.
a = Cross sectional flow area. — ft.^{2}
T = Width of water surface. — ft.
g = 32.16 ft./sec.^{2}

SCALE DESIGNATION USED IN KEY

FORMULA

$$\frac{Q_c^2}{g} = \frac{a^3}{T}$$



REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-75
SHEET 2 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF GENERAL EQUATION FOR CRITICAL FLOW

T = 23.4 to 2400
a = 305 to 11,200
Q_c = 1340 to 139,000

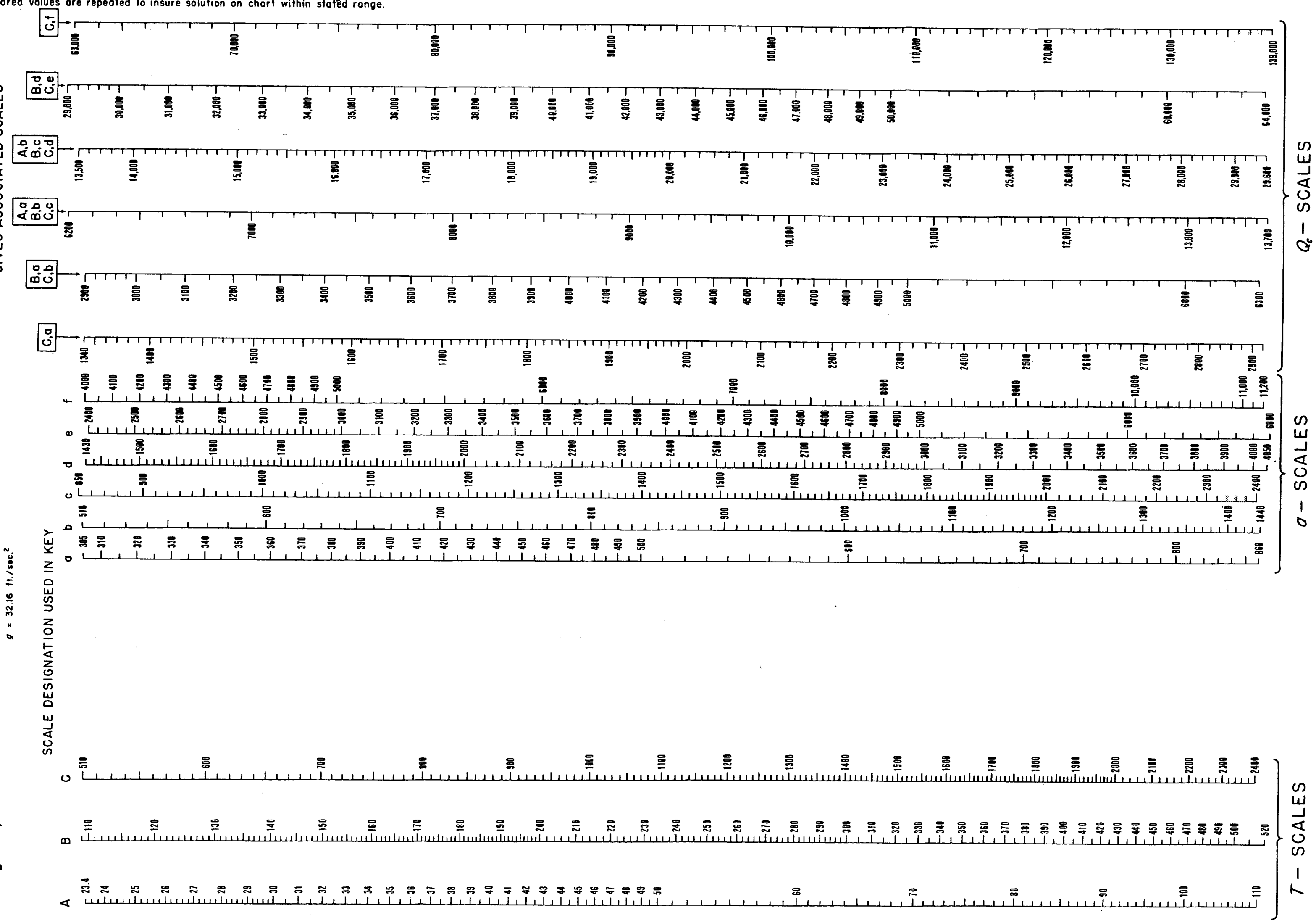
Note that area values are repeated to insure solution on chart within stated range.

FORMULA

$$\frac{Q_c^2}{g} = \frac{a^3}{T}$$

DEFINITION OF SYMBOLS

Q_c = Critical Discharge. — cfs.
a = Cross sectional flow area. — ft.^{2}
T = Width of water surface. — ft.
g = 32.16 ft./sec.^{2}



REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

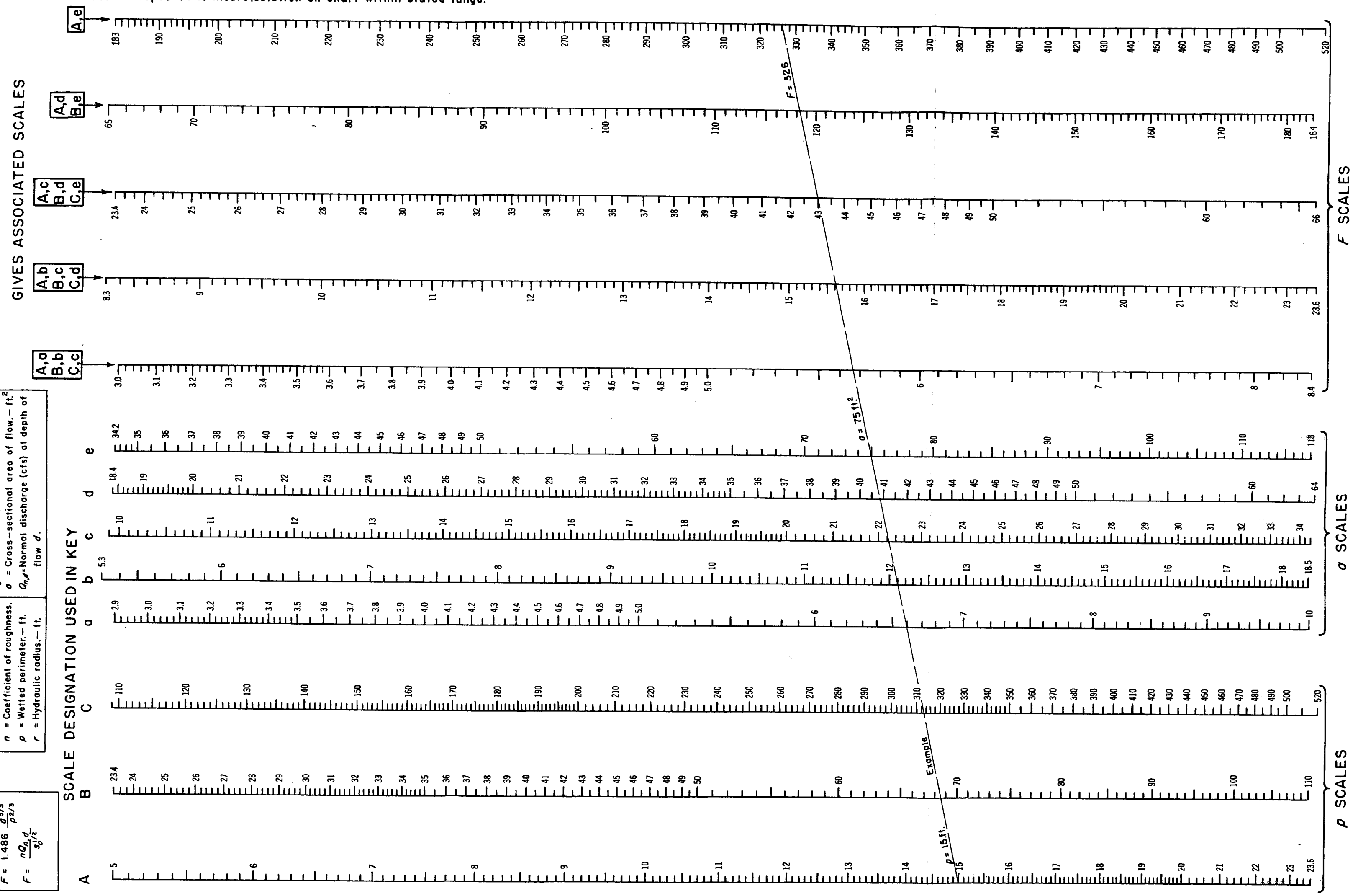
U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-75
SHEET 3 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF CROSS SECTION FACTOR $F = \frac{nQ_{n,d}}{S_o^{1/2}} = 1.486\, a r^{2/3}$

p = 5 to 520
a = 2.9 to 118
F = 3 to 520

Note that area values are repeated to insure solution on chart within stated range.



FORMULA

$F = 1.486\, a r^{2/3}$

$F = 1.486 \frac{a^{5/3}}{p^{2/3}}$

$F = \frac{nQ_{n,d}}{S_o^{1/2}}$

DESIGNATION OF SYMBOLS

F = Cross-section factor.
n = Coefficient of roughness.
p = Wetted perimeter.– ft.
r = Hydraulic radius.– ft.
S_o = Bottom slope of channel.
a = Cross-sectional area of flow.– ft.²
$Q_{n,d}$ = Normal discharge (cfs) at depth of flow d.

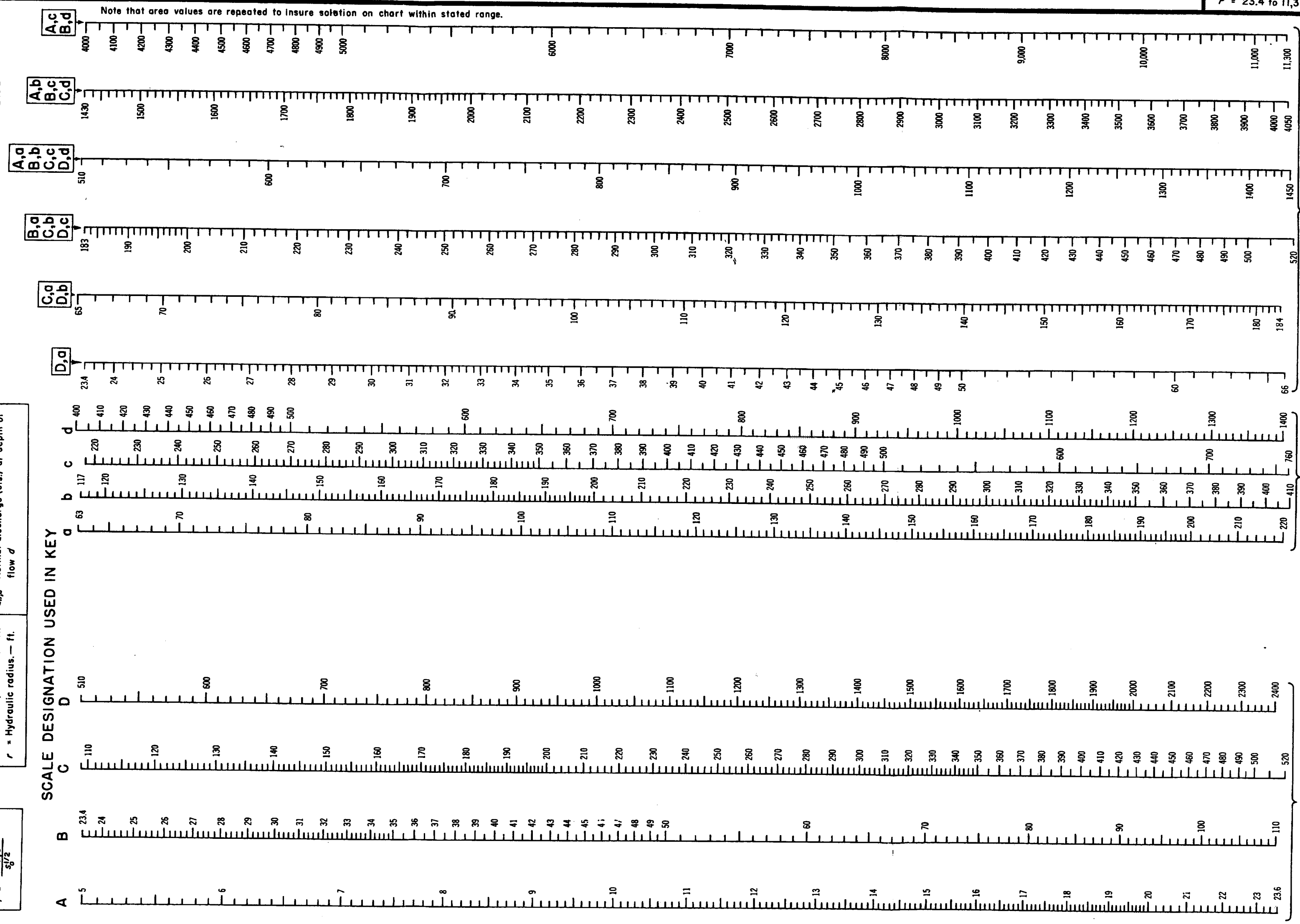
REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES 76
SHEET 1 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF CROSS SECTION FACTOR $F = \frac{nQ_{n,d}}{S_o^{1/2}} = 1.486\,a\,r^{2/3}$

p = 5 to 2400
a = 63 to 1400
F = 23.4 to 11,300



FORMULA

$F = 1.486\,a\,r^{2/3}$

$F = 1.486\,\frac{a^{5/3}}{p^{2/3}}$

$F = \frac{nQ_{n,d}}{S_o^{1/2}}$

DESIGNATION OF SYMBOLS

F = Cross-section factor.
n = Coefficient of roughness.
p = Wetted perimeter.— ft.
r = Hydraulic radius.— ft.
S_o = Bottom slope of channel.
a = Cross-sectional area of flow.— ft.^{2}
$Q_{n,d}$ = Normal discharge (cfs.) at depth of flow d

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

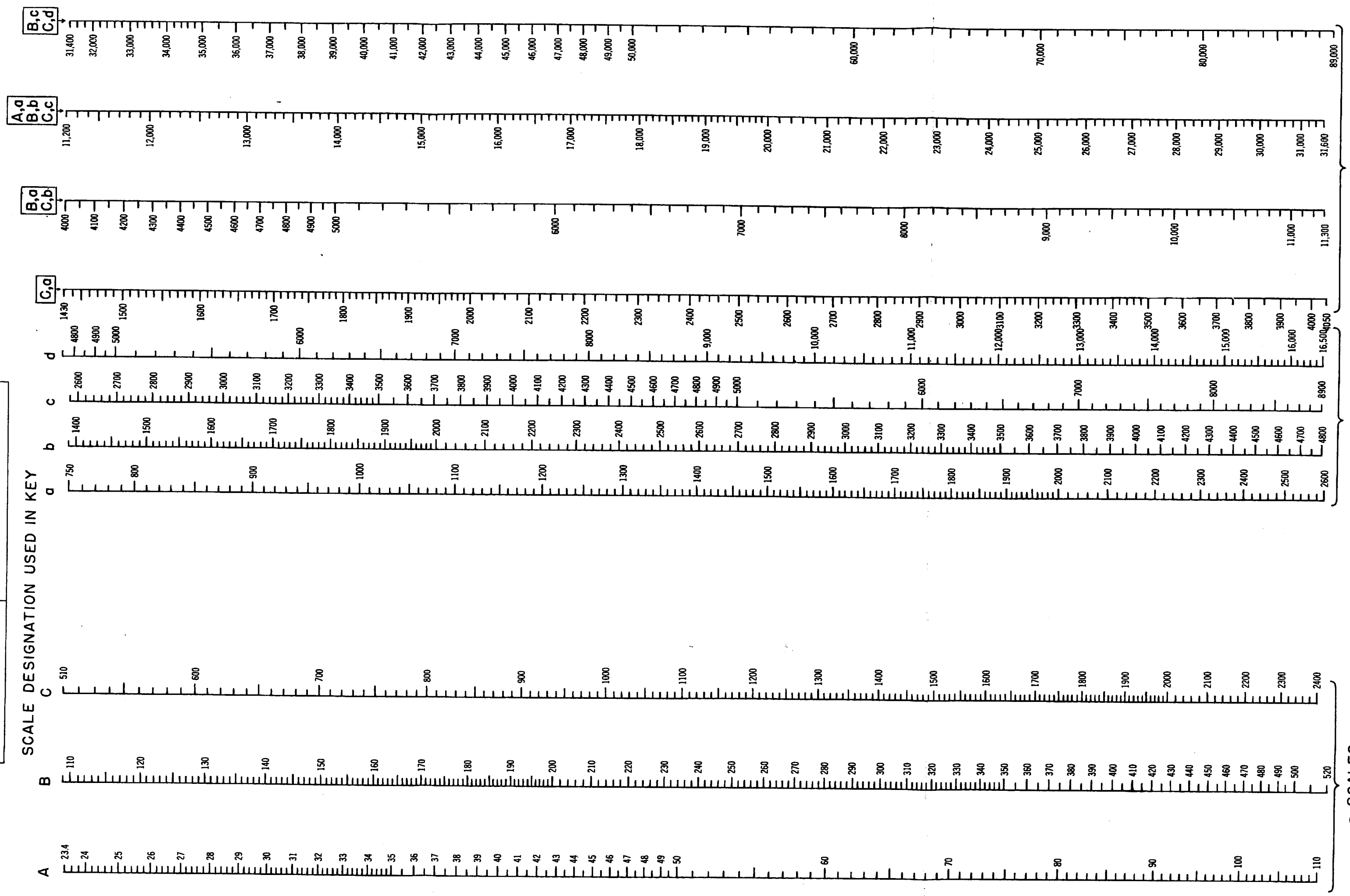
STANDARD DWG. NO.
ES 76
SHEET 2 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF CROSS SECTION FACTOR $F = \frac{nQ_{n,d}}{s_o^{1/2}} = 1.486\, a r^{2/3}$

p = 23.4 to 2400
a = 750 to 16,500
F = 1430 to 89,000

Note that area values are repeated to insure solution on chart within stated range.

KEY
GIVES ASSOCIATED SCALES



SCALE DESIGNATION USED IN KEY

DESIGNATION OF SYMBOLS

F = Cross-section factor.
n = Coefficient of roughness.
p = Wetted perimeter.— ft.
r = Hydraulic radius.— ft.
s_o = Bottom slope of channel.
a = Cross-sectional area of flow.— ft.^{2}
$Q_{n,d}$ = Normal discharge (cfs.) at depth of flow d

FORMULA

$F = 1.486\, a r^{2/3}$
$F = 1.486 \frac{a^{5/3}}{p^{2/3}}$
$F = \frac{nQ_{n,d}}{s_o^{1/2}}$

REFERENCE This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

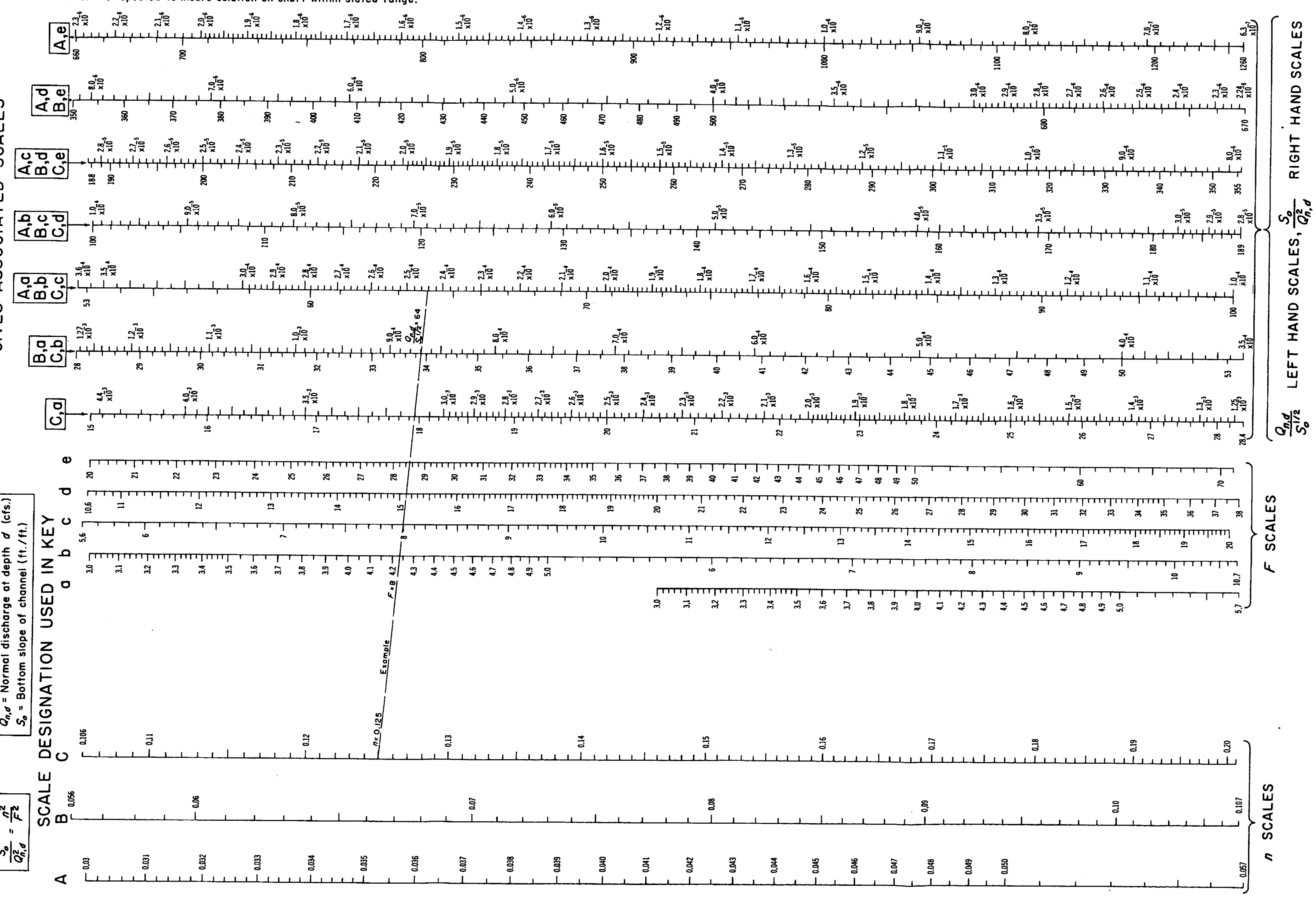
STANDARD DWG. NO.
ES 76
SHEET 3 OF 3
DATE May 1953

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} a r^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 F = 3.0 to 70

$\frac{Q_{n,d}}{S_o^{1/2}}$ = 15 to 1260 $\frac{S_o}{Q^2_{n,d}}$ = 6.3×10^{-7} to 4.4×10^{-3}

Note that factor F values are repeated to insure solution on chart within stated range.



DESIGNATION OF SYMBOLS

F = Cross-section factor (see ES 76)
n = Roughness coefficient
$Q_{n,d}$ = Normal discharge at depth d (cfs.)
S_o = Bottom slope of channel (ft./ft.)

SCALE DESIGNATION USED IN KEY

FORMULA

$$\frac{Q_{n,d}}{S_o^{1/2}} = \frac{F}{n}$$

$$\frac{S_o}{Q^2_{n,d}} = \frac{n^2}{F^2}$$

REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

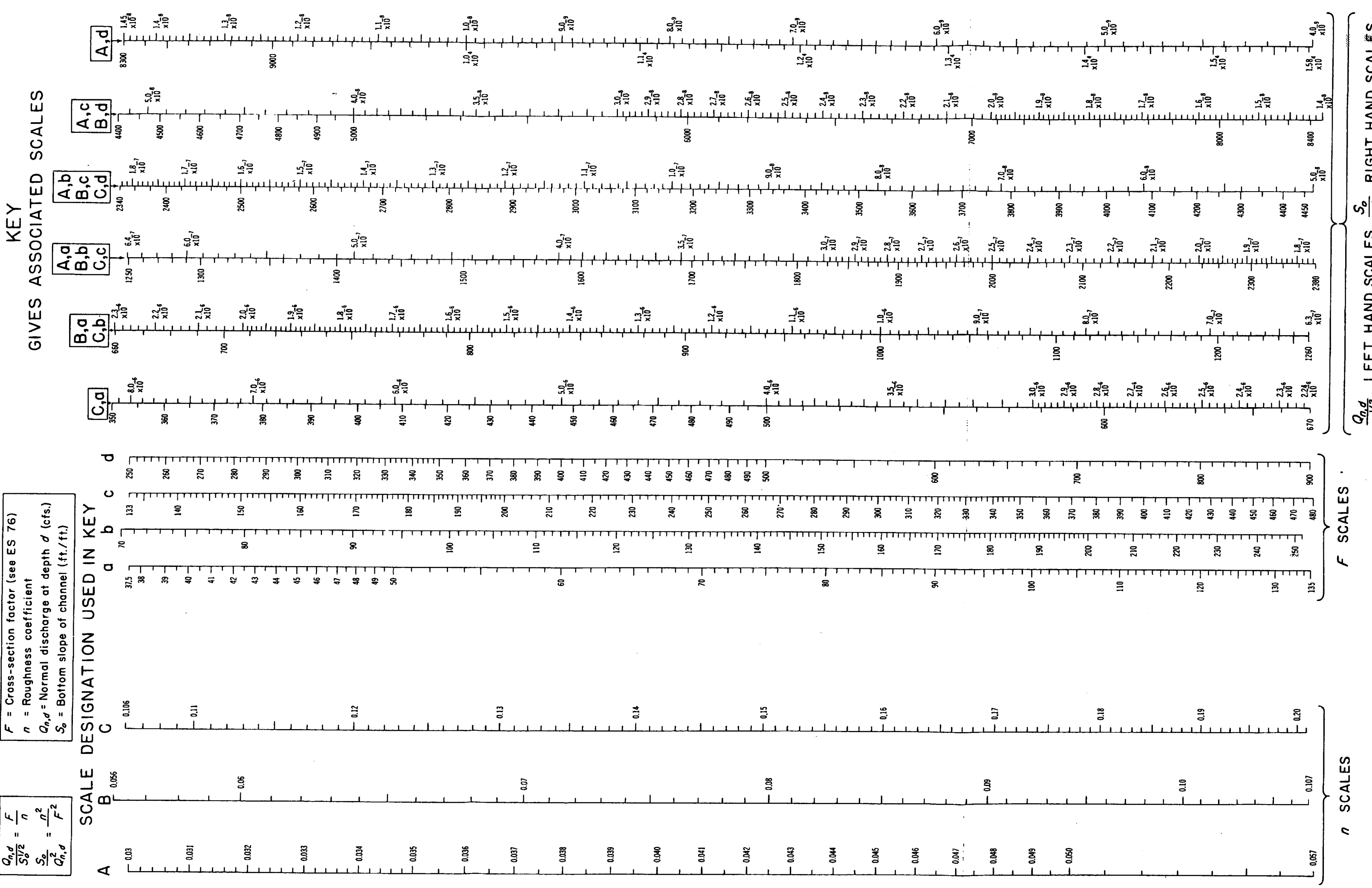
STANDARD DWG. NO.
ES-77
SHEET 1 OF 4
DATE JULY 1953

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} ar^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 F = 38 to 900

$\frac{Q_{n,d}}{S_o^{1/2}}$ = 350 to 1.58x10^4 $\frac{S_o}{Q^2_{n,d}}$ = 4.0x10^{-9} to 8.0x10^{-6}

Note that factor F values are repeated to insure solution on chart within stated range.



FORMULA

$\frac{Q_{n,d}}{S_o^{1/2}} = \frac{F}{n}$

$\frac{S_o}{Q^2_{n,d}} = \frac{n^2}{F^2}$

DESIGNATION OF SYMBOLS

F = Cross-section factor (see ES 76)
n = Roughness coefficient
$Q_{n,d}$ = Normal discharge at depth d (cfs.)
S_o = Bottom slope of channel (ft./ft.)

REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-77
SHEET 2 OF 4
DATE JULY 1953

A.29

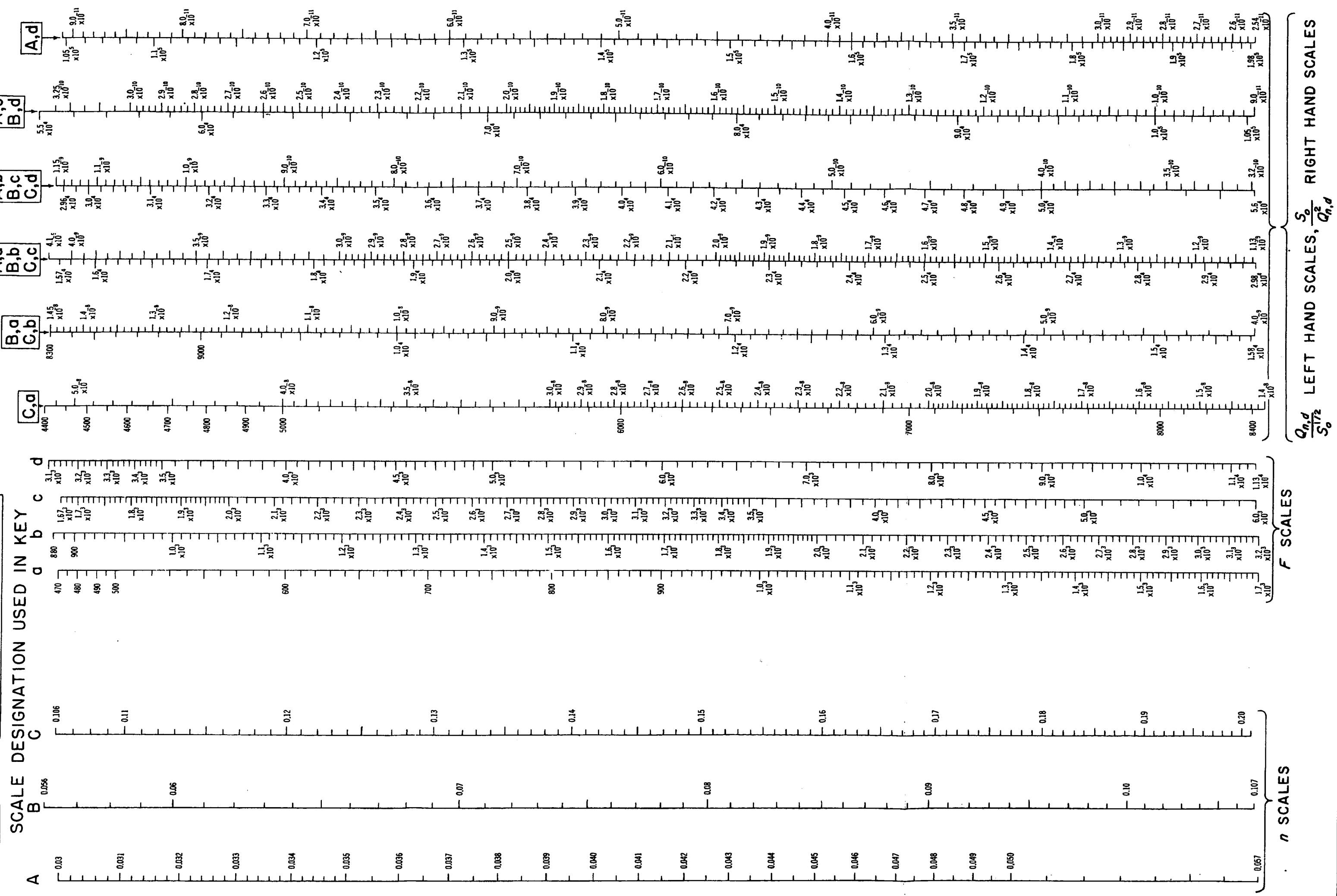
HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n} a r^{2/3} = \frac{F}{n}$

n = 0.03 to 0.20 F = 470 to 1.1×10^4

$\frac{Q_{n,d}}{S_o^{1/2}}$ = 4400 to 1.98×10^5 $\frac{S_o}{Q_{n,d}^2} = 2.54 \times 10^{-11}$ to 5.0×10^{-8}

Note that factor F values are repeated to insure solution on chart within stated range.

KEY
GIVES ASSOCIATED SCALES



FORMULA

$\frac{Q_{n,d}}{S_o^{1/2}} = \frac{F}{n}$

$\frac{S_o}{Q_{n,d}^2} = \frac{n^2}{F^2}$

DESIGNATION OF SYMBOLS

F = Cross-section factor (see ES 76)
n = Roughness coefficient
$Q_{n,d}$ = Normal discharge at depth d (cfs.)
S_o = Bottom slope of channel (ft./ft.)

SCALE DESIGNATION USED IN KEY

REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO.
ES-77
SHEET 3 OF 4
DATE JULY 1953

A.40

HYDRAULICS: SOLUTION OF MANNING'S FORMULA $\frac{Q_{n,d}}{S_o^{1/2}} = \frac{1.486}{n}\, a r^{2/3} = \frac{F}{n}$

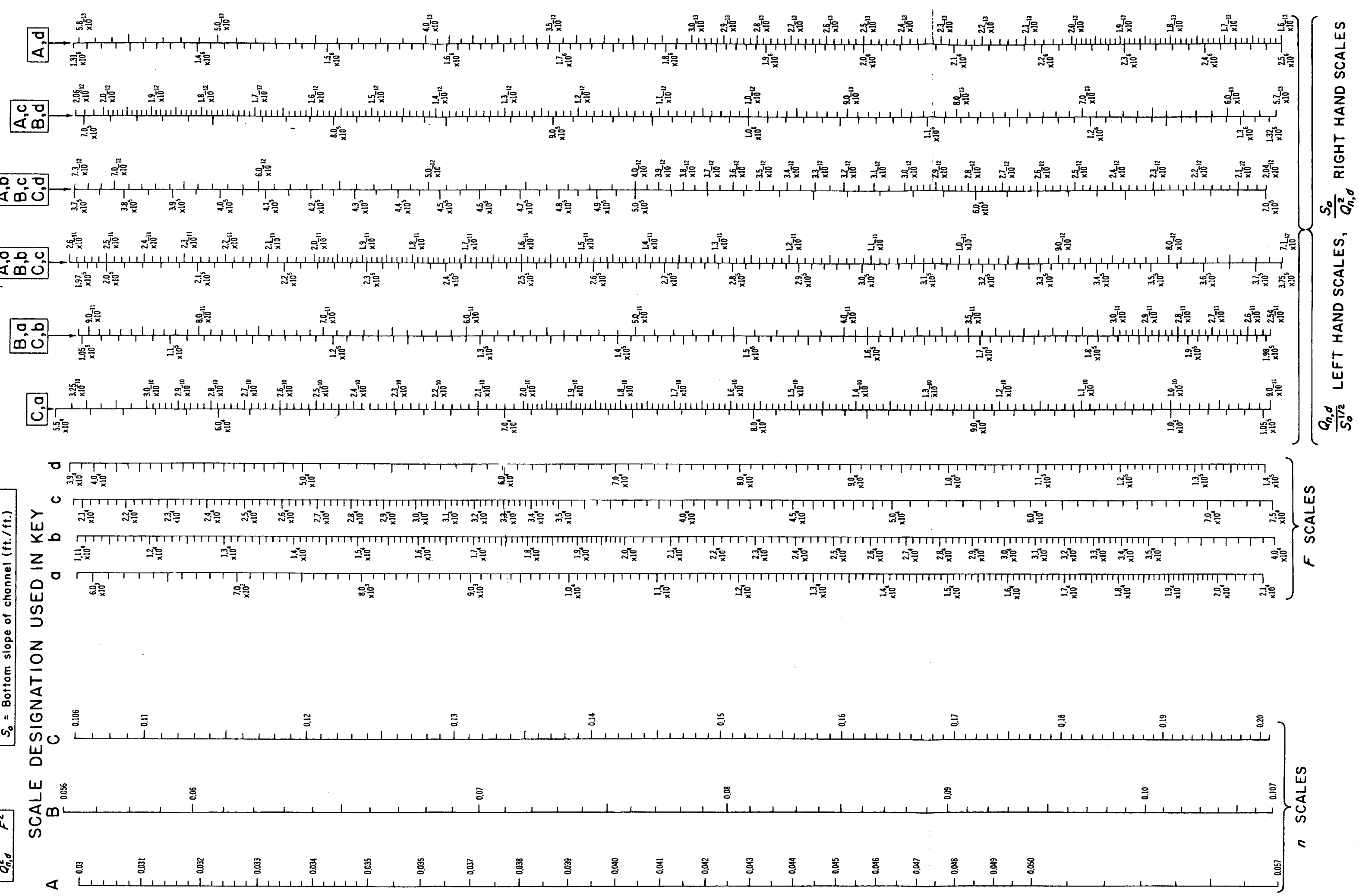
n = 0.03 to 0.20 F = 6.0x10³ to 1.4x10⁵

$\frac{Q_{n,d}}{S_o^{1/2}} = 5.5\times10^4$ to 2.5×10^6 $\frac{S_o}{Q^2_{n,d}} = 1.6\times10^{-13}$ to 3.25×10^{-10}

Note that factor F values are repeated to insure solution on chart within stated range.

KEY

GIVES ASSOCIATED SCALES



FORMULA

$\frac{Q_{n,d}}{S_o^{1/2}} = \frac{F}{n}$

$\frac{S_o}{Q^2_{n,d}} = \frac{n^2}{F^2}$

DESIGNATION OF SYMBOLS

F = Cross-section factor (see ES 76)
n = Roughness coefficient
$Q_{n,d}$ = Normal discharge at depth d (cfs.)
S_o = Bottom slope of channel (ft./ft.)

SCALE DESIGNATION USED IN KEY

REFERENCE

This nomogram was developed by Paul D. Doubt of the Design Section.

SOIL CONSERVATION SERVICE
U. S. DEPARTMENT OF AGRICULTURE
ENGINEERING DIVISION - DESIGN SECTION

STANDARD DWG. NO. ES-77
SHEET 4 OF 4
DATE JULY 1953

PREFACE

SUPPLEMENT B

HYDRAULICS

This supplement expands and augments subsection 4.4 of the Hydraulics Section of the Engineering Handbook.

The objective of Supplement B is to present a systematic procedure for the estimation of n values for use in hydraulic computations associated with natural streams, floodways and drainage channels.

This method of estimating roughness coefficients was developed by Woody L. Cowan. Mrs. Vivian Edwards typed the manuscript.

August 1, 1956

This supplement describes a method for estimating the roughness coefficient n for use in hydraulic computations associated with natural streams, floodways and similar streams. The procedure proposed applies to the estimation of n in Manning's formula. This formula is now widely used, it is simpler to apply than other widely recognized formulas and has been shown to be reliable.

Manning's formula is empirical. The roughness coefficient n is used to quantitatively express the degree of retardation of flow. The value of n indicates not only the roughness of the sides and bottom of the channel, but also all other types of irregularities of the channel and profile. In short, n is used to indicate the net effect of all factors causing retardation of flow in a reach of channel under consideration.

There seems to have developed a tendency to regard the selection of n for natural channels as either an arbitrary or an intuitive process. This probably results from the rather cursory treatment of the roughness coefficient in most of the more widely used hydraulic textbooks and handbooks. The fact is that the estimation of n requires the exercise of critical judgment in the evaluation of the primary factors affecting n. These factors are: irregularity of the surfaces of the channel sides and bottom; variations in shape and size of cross sections; obstructions; vegetation; meandering of the channel.

The need for realistic estimates of n justifies the adoption of a systematic procedure for making the estimates.

Procedure for estimating n. The general procedure for estimating n involves; first, the selection of a basic value of n for a straight, uniform, smooth channel in the natural materials involved; then, through critical consideration of the factors listed above, the selection of a modifying value associated with each factor. The modifying values are added to the basic value to obtain n for the channel under consideration.

In the selection of the modifying values associated with the 5 primary factors it is important that each factor be examined and considered indepéndently. In considering each factor, it should be kept in mind that n represents a quantitative expression of retardation of flow. Turbulence of flow can, in a sense, be visualized as a measure or indicator of retardance. Therefore, in each case, more critical judgment may be exercised if it is recognized that as conditions associated with any factor change so as to induce greater turbulence, there should be an increase in the modifying value. A discussion and tabulated guide to the selection of modifying values for each factor is given under the following procedural steps.

1st step. Selection of basic n value. This step requires the selection of a basic n value for a straight, uniform, smooth channel in the natural materials involved. The selection involves consideration of what may be regarded as a hypothetical channel. The conditions of straight alignment, uniform cross section, and smooth side and bottom surfaces without vegetation should be kept in mind. Thus the basic n will be visualized as varying only with the materials forming the sides and bottom of the channel. The minimum values of n shown by reported test results for the best channels in earth are in the range from 0.016 to 0.018. Practical limitations associated with maintaining smooth and uniform channels in earth for any appreciable period indicate that 0.02 is a realistic basic n. The basic n, as it is intended for use in this procedure, for natural or excavated channels, may be selected from the table below. Where the bottom and sides of a channel are of different materials this fact may be recognized in selecting the basic n.

Character of channel	Basic n
Channels in earth	0.02
Channels cut into rock	0.025
Channels in fine gravel	0.024
Channels in coarse gravel	0.028

2nd step. Selection of modifying value for surface irregularity. The selection is to be based on the degree of roughness or irregularity of the surfaces of channel sides and bottom. Consider the actual surface irregularity; first, in relation to the degree of surface smoothness obtainable with the natural materials involved, and second, in relation to the depths of flow under consideration. Actual surface irregularity comparable to the best surface to be expected of the natural materials involved calls for a modifying value of zero. Higher degrees of irregularity induce turbulence and call for increased modifying values. The table below may be used as a guide to the selection.

Degree of irregularity	Surfaces comparable to	Modifying value
Smooth	The best obtainable for the materials involved.	0.000
Minor	Good dredged channels; slightly eroded or scoured side slopes of canals or drainage channels.	0.005
Moderate	Fair to poor dredged channels; moderately sloughed or eroded side slopes of canals or drainage channels.	0.010
Severe	Badly sloughed banks of natural channels; badly eroded or sloughed sides of canals or drainage channels; unshaped, jagged and irregular surfaces of channels excavated in rock.	0.020

3rd step. Selection of modifying value for variations in shape and size of cross sections. In considering changes in size of cross sections judge the approximate magnitude of increase and decrease in successive cross sections as compared to the average. Changes of considerable magnitude, if they are gradual and uniform, do not cause significant turbulence. The greater turbulence is associated with alternating large and small sections where the changes are abrupt. The degree of effect of size changes may be best visualized by considering it as depending primarily on the frequency with which large and small sections alternate and secondarily on the magnitude of the changes.

In the case of shape variations, consider the degree to which the changes cause the greatest depth of flow to move from side to side of the channel. Shape changes causing the greatest turbulence are those for which shifts of the main flow from side to side occur in distances short enough to produce eddies and upstream currents in the shallower portions of those sections where the maximum depth of flow is near either side. Selection of modifying values may be based on the following guide:

Character of variations in size and shape of cross sections	Modifying value
Changes in size or shape occurring gradually	0.000
Large and small sections alternating occasionally or shape changes causing occasional shifting of main flow from side to side	0.005
Large and small sections alternating frequently or shape changes causing frequent shifting of main flow from side to side	0.010 to 0.015

4th step. Selection of modifying value for obstructions. The selection is to be based on the presence and characteristics of obstructions such as debris deposits, stumps, exposed roots, boulders, fallen and lodged logs. Care should be taken that conditions considered in other steps are not re-evaluated or double-counted by this step.

In judging the relative effect of obstructions, consider: the degree to which the obstructions occupy or reduce the average cross sectional area at various stages; the character of obstructions, (sharp-edged or angular objects induce greater turbulence than curved, smooth-surfaced objects); the position and spacing of obstructions transversely and longitudinally in the reach under consideration. The following table may be used as a guide to the selection.

Relative effect of obstructions	Modifying value
Negligible	0.000
Minor	0.010 to 0.015
Appreciable	0.020 to 0.030
Severe	0.040 to 0.060

5th step. Selection of modifying value for vegetation. The retarding effect of vegetation is probably due primarily to the turbulence induced as the water flows around and between the limbs, stems and foliage, and secondarily to reduction in cross section. As depth and velocity increase, the force of the flowing water tends to bend the vegetation. Therefore, the ability of vegetation to cause turbulence is partly related to its resistance to bending force. Furthermore, the amount and character of foliage; that is, the growing season condition versus dormant season condition is important. In judging the retarding effect of vegetation, critical consideration should be given to the following: the height in relation to depth of flow; the capacity to resist bending; the degree to which the cross section is occupied or blocked out; the transverse and longitudinal distribution of vegetation of different types, densities and heights in the reach under consideration. The following table may be used as a guide to the selection:

Vegetation and flow conditions comparable to:	Degree of effect on n	Range in modifying value
Dense growths of flexible turf grasses or weeds, of which Bermuda and blue grasses are examples, where the average depth of flow is 2 to 3 times the height of vegetation.	Low	0.005 to 0.010
Supple seedling tree switches such as willow, cottonwood or salt cedar where the average depth of flow is 3 to 4 times the height of the vegetation.		
Turf grasses where the average depth of flow is 1 to 2 times the height of vegetation.		
Stemmy grasses, weeds or tree seedlings with moderate cover where the average depth of flow is 2 to 3 times the height of vegetation.	Medium	0.010 to 0.025
Brushy growths, moderately dense, similar to willows 1 to 2 years old, dormant season, along side slopes of channel with no significant vegetation along the channel bottom, where the hydraulic radius is greater than 2 feet.		

Turf grasses where the average depth of flow is about equal to the height of vegetation.		
Dormant season, willow or cottonwood trees 8 to 10 years old, intergrown with some weeds and brush, none of the vegetation in foliage, where the hydraulic radius is greater than 2 feet.	High	0.025 to 0.050
Growing season, bushy willows about 1 year old intergrown with some weeds in full foliage along side slopes, no significant vegetation along channel bottom, where hydraulic radius is greater than 2 feet.		
Turf grasses where the average depth of flow is less than one half the height of vegetation.		
Growing season, bushy willows about 1 year old, intergrown with weeds in full foliage along side slopes; dense growth of cattails along channel bottom; any value of hydraulic radius up to 10 or 15 feet.	Very high	0.050 to 0.100
Growing season; trees intergrown with weeds and brush, all in full foliage; any value of hydraulic radius up to 10 or 15 feet.		

A further basis for judgment in the selection of the modifying value for vegetation may be found in Table 1 which contains descriptions and data for actual cases where n has been determined. In each of the cases listed in Table 1 the data were such that the increase in n due to vegetation could be determined within reasonably close limits.

6th step. Determination of the modifying value for meandering of channel. The modifying value for meandering may be estimated as follows: Add the basic n for Step 1 and the modifying values of Steps 2 through 5 to obtain the subtotal of n_s.

Let ℓ_s = the straight length of the reach under consideration.

ℓ_m = the meander length of the channel in the reach.

Compute modifying value for meandering in accordance with the following table:

Ratio ℓ_m/ℓ_s	Degree of meandering	Modifying value
1.0 to 1.2	Minor	0.000
1.2 to 1.5	Appreciable	$0.15\ n_s$
1.5 and greater	Severe	$0.30\ n_s$

Where lengths for computing the approximate value of ℓ_m/ℓ_s are not readily obtainable the degree of meandering can usually be judged reasonably well.

7th step. Computation of n for the reach. The value of n for the reach is obtained by adding the values determined in Steps 1 through 6. An illustration of the estimation of n is given in Example 1.

Dealing with cases where both channel and flood plain flow occurs.

Work with natural streams and floodways often requires consideration of a wide range of discharges. At the higher stages both channel and overbank or flood plain flow are involved. Usually the conditions are such that the channel and flood plain will have different degrees of retardance and, therefore, different n values. In such cases the hydraulic computations will be improved by dividing the cross sections into parts or subdivisions having different n values.

The reason for and effect of subdividing cross sections is to permit the composite n for the reach to vary with stage above the bankfull stage. This effect is illustrated by Example 2. The usual practice is to divide the cross section into two parts; one subdivision being the channel portion and the other the flood plain. More than two subdivisions may be made if conditions indicate wide variations of n. However, in view of the practical aspects of the problem, more than three subdivisions would not normally be justified.

In estimating n for the channel subdivision, all of the factors discussed above and all of the procedural steps would be considered. Although conditions might indicate some variation of n with stage in the channel, it is recommended that an average value of n be selected for use in the hydraulic computations for all stages.

In the case of flood plain subdivisions, the estimate of n would consider all factors except meandering. That is, the estimate would employ all of the procedural steps except Step 6. Flood plain n values will normally be somewhat greater than the channel values. Agricultural flood plain conditions are not likely to indicate an n less than 0.05 to 0.06. Many cases will justify values in the 0.07 to 0.09 range and cases calling for values as high as 0.15 to 0.20 may be encountered. These higher values apply primarily because of the relatively shallow depths of flow. The two factors requiring most careful consideration are obstructions and vegetation. Many agricultural flood plains have fairly dense networks of fences to be evaluated as obstructions in Step 4. Vegetation probably would be judged on the basis of growing season conditions.

Field and office work.

It is suggested that field parties record adequate notes on field conditions pertinent to the five factors affecting n at the time cross section surveys are being made. The actual estimates of n may then be made in the office. This will require training of both field and office personnel. The conditions to be covered by field notes and considered in the estimate of n apply to a reach of channel and flood plain. It is not adequate to consider only those conditions in the immediate vicinity of a cross section. Note the sketch on Figure B.1. With cross sections located as shown, field notes should describe the channel and flood plain conditions through the reach indicated as a basis for estimating the n values (assuming subdivided sections) to be incorporated in the hydraulic computations at Section 2.

Figure B.2 shows a sample set of notes that illustrate the type of field information to be recorded as a basis for estimating n. Field men should be trained to recognize and record in brief statements those conditions that are necessary for realistic evaluation of the five factors discussed under procedural Steps 1 to 6.

Example 1. Estimation of n for a reach.

This example is based on a case where n has been determined so that comparison between the estimated and actual n can be shown.

Channel: Camp Creek dredged channel near Seymour, Illinois; see USDA Technical Bulletin No. 129, Plate 29-C for photograph and Table 9, page 86, for data.

Description: Course straight; 661 feet long. Cross section, very little variation in shape; variation in size moderate, but changes not abrupt. Side slopes fairly regular, bottom uneven and irregular. Soil, lower part yellowish gray clay; upper part, light gray silty clay loam. Condition, side slopes covered with heavy growth of poplar trees 2 to 3 inches in diameter, large willows and climbing vines; thick growth of water weed on bottom; summer condition with vegetation in full foliage.

Average cross section approximates a trapezoid with side slopes about 1.5 to 1 and bottom width about 10 feet. At bankfull stage, average depth and surface width are about 8.5 and 40 feet respectively.

Step	Remarks	Modifying values
1	Soil materials indicate minimum basic n.	0.02
2	Description indicates moderate irregularity.	0.01
3.	Changes in size and shape judged insignificant.	0.00
4.	No obstructions indicated.	0.00
5.	Description indicates very high effect of vegetation.	0.08
6.	Reach described as straight.	0.00
	Total estimated n	0.11

USDA Technical Bulletin No. 129, Table 9, page 96, gives the following determined values for n for this channel: for average depth of 4.6 feet n = 0.095; for average depth of 7.3 feet n = 0.104.

Example 2. Effect of subdividing cross sections.

The sole purpose of this example is to illustrate the effect of subdividing sections on the value of n for the complete section. It is not an illustration of hydraulic computations for determining water surface profiles or stage-discharge relationships.

This illustration is based on the following:

1. An actual stream cross section for which curves showing depth versus area and depth versus hydraulic radius for the channel and flood plain subdivisions and for the complete section are plotted on Figure B.3. Values of n are: for the channel subdivision 0.04; for the flood plain subdivision 0.08.

2. The conditions of uniform, steady flow are assumed.

   Manning's formula is handled in accordance with Leach's method. See Handbook of Hydraulics, McGraw-Hill Book Company, 3rd edition, page 534; 4th edition, page 8-65.

Notation:

Q = discharge - cfs

a = cross section area - ft.^{2}

r = hydraulic radius, ft.

p = wetted perimeter, ft.

s_o = channel slope, ft. per ft.

n = roughness coefficient

$$Q = \frac{1.486}{n} \ a \ r^{2/3} \ s_o^{\ 1/2} \qquad (B.1)$$

Let $K_d = \frac{1.486}{n} \ a \ r^{2/3}$, then

$$Q = K_d \ s_o^{\ 1/2} \qquad (B.2)$$

Assume the conditions are such that it is desirable to recognize more than one subdivision, each having a different n. Let subscripts 1, 2, and 3 refer to the section subdivisions and subscript t to the total section.

From equation B.2

$$Q = \left(K_{d1} + K_{d2} + K_{d3} - - - - + K_{dn}\right) s_o^{\ 1/2} = \Sigma K_d \ s_o^{\ 1/2} \qquad (B.3)$$

Also: $\frac{Q}{s_o^{\ 1/2}} = \Sigma K_d = \frac{1.486}{n_t} \ a_t \ r_t^{\ 2/3}$; therefore

$$n_t = \frac{1.486 \ a_t \ r_t^{\ 2/3}}{\Sigma K_d} \qquad (B.4)$$

Table B.2 shows the computations for Example 2 and Figure B.3 shows a plot of roughness coefficient for the complete section versus depth.

In natural streams n normally shows a minor decrease as stage increases up to, or somewhat above, the bankfull stage, then appreciably increases as overbank stage increases. When n is significantly different for different parts of the cross section, subdivision of the cross section, as a basis for making the computations, automatically causes n_t to vary with stage above the bankfull stage. This is true although n_t is not computed in methods for determining water surface profiles. Note on Figure B.3 that n_t, which has been computed in Example 2 for illustrative purposes, shows considerable increase with stage above the 10-foot depth and that this increase is automatically recognized by subdivision of the cross section.

The plot of hydraulic radius on Figure B.3 illustrates a typical characteristic of natural streams. Note that the hydraulic radius for the complete section increases up to bankfull depth, then decreases through a limited range of depth, and again increases as depth of overbank flow increases.

This example also illustrates that recognition of high retardance for flood plain subdivisions by the use of relatively high n values does not cause n for the complete section, n_t, to be unreasonably high. In this case, the channel and flood plain are assigned n values of 0.04 and 0.08. The value of n_t ranges up to 0.072 as shown by Table B.2 and Figure B.3.

Table B.1 Examples of effect of vegetation on n. (Sheet 1 of 3)

Example No.	Names and Descriptions of Channels. Names, Plates and Tables Refer to USDA Technical Bulletin No. 129, November 1929	Range in mean velocity	Range in hydraulic radius	Average value n	Modifying value
1.	Fountain Head dredged channel near Champaign, Illinois; Plate 31-B and C, Table 9. Average cross section of channel resembles a parabola. At bankfull stage depth about 8 ft., top width about 30 feet.	2.09 to 2.59	1.73 to 2.42		
	a. Dormant season. Dry weeds on side slopes, no vegetation on bottom. Retarding effect of vegetation negligible.			0.031	
	b. Growing season, otherwise vegetation same as above. Heavy growth of weeds and grass in full foliage on side slopes.			0.037	0.006
2.	Cummins Lake dredged channel near Gould, Arkansas; Plate 18-B and C, Table 7. Average cross section of channel resembles a parabola. At bankfull stage depth about 13 ft., top width about 75 feet.	0.53 to 1.82	2.41 to 6.23		
	a. Side slopes moderately irregular from erosion and sloughing; estimated n for channel without vegetation 0.035.				
	b. Dormant season. Willows about one year old and 6 to 10 feet high continuous along side slopes except for about the upper third of sides. No growth in a strip about 20 feet wide along bottom. No foliage.			0.056	0.021
	c. Growing season, otherwise vegetation same as above. Willows and some weeds in full foliage. No vegetation along bottom.			0.072	0.037

Table B.1 Examples of effect of vegetation on n. (Sheet 2 of 3)

Example No.	Names and Descriptions of Channels. Names, Plates and Tables Refer to USDA Technical Bulletin No. 129, November 1929	Range in mean velocity	Range in hydraulic radius	Average value n	Modifying value
3.	Lateral Ditch No. 15 near Bement, Illinois; Plate 30-A, B and C, Table 9. Average cross section is practically a trapezoid with wide slopes about 1.1 and bottom width and depth each about 10 feet.	0.28 to 1.71	1.16 to 5.61		
	a. Dormant season. Dead weeds practically flat on side slopes; no dead growth in bottom.			0.033	
	b. Dormant season. Bushy willows about 1 year old and dead weeds on side slopes. No vegetation along bottom of channel. No foliage.			0.055	0.022
	c. Growing season. Vegetation same as b, above, except willows and weeds in full foliage. No vegetation on bottom.			0.072	0.039
	d. Growing season. Bushy willows and weeds in full foliage along side slopes. Dense growth of cattails along bottom.			0.119	0.086
	Ditch No. 18 of Cypress Creek drainage district near Arkansas City, Arkansas; Plate 17-B and C, Table 7. Average cross section is approximately triangular; at bankfull stage depth about 13 ft., top width about 70 feet.	0.47 to 1.08	1.91 to 6.23		
	a. Dredged channel about 8 years old. Side slopes moderately irregular. Estimated n for the channel without vegetation 0.035.				
	b. Dormant season. Practically the entire reach covered with trees, mostly willows and cottonwoods. Some dry weeds and brush. No foliage.			0.061	0.026

Table B.1 Examples of effect of vegetation on n. (Sheet 3 of 3)

Example No.	Names and Descriptions of Channels. Names, Plates and Tables Refer to USDA Technical Bulletin No. 129, November 1929.	Range in mean velocity	Range in hydraulic radius	Average value n	Modifying value
	c. Growing season. Vegetation described under b; in full foliage.			0.103	0.068
5.	Lake Fork special dredged channel near Bement, Illinois; Plate 25-A, B, and C, Table 9. Average cross section is approximately parabolic; at bankfull stage depth about 13 ft., top width about 65 to 70 feet.	0.79 to 1.65	3.55 to 7.33		
	a. Dormant season. Channel cleared; practically no vegetation of any type in channel.			0.031	
	b. Dormant season. Densely growing, bushy willows continuous along side slopes; some poplar saplings scattered among willows. No growth in a strip 20 to 30 feet wide along bottom. No foliage.			0.071	0.040
	c. Growing season. Vegetation described under b; in full foliage.			0.092	0.061
6.	Ditch No. 1 of Little River drainage district near Chafee, Missouri; Plate 21-B and C, Table 8. Average cross section trapezoidal, side slopes about 1.1, bottom width about 10 ft., depth about 8 feet.	0.68 to 1.51	2.00 to 4.26		
	a. Channel newly cleared, practically no vegetation			0.031	
	b. Dormant season. Dense, bushy willows continuous along side slopes; no foliage. No vegetation along bottom of channel.			0.071	0.040

Table B.2 Computations for Example 2.

Depth	a_1	r_1	$r_1^{2/3}$	K_{d1}	a_2	r_2	$r_2^{2/3}$	K_{d2}	ΣK_d	a_t	r_t	$r_t^{2/3}$	K	n_t
0.0	0	0.00	0.000	0	0	0.00	0.000	0	0	0	0.00	0.000	0	0.000
4.7	90	3.33	2.230	7450	0	0.00	0.000	0	7450	90	3.33	2.230	298	0.040
7.8	180	5.29	3.036	20250	0	0.00	0.000	0	20250	180	5.29	3.036	810	0.040
9.7	240	7.06	3.680	31900	750	1.06	1.040	14500	46400	990	1.31	1.197	1760	0.038
11.7	300	8.82	4.269	47500	2238	2.88	2.024	84000	131500	2538	3.14	2.144	8090	0.062
13.7	360	10.59	4.822	64400	3853	4.58	2.758	197000	261400	4213	4.82	2.854	17850	0.069
16.7	450	13.22	5.591	93400	6488	7.08	3.687	444000	537400	6938	7.30	3.763	38750	0.072

$$K_{d1} = \frac{1.486}{0.040} a_1 r_1^{2/3} = 37.15 a_1 r_1^{2/3}$$

$$K_{d2} = \frac{1.486}{0.080} a_2 r_2^{2/3} = 18.58 a_2 r_2^{2/3}$$

$$K = 1.486 a_t r_t^{2/3}$$

$$n_t = \frac{K}{K_d}$$

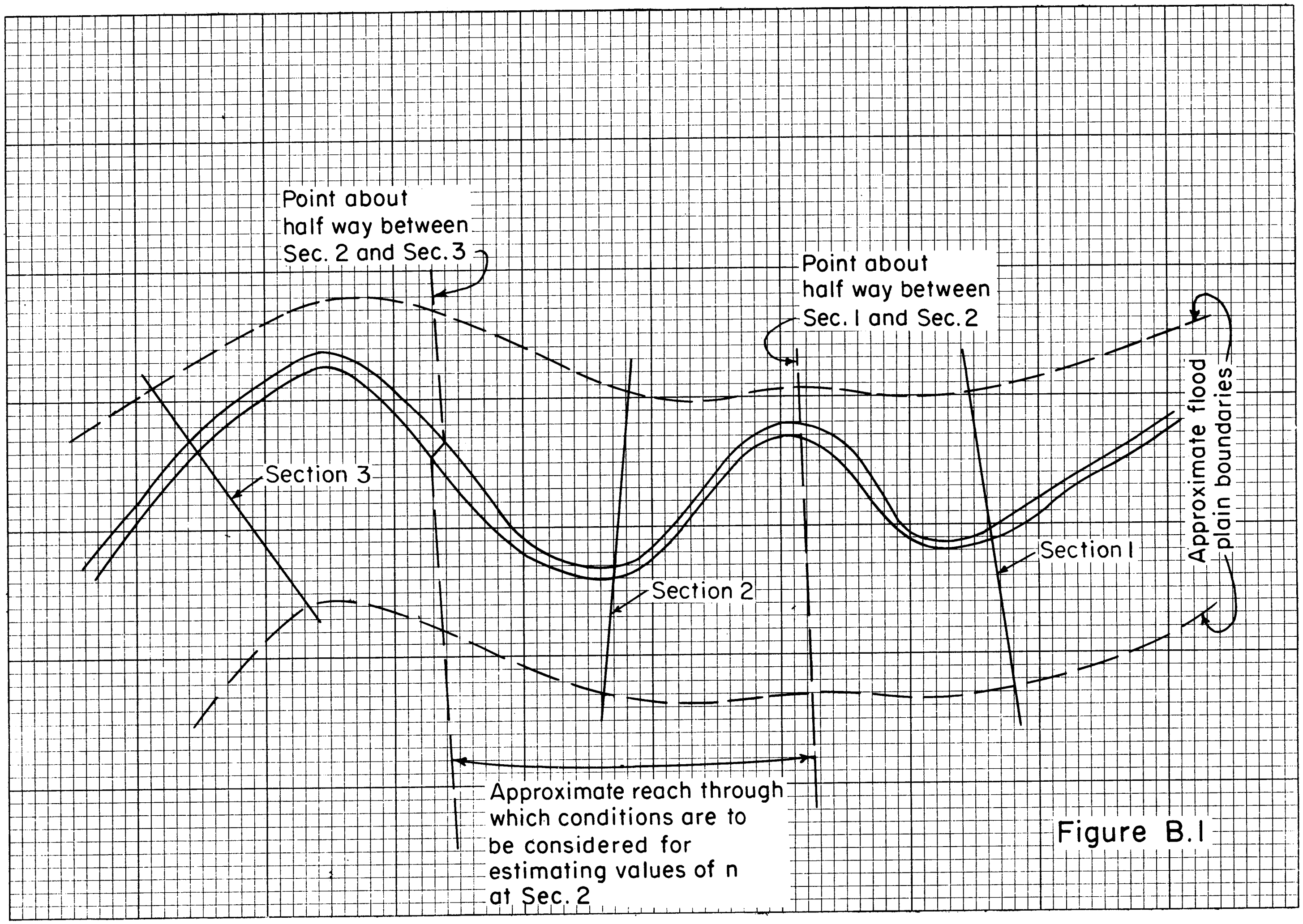
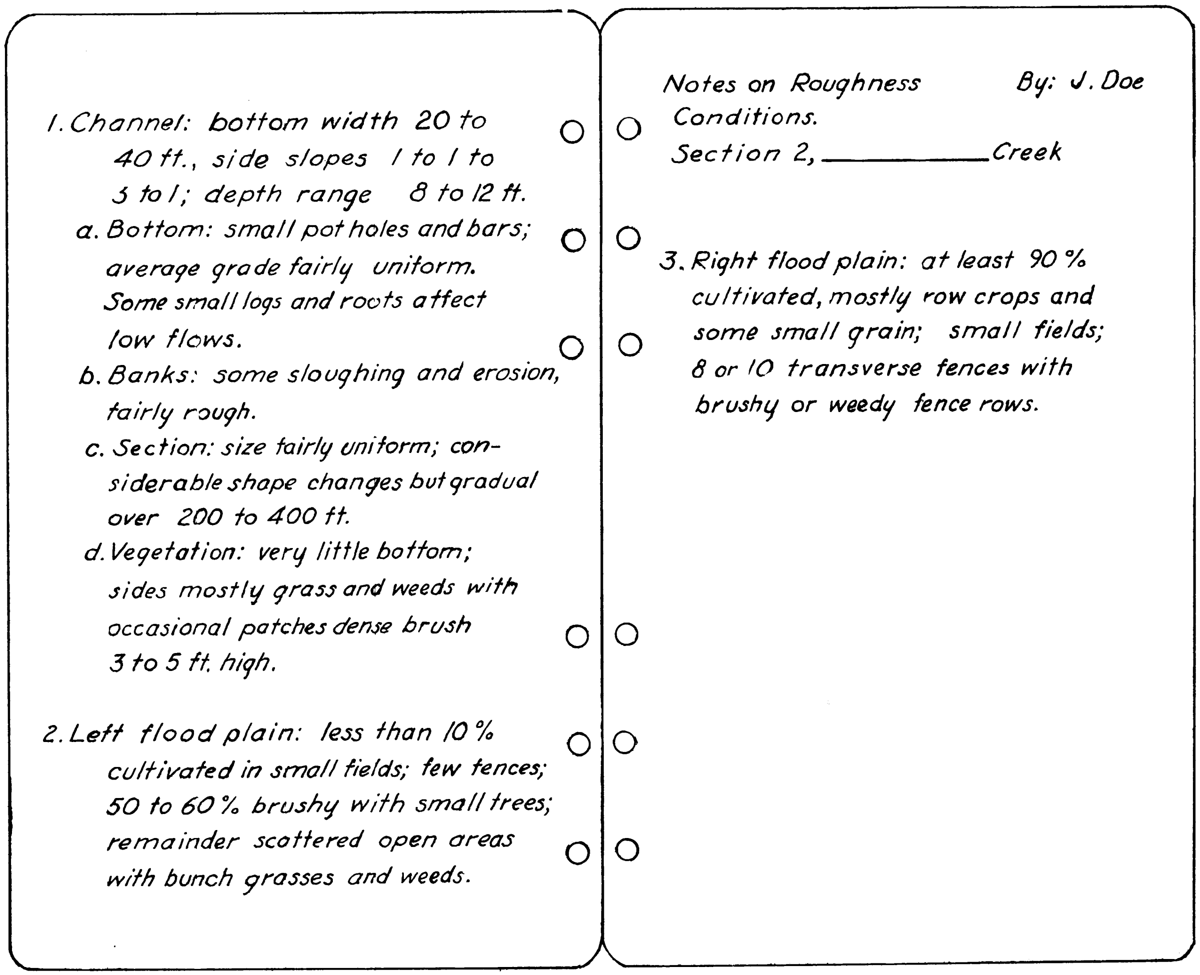

Point about
half way between
Sec. 2 and Sec. 3
Point about
half way between
Sec. 1 and Sec. 2
Approximate flood
plain boundaries
Section 3
Section 2
Section 1
Approximate reach through
which conditions are to
be considered for
estimating values of n
at Sec. 2
Figure B.1

Figure B.2 Sample notes on roughness conditions.



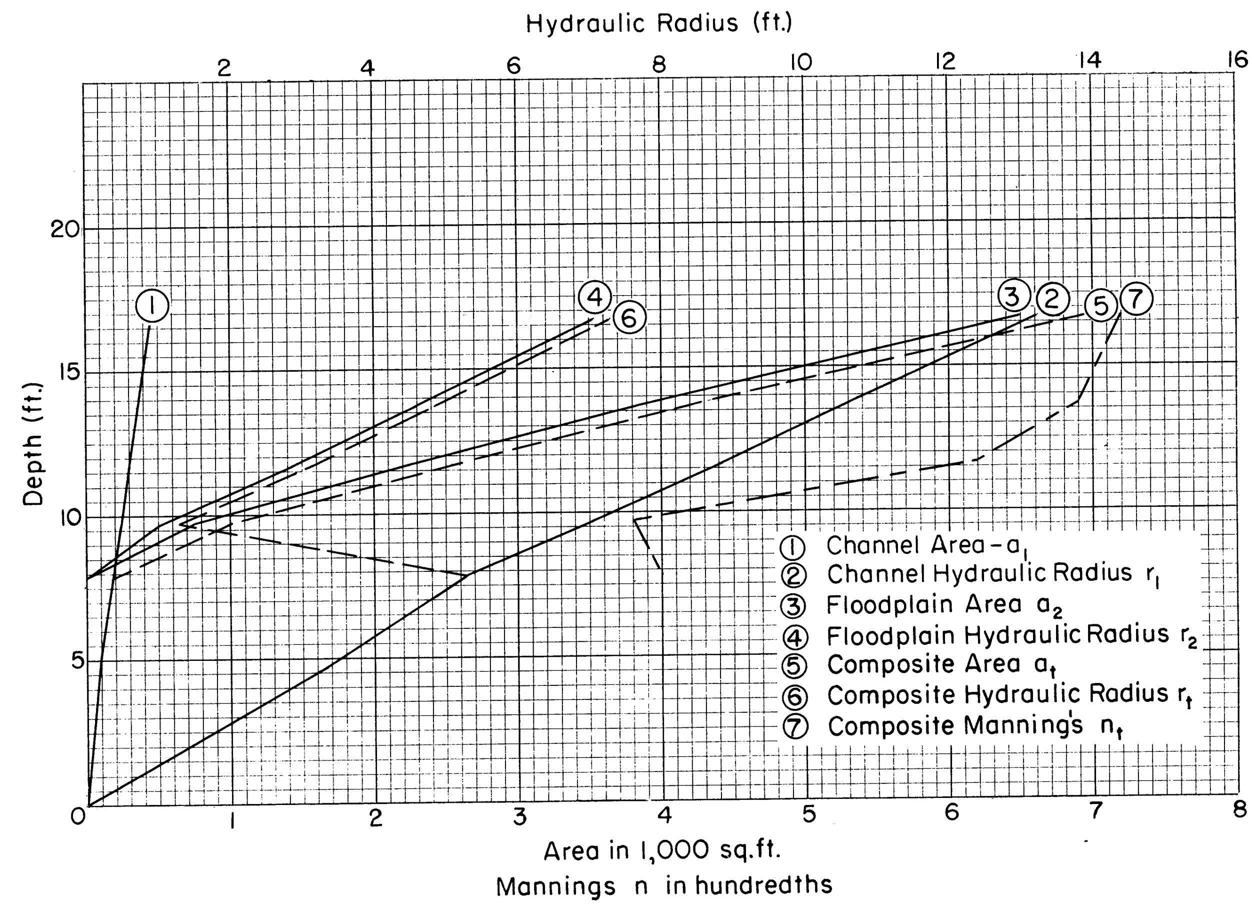


Figure B.3

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